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Exercise 1.1

1. Find and circle all the factors of 114.

1 2 3 4 6 7 14 19 28 38 42 57 114

Solution:

Because, we can divide 114 by 1, 2, 3, 6, 19, 38, 57, and 114 completely without any remainder. So, circle the numbers as factors of 114 are:

$\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{6}$ $\boxed{19}$ $\boxed{38}$ $\boxed{114}$

2. Write all the factors and factor pairs of the given numbers.

a) 78	**b) 91**	**c) 124**	**d) 147**	**e) 429**
f) 522	**g) 69**	**h) 42**	**i) 425**	**j) 153**

Solution:

a) We can start enlisting all the factors of 78 that are given below.

$$1 \times 78 = 78$$
$$2 \times 39 = 78$$
$$3 \times 26 = 78$$
$$6 \times 13 = 78$$

So, the factors of 78 are: 1, 2, 3, 6, 13, 26, 39, and 78.
Also, factor pairs of 78 are:
(1, 78), (2, 39), (3, 26) and (6, 13)

b) We can start enlisting all the factors of 91 that are given below.

$$1 \times 91 = 91$$
$$7 \times 13 = 91$$

So, the factors of 91 are: 1, 7, 13, and 91.
Also, factor pairs of 91 are:
(1, 91), (7, 13)

c) We can start enlisting all the factors of 124 that are given below.

$$1 \times 124 = 124$$
$$2 \times 62 = 124$$
$$4 \times 31 = 124$$

So, the factors of 124 are: 1, 2, 4, 31, 62, 124.
Also, factor pairs of 124 are:
(1, 124), (2, 62), (4, 31)

d) We can start enlisting all the factors of 147 that are given below.

$$1 \times 147 = 147$$
$$3 \times 49 = 147$$
$$7 \times 21 = 147$$

So, the factors of 147 are: 1, 3, 7, 21, 49, 147.
Also, factor pairs of 147 are:
(1, 147), (3, 49), (7, 21)

e) We can start enlisting all the factors of 429 that are given below.

$$1 \times 429 = 429$$
$$3 \times 143 = 429$$
$$11 \times 39 = 429$$
$$13 \times 33 = 429$$

So, the factors of 429 are: 1, 3, 11, 13, 33, 39, 143, and 429.
Also, factor pairs of 429 are:
(1, 429) (3, 143) (11, 39) (13, 33)

f) We can start enlisting all the factors of **522** that are given below.

$$1 \times \mathbf{522} = \mathbf{522}$$
$$2 \times 261 = \mathbf{522}$$
$$3 \times 174 = \mathbf{522}$$
$$6 \times 87 = \mathbf{522}$$
$$9\times 58 = \mathbf{522}$$
$$18 \times 29 = \mathbf{522}$$

So, the factors of **522** are: 1, 2, 3, 6, 9, 18, 29, 58, 87, 174, 261, and 522.
Also, factor pairs of **522** are:
(1, 522) (2, 261) (3, 174) (6, 87) (9, 58) (18, 29)

g) We can start enlisting all the factors of 69 that are given below.

$$1 \times 69 = 69$$
$$3\times 23 = 69$$

So, the factors of 69 are: 1, 3, 23, and 69.
Also, factor pairs of 69 are:
(1, 69) (3, 23)

h) We can start enlisting all the factors of 42 that are given below.

$$1 \times 42 = 42$$
$$2 \times 21 = 42$$
$$3 \times 14 = 42$$
$$6 \times 7 = 42$$

So, the factors of 42 are: 1, 2, 3, 6, 7, 14, 21, and 42.
Also, factor pairs of 42 are:
(1, 42) (2, 21) (3, 14) (6, 7)

i) We can start enlisting all the factors of 425 that are given below.

$$1 \times 425 = 425$$
$$5 \times 85 = 425$$
$$17 \times 25 = 425$$

So, the factors of 425 are: 1, 5, 17, 25, 85, and 425.
Also, factor pairs of 425 are:
(1, 425) (5, 85) (17, 25)

j) We can start enlisting all the factors of 153 that are given below.

$$1 \times 153 = 153$$
$$3 \times 51 = 153$$

$$9 \times 17 = 153$$

So, the factors of 153 are: 1, 3, 9, 17, 51, and 153.
Also, factor pairs of 153 are:
(1, 153) (3, 51) (9, 17)

3. **Is 29 a factor of 261?**
   Solution:
   yes, 29 is a factor of 261.
   Reason:
   We can start enlisting all the factors of 261 that are given below.

$$1 \times 261 = 261$$
$$3 \times 87 = 261$$
$$9 \times 29 = 261$$

   So, the factors of 261 are: 1, 3, 9, 29, 87, and 261.
   Also, factor pairs of 261 are:
   (1, 261) (3, 87) (9, 29)
   Thus, 29 is a factor of 261.

4. **Find the multiples of 48 greater than 50 but less than 300.**
   Solution:
   The multiples of 48 greater than 50 but less than 300 are:

$$2 \times 48 = 96$$
$$3 \times 48 = 144$$
$$4 \times 48 = 192$$
$$5 \times 48 = 240$$
$$6 \times 48 = 288$$

   Therefore, the multiples of 48 greater than 50 but less than 300 are: 48, 96, 144, 192, 240, 288.

5. **Find the multiples of 56 less than 400 but greater than 100.**
   Solution:
   The multiples of 56 less than 400 but greater than 100 are:

$$2 \times 56 = 112$$
$$3 \times 56 = 168$$
$$4 \times 56 = 224$$
$$5 \times 56 = 280$$
$$6 \times 56 = 336$$
$$7 \times 56 = 392$$

   Therefore, the multiples of 56 that are between 100 and 400 are: 112, 168, 224, 280, 336, and 392.

6. **Find who I am!**
   a) I am a multiple of 8 but I am not a multiple of 6. I am a factor of 48.

12	8	2	24	4	36	7	6

b) I am an odd number and a multiple of 5. I am not a multiple of 10 but a factor of 80.

10	15	16	8	2	40	5	20

Solution:

a) The factors of 48 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, and 48.
We know that the number is a multiple of 8 and a factor of 48, which means it must be one of: 8, 16, or 24.
We also know that the number is not a multiple of 6. The only number among 8, 16, and 24 that satisfies this condition is 8.
Therefore, the number in question is 8. It is a multiple of 8, a factor of 48, and not a multiple of 6.

b) The factors of 80 are: 1, 2, 4, 5, 8, 10, 16, 20, 40, and 80.
We know that the number is odd and a multiple of 5, so it must end in a 5.
The only factor of 80 that ends in 5 is 5 itself. So, the number in question must be 5.
Therefore, the number is 5. It is an odd number, a multiple of 5, not a multiple of 10, and a factor of 80.

7. Find the first 12 multiples of the given numbers.

a) 9 **b) 23** **c) 31** **d) 38** **e) 46** **f) 54**

Solution:

a) To find the multiples of 9, recall the table of 9 up to 12.

9 x 1 = 9

9 x 2 = 18

9 x 3 = 27

9 x 4 = 36

9 x 5 = 45

9 x 6 = 54

9 x 7 = 63

9 x 8 = 72

9 x 9 = 81

9 x 10 = 90

9 x 11 = 99

9 x 12 = 108

So, the first 12 multiples of 9 are: 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108.

b) Multiples of 23:

To find the multiples of 23, recall the table of 9 up to 23.

23 x 1 = 23

23 x 2 = 46

23 x 3 = 69

23 x 4 = 92

23 x 5 = 115

23 x 6 = 138

23 x 7 = 161

23 x 8 = 184

23 x 9 = 207

23 x 10 = 230

23 x 11 = 253

23 x 12 = 276

So, the first 12 multiples of 23 are: 23, 46, 69, 92, 115, 138, 161, 184, 207, 230, 253, 276.

c) Multiples of 31:

To find the multiples of 31, recall the table of 31 up to 12.

31 x 1 = 31

31 x 2 = 62

31 x 3 = 93

31 x 4 = 124

31 x 5 = 155

31 x 6 = 186

31 x 7 = 217

31 x 8 = 248

31 x 9 = 279

31 x 10 = 310

31 x 11 = 341

31 x 12 = 372

So, the first 12 multiples of 31 are: 31, 62, 93, 124, 155, 186, 217, 248, 279, 310, 341, 372.

d) Multiples of 38:

To find the multiples of 38, recall the table of 38 up to 12.

38 x 1 = 38

38 x 2 = 76

38 x 3 = 114

38 x 4 = 152

38 x 5 = 190

38 x 6 = 228

38 x 7 = 266

38 x 8 = 304

38 x 9 = 342

38 x 10 = 380

38 x 11 = 418

38 x 12 = 456

So, the first 12 multiples of 38 are: 38, 76, 114, 152, 190, 228, 266, 304, 342, 380, 418, 456.

e) Multiples of 46:

To find the multiples of 46, we add 46 to itself repeatedly.

46 x 1 = 46

46 x 2 = 92

46 x 3 = 138

46 x 4 = 184

46 x 5 = 230

46 x 6 = 276

46 x 7 = 322

46 x 8 = 368

46 x 9 = 414

46 x 10 = 460

46 x 11 = 506

46 x 12 = 552

So, the first 12 multiples of 46 are: 46, 92, 138, 184, 230, 276, 322, 368, 414, 460, 506, 552.

f) Multiples of 54:

To find the multiples of 54, we add 54 to itself repeatedly.

54 x 1 = 54

54 x 2 = 108

54 x 3 = 162

54 x 4 = 216

54 x 5 = 270

54 x 6 = 324

54 x 7 = 378

54 x 8 = 432

54 x 9 = 486

54 x 10 = 540

54 x 11 = 594

54 x 12 = 648

So, the first 12 multiples of 54 are: 54, 108, 162, 216, 270, 324, 378, 432, 486, 540, 594, 648.

8. **The teacher wants to divide 112 students from three sections of grade 6 into groups so that there are the same number of students in each group. In how many ways can the teacher form the groups?**
   Solution:
   Since we want to divide the students into three equal groups, we need to choose a factor that is divisible by 3. The only factor of 112 that satisfies this condition is 56.

   56 x 2 = 112

   28 x 4 = 112

   16 x 7 = 112
   Therefore, we can form 56 groups of 2 students each or 28 groups of 4 students each or 16 groups of 7 students each. There is a total of 3 ways to form the groups.
9. **How many packets can 314 kg of dates be packed into so that the quantity of dates in each packet is the same? How many different ways can we make the packets?**
   Solution:
   Weight of dates = 314 kg
   We can start enlisting all the factors of 314 that are given below.
   1 × 314= 314
   2 × 157 = 314
   The factors of 314 are: 1, 2, 157, 314.
   So, there are 4 different ways we can make the packets: 314 packets of 1 kg each, 157 packets of 2 kg each, 2 packets of 157 kg each, or 1 packet of 314 kg.

Leaning Check 1.2

1. **Find the missing numbers in the given table.**

	Numbers	**Prime factors**	**Index form**
a)	1225	**5 × 5 × 7 × 7**	**$5^2 \times 7^2$**
b)	324	**2 × 2 × 3 × 3 × 3 × 3**	$2^2 \times 3^4$
c)	2500	2 × 2 × 5 × 5 × 5 × 5	**$2^2 \times 5^4$**
d)	6174	**2 × 3 × 3 × 7 × 7 × 7**	**$2 \times 3^2 \times 7^3$**
e)	7875	**3 × 3 × 5 × 5 × 5 × 7**	**$3^2 \times 5^3 \times 7$**

2. **Find the prime factors of the given numbers and write them in index form.**

a) 482	b) 550	c)7845	d) 9340
e) 1050	f) 7252	g) 3216	h) 8310

Solution:

a) Prime factors of 482 are: 2×241.
Index form: $2^1 \times 241^1$

2	482
	241

b) Prime factors of 550 are: $2 \times 5 \times 5 \times 11$.
Index form: $2^1 \times 5^2 \times 11^1$

2	550
5	275
5	55
	11

c) Prime factors of 7845 are $3 \times 5 \times 523$.
Index form: $3^1 \times 5^1 \times 523^1$

3	7845
5	2615
	523

d) Prime factors of 9340 are $2 \times 2 \times 5 \times 467$.
Index form: $2^2 \times 5^1 \times 467^1$

2	9340
2	4670
5	2335
	467

e) Prime factors of 1050 are $2 \times 3 \times 5 \times 5 \times 7$
Index form: $2 \times 3 \times 5^2 \times 7$

2	1050
3	525
5	175
5	35
	7

f) Prime factors of 7252 are $2 \times 2 \times 7 \times 7 \times 37$.
Index form: $2^2 \times 7^2 \times 37^1$

2	7252
2	3626
7	1813
7	259
	37

g) Prime factors of 3216 are $2 \times 2 \times 2 \times 2 \times 3 \times 67$.
Index form: $2^4 \times 3^1 \times 67^1$

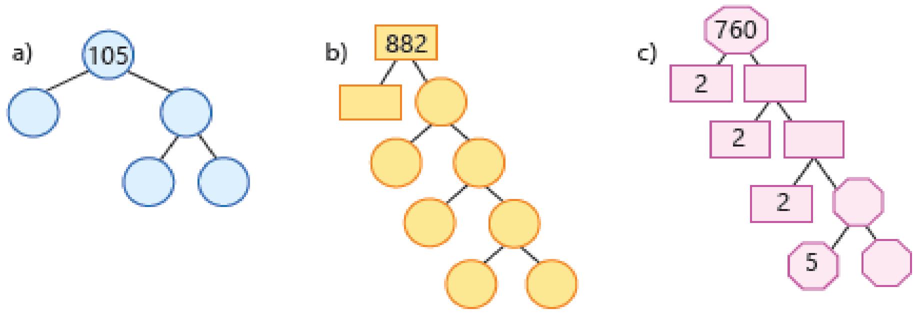
2	3216
2	1608
2	804
2	402
3	201
	67

h) Prime factors of 8310 are $2 \times 3 \times 5 \times 277$
Index form: $2^1 \times 3^1 \times 5^1 \times 277^1$

2	8310
3	4155
5	1385
	277

3. Complete the following.

a)



Solution:

a) Let's find the prime factors of 105 using the factor tree method.

```
105
|  \
35  3
|  \
7   5
```

The prime factors of 105 are 3, 5, 7.

The prime factorization of 105 = 3 × 5 × 7

b) Let's find the prime factors of 882 using the factor tree method.

```
882
|  \
441 2
|  \
147 3
|  \
49  3
|  \
7   7
```

The prime factors of 882 are 2, 3, 3, 7, 7.

The prime factorization of 882 = 2 × 3 × 3 × 7 × 7

c) Let's find the prime factors of 760 using the factor tree method.

```
760
|  \
380 2
|  \
190 2
|  \
95  2
|  \
19  5
```

The prime factors of 760 are 2, 2, 2, 5, 19.

The prime factorization of 760 = 2 × 2 × 2× 5 × 19

4. Which of these are the prime factors of 4356?

a) $2^4 \times 3 \times 11^2$ b) $2^2 \times 3^2 \times 11^2$

Solution:

2	4356
2	2178
3	1089
3	363
11	121
	11

Prime factors of 4356 are 2 × 2 × 3 × 3 × 11 × 11.

Index form: $2^2 \times 3^2 \times 11^2$

So, option (b) is a correct answer.

5. Write the given prime factors in index form. Also mention the base and exponent of each index form.

a) 2 × 2 × 2 × 7 × 7 × 11 **b) 3 × 5 × 5 × 11 × 11**

c) 2 × 2 × 13 × 13 × 13 **d) 2 × 2 × 2 × 2 × 2 × 7 × 7**

Solution:

a) Index form: $2^3 \times 7^2 \times 11$ Bases: 2, 3, 11 Exponents: 3, 2, 1

b) Index form: $3 \times 5^2 \times 11^2$ Bases: 3, 5, 11 Exponents: 1, 2, 2

c) Index form: $2^2 \times 13^3$ Bases: 2, 13 Exponents: 2, 3

d) Index form: $2^5 \times 7^2$ Bases: 2, 7 Exponents: 5, 2

6. Write each in expanded form.

a) $2^4 \times 3^2 \times 7^2$ **b) $3^3 \times 5^3$** **c) $2^5 \times 5^2 \times 7$** **d) $3^4 \times 11^2$**

Solution:

a) Expanded form: 2 × 2 × 2 × 2 × 3 × 3 × 7 × 7

b) Expanded form: 3 × 3 × 3 × 5 × 5 × 5

c) Expanded form: 2 × 2 × 2 × 2 × 2 × 5 × 5 × 7

d) Expanded form: 3 × 3 × 3 × 3 × 11 × 11

Learning Check 1.3

1. Find the HCF of the following numbers by the prime factorization method.

a) 56, 178 b) 624, 198, 200 c) 516, 432, 888

d) 24, 486, 245 e) 114, 386, 543 f) 100, 300, 500

Solution:

a) 56, 178

Let's find the prime factors first.

Now enlist them in factorization form and circle the common factors.

2	56
2	28
2	14
7	7
	1

Prime factorization of 56 = **2** × 2× 2 × 7

Prime factorization of 178 = **2** × 89

2	178
	89

Common factors = 2

HCF = common factors = 2

b) 624, 198, 200

Let's find the prime factors first.

Now enlist them in factorization form and circle the common factors.

Prime factorization of 624 = **2** × 2 × 2 × 2 × 3 × 13

Prime factorization of 198 = **2** × 3 × 3 × 11
Prime factorization of 200 = **2** × 2 × 2 × 5 × 5

Common factors = 2

HCF = common factors = 2

2	198
3	99
3	33
11	11
	1

2	200
2	100
2	50
5	25
5	5
	1

2	624
2	312
2	156
2	78
3	39
13	13
	1

c) 516, 432, 888
Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.
Prime factorization of 516 = **2** × **2**× **3** × 43
Prime factorization of 432 = **2** × **2** × 2 × 2 × **3** × 3 × 3
Prime factorization of 888 = **2** × **2** × 2 × **3** × 37
Common factors = 2 × 2 × 3
HCF = common factors = 12

2	516
2	258
3	129
43	43
	1

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

2	888
2	444
2	222
3	111
37	37
	1

d) 24, 486, 245
Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.
Prime factorization of 24 = 2 × 2× 2 × 3
Prime factorization of 486 = 2 × 3 × 3 × 3 × 3 × 3
Prime factorization of 245 = 5 × 7 × 7
Common factors = 1
HCF = common factors = 1

2	24
2	12
2	6
3	3
	1

2	486
3	243
3	81
3	27
3	9
3	3
	1

5	245
7	49
7	7
	1

e) 114, 386, 543
Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.
Prime factorization of 114 = 2 × 3 × 19
Prime factorization of 386 = 2 × 193
Prime factorization of 543 = 3× 181
Common factors = 1
HCF = common factors = 1

2	114
3	57
19	19
	1

2	386
193	193
	1

3	543
181	181
	1

f) 100, 300, 500

Let's find the prime factors first.

Now enlist them in factorization form and circle the common factors.

2	100
2	50
5	25
5	5
	1

2	300
2	150
3	75
5	25
5	5
	1

2	500
2	250
5	125
5	25
5	5
	1

Prime factorization of 100 = **2** × **2**× **5** × **5**

Prime factorization of 300 = **2** × **2**× 3 × **5** ×**5**

Prime factorization of 500 = **2** × **2** × 5 × **5** × **5**

Common factors = 2 × 2× 5 × 5

HCF = common factors = 100

2. Find the HCF of the following numbers by the division method.

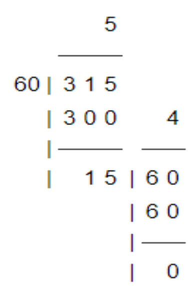
a) 60, 315 b) 78, 422, 118 c) 645, 195, 760

d) 87, 54, 118 e) 224, 324, 424 f) 23, 127, 223

Solution:

a) 60, 315

Let's take the greatest number 315 as a dividend and the smallest value 60 as a divisor.

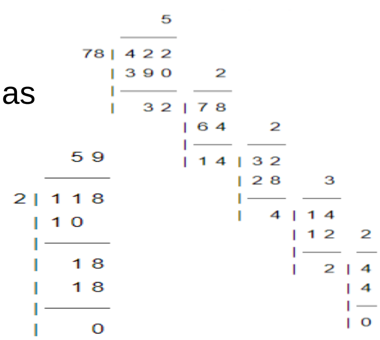


Since the remainder is 0 and the last divisor is 15.

Highest common factor (HCF) of 782, 624, and 544 = 15

b) 78, 422, 118

Let's take the greatest number 422 as a dividend and the smallest value 78 as a divisor.



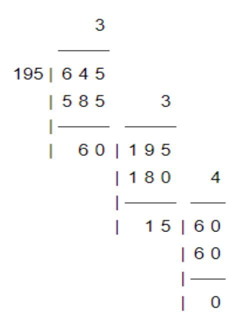
Now, 2 (the HCF of 422and 78) will be taken as the divisor, and the remaining number 118 will be taken as a dividend.

Since the remainder is 0 and the last divisor is 2.

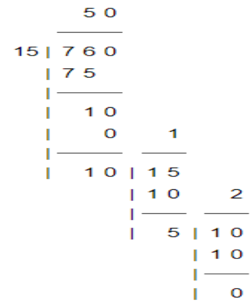
Highest common factor (HCF) of 78, 422, and 118 = 2

c) 645, 195, 760

Let's take the greatest number 645 as a dividend and the smallest value 195 as a divisor.

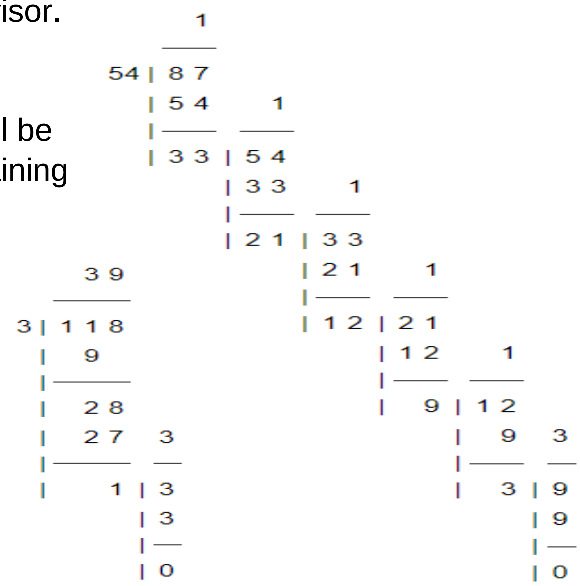


Now, 15 (the HCF of 645 and 195) will be taken as the divisor, and the remaining number 760 will be taken as a dividend.



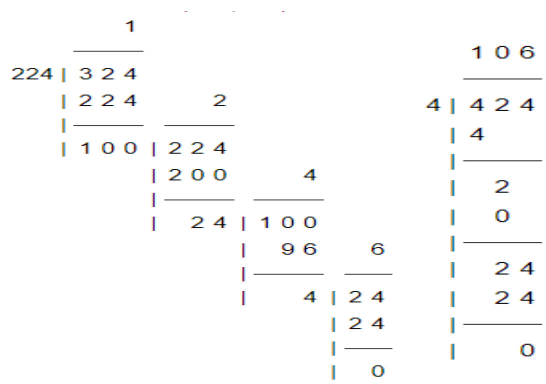
Since the remainder is 0 and the last divisor is 5.
Highest common factor (HCF) of 645, 195, and 760 = 5

d) 87, 54, 118
Let's take the greatest number 87 as a dividend and the smallest value 54 as a divisor.



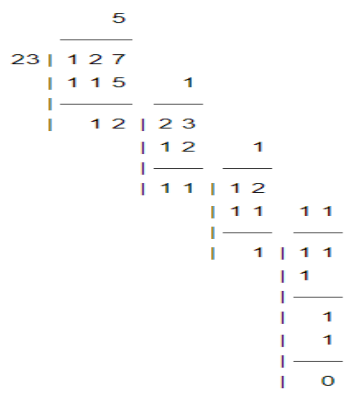
Now, 3 (the HCF of 87 and 54) will be taken as the divisor, and the remaining number 118 will be taken as a dividend.
Since the remainder is 0 and the last divisor is 1.
Highest common factor (HCF) of 87, 54, and 118 = 1

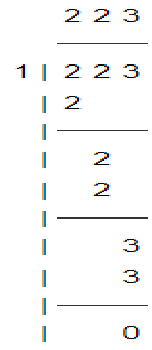
e) 224, 324, 424
Let's take the greatest number 324 as a dividend and the smallest value 224 as a divisor.



Now, 2 (the HCF of 324 and 224) will be taken as the divisor, and the remaining number 424 will be taken as a dividend.
Since the remainder is 0 and the last divisor is 4.
Highest common factor (HCF) of 224, 324, and 424 = 4.

f) 23, 127, 223
Let's take the greatest number 127 as a dividend and the smallest value 23 as a divisor.





Now, 1 (the HCF of 127 and 23) will be taken as the divisor, and the remaining number 223 will be taken as a dividend.
Since the remainder is 0 and the last divisor is 1.
Highest common factor (HCF) of 23, 127, and 223 = 1.

3. What is the length of the longest metre tape that can exactly measure fabrics with lengths of 150 m, 125 m, and 120 m?

Solution:

2	150
3	75
5	25
5	5
	1

5	125
5	25
5	5
	1

2	120
2	60
2	30
3	15
5	5
	1

Let's find the prime factors first.
Prime factorization of 150 = 2 × 3 × 5 × 5
Prime factorization of 125 = 5 × 5 × 5
Prime factorization of 120 = 2 × 2 × 2 × 3 × 5
HCF = common factors = 5
Therefore, the longest metre tape that can exactly measure fabrics with lengths of 150 m, 125 m, and 120 m is 5 meters long.

4. Three oil drums have capacities of 380*l*, 465*l* and 405*l*. Find the maximum capacity of a drum that can exactly measure the capacity of the three drums.
Solution:

2	380
2	190
5	95
19	19
	1

3	465
5	155
31	31
	1

3	405
3	135
3	45
3	15
5	5
	1

Let's find the prime factors first.
Prime factorization of 380 = 2 × 2 × 5 × 19
Prime factorization of 465 = 3 × 5 × 31
Prime factorization of 405 = 3 × 3 × 3 × 3 × 5
HCF = common factors = 5
Therefore, the maximum capacity of a drum that can exactly measure the capacity of the three drums is 5 liters.

5. **A baker baked 110 cupcakes, 215 pastries and 80 donuts. He wants to make packs so that the number of cupcakes, pastries and donuts in each pack is equal. What is the greatest number of packs he must make so that no cupcake, pastry or donut is left unused?**
   Solution:
   Let's find the prime factors first.
   Prime factorization of 110 = 2 × 5 × 11
   Prime factorization of 215 = 5 × 43
   Prime factorization of 80 = 2 × 2 × 2 × 2 × 5
   HCF = common factors
   = 5

2	110
5	55
11	11
	1

5	215
43	43
	1

2	80
2	40
2	20
2	10
5	5
	1

   Therefore, the baker must make 5 packs to use all the cupcakes, pasties, and donuts without leaving any unused.

6. **Ali has two pieces of rope. One is 18 m long and the other is 112 m long. He wants to cut them into pieces that are all the same length, without any remainder. What is the longest length that he can cut them into?**
   Solution:
   Let's divide the length 112 m by 18 m.
   Therefore, Ali can cut the ropes into pieces of 2 meters each, which is the longest length that he can cut them into without any remainder.

```
          6
       ------
    18 | 1 1 2
       | 1 0 8     4
       | -----   -----
       |     4 | 1 8
               | 1 6    2
               | ---   ---
               |   2 | 4
                     | 4
                     | -
                     | 0
```

Learning Check 1.4

1. **Find the LCM of the following numbers by the prime factorization method.**

a) 17, 102	b) 212, 115, 120	c) 344, 444, 544
d) 180, 218, 542	e) 670, 540, 830	f) 728, 166, 198

   Solution:
   a) 17, 102
   Let's find the prime factors first.
   Prime factorization of 17 = 17
   Prime factorization of 102 = 2 × 3× 17
   Product of common prime factors of 2 or more numbers = 17
   Product of non-common prime factors = 2 × 3
   LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
   = 2 × 3 × 17 = 102

17	17
	1

2	102
3	51
17	17
	1

b) 212, 115, 120

Let's find the prime factors first.

2	212
2	106
53	53
	1

5	115
23	23
	1

2	120
2	60
2	30
3	15
5	5
	1

Prime factorization of 212 = 2 × 2 × 53

Prime factorization of 115 = 5 × 23

Prime factorization of 120 = 2 × 2 × 2 × 3 × 5

Product of common prime factors of 2 or more numbers = 2 × 2 ×5

Product of non-common prime factors = 2 × 3 × 23 × 53

LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors

= 2 × 2 × 2 × 3 × 5 × 23 × 53 = 146280

c) Let's find the prime factors first.

2	344
2	172
2	86
43	43
	1

2	444
2	222
3	111
37	37
	1

2	544
2	272
2	136
2	68
2	34
17	17
	1

Prime factorization of 344 = 2 × 2 × 2 × 43

Prime factorization of 444 = 2 × 2 × 3 × 37

Prime factorization of 544 = 2 × 2 × 2 × 2 × 2 × 17

Product of common prime factors of 2 or more numbers = 2 × 2 × 2

Product of non-common prime factors = 2 × 2 × 3 × 17 × 37 × 43

LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors

= 2 × 2 × 2 × 2 × 2 × 3 × 17 × 37 × 43

= 2596512

d) Let's find the prime factors first.

2	180
2	90
3	45
3	15
5	5
	1

2	218
109	109
	1

2	542
271	271
	1

Prime factorization of 180 = 2 × 2 × 3 × 3 × 5

Prime factorization of 218 = 2 × 109

Prime factorization of 542 = 2 × 271

Product of common prime factors of 2 or more numbers = 2

Product of non-common prime factors = 2 × 3 × 3 × 5 × 109 × 271

LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors

= 2 × 2 × 3 × 3 × 5 × 109 × 271

= 5317020

e) Let's find the prime factors first.

2	670
5	335
67	67
	1

2	540
2	270
3	135
3	45
3	15
5	5
	1

2	830
5	415
83	83
	1

Prime factorization of 670 = 2 × 5 × 67

Prime factorization of 540 = 2 × 2 × 3 × 3 × 3 × 5

Prime factorization of 830 = 2 × 5 × 83

Product of common prime factors of 2 or more numbers = 2 × 5

Product of non-common prime factors = 2 × 3 × 3 × 3 × 67 × 83
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2 × 2 × 3 × 3 × 3 × 5 × 67 × 83
= 3002940

f) Let's find the prime factors first.
Prime factorization of 728 = 2 × 2 × 2 × 7 × 13
Prime factorization of 166 = 2 × 83
Prime factorization of 198 = 2 × 3 × 3 × 11
Product of common prime factors of 2 or more numbers = 2
Product of non-common prime factors = 2 × 2 × 3 × 3 × 7 × 11 × 13 × 83 LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2 × 2 × 2 × 3 × 3 × 7 × 11 × 13 × 83
= 5981976

2	728
2	364
2	182
7	91
13	13
	1

2	166
83	83
	1

2	198
3	99
3	33
11	11
	1

2. Find the LCM of the following numbers by the division method.

a) 515, 710 b) 834, 724, 614 c) 400, 260, 320
d) 608, 726, 614 e) 44, 170, 240 f) 540, 378, 265

Solution:

a) 515, 710
Let's start the solution for LCM

2	515	710
5	515	355
71	103	71
103	103	1
	1	1

So, LCM of (515, 710) = 2 × 5 × 71 × 103
= 73,130

b) 834, 724, 614
Let's start the solution for LCM

2	834	724	614
2	417	362	307
3	417	181	307
139	139	181	307
181	1	181	307
307	1	1	307
	1	1	1

So, LCM of (834, 724, 614) = 2 × 2 × 3 139 × 181 × 307

= 92685756

c) 400, 260, 320

Let’s start the solution for LCM

2	400	260	320
2	200	130	160
2	100	65	80
2	50	65	40
2	25	65	20
2	25	65	10
5	25	65	5
5	5	13	1
13	1	13	1
	1	1	1

LCM of (400, 260, 320) = 2 × 2 × 2 × 2 × 2 × 2 × 5 × 5 × 13

= 20,800

d) 608, 726, 614

Let’s start the solution for LCM

2	608	726	614
2	304	363	307
2	152	363	307
2	76	363	307
2	38	363	307
3	19	363	307
11	19	121	307
11	19	11	307
19	19	1	307
307	1	1	307
	1	1	1

LCM of (608, 726, 614) = 2 × 2 × 2 × 2 × 2 × 3 × 11 × 11 19 × 307

= 67,756,128

e) 44, 170, 240

Let’s start the solution for LCM

2	44	170	240
2	22	85	120
2	11	85	60
2	11	85	30
3	11	85	15
5	11	85	5
11	11	17	1
17	1	17	1
	1	1	1

LCM of (44, 170, 240) = 2 × 2 × 2 × 2 × 3 × 5 × 11 × 17

= 44,880

f) 540, 378, 265

Let’s start the solution for LCM

2	540	378	265
2	270	189	265
3	135	189	265
3	45	63	265
3	15	21	265
5	5	7	265
7	1	7	53
53	1	1	53
	1	1	1

LCM of (540, 378, 265) = 2 × 2 × 3 × 3 × 3 × 5 × 7 × 53

= 200,340

3. **Find the least number of labourers that can be sent to workplaces in groups of 54, 102 and 200 labourers.**
   Solution:
   Let's find the prime factors first.
   Prime factorization of 54 = $2 \times 3 \times 3 \times 3$
   Prime factorization of 102 = $2 \times 3 \times 17$
   Prime factorization of 200 = $2 \times 2 \times 2 \times 5 \times 5$
   Product of common prime factors of 2 or more numbers = 2×3
   Product of non-common prime factors = $2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 17$

2	54
3	27
3	9
3	3
	1

2	102
3	51
17	17
	1

2	200
2	100
2	50
5	25
5	5
	1

   LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
   $= 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 17$
   $= 91{,}800$
   Therefore, the least number of laborers that can be sent to workplaces in groups of 54, 102, and 200 laborers is 91,800.
4. **Find the least number of plants that can be planted in rows containing 100 and 165 plants in each row.**
   Solution:
   Let's find the prime factors first.
   Prime factorization of 100 = $2 \times 2 \times 5 \times 5$
   Prime factorization of 165 = $3 \times 5 \times 11$
   Product of common prime factors of 2 or more numbers = 3×5
   Product of non-common prime factors = $2 \times 2 \times 5 \times 11$

2	100
2	50
5	25
5	5
	1

3	165
5	55
11	11
	1

   LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
   $= 2 \times 2 \times 3 \times 5 \times 5 \times 11$
   $= 3{,}300$
   Therefore, the least number of plants that can be planted in rows containing 100 and 165 plants in each row is 3,300.
5. **Find the length of the shortest ribbon that can be cut into pieces of 108 cm, 45 cm and 112 cm, respectively without leaving any leftover ribbon.**
   Solution:
   Let's find the prime factors first.
   Prime factorization of 108 = $2 \times 2 \times 3 \times 3 \times 3$
   Prime factorization of 45 = $3 \times 3 \times 5$
   Prime factorization of 112 = $2 \times 2 \times 2 \times 2 \times 7$
   Product of common prime factors of 2 or more numbers = $2 \times 2 \times 3 \times 3$

2	108
2	54
3	27
3	9
3	3
	1

3	45
3	15
5	5
	1

2	112
2	56
2	28
2	14
7	7
	1

Product of non-common prime factors = 2 × 2 × 3 × 5 × 7
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors = 2 × 2 × 2 × 2 × 3 × 3 × 3 × 5 × 7
= 15,120
Therefore, the length of the shortest ribbon that can be cut into pieces of 108 cm, 45 cm, and 112 cm without leaving any leftover ribbon is 15,120 cm.

6. **Find the least number of blocks that can be exactly packed in boxes having a capacity of 45, 115 or 60 blocks per box.**
Solution:
Let's find the prime factors first.
Prime factorization of 45 = 3 × 3 × 5
Prime factorization of 115 = 5 × 23
Prime factorization of 60 = 2 × 2 × 3 × 5

3	45
3	15
5	5
	1

5	115
23	23
	1

2	60
2	30
3	15
5	5
	1

Product of common prime factors of 2 or more numbers = 3 × 5
Product of non-common prime factors = 2 × 2 × 3 × 23
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2 × 2 × 3 × 3 × 5 × 23
= 4,140
This means that the least number of blocks that can be exactly packed in boxes having a capacity of 45, 115, or 60 blocks per box is 4,140 blocks.

Learning Check 1.5

1. **Calculate the square of the following numbers.**

a) 14	b) 56	c) 61	d) 23	e) 78
f) 35	g) 47	h) 13	i) 64	j) 94

Solution:

a) To calculate the square of 14, we multiply 14 by itself:
14 × 14 = 196
Therefore, the square of 14 is 196.

b) To calculate the square of 56, we multiply 56 by itself:
56 × 56 = 3,136
Therefore, the square of 56 is 3,136.

c) To calculate the square of 61, we multiply 61 by itself:
61 × 61 = 3,721
Therefore, the square of 61 is 3,721.

d) To calculate the square of 23, we multiply 23 by itself:
23 × 23 = 529
Therefore, the square of 23 is 529.

e) To calculate the square of 78, we multiply 78 by itself:
78 × 78 = 6,084

Therefore, the square of 78 is 6,084.

f) To calculate the square of 35, we multiply 35 by itself:

$35 \times 35 = 1{,}225$

Therefore, the square of 35 is 1,225.

g) To calculate the square of 47, we multiply 47 by itself:

$47 \times 47 = 2{,}209$

Therefore, the square of 47 is 2,209.

h) To calculate the square of 13, we multiply 13 by itself:

$13 \times 13 = 169$

Therefore, the square of 13 is 169.

i) To calculate the square of 64, we multiply 64 by itself:

$64 \times 64 = 4{,}096$

Therefore, the square of 64 is 4,096.

j) To calculate the square of 94, we multiply 94 by itself:

$94 \times 94 = 8{,}836$

Therefore, the square of 94 is 8,836.

2. A square-shaped floor of a masjid has a length of 12 metres. What is the area of the masjid?

Solution:

Since, the floor of the masjid is square-shaped with a length of 12 meters, then the area of the masjid can be calculated as:

Area = $(\text{Length})^2$

Therefore, the area of the masjid is:

Area = $(\text{Length})^2$

= $(12)^2 = 144$ square meters

So, the area of the masjid is 144 square meters.

3. A square-shaped playground has a side length of 9.5 metres. Find the area of the playground.

Solution:

Since the playground is a square-shaped with a side length of 9.5 meters, the area of the playground can be calculated as:

Area = $(\text{side length})^2$

Substituting the value of the side length:

Area = $(9.5)^2$

Area = 9.5×9.5

= 90.25 square meters

Therefore, the area of the playground is 90.25 square meters.

Unit Check 1

1. Encircle the correct answer.

a) The smallest factor of any number is:

i) 1 ii) 0 iii) 10 iv) number itself

b) The 12th multiple of 16 is:

i) 160 ii) 148 iii) 176 iv) 192

c) The prime factors of 196 are:

i) $2 \times 2 \times 5 \times 5$ ii) $2 \times 2 \times 7 \times 7$ iii) $2 \times 2 \times 5 \times 7$ iv) $2 \times 3 \times 5 \times 7$

d) The LCM of 54, 60 and 24 is:

i) 2 ii) 6 iii) 540 iv) 1080

e) The square of 43 is:

i) 1844 ii) 1744 iii) 1849 iv) 1749

2. Fill in the blanks.

a) A number that divides another number exactly without any leftover is called its **factor.**

b) A multiple of a number is the **product** of that number with any other number.

c) The process of writing prime factors of any number is called **prime factorization.**

d) The index form of $5 \times 5 \times 5 \times 5$ is $\mathbf{5^4}$**.**

e) The greatest common factor of two or more numbers is called **HCF.**

f) The square of the greatest 2-digit number is **9801.**

3. Find the factors and factor pairs of the following numbers.

a) 192 b) 358 c) 298 d) 123 e) 975

Solution:

a) We can start enlisting all the factors of 192 that are given below.

$$1 \times 192 = 192$$
$$2 \times 96 = 192$$
$$3 \times 64 = 192$$
$$4 \times 48 = 192$$
$$6 \times 32 = 192$$
$$8 \times 24 = 192$$
$$12 \times 16 = 192$$

So, the factors of 192 are: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 64, 96, and 192.

Also, factor pairs of 192 are:

(1, 192), (2, 96), (3, 64), (4, 48), (6, 32), (8, 24), and (12, 16).

b) We can start enlisting all the factors of 358 that are given below.

$$1 \times 358 = 358$$
$$2 \times 179 = 358$$

So, the factors of 358 are: 1, 2, 179, and 358.

Also, factor pairs of 358 are:

(1, 358), (2, 179)

c) We can start enlisting all the factors of 298 that are given below.

$$1 \times 298 = 298$$

$$2 \times 149 = 298$$

So, the factors of 298 are: 1, 2, 149, and 298.
Also, factor pairs of 298 are:
(1, 298), (2, 149)

d) We can start enlisting all the factors of 123 that are given below.

$$1 \times 123 = 123$$
$$3 \times 41 = 123$$

So, the factors of 123 are: 1, 3, 41, and 123.
Also, factor pairs of 123 are:
(1, 123), (3, 41)

e) We can start enlisting all the factors of 975 that are given below.

$$1 \times 975 = 975$$
$$3 \times 325 = 975$$
$$5 \times 195 = 975$$
$$13 \times 75 = 975$$
$$15 \times 65 = 975$$
$$25 \times 39 = 975$$

So, the factors of 975 are: 1, 3, 5, 13, 15, 25, 39, 65, 75, 195, 325, and 975
Also, factor pairs of 975 are:
(1, 975) (3, 325) (5, 195) (13, 75) (15, 65) (25, 39)

4. Find the factors of the following numbers by the prime factorization method. Also write them in index form.

a) 4566 b) 6172 c) 7183 d) 9880 e) 1244

Solution:

a) First, break 4566 into its factors.

$$2 \times 2283$$
$$3 \times 761$$

Now 3 and 761 are prime factors, so we stop here.
$4566 = 2 \times 3 \times 761$
Index form: $2^1 \times 3^1 \times 761^1$

2	4566
3	2283
761	761
	1

b) First, break 6172 into its factors.

$$2 \times 3086$$
$$2 \times 1543$$

Now 2 and 1543 are prime factors, so we stop here.
$6172 = 2 \times 2 \times 1543$
Index form: $2^2 \times 1543^1$

2	6172
2	3086
1543	1543
	1

c) First, break 7183 into its factors.

$$11 \times 653$$

Now 11 and 653 are prime factors, so we stop here.
$7183 = 11 \times 653$
Index form: $11^1 \times 653^1$

11	7183
653	653
	1

d) First, break 9880 into its factors.

2×4940

2×2470

2×1235

5×247

13×19

2	9880
2	4940
2	2470
5	1235
13	247
19	19
	1

Now 13 and 19 are prime factors, so we stop here.

$9880 = 2 \times 2 \times 2 \times 5 \times 13 \times 19$

Index form: $2^3 \times 5^1 \times 13^1 \times 19^1$

e) First, break 1244 into its factors.

2×622

2×311

2	1244
2	622
311	311
	1

Now 2 and 311 are prime factors, so we stop here.

$1244 = 2 \times 2 \times 311$

Index form: $2^2 \times 311^1$

5. Find the first 10 multiples of the given numbers.

a) 37 b) 89 c) 15 d) 55 e) 97 f) 72

Solution:

a) The products that result from multiplying a number by 37 are the multiples of 37. To find the first 10 multiples of 37, recall the table of 37 up to 10.

1 x 37 = 37
2 x 37 = 74
3 x 37 = 111
4 x 37 = 148
5 x 37 = 185
6 x 37 = 222
7 x 37 = 259
8 x 37 = 296
9 x 37 = 333
10 x 37 = 370

So, the first 10 multiples of 37 are: 37, 74, 111, 148, 185, 222, 259, 296, 333 and 370.

b) The products that result from multiplying a number by 89 are the multiples of 89. To find the first 10 multiples of 89, recall the table of 89 up to 10.

1 x 89 = 89
2 x 89 = 178
3 x 89 = 267
4 x 89 = 356
5 x 89 = 445
6 x 89 = 534
7 x 89 = 623
8 x 89 = 712

9 x 89 = 801
10 x 89 = 890
So, the first 10 multiples of 89 are: 89, 178, 267, 356, 445, 534, 623, 712, 801 and 890.

c) The products that result from multiplying a number by 15 are the multiples of 15. To find the first 10 multiples of 15, recall the table of 15 up to 10.
1 x 15 = 15
2 x 15 = 30
3 x 15 = 45
4 x 15 = 60
5 x 15 = 75
6 x 15 = 90
7 x 15 = 105
8 x 15 = 120
9 x 15 = 135
10 x 15 = 150
So, the first 10 multiples of 15 are: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150

d) The products that result from multiplying a number by 55 are the multiples of 55. To find the first 10 multiples of 55, recall the table of 55 up to 10.
1 x 55 = 55
2 x 55 = 110
3 x 55 = 165
4 x 55 = 220
5 x 55 = 275
6 x 55 = 330
7 x 55 = 385
8 x 55 = 440
9 x 55 = 495
10 x 55 = 550
So, the first 10 multiples of 55 are: 55, 110, 165, 220, 275, 330, 385, 440, 495 and 550.

e) The products that result from multiplying a number by 97 are the multiples of 97. To find the first 10 multiples of 97, recall the table of 97 up to 10.
1 x 97 = 97
2 x 97 = 194
3 x 97 = 291
4 x 97 = 388
5 x 97 = 485
6 x 97 = 582
7 x 97 = 679
8 x 97 = 776

9 x 97 = 873
10 x 97 = 970
So, the first 10 multiples of 97 are: 97, 194, 291, 388, 485, 582, 679, 776, 873 and 970.

f) The products that result from multiplying a number by 72 are the multiples of 72. To find the first 10 multiples of 72, recall the table of 72 up to 10.
1 x 72 = 72
2 x 72 = 144
3 x 72 = 216
4 x 72 = 288
5 x 72 = 360
6 x 72 = 432
7 x 72 = 504
8 x 72 = 576
9 x 72 = 648
10 x 72 = 720
So, the first 10 multiples of 72 are: 72, 144, 216, 288, 288, 432, 504, 576, 648 and 720.

6. Find the HCF and LCM of the following numbers by the prime factorization method.

a) 414, 614 b) 64, 188, 924 c) 388, 545, 255 d) 728, 432, 195

Solution:

a) Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.
Prime factorization of 414 = **2** × 3 × 3 × 23
Prime factorization of 614 = **2** × 307
Common factors = 2
Product of common prime factors of 2 or more numbers = 2
Product of non-common prime factors = 3 × 3 × 23 × 307
HCF = common factors = 2
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2 × 3 × 3 × 23 × 307 = 127098

b) Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.
Prime factorization of 64 = **2** × **2** × 2× 2× 2× 2
Prime factorization of 188 = **2** × **2**× 47
Prime factorization of 924 = **2** × **2** × 3 × 7× 11
Common factors = 2 × 2
Product of common prime factors of 2 or more numbers
= 2 × 2

2	64
2	32
2	16
2	8
2	4
2	2
	1

2	188
2	94
47	47
	1

2	924
2	462
3	231
7	77
11	11
	1

Product of non-common prime factors = 2× 2× 2× 2 × 3 × 7× 11 × 47
HCF = common factors = 2 × 2 = 4
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2 × 2 × 2 × 2× 2× 2 × 3 × 7× 11 × 47 = 694848

c) Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.

2	388
2	194
97	97
	1

5	545
109	109
	1

3	255
5	85
17	17
	1

Prime factorization of 388 = 2 × 2 × 97
Prime factorization of 545 = 5 × 109
Prime factorization of 255 = 3 × 5 × 17
Common factors = 1
Product of common prime factors of 2 or more numbers = 5
Product of non-common prime factors = 2× 2× 3 × 5 × 17× 97 × 109
HCF = Common factors = 1
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2× 2× 3 × 5 × 17× 97 × 109 = 10784460

d) Let's find the prime factors first.
Now enlist them in factorization form and circle the common factors.
Prime factorization of 728 = 2 × 2 × 2 × 7 × 13
Prime factorization of 432 = 2× 2 × 2× 2 × 3 × 3 × 3
Prime factorization of 195 = 3 × 5 × 13

2	728
2	364
2	182
7	91
13	13
	1

2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

3	195
5	65
13	13
	1

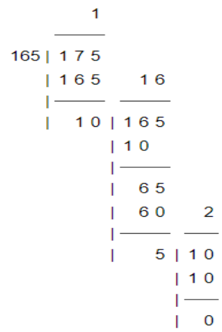
Common factors = 1
Product of common prime factors of 2 or more numbers = 2× 3 × 13
Product of non-common prime factors = 2× 2 × 2 × 3 × 3 × 5 × 7
HCF = Common factors = 1
LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors
= 2× 3 × 13 × 2× 2 × 2 × 3 × 3 × 5 × 7 = 196560

7. Find the HCF and LCM of the following numbers by the division method.

a) 165, 175 b) 782, 624, 544 c) 900, 912, 856 d) 23, 127, 143

Solution:

a) Let's take the greatest number 175 as a dividend and the smallest value 165 as a divisor.



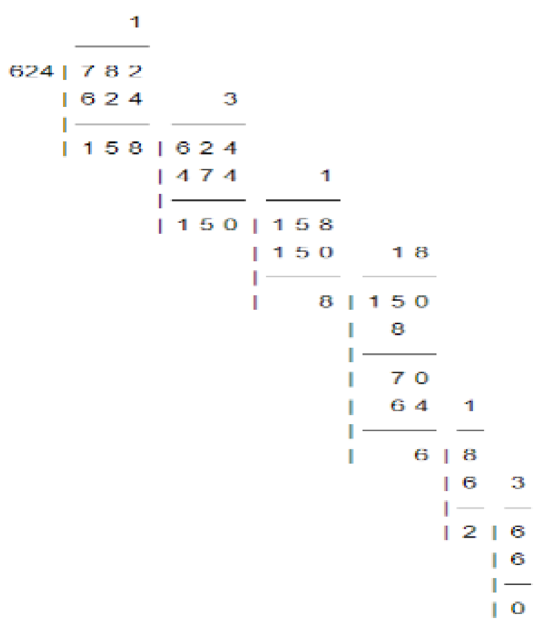
Since the remainder is 0 and the last divisor is 5, the highest common factor of 165 and 175 is 5.

Now, let's start the solution for LCM.

3	165	175
5	55	175
5	11	35
7	11	7
11	11	1
	1	1

So, LCM of 165 and 175 = 3 × 5 × 5 × 7 × 11 = 5775

b) Let's take the greatest number 782 as a dividend and the smallest value 624 as a divisor.



Now, 2 (the HCF of 782 and 624) will be taken as the divisor, and the remaining number 544 will be taken as a dividend.

```
     2 7 2
    ------
2 | 5 4 4
  | 4
  | ------
  | 1 4
  | 1 4
  | ------
  |     4
  |     4
  | ------
  |     0
```

Since the remainder is 0 and the last divisor is 2, the highest common factor of 782, 624, and 544 is 2.

Now, let's start the solution for LCM

2	782	624	544
2	391	312	272
2	391	156	136
2	391	78	68
2	391	39	34
3	391	39	17
13	391	13	17
17	391	1	17
23	23	1	1
	1	1	1

So, LCM of 782, 624, and 544 = 2 × 2 × 2 × 2 × 2 × 3 × 13 × 17 × 23

= 487968

c) Let's take the greatest number 912 as a dividend and the smallest value 900 as a divisor.

```
          1
       ------
900 | 9 1 2
    | 9 0 0     7 5
    | ------   ------
    |   1 2 | 9 0 0
            | 8 4
            | ------
            |   6 0
            |   6 0
            | ------
            |     0
```

Now, 12 (the HCF of 900 and 912) will be taken as the divisor, and the remaining number 856 will be taken as a dividend.

```
         7 1
        ─────
   12 | 8 5 6
      | 8 4
      |─────
      |   1 6
      |   1 2      3
      |─────     ────
      |       4 | 1 2
                | 1 2
                |────
                |   0
```

Since the remainder is 0 and the last divisor is 4, the highest common factor of 900, 912, and 856 is 4.
Now, let's start the solution for LCM

2	900	912	856
2	450	456	428
2	225	228	214
2	225	114	107
3	225	57	107
3	75	19	107
5	25	19	107
5	5	19	107
19	1	19	107
107	1	1	107
	1	1	1

So, LCM of 782, 624, and 544 = 2 × 2 × 2 × 2 × 3 × 3 × 5 × 5 × 19 × 107 = 7318800

d) Let's take the greatest number 127 as a dividend and the smallest value 23 as a divisor

```
          5
       ──────
  23 | 1 2 7
     | 1 1 5      1
     |──────    ─────
     |    1 2 | 2 3
              | 1 2       1
              |─────    ─────
              | 1 1 | 1 2
                    | 1 1     1 1
                    |─────   ─────
                    |    1 | 1 1
                           | 1
                           |─────
                           |   1
                           |   1
                           |─────
                           |   0
```

Now, 1 (the HCF of 23 and 127) will be taken as the divisor, and the remaining number 143 will be taken as a dividend.

```
    1 4 3
    -----
1 | 1 4 3
  | 1
  | -----
  |   4
  |   4
  | -----
  |     3
  |     3
  | -----
  |     0
```

Since the remainder is 0 and the last divisor is 1, the highest common factor of 23, 127, and 143 is 1.
Now, let's start the solution for LCM

11	23	127	143
13	23	127	13
23	23	127	1
127	1	127	1
	1	1	1

So, LCM of 23, 127, and 143 = 11 × 13 × 23 × 127 = 417703

8. Find the square of the following numbers.

a) 34 b) 76 c)67 d) 81 e) 53

Solution:

a) To calculate the square of 34, we multiply 34 by itself:
   34 × 34 = 1,156
   Therefore, the square of 34 is 1,156.
b) To calculate the square of 76, we multiply 76 by itself:
   76 × 76 = 5,776
   Therefore, the square of 76 is 5,776.
c) To calculate the square of 67, we multiply 67 by itself:
   67 × 67 = 4,489
   Therefore, the square of 67 is 4,489.
d) To calculate the square of 81, we multiply 81 by itself:
   81 × 81 = 6,561
   Therefore, the square of 81 is 6,561.
e) To calculate the square of 53, we multiply 53 by itself:
   53 × 53 = 2,809
   Therefore, the square of 53 is 2,809.

9. Find the length of the shortest rope that can be exactly cut into pieces of 15 cm, 25 cm and 90 cm, respectively, without leaving any remainder.

Solution:

To find the length of the shortest rope that can be cut into pieces of 15 cm, 25 cm, and 90 cm, we need to find the least common multiple (LCM) of these three numbers.

Let's find the prime factors first.

Prime factorization of 15 = 3 × 5

Prime factorization of 25 = 5× 5

Prime factorization of 90 = 2 × 3 × 3 × 5

3	15
5	5
	1

5	25
5	5
	1

2	90
3	45
3	15
5	5
	1

Product of common prime factors of 2 or more numbers = 3× 5

Product of non-common prime factors = 2× 3 × 5

LCM = Product of common prime factors of 2 or more numbers × Product of non-common prime factors = 3 × 5 × 2 × 3 × 5 = 450

Therefore, the length of the shortest rope that can be cut into pieces of 15 cm, 25 cm, and 90 cm is 450 cm.

10. Sania and Areeba go to painting class every 6 days and 8 days, respectively. If today both are in class, then after how many days will they again be in the same class simultaneously?

Solution:

To find out when Sania and Areeba will be in class simultaneously again, we need to find the smallest common multiple (LCM) of 6 and 8.

The multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, ...

The multiples of 8 are: 8, 16, 24, 32, 40, 48, 56, 64, ...

We can see that the smallest number that appears in both lists is 24, so the LCM of 6 and 8 is 24.

This means that Sania and Areeba will be in class together again after 24 days.

11. The length, width and height of a box are 680 cm, 355 cm and 200 cm, respectively. Find the longest tape that can measure the dimensions of the box exactly.

Solution:

To find the longest tape that can measure the dimensions of the box exactly, we need to find (HCF) of its dimensions.

Let's find the prime factors first.

Prime factorization of 680 = 2 × 2 × 2 × **5** × 17

Prime factorization of 355 = **5** × 71

Prime factorization of 200 = 2 × 2 × 2 × **5** × 5

Common factors = 5

HCF = Highest Common Factor = 5

Therefore, the longest tape that can measure the dimensions of the box exactly is 5 cm.

2	680
2	340
2	170
5	85
17	17
	1

5	355
71	71
	1

2	200
2	100
2	50
5	25
5	5
	1

Learning Check 2.1

1. Positive integers: +5, +34, +78, +49, +19, +56, +98
Negative integers: −3, −6, −19, −45, −17

2. a) Since this is a withdrawal of money, we can represent this statement as a negative integer. As, - 67000.

b) Since the temperature has decreased, we can represent this statement as a negative integer. Therefore, the integer representation of this statement is -7.

c) Since losing a match has a negative connotation, we can represent this statement as a negative integer. Therefore, the integer representation of this statement is -3.

d) Since, both moves 3 km, but in opposite direction. As

$$3 + (-3) = 0$$

e) Since this is a profit, we can represent this statement as a positive integer. Therefore, the integer representation of this statement is 50.

3. Write the opposite of each of the following integers.

a) +15 b) −20 c) +34 d) −7 e) −33 f) +1 g) −67

Solution:

a) The opposite of +15 is -15. This is because the opposite of a positive integer is a negative integer.
b) The opposite of -20 is +20. This is because the opposite of a negative integer is a positive integer.
c) The opposite of +34 is -34. This is because the opposite of a positive integer is a negative integer.
d) The opposite of -7 is +7. This is because the opposite of a negative integer is a positive integer.
e) The opposite of -33 is +33. This is because the opposite of a negative integer is a positive integer.
f) The opposite of +1 is -1. This is because the opposite of a positive integer is a negative integer.
g) The opposite of -67 is +67. This is because the opposite of a negative integer is a positive integer.

4. Write five integers that are greater than −7 but smaller than 7.

Solution:

Since, integers between – 7 and + 7 are given below.

-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6

So, we can choose any five integers from this list, such as -6, -2, 0, 3, 5.

5. Write all the integers that come between the given integers.

a) +8 and +14 b) −3 and +5 c) −16 and 0 d) −11 and +4
e) −1 to +1 f) −17 to −11 g) −5 to +9 h) −3 to +3

Solution:

a) To find the integers between +8 and +14, we need to list all integers that are greater than +8 but less than +14. These are: +9, +10, +11, +12, +13.

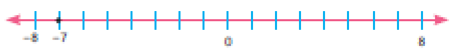
b) To find the integers between -3 and +5, we need to list all integers that are greater than −3 but less than +5. These are: −2, −1, 0, 1, 2, 3, and 4.
c) To find the integers between -16 and 0, we need to list all integers that are greater than −16 but less than 0. These are: −15, −14, −13, −12, −11, −10, −9, −8, −7, −6, −5, −4, −3, −2, and −1.
d) To find the integers between −11 and +4, we need to list all integers that are greater than −11 but less than +4. These are: −10, −9, −8, −7, −6, −5, −4, −3, −2, −1, 0, 1, 2, and 3.
e) To find the integers between −1 and +1, we need to list all integers that are greater than −1 but less than +1. There is only one such integer, which is 0.
f) To find the integers between −17 and −11, we need to list all integers that are greater than −17 but less than -11. These are: −16, −15, −14, −13, and −12.
g) To find the integers between -5 and +9, we need to list all integers that are greater than -5 but less than +9. These are: -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, and 8.
h) To find the integers between −3 and +3, we need to list all integers that are greater than −3 but less than +3. These are: −2, −1, 0, 1, and 2.

6. Represent the given integers on a number line.

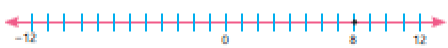
a) −7 b) +8 c) −12 d) +2 e) −15 f) −19

Solution:

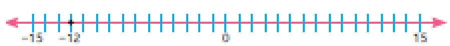
a) The number -7 is represented on a number line as follows:



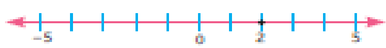
b) The number +8 is represented on a number line as follows:



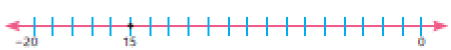
c) The number -12 is represented on a number line as follows:



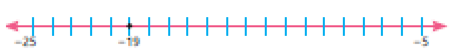
d) The number +2 is represented on a number line as follows:



e) The number -15 is represented on a number line as follows:



f) The number -19 is represented on a number line as follows:



7. Find the absolute value of the given integers.

a) −2 b) +4 c) −9 d) −17 e) +44 f) −51

Solution:

a) The absolute value of -2 is |-2| = 2. This is because the absolute value of a number is its distance from zero on the number line, and |-2| represents the distance between -2 and 0, which is 2.

b) The absolute value of +4 is |+4| = 4. This is because the absolute value of a number is its distance from zero on the number line, and |+4| represents the distance between +4 and 0, which is 4.

c) The absolute value of -9 is |-9| = 9. This is because the absolute value of a number is its distance from zero on the number line, and |-9| represents the distance between -9 and 0, which is 9.

d) The absolute value of -17 is |-17| = 17. This is because the absolute value of a number is its distance from zero on the number line, and |-17| represents the distance between -17 and 0, which is 17.

e) The absolute value of +44 is |+44| = 44. This is because the absolute value of a number is its distance from zero on the number line, and |+44| represents the distance between +44 and 0, which is 44.

f) The absolute value of -51 is |-51| = 51. This is because the absolute value of a number is its distance from zero on the number line, and |-51| represents the distance between -51 and 0, which is 51.

8. Write the next three integers for each of the following sequences.

a) −3, −6, −9 b) −1, 0, +1 c) −4, −8, −12 d) −8, −12, −16

e) −5, −3, −1 f) −6, −1, +4 g) +6, +9, +12 h) −2, −3, −4

Solution:

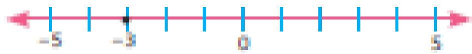
a) The sequence is decreasing by 3 each time, so to find the next three integers we just subtract 3 from the previous term: -9 - 3 = -12, -12 - 3 = –15, and 15 – 3 = –18.
b) The sequence is increasing by 1 each time, so to find the next three integers we just add 1 to the previous term: +1 + 1 = +2, +2 + 1 = +3, and +3 + 1 = +4.
c) The sequence is decreasing by 4 each time, so to find the next three integers we just subtract 4 from the previous term: -12 - 4 = -16, -16 - 4 = –20, and 20 – 4 = – 24.
d) The sequence is decreasing by 4 each time, so to find the next three integers we just subtract 4 from the previous term: -16 - 4 = -20, -20 - 4 = –24, and 24 – 4 = –28.
e) The sequence is increasing by 2 each time, so to find the next three integers we just add 2 to the previous term: -1 + 2 = +1, +1 + 2 = +3, and +3 + 2 = + 5.
f) The sequence is increasing by 5 each time, so to find the next three integers we just add 5 to the previous term: +4 + 5 = +9, +9 + 5 = +14, and +14 + 5 = + 19.
g) The sequence is increasing by 3 each time, so to find the next three integers we just add 3 to the previous term: +12 + 3 = +15, +15 + 3 = +18, and +18 + 3 = + 21.

h) The sequence is decreasing by 1 each time, so to find the next three integers we just subtract 1 from the previous term: −4 −1 = −5, −5 − 1 = −6, and – 6 −1 = −7.

9. The temperature in Nathia Gali is 6°C. It drops by 9°C at night. Represent the temperature in the form of an integer and then represent it on a number line.

Solution:

If the temperature in Nathia Gali is 6°C and it drops by 9°C at night, then the temperature can be represented as:



6°C − 9°C = −3°C

So, the temperature at night is −3°C.

To represent this on a number line, we would draw a horizontal line and label 0 at the center. Then, we would mark -3 to the left of 0 to indicate the temperature of Nathia Gali at night.

Learning Check 2.2

1. Compare and tell which one is the greater integer.

a) +16, +56 b) −15, +1 c) −2, +2 d) +18, −13

e) −6 and +9 f) −4 and 0 g) +7 and 0 h) −5 and +6

Solution:

a) +16 and +56

The given integers are both positive. When comparing two positive integers, the greater one is the one that is farther to the right on the number line. In this case, +56 is farther to the right than +16.

So, +56 is the greater integer.

b) −15 and +1

+1 is the greater integer because it is located to the right of zero on the number line while −15 is located to the left of zero. We can also see that the difference between the two numbers +1 – (-15) = 1+ 15 = 16 is positive, which confirms that +1 is greater.

c) −2 and +2

+2 is the greater integer because it is located to the right of zero on the number line while −2 is located to the left of zero. We can also see that the difference between the two numbers 2 – (-2) = 2 + 2 = 4 is positive, which confirms that +2 is greater.

d) +18 and −13

+18 is the greater integer because it is located to the right of zero on the number line while −13 is located to the left of zero. We can also see that the difference between the two numbers +18 – (-13) = 18 + 13 = 31 is positive, which confirms that +18 is greater.

e) −6 and +9

+9 is the greater integer because it is located to the right of zero on the number line while −6 is located to the left of zero. We can also see that the difference between the two numbers +9 – (-6) = 9 + 6 = +15 is positive, which confirms that +9 is greater.

f) −4 and 0

0 is the greater integer because it is located at zero on the number line while −4 is located to the left of zero. We can also see that the difference between the two numbers 0 – (-4) = 0 + 4 = +4 is positive, which confirms that 0 is greater.

g) +7 and 0

0 is the greater integer because it is located at zero on the number line while +7 is located to the right of zero. Since, + 7 is farther than 0, So greater number is +7.

h) −5 and +6

+6 is the greater integer because it is located to the right of zero on the number line while −5 is located to the left of zero. We can also see that the difference between the two numbers +6 – (-5) = 6 + 5 = 11 is positive, which confirms that +6 is greater.

2. Arrange the following integers in ascending order.

a) −4, −2, +6, −5 b) −2, −1, +4, −5 c) +9, −6, −8, −3
d) −11, +7, −15, −5 e) +17, +12, −15, −6 f) −7, −12, −1, +1

Solution:

a) −4, −2, +6, - 5

Compare the negative numbers separately.

−5 < −4 < - 2 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Since, + 6 is a positive integer.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-5, -4, -2, +6

b) −2, −1, +4, −5

Compare the negative numbers separately.

−5 < −2 < -1 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Since, +4 is a positive integer.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-5, -2, -1, +4

c) +9, −6, −8, −3

Compare the negative numbers separately.

−8 < −6 < -3 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Since, +9 is a positive integer.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-8, -6, -3, 9

d) −11, +7, −15, −5

Compare the negative numbers separately.

−15 < −11 < -5 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Since, +7 is a positive integer.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-15, -11, -5, 7

e) +17, +12, −15, −6

Compare the negative numbers separately.

−15 < −6 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Similarly, compare the positive integers separately.

17 is the greatest positive integer here and 12 is the smallest positive integer.

So, 12 < 17.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-15, -6, 12, 17

f) −7, −12, −1, +1

Compare the negative numbers separately.

−12 < −7 < -1 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Since, +1 is a positive integer.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-12, -7, -1, +1

3. Arrange the given integers in descending order.

a) +7, +12, −17, −3 b) −6, +15, +9, −4 c) −18, +15, −6, +13

d) −21, +23, +14, −32 e) −23, −45, −20, +15 f) −7, −4, −10, −2

Solution:

a) +7, +12, −17, −3

Compare the negative numbers separately.

−17 < −3 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Similarly, compare the positive integers separately.

12 is the greatest positive integer here and 7 is the smallest positive integer.

So, 7 < 12.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.
Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:
12, 7, - 3, - 17

b) −6, +15, +9, −4

Compare the negative numbers separately.
−6 < −4 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)
Similarly, compare the positive integers separately.
15 is the greatest positive integer here and 9 is the smallest positive integer.
So, 15 < 9.
0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.
Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:
15, 9, -4, -6

c) −18, +15, −6, +13

Compare the negative numbers separately.
−18 < −6 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)
Similarly, compare the positive integers separately.
15 is the greatest positive integer here and 13 is the smallest positive integer.
So, 13 < 15.
0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.
Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:
15, 13, -6, -18

d) −21, +23, +14, −32

Compare the negative numbers separately.
−32 < −21 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)
Similarly, compare the positive integers separately.
23 is the greatest positive integer here and 14 is the smallest positive integer.
So, 14 < 23.
0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.
Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:
23, 14, -21, -32

e) −23, −45, −20, +15

Compare the negative numbers separately.
−45 < −23 < -20 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

here, +15 is a positive number.
0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.
Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:
15, -20, -23, -45

f) −7, −4, −10, −2

Compare the negative numbers separately.
−10 < −7 < - 4 < - 2 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)
Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:
-2, -4, -7, -10

4. Compare the absolute values of the given integers and find the smallest and largest integer.

a) +7, −16, +13, −23 b) +34, −76, −44, −32 c) +12, −56, −39, +19
d) −28, −33, +5, +1 e) −45, −31, +32, −44 f) −18, −24, +41, +62

Solution:

a) +7, −16, +13, −23

The smallest integer is -23, and the largest integer is +13. In absolute value notation, we can write:
|+7| = 7, |-16| = 16, |+13| = 13, |-23| = 23
Therefore, the smallest integer is -23, which has an absolute value of 23, and the largest integer is +13, which has an absolute value of 13.

b) +34, −76, −44, −32

The smallest integer is -76, and the largest integer is +34. In absolute value notation, we can write:
|+34| = 34, |-76| = 76, |-44| = 44, |-32| = 32
Therefore, the smallest integer is -76, which has an absolute value of 76, and the largest integer is +34, which has an absolute value of 34.

c) +12, −56, −39, +19

The smallest integer is -56, and the largest integer is +19. In absolute value notation, we can write:
|+12| = 12, |-56| = 56, |-39| = 39, |+19| = 19
Therefore, the smallest integer is -56, which has an absolute value of 56, and the largest integer is +19, which has an absolute value of 19.

d) −28, −33, +5, +1

The smallest integer is -33, and the largest integer is +5. In absolute value notation, we can write:

|-28| = 28, |-33| = 33, |+5| = 5, |+1| = 1

Therefore, the smallest integer is -33, which has an absolute value of 33, and the largest integer is +5, which has an absolute value of 5.

e) −45, −31, +32, −44

The smallest integer is -45, and the largest integer is +32. In absolute value notation, we can write:

|-45| = 45, |-31| = 31, |+32| = 32, |-44| = 44

Therefore, the smallest integer is -45, which has an absolute value of 45, and the largest integer is +32, which has an absolute value of 32.

f) −18, −24, +41, +62

The smallest integer is -24, and the largest integer is +62. In absolute value notation, we can write:

|-18| = 18, |-24| = 24, |+41| = 41, |+62| = 62

Therefore, the smallest integer is -24, which has an absolute value of 24, and the largest integer is +62, which has an absolute value of 62.

5. Ahad and Arham start at the same point. Ahad takes 9 steps upstairs and Arham comes 4 steps down the stairs. Write the integers for the two situations. Also, write the absolute value of the integers.so, write the absolute value of the integers.

Solution:

Let's assume that going upstairs is represented by positive integers, and going downstairs is represented by negative integers. Then, the situation can be represented as follows:

Ahad takes 9 steps upstairs: +9

Arham comes 4 steps down the stairs: -4

In absolute value notation, we can write:

|+9| = 9 and |-4| = 4

Therefore, the absolute value of Ahad's position is 9 steps away from the starting point, and the absolute value of Arham's position is 4 steps away from the starting point in the opposite direction.

6. The table shows the temperatures of the different cities in Pakistan at night. Find the absolute values and then arrange them in ascending and descending

Cities	Lahore	Islamabad	Murree	Chakwal	Gilgit	Chitral
Temperature	11ºC	4ºC	-1ºC	3ºC	-7ºC	-2ºC

Solution:

In absolute notation, we can write as:

|11| = 11, |4| = 4, |-1| = 1, |3| = 3, |-7| = 7, |-2| = 2

Arranging the temperature from lowest to highest degree Celsius, i.e., in ascending order, we get:

1, 2, 3, 4, 7, 11

Arranging the temperature from highest to lowest degree Celsius, i.e., in descending order, we get:
11, 7, 4, 3, 2, 1

Learning Check 2.3

1. **Add the following.**

a) −3 and +4 b) +12 and −14 c) −18 and −21 d) −11 and −5
e) +17 and −17 f) −13 and +15 g) −23 and −34 h) +12 and +17

Solution:

a) −3 and +4

To add the integers, first we take the absolute values:
|−3| = 3 and |+4| = 4
Now subtract the absolute values.
4 − 3 = 1
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
−3 + (+4) = +1

b) +12 and −14

To add the integers, first we take the absolute values:
|+12| = 12 and |-14| = 14
Now subtract the absolute values.
14 − 12 = 2
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
+12 + (-14) = -2

c) −18 and −21

To add the integers, first we take the absolute values:
|−18| = 18 and |-21| = 21
Now add the absolute values.
18 + 21 = 39
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
−18 + (-21) = -39

d) −11 and −5

To add the integers, first we take the absolute values:
|−11| = 11 and |-5| = 5
Now add the absolute values.
11 + 5 = 16
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
−11 + (-5) = -16

e) +17 and −17

To add the integers, first we take the absolute values:
|+17| = 17 and |-17| = 17
Now subtract the absolute values.

17 − 17 = 0
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
+17 + (-17) = 0

f) −13 and +15
To add the integers, first we take the absolute values:
|−13| = 13 and |+15| = 15
Now subtract the absolute values.
15 − 13 = 2
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
−13 + (+15) = +2

g) −23 and −34
To add the integers, first we take the absolute values:
|−23| = 23 and |+34| = 34
Now add the absolute values.
23 + 34 = 57
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
−23 + (-34) = -57

h) +12 and +17
To add the integers, first we take the absolute values:
|+12| = 12 and |+17| = 17
Now add the absolute values.
12 + 17 = 29
Because the signs of the two integers are opposite, the sign with the greater integer is with the result.
+12 + (+17) = +29

2. Subtract the following.

a) −11 from −17 b) −2 from +8 c) +14 from −16 d) −94 from −84
e) +54 from −66 f) −5 from +5 g) +45 from −32 h) −23 from −62

Solution:

a) – 11 from – 17
The first number – 17 remains unchanged. Change the operation from subtraction to addition and change the sign of the number −11 that is to be subtracted. It will become +11. Then add according to the rules for adding integers.
(-17) − (−11) = (-17) + (+11)
Take the absolute values.
|-17| = 17 and |-11| = 11
Subtract the absolute values.
17 – 11 = 6
Put the common sign with the answer.
So, (-17) − (−11) = - 6.

b) −2 from +8

The first number + 8 remains unchanged. Change the operation from subtraction to addition and change the sign of the number – 2 that is to be subtracted. It will become + 2. Then add according to the rules for adding integers.

(+8) − (−2) = (+8) + (+2)

Take the absolute values.

|+8| = 8 and |- 2| = 2

Add the absolute values.

8 + 2 = 10

Put the common sign with the answer.

So, (8) − (−2) = + 10.

c) +14 from −16

The first number – 16 remains unchanged. Change the operation from subtraction to addition and change the sign of the number +14 that is to be subtracted. It will become – 14. Then add according to the rules for adding integers.

(- 16) − (+14) = (- 16) - (+14)

Take the absolute values.

|- 16| = 16 and |+14| = 14

Add the absolute values.

16 + 14 = 30

Put the common sign with the answer.

So, (- 16) − (+14) = - 30.

d) −94 from −84

The first number – 84 remains unchanged. Change the operation from subtraction to addition and change the sign of the number −94 that is to be subtracted. It will become +94. Then add according to the rules for adding integers.

(- 84) − (−94) = (- 84) + (+94)

Take the absolute values.

|- 84| = 84 and |- 94| = 94

Subtract the absolute values.

94 - 84 = 10

Put the common sign with the answer.

So, (- 84) − (−94) = + 10.

e) +54 from −66

The first number – 66 remains unchanged. Change the operation from subtraction to addition and change the sign of the number +54 that is to be subtracted. It will become – 54. Then add according to the rules for adding integers.

(-66) − (+54) = (-66) - (+54)

Take the absolute values.

|-66| = 66 and |+54| = 54

Add the absolute values.

66 + 54 = 120

Put the common sign with the answer.

So, (-66) − (+54) = - 120.

f) −5 from +5

The first number +5 remains unchanged. Change the operation from subtraction to addition and change the sign of the number −5 that is to be subtracted. It will become +5. Then add according to the rules for adding integers.

(+5) − (−5) = (+5) + (+5)

Take the absolute values.

|+5| = 5 and |-5| = 5

Add the absolute values.

5 + 5 = 10

Put the common sign with the answer.

So, (+5) − (−5) = + 10.

g) +45 from −32

The first number – 32 remains unchanged. Change the operation from subtraction to addition and change the sign of the number +45 that is to be subtracted. It will become – 45. Then add according to the rules for adding integers.

(- 32) − (+45) = (- 32) - (+45)

Take the absolute values.

|- 32| = 32 and |+45| = 45

Add the absolute values.

32 + 45 = 77

Put the common sign with the answer.

So, (- 32) − (+45) = - 77.

h) −23 from −62

The first number – 62 remains unchanged. Change the operation from subtraction to addition and change the sign of the number −23 that is to be subtracted. It will become +23. Then add according to the rules for adding integers.

(- 62) − (−23) = (- 62) + (+23)

Take the absolute values.

|-62| = 62 and |-23| = 23

Subtract the absolute values.

62 – 23 = 39

Put the common sign with the answer.

So, (- 62) − (−23) = - 39.

3. Find the missing term in each of the following.

a) ______ −(−5) = 4 b) −14 + ______ = −16 c) +34 + _______ = −45

d) −7 + (−15) = ____ e) _______+ 7 = −11 f) 16 + ________ = −15

Solution:

a) ______ −(−5) = 4

Let x be the missing number, then

$x - (-5) = 4$

$x + 5 = 4$

$x = 4 - 5$

$x = -1$

Thus, the missing number is -1.

b) $-14 + \_\_\_\_ = -16$

Let x be the missing number, then
$-14 + x = -16$
$x = -16 + 14$
$x = -2$
Thus, the missing number is -2.

c) $+34 + \_\_\_\_\_\_ = -45$
Let x be the missing number, then
$+34 + x = -45$
$x = -45 - 34$
$x = -79$
Thus, the missing number is -1.

d) $-7 + (-15) = \_\_\_$
Let x be the missing number, then
$-7 + (-15) = x$
$-7 - 15 = x$
$-22 = x$
Thus, the missing number is -22.

e) $\_\_\_\_\_\_ + 7 = -11$
Let x be the missing number, then
$x + 7 = -11$
$x = -11 - 7$
$x = -18$
Thus, the missing number is -18.

f) $16 + \_\_\_\_\_\_ = -15$
Let x be the missing number, then
$16 + x = -15$
$x = -15 - 16$
$x = -31$
Thus, the missing number is -31.

4. Complete the given tables.

a)

+	-13	-21
+32	+19	+11
-11	-24	-32
+27	+14	+6

b)

-	-47	-72
-16	+31	+56
+24	+71	+96
-39	+8	+33

5. **The result of the subtraction of two integers is −63. If the subtrahend is −17, then find the minuend.**
Solution:
Difference = - 63
Subtrahend = −17
Minuend =?
Suppose the minuend is x. Then:
minuend – subtrahend = difference
$x - (-17) = -63$
$x + 17 = -63$
$x = -63 - 17$
$x = -80$
So, the minuend is −80.

6. **The sum of two integers is −2. If one number is −65, then find the other one.**
Solution:
Sum of two integers = - 2
First integer = −65
Second integer =?
Suppose the second integer is x. Then:
$x + (-65) = -2$
$x - 65 = -2$
$x = -2 + 65$
$x = 63$
So, the second integer is 63.

7. **An elevator goes down 4 floors from the ground floor. Then it goes up 3 floors. What floor is the elevator on now?**
Solution:
Since, elevator started on the ground floor, which we can call floor 0.
Elevator goes down from the floor = 0 – 4 = - 4
Elevator goes up from the floor = 0 + 3 = +3
Position of elevator = down floor + up floor
= - 4 + 3 = - 1
Therefore, the elevator is now on the 1st floor down from the ground floor.

8. **The temperature of a mountainous area is −15°C. After three hours, the temperature has dropped by −5°C. What is the new temperature?**
Solution:
The temperature of the mountain area = - 15^{0}C
After three hours,
Temperature drop = - 5^{0}C
New temperature = ?
Let x be the new temperature of the mountain, then:
New temperature = Initial temperature - Drop in temperature
x = (-15°C) - (-5°C)
New temperature = -15°C + 5°C

New temperature = -10°C
Therefore, the new temperature is -10°C.

9. Two cars started from the same point. The first car went east and covered 78 km in an hour. The second car went west and covered 90 km in an hour. Find the distance between the two cars after an hour.

Solution:

Since, the two cars started from the same point.
Distance covered by the first car = 78 km
Distance covered by the second car = 90 km
Total distance covered by both cars = 78 km + 90 km = 168 km
Therefore, the distance between the two cars after an hour is 168 km.

10. A place is 226 feet above sea level and another is 100 feet below sea level. What is the distance between the two places?

Solution:

1st place above the sea level = 226 feet
2nd place below the sea level = 100 feet
Total distance between two places = 226 feet + 100 feet
= 326 feet
Therefore, the distance between the two places is 326 feet.

Learning Check 2.4

1. Identify the properties and complete the following.

a) $-8 + (-4) = (-4) + (-8)$
Commutative property w.r.t "+"

b) $-9 + (-4) = (-4) + (-9)$
Commutative property w.r.t "+"

c) $-13 + (-7) = -7 + (-13)$
Commutative property w.r.t "+"

d) $[-17 + (-5)] + (-12) = -17 + [(-5) + (-12)]$
Associative property w.r.t "+"

e) $+3 + (-3) = 0$
Additive inverse

f) $50 + [(-13) + (-16)] = [50 + (-13)] + (-16)$
Associative property w.r.t "+"

2. Verify the closure law for the given integers.

a) −7 and +12 b) −3 and −2 c) +9 and − 9 d) −15 and +18

Solution:

a) – 7 and + 12
Let's add −7 and +12.
$-7 + (+12) = -7 + 12 = +5$
5 is also an integer.

b) -3 and −2
Let's add −3 and – 2.
$-3 + (-2) = -3 - 2 = -5$

- 5 is also an integer.

c) +9 and − 9

Let's add +9 and −9.

$+9 + (-9) = +9 - 9 = 0$

0 is also an integer.

d) $-15\ and\ +18$

Let's add −15 and +18.

$-15 + (+18) = -15 + 18 = +3$

3 is also an integer.

3. Verify the commutative law for the given integers.

a) 3 and −4 b) −8 and −5 c) −22 and −12 d) −16 and +21

Solution:

a) 3 and −4

Let's add +3 and −4.

$+3 + (-4) = +3 - 4 = -1$

$-4 + (3) = -4 + 3 = -1$

Observe that the change in the order of +3 and −4 does not affect the result.

b) −8 and −5

Let's add −8 and −5.

$-8 + (-5) = -8 - 5 = -13$

$-5 + (-8) = -5 - 8 = -13$

Observe that the change in the order of −8 and −5 does not affect the result.

c) −22 and −12

Let's add −22 and −12.

$-22 + (-12) = -22 - 12 = -34$

$-12 + (-22) = -12 - 22 = -34$

Observe that the change in the order of −22 and −12 does not affect the result.

d) −16 and +21

Let's add −16 and +21.

$-16 + (+21) = -16 + 21 = +5$

$+21 + (-16) = +21 - 16 = +5$

Observe that the change in the order of −16 and + 21 does not affect the result.

4. Verify the associative law for the given integers.

a) −3, −6, −8 b) −7, +5, −12 c) +11, +17, −15 d) +21, +32, +10

Solution:

a) −3, −6, −8

$[(-4 + (+5)] + (-3) = -4 + [(+5) + (-3)]$

$\text{L.H.S} = [(-3 + (-6)] + (-8)$

$= [(-3 - 6)] + (-8)$

$= -9 + (-8)$

$= -9 - 8$

$= -17$
$\text{R.H.S} = -3 + [(-6) + (-8)]$
$= -3 + [(-6) + (-3)]$
$= -3 + [-6 - 8]$
$= -3 - 14$
$= -17$
So, L.H.S = R.H.S.
$[(-3 + (-6)] + (-8) = -3 + [(-6) + (-8)]$
$-17 = -17$

b) −7, +5, −12
$[(-7 + (+5)] + (-12) = -7 + [(+5) + (-12)]$
$\text{L.H.S} = [(-7 + (+5)] + (-12)$
$= [(-7 + 5)] + (-12)$
$= -2 + (-12)$
$= -2 - 12$
$= -14$
$\text{R.H.S} = -7 + [(+5) + (-12)]$
$= -7 + [(+5) + (-12)]$
$= -7 + [5 - 12]$
$= -7 - 7$
$= -14$
So, L.H.S = R.H.S.
$[(-7 + (+5)] + (-12) = -7 + [(+5) + (-12)]$
$-14 = -14$

c) +11, +17, −15
$[(+11 + (+17)] + (-15) = +11 + [(+17) + (-15)]$
$\text{L.H.S} = [(+11 + (+17)] + (-15)$
$= [(+11 + (+17)] + (-15)$
$= +28 + (-15)$
$= +28 - 15$
$= +13$
$\text{R.H.S} = +11 + [(+17) + (-15)]$
$= +11 + [(+17) + (-15)]$
$= +11 + [17 - 15]$
$= +11 + 2$
$= +13$
So, L.H.S = R.H.S.
$[(+11 + (+17)] + (-15) = +11 + [(+17) + (-15)]$
$+13 = +13$

d) +21, +32, +10
$[(+21 + (+32)] + (+10) = +21 + [(+32) + (+10)]$
$\text{L.H.S} = [(+21 + (+32)] + (+10)$
$= [(+21 + 32)] + (+10)$
$= +53 + (+10)$

$= +53 + 10$
$= +63$
$\text{R.H.}S = +21 + [(+32) + (+10)]$
$= +21 + [(+32) + (+10)]$
$= +21 + [32 + 10]$
$= +21 + 42$
$= +63$
So, $\text{L.H.}S = \text{R.H.S.}$
$[(+21 + (+32)] + (+10) = +21 + [(+32) + (+10)]$
$+63 = +63$

5. Find the additive inverse of the given integers.

a) −17 b) +45 c) −22 d) −37 e) +52

Solution:

a) −17
Add −17 and + 17.
−17 + (+17) = −17 + 17 = 0
Here, 17 and −17 are called the additive inverse of each other.

b) + 45
Add + 45 and −45.
+45 + (−45) = 45 – 45 = 0
Here, +45 and −45 are called the additive inverse of each other.

c) − 22
Add – 22 and + 22.
−22 + (+22) = −22 + 22 = 0
Here, 22 and −22 are called the additive inverse of each other.

d) − 37
Add −37 and + 37.
−37 + (+37) = −37 + 37 = 0
Here, 37 and −37 are called the additive inverse of each other.

e) + 52
Add + 52 and −52.
+52 + (−52) = +52 − 52 = 0
Here, 52 and −52 are called the additive inverse of each other.

6. What is the additive identity of the integers?

The additive identity of an integer is 0.

7. Verify the following.

a) (−7) + 10 = 10 + (−7) b) 15 + (−13) = −13 + 15
c) [(2 + (−11)] + 23 = 2 + [(−11 + 23)] d) (−21 + 9) + (−12) = −21+ [(9 + (−12)]

Solution:

a) $(-7) + 10 = 10 + (-7)$
$LHS = (-7) + 10$
$= -7 + 10$
$= +3$
$R.H.S = 10 + (-7)$
$= 10 - 7$

$= +3$
$So, L.H.S = R.H.S.$
$(-7) + 10 = 10 + (-7)$
$+3 = +3$

b) $15 + (-13) = (-13) + 15$
$LHS = 15 + (-13)$
$= +15 - 13$
$= +2$
$R.H.S = (-13) + 15$
$= -13 + 15$
$= +2$
$So, L.H.S = R.H.S.$
$15 + (-13) = (-13) + 15$
$+2 = +2$

c) $[(2 + (-11)] + 23 = 2 + [(-11 + 23)]$
$L.H.S = [(2 + (-11)] + 23$
$= [(2 + (-11)] + 23$
$= -9 + (+23)$
$= -9 + 23$
$= +14$
$R.H.S = 2 + [(-11 + 23)]$
$= 2 + [(-11 + 23)]$
$= 2 + [-11 + 23]$
$= 2 + 12$
$= +14$
$So, L.H.S = R.H.S.$
$[(2 + (-11)] + 23 = 2 + [(-11 + 23)]$
$+14 = +14$

d) $[(-21) + (+9)] + (-12) = -21 + [(+9) + (-12)]$
$L.H.S = [(-21) + (+9)] + (-12)$
$= [(-21) + (+9)] + (-12)$
$= -12 + (-12)$
$= -12 - 12$
$= -24$
$R.H.S = -21 + [(+9) + (-12)]$
$= -21 + [(+9) + (-12)]$
$= -21 + [+9 - 12]$
$= -21 - 3$
$= -24$
$So, L.H.S = R.H.S.$
$[(-21) + (+9)] + (-12) = -21 + [(+9) + (-12)]$
$24 = -24$

8. Write the name of the given properties and also write one example for each.
Note: a, b, and c are elements of a set of integers.

No	Properties	Property Name	Example
a)	a + b = c	Closure property	2 + 3 = 5
b)	a + 0 = a	Additive identity	2 + 0 = 2
c)	a + (- a) = 0	Additive inverse	2 + (-2) = 0
d)	a + (b + c) = (a + b) + c	Associative property	2 + (3 + 4) = (2 + 3) + 4
e)	a + b = b + a	Commutative property	2 + 3 = 3 + 2

Learning Check 2.5

1. Multiply the following and complete the table.

×	−3	−17	+21	+2	−1	0	+7	−11	−6
−6	+18	+102	−126	−12	+6	0	−42	+66	+36
+13	−39	−221	+273	+26	−13	0	+91	−143	−78
−23	+69	+391	−483	−46	+23	0	−161	−253	+138

2. Solve the following.

$a)\ (-3) \times (-4) \times (-2)$ $b)\ (-3) \times (-5) \times (+4)$
$c)\ (+7) \times (-7) \times (-3)$ $d)\ (-81) \times (5) \times (6)$
$e)\ (65) \times (-72) \times (-36)$ $f)\ (-10) \times (-20) \times (30)$

Solution:

a) $(-3) \times (-4) \times (-2)$

Multiplying three negative numbers together gives a negative result, so:

$(-3) \times (-4) = 12$

Now, we can multiply that result by the final factor:

$12\ x\ (-2) = -24$

Therefore,

$(-3) \times (-4) \times (-2) = -24$

b) $(-3) \times (-5) \times (+4)$

Multiplying two negative numbers together gives a positive result, so:

$(-3) \times (-5) = 15$

Now, we can multiply that result by the final factor:

$15\ x\ 4 = 60$

Therefore,

$(-3) \times (-5) \times (+4) = 60$

c) $(+7) \times (-7) \times (-3)$

Multiplying two negative numbers together gives a positive result, so:

$-3 \times -7 = +21$

Now, we can multiply that result by the final factor:

$21 \times 7 = +147$

Therefore,

$(+7) \times (-7) \times (-3) = +147$

d) $(-81) \times (5) \times (6)$

Multiplying one negative and two positive numbers together gives a negative result, so:

$(-81) \times 5 = -405$

Now, we can multiply that result by the final factor:

$-405 \times 6 = -2430$

Therefore,

$(-81) \times (5) \times (6) = -2430$

e) $(65) \times (-72) \times (-36)$

Multiplying two negative numbers together gives a positive result, so:

$(-72) \times (-36) = 2592$

Now, we can multiply that result by the first factor:

$65\, x\, 2592 = 168480$

Therefore,

$(65) \times (-72) \times (-36) = 168480$

f) $(-10) \times (-20) \times (30)$

Multiplying two negative numbers together gives a positive result, so:

$(-10) \times (-20) = 200$

Now, we can multiply that result by the final factor:

$200 \times 30 = 6000$

Therefore,

$(-10) \times (-20) \times (30) = 6000$

3. Solve the following.

a) 56 ÷ 7 b) − 84 ÷ (−2) c) (−33) ÷ 11 d) +21 ÷ (−3) e) −78 ÷ (−13)
f) 99 ÷ (−9) g) −72 ÷ (+8) h) −80 ÷ (−5) i) +45 ÷ (+9) j) (−24) ÷ (−2)

Solution:

a) $56 \div 7$

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.

$(+56) \div (+7) = |+56| \div |+7| = 56 \div 7 = 8$

$(+56) \div (+7) = +8$

b) $-84 \div (-2)$

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.

$(-84) \div (-2) = |-84| \div |-2| = 84 \div 2 = 42$

$(-84) \div (-2) = +42$

c) $(-33) \div 11$

When we divide two integers with opposite sign, simply divide their absolute value and then put the negative sign with the answer.

$(-33) \div (+11) = |-33| \div |+11| = 33 \div 11 = 3$

$(-33) \div (+11) = -3$

d) $+21 \div (-3)$

When we divide two integers with opposite sign, simply divide their absolute value and then put the negative sign with the answer.
$(+21) \div (-3) = |+21| \div |-3| = 21 \div 3 = 7$
$(+21) \div (-3) = -7$

e) $-78 \div (-13)$

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.
$(-78) \div (-13) = |-78| \div |-13| = 78 \div 13 = 6$
$(-78) \div (-13) = +6$

f) $+99 \div (-9)$

When we divide two integers with opposite sign, simply divide their absolute value and then put the negative sign with the answer.
$(+99) \div (-9) = |+99| \div |-9| = 99 \div 9 = 11$
$(+99) \div (-9) = -11$

g) $-72 \div (+8)$

When we divide two integers with opposite sign, simply divide their absolute value and then put the negative sign with the answer.
$(-72) \div (+8) = |-72| \div |+8| = 72 \div 8 = 9$
$(-72) \div (+8) = -9$

h) $-80 \div (-5)$

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.
$(-80) \div (-5) = |-80| \div |-5| = 80 \div 5 = 16$
$(-80) \div (-5) = +16$

i) $+45 \div (+9)$

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.
$(+45) \div (+9) = |+45| \div |+9| = 45 \div 9 = 5$
$(+45) \div (+9) = +5$

j) $(-24) \div (-2)$

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.
$(-24) \div (-2) = |-24| \div |-2| = 24 \div 2 = 12$
$(-24) \div (-2) = +12$

4. **The product of two integers is −49. If one integer is −7, find the other.**
Solution:
Product of two integers = −49
First integer = −7
Other integer = ?
Let x be the other integer, then:
$x \times (-7) = -49$
$x = (-49) \div (-7)$
When we divide the integers with the same sign, then put the positive sign with the result.

$x = +7$
Thus, the other integer is + 7.

5. **Find the product of:**

a) The greatest negative integer and the smallest positive integer.
b) The first 4 negative integers. c) +90 and −90

Solution:

a) Since, the greatest negative integer is -1 and the smallest positive integer is 1. We know that the product of a positive and a negative integer is a negative integer.
$(-1) \times (+1) = -1$

b) The first 4 negative integers are -1, -2, -3, and -4.
We know that the product of even negative integers is a positive integer.
$(-1) \times (-2) \times (-3) \times (-4)$
$= 1 \times 2 \times 3 \times 4$
$= 24$

c) +90 and -90
We know that the product of a positive and a negative integer is a negative integer.
$(+90) \times (-90) = -8100$

Learning Check 2.6

1. **Identify and write the name of the properties.**

No	Property	Property Name
a)	$-6 \times 1 = -6$	Multiplicative identity
b)	$-5 \times (-14) = +70$	Closure property
c)	$-4 \times \frac{1}{-4} = 1$	Multiplicative inverse
d)	$-11 \times (-15) = -15 \times (-11)$	Commutative property
e)	$-17 \times [(-6 \times 19)]$ $= [(-17) \times (-6)] \times 19$	Associative property
f)	$-7 \times [(-9) + (-12)]$ $= [(-7) \times (-9)] + [(-7) \times (-12)]$	Over addition
g)	$-13 \times [4 - (-15)]$ $= [(-13) \times 4] - [(-13) \times (-15)]$	Over subtraction

2. **Verify the closure law for the given integers under multiplication.**

a) −7, −18 b) +11, +12 c) −21, −3 d) −2 and −1

Solution:

a) −7, −18
Let's multiply -7 and -18.
−7 × (−18) = +126
26 is also an integer.

b) +11, +12
Let's multiply +11 and +12.
+11 × (12) = +132
132 is also an integer.

c) −21, −3
Let's multiply -21 and -3.
−21 × (−3) = +63
63 is also an integer.

d) −2 and −1
Let's multiply -2 and -1.
−2 × (−1) = +2
2 is also an integer.

3. Verify the commutative law for the given integers under multiplication.

a) −23, −25 b) −4, +31 c) −10, −20 d) +17, +24

Solution:

a) −23, −25
Let's multiply −23 and −25.
$-23 \times (-25) = -23 \times (-25) = +575$
$-25 \times (-23) = -25 \times (-23) = +575$
Observe that the change in the order of −23 and +8 does not affect the result.

b) $-4, +31$
Let's multiply −4 and +31.
$-4 \times (+31) = -4 \times 31 = -124$
$+31 \times (-4) = +31 \times (-4) = -124$
Observe that the change in the order of −4 and +31 does not affect the result.

c) $-10, -20$
Let's multiply −10 and −20.
$-10 \times (-20) = -10 \times (-20) = +200$
$-20 \times (-10) = -20 \times (-10) = +200$
Observe that the change in the order of −10 and −20 does not affect the result.

d) $+17, +24$
Let's multiply +17 and +24.
$+17 \times (+24) = 17 \times 24 = +408$
$+24 \times (+17) = 24 \times 17 = +408$
Observe that the change in the order of +17 and +24 does not affect the result.

4. Verify the associative law for the given integers under multiplication.

a) −6, −4, −3 b) −17, +15, −10 c) +11, −20, −14 d) +5, +7, −12

Solution:

a) $-6, -4, -3$
$[(-6) \times (-4)] \times (-3) = -6 \times [(-4) \times (-3)]$
$L.H.S = [(-6) \times (-4)] \times (-3)$
$= [6 \times 4] \times (-3)$
$= 24 \times (-3)$
$= -72$

$R.H.S = -6 \times [(-4) \times (-3)]$
$= -6 \times [(-4) \times (-3)]$
$= -6 \times [4 \times 3]$
$= -6 \times 12$
$= -72$
$So, L.H.S = R.H.S.$
$[(-6) \times (-4)] \times (-3) = -6 \times [(-4) \times (-3)]$
$72 = -72$

b) $-17, +15, -10$
$[(-17 \times (+15)] \times (-10) = -17 \times [(+15) \times (-10)]$
$L.H.S = [(-17 \times (+15)] \times (-10)$
$= [-17 \times 15] \times (-10)$
$= -255 \times (-10)$
$= +2550$
$R.H.S = -17 \times [(+15) \times (-10)]$
$= -17 \times [(+15) \times (-10)]$
$= -17 \times [-150]$
$= +2550$
$So, L.H.S = R.H.S.$
$[(-17 \times (+15)] \times (-10) = -17 \times [(+15) \times (-10)]$
$+2550 = +2550$

c) $+11, -20, -14$
$[(+11) \times (-20)] \times (-14) = +11 \times [(-20) \times (-14)]$
$L.H.S = [(+11) \times (-20)] \times (-14)$
$= [11 \times (-20)] \times (-14)$
$= -220 \times (-14)$
$= +3080$
$R.H.S = +11 \times [(-20) \times (-14)]$
$= +11 \times [(-20) \times (-14)]$
$= +11 \times [280]$
$= +3080$
$So, L.H.S = R.H.S.$
$[(+11) \times (-20)] \times (-14) = +11 \times [(-20) \times (-14)]$
$+3080 = +3080$

d) $+5, +7, -12$
$[(+5) \times (+7)] \times (-12) = +5 \times [(7) \times (-12)]$
$L.H.S = [(+5) \times (+7)] \times (-12)$
$= [(+5) \times (+7)] \times (-12)$
$= +35 \times (-12)$
$= -420$
$R.H.S = +5 \times [(7) \times (-12)]$
$= +5 \times [(7) \times (-12)]$
$= +5 \times [-84]$
$= -420$

$So, L.H.S = R.H.S.$
$[(+5) \times (+7)] \times (-12) = +5 \times [(7) \times (-12)]$
$-420 = -420$

5. Verify the distributive laws of multiplication under addition for the given integers.

a) −11, −12, −13 b) −24, +29, −13 c) −1, −4, +9 d) +13, +15, −6

Solution:

a) $-11, -12, -13$
$-11 \times [(-12) + (-13)] = [(-11) \times (-12)] + [(-11) \times (-13)]$
$Look\ below$:
$Left\ Hand\ Side\ (L.H.S)$
$-11 \times [(-12) + (-13)]$
$-11 \times [-12 - 13]$
$= -11 \times (-25)$
$= +275$
$Right\ Hand\ Side\ (R.H.S)$
$[(-11) \times (-12)] + [(-11) \times (-13)]$
$= +132 + (+143)$
$= +275$
$We\ observe\ that\ L.H.S = R.H.S.$
$-11 \times [(-12) + (-13)] = [(-11) \times (-12)] + [(-11) \times (-13)]$

b) $-24, +29, -13$
$-24 \times [(+29) + (-13)] = [(-24) \times (+29)] + [(-24) \times (-13)]$
$Look\ below$:
$Left\ Hand\ Side\ (L.H.S)$
$-24 \times [(+29) + (-13)]$
$= -24 \times (+16)$
$= -384$
$Right\ Hand\ Side\ (R.H.S)$
$[(-24) \times (+29)] + [(-24) \times (-13)]$
$= -696 + (+312)$
$= -384$
$We\ observe\ that\ L.H.S = R.H.S.$
$-24 \times [(+29) + (-13)] = [(-24) \times (+29)] + [(-24) \times (-13)]$

c) $-1, -4, +9$
$-1 \times [(-4) + (+9)] = [(-1) \times (-4)] + [(-1) \times (+9)]$
$Look\ below$:
$Left\ Hand\ Side\ (L.H.S)$
$-1 \times [(-4) + (+9)]$
$= -1 \times (+5)$
$= -5$
$Right\ Hand\ Side\ (R.H.S)$
$[(-1) \times (-4)] + [(-1) \times (+9)]$
$= +4 + (-9)$

$= -5$
$We\ observe\ that\ L.H.S\ =\ R.H.S.$
$-1 \times [(-4) + (+9)] = [(-1) \times (-4)] + [(-1) \times (+9)]$

d) $+13, +15, -6$
$+13 \times [(+15) + (-6)] = [(+13) \times (+15)] + [(+13) \times (-6)]$
$Look\ below$:
$Left\ Hand\ Side\ (L.H.S)$
$+13 \times [(+15) + (-6)]$
$= +13 \times (+9)$
$= +117$
$Right\ Hand\ Side\ (R.H.S)$
$[(+13) \times (+15)] + [(+13) \times (-6)]$
$= +195 - 78$
$= +117$
$We\ observe\ that\ L.H.S\ =\ R.H.S.$
$+13 \times [(+15) + (-6)] = [(+13) \times (+15)] + [(+13) \times (-6)]$

6. Verify the distributive laws of multiplication under subtraction for the given integers.

a) −16, +18, +23 b) −21, −40, −20 c) −6, −7, +11 d) +9, +8, −2

Solution:

a) $-16, +18, +23$
$[(-16) - (+18)] \times (+23) = [(-16) \times (+23)] - [(+18) \times (+23)]$
$Left\ Hand\ Side\ (L.H.S)$
$[(-16) - (+18)] \times (+23)$
$= [-16 - 18] \times (+23)$
$= -34 \times (+23)$
$= -782$
$Right\ Hand\ Side\ (R.H.S)$
$[(-16) \times (+23)] - [(+18) \times (+23)]$
$= [-16 \times 23] - [18 \times 23]$
$= -368 - 414$
$= -782$
$We\ observe\ that\ L.H.S\ =\ R.H.S.$
$Thus, [(-16) - (+18)] \times (+23) = [(-16) \times (+23)] - [(+18) \times (+23)]$

b) $-21, -40, -20$
$[(-21) - (-40)] \times (-20) = [(-21) \times (-20)] - [(-40) \times (-20)]$
$Left\ Hand\ Side\ (L.H.S)$
$[(-21) - (-40)] \times (-20)$
$= [-21 + 40] \times (-20)$
$= +19 \times (-20)$
$= -380$
$Right\ Hand\ Side\ (R.H.S)$
$[(-21) \times (-20)] - [(-40) \times (-20)]$
$= [-21 \times (-20)] - [-40 \times (-20)]$

$= +420 - (+800)$
$= +420 - 800$
$= -380$
$We\ observe\ that\ L.H.S = R.H.S.$
$Thus, [(-21) - (-40)] \times (-20) = [(-21) \times (-20)] - [(-40) \times (-20)].$

c) $-6, -7, +11$
$[(-6) - (-7)] \times (+11) = [(-6) \times (+11)] - [(-7) \times (+11)]$
$Left\ Hand\ Side\ (L.H.S)$
$[(-6) - (-7)] \times (+11)$
$[-6 + 7] \times (+11)$
$= +1 \times (+11)$
$= +11$
$Right\ Hand\ Side\ (R.H.S)$
$[(-6) \times (+11)] - [(-7) \times (+11)]$
$[-6 \times 11] - [-7 \times 11]$
$= -66 - (-77)$
$= -66 + 77$
$= +11$
$We\ observe\ that\ L.H.S = R.H.S.$
$Thus, [(-6) - (-7)] \times (+11) = [(-6) \times (+11)] - [(-7) \times (+11)].$

d) $+9, +8, -2$
$[(+9) - (+8)] \times (-2) = [(+9) \times (-2)] - [(+8) \times (-2)]$
$Left\ Hand\ Side\ (L.H.S)$
$[(+9) - (+8)] \times (-2)$
$= [9 - 8] \times (-2)$
$= +1 \times (-2)$
$= -2$
$Right\ Hand\ Side\ (R.H.S)$
$[(+9) \times (-2)] - [(+8) \times (-2)]$
$= [9 \times (-2)] - [8 \times (-2)]$
$= -18 - (-16)$
$= -18 + 16$
$= -2$
$We\ observe\ that\ L.H.S = R.H.S.$
$Thus, [(+9) - (+8)] \times (-2) = [(+9) \times (-2)] - [(+8) \times (-2)].$

7. Write the multiplicative inverse of each of the given integers.

a) −18 b) +29 c) −13 d) +27 e) −54

Solution:

a) Multiply −18 and $\frac{1}{-18}$

$-18 \times \left(\frac{1}{-18}\right) = 1$

Here, −18 and $\frac{1}{-18}$ are called the multiplicative inverse of each other.

b) Multiply +29 and $\frac{1}{+29}$

$+29 \times (\frac{1}{+29}) = 1$

Here, +29 and $\frac{1}{+29}$ are called the multiplicative inverse of each other.

c) Multiply −13 and $\frac{1}{-13}$

$-13 \times (\frac{1}{-13}) = 1$

Here, −13 and $\frac{1}{-13}$ are called the multiplicative inverse of each other.

d) Multiply +27and $\frac{1}{+27}$

$+27 \times (\frac{1}{+27}) = 1$

Here, +27 and $\frac{1}{+27}$ are called the multiplicative inverse of each other.

e) Multiply −54 and $\frac{1}{-54}$

$-54 \times (\frac{1}{-54}) = 1$

Here, −54 and $\frac{1}{-54}$ are called the multiplicative inverse of each other.

9. What is the multiplicative identity of an integer?

Solution:

The multiplicative identity of an integer is the number " 1 ".

Learning Check 2.7

1. Simplify the following.

a) 207− (102 + 56) − 45÷ 9

b) 371 + (108 −50) − 48 ÷ 6 × 12

c) $2\frac{6}{7} \times (\frac{3}{5} - \frac{1}{2}) + 2\frac{3}{10} - \frac{4}{5}$

d) $5\frac{3}{4} - \left(7\frac{3}{4} - 6\frac{1}{2}\right) - \frac{11}{15} \div \frac{2}{3}$

e) 408 + 45.15 ÷15 −22 × 12

f) 25.75 – (18.20 + 4.45) +1.32 × 7.5 ÷2.5

g) 84.66 + 7.2 ÷1.8 −75.99 ÷3

h) $\frac{3}{2}$ × 597 − 16 × 120 ÷ 10 +14

i) $\frac{4}{5}$× 120 − 15 +56 ÷ 8 −10

j) $\frac{3}{10}$ × 150 +12 × 49 ÷7 −3

Solution:

a) 207− (102 + 56) − 45÷ 9

Step 1: Add within the parentheses.

= 207− 58 − 45÷ 9

Step 2: Divide

= 207− 58 − 5

Step 3: Subtract

= 144

b) 371 + (108 – 50) – 48 ÷ 6 × 12

Step 1: Subtract within the parentheses.

= 371 + 58 – 48 ÷ 6 × 12

Step 2: Divide

= 371 + 58 – 8 × 12

Step 3: Multiply

= 371 + 58 – 96
Step 4: Add
= 429 – 96
Step 5: Subtract
= 333

c) $2\frac{6}{7}\times(\frac{3}{5}-\frac{1}{2}) + 2\frac{3}{10}-\frac{4}{5}$
Step 1: Convert mixed fraction into improper fraction.
$= \frac{20}{7} - \left(\frac{3}{5} - \frac{1}{2}\right) - \frac{23}{10} \div \frac{4}{5}$
Step 2: Subtract within the parentheses.
$= \frac{20}{7} - \frac{1}{10} - \frac{23}{10} \div \frac{4}{5}$
Step 3: Divide
$= \frac{20}{7} - \frac{1}{10} - \frac{23}{8}$
Step 4: Subtract
$= -\frac{33}{280}$

d) $5\frac{3}{4} - \left(7\frac{3}{4} - 6\frac{1}{2}\right) - \frac{11}{15} \div \frac{2}{3}$
Step 1: Convert mixed fraction into improper fraction.
$= \frac{23}{4} - \left(\frac{31}{4} - \frac{13}{2}\right) - \frac{11}{15} \div \frac{2}{3}$
Step 2: Subtract within the parentheses.
$= \frac{23}{4} - \frac{5}{4} - \frac{11}{15} \div \frac{2}{3}$
Step 3: Divide
$= \frac{23}{4} - \frac{5}{4} - \frac{11}{10}$
Step 4: Subtract
$= 3\frac{22}{5}$

e) 408 + 45.15 ÷15 − 22 × 12
Step 1: Divide
= 408 + 3.01 − 22 × 12
Step 2: Multiply
= 408 + 3.01 − 264
Step 3: Add
= 411.01 − 264
Step 4: Subtract
= 147.01

f) 25.75 – (18.20 + 4.45) +1.32 × 7.5 ÷2.5
Step 1: Add within the parentheses.
= 25.75 – 22.65 +1.32 × 7.5 ÷2.5
Step 2: Divide
= 25.75 – 22.65 +1.32 × 3
Step 3: Multiply
= 25.75 – 22.65 +3.96
Step 4: Add

= 29.71 – 22.65
Step 5: Subtract
= 7.06

g) 84.66 + 7.2 ÷1.8 −75.99 ÷3
Step 1: Divide
= 84.66 + 4 −25.33
Step 2: Add
= 88.66 −25.33
Step 3: Subtract
= 63.33
84.66 + 4 −25.33

h) $\frac{3}{2}$ × 597 − 16 × 120 ÷ 10 +14
Step 1: Divide
1.5× 597 − 16 × 12 +14
Step 2: Multiply
= 895.5 − 192 +14
Step 3: Add
= 909.5 − 192
Step 4: Subtract
= 717.5

i) $\frac{4}{5}$× 120 − 15 +56 ÷ 8 −10
Step 1: Divide
= 0.8× 120 − 15 +7 −10
Step 2: Multiply
= 96 − 15 +7 −10
Step 3: Add
= 103 – 15 −10
Step 4: Subtract
= 78

j) $\frac{3}{10}$ × 150 +12 × 49 ÷7 −3
Step 1: Divide
= 0.3 × 150 +12 × 7 −3
Step 2: Multiply
= 45 + 84 − 3
Step 3: Add
= 129 – 3
Step 4: Subtract
= 126

2.

The cost of one pencil, one eraser and one notebook is Rs 15, Rs 9 and Rs 45, respectively. If Salman bought 5 pencils, 10 erasers and 11 notebooks, what is the total amount that Salman paid to the shopkeeper?

Solution:

The cost of one pencil = Rs. 15
The cost of one eraser = Rs. 9
The cost of one notebook = Rs. 45
The cost of 5 pencils = 5 × Rs. 15
The cost of 10 erasers = 10 x Rs 9
The cost of 11 notebooks = 11 ×Rs. 45
Total cost = (5 × 15) + (10 × 9) + (11 × 45)
= 75 + 90 + 495 = 660

Therefore, the total amount that Salman paid to the shopkeeper is Rs 660.

3. **In a restaurant, a family ordered 5 sandwiches for Rs 350 each, 4 glasses of cold drinks for Rs 280 each and 3 cups of coffee for Rs 410. They paid an additional Rs 234 tax. How much was their total bill?**

   Solution:

   The cost of 5 sandwiches each = 5 × Rs.350
   The cost of 4 glasses of cold drinks each = 4 × Rs.280
   The cost of 3 cups of coffee = Rs.410
   Additional tax = Rs. 234
   Total cost = (5 × 350) + (4 × 280) + 410 + 234
   = 1750 + 1120 + 410 + 234
   = 3,514

   Therefore, the total bill for the family's meal in the restaurant is Rs 3514.

4. **Aman stitched 4 scarves, 3 shirts and 5 abayas. She used 1.5 m of cloth for each scarf, 2.75 m of cloth for each shirt and 5.75 m for each abaya. How much cloth did she use in all?**

   Solution:

   Amount of cloth used for 4 scarves = 4 x 1.5 m
   Amount of cloth used for 3 shirts = 3 x 2.75 m
   Amount of cloth used for 5 abayas = 5 x 5.75 m
   Total amount of cloth used = (4 x 1.5) + (3 x 2.75) + (5 x 5.75)
   = 6 + 8.25 + 28.75
   = 43

   Therefore, Aman used a total of 43 meters of cloth for 4 scarves, 3 shirts, and 5 abayas.

Unit Check 2

1. **Encircle the correct answer.**

   a) All positive numbers, negative numbers and the number 0 together make the set of:
   i) whole numbers   ii) natural numbers   iii) integers   iv) odd numbers

b) The integer that is greater than −17 is:
i) −12 ii) −18 iii) −20 iv) −25

c) The sum of −6 and +13 is:
i) −19 ii) +7 iii) −7 iv) +19

d) The result of the subtraction of −19 from −21 is:
i) −40 ii) +3 iii) −2 iv) +21

e) The greatest negative integer is:
i) −2 ii) 0 iii) −10 iv) −1

2. Fill in the blanks.

a) The smallest positive integer is **1**.
b) The product of two negative integers is always **positive** integers.
c) The multiplicative identity for integers is **1**.
d) In the BODMAS rule, first we solve **division** operations.
e) According to **closure** law, the product of two integers is always an integer.

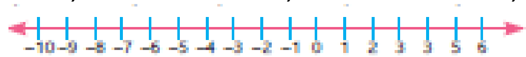
3. Write the set of integers between −11 to +7 and then show those integers on a number line.

Solution:

The set of integers between -11 to +7 is:
$\{-11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\}$
To represent these integers on a number line, we can draw a horizontal line and mark $-11, -10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6$, and 7 on it.

4. Show the given numbers on a number line.

a) −18 b) −6 c) +12 d) −23 e) 13



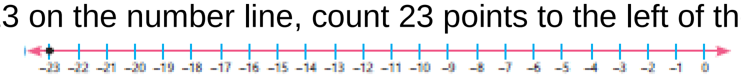
Solution:

a) – 18
To represent −18 on the number line, count 18 points to the left of the zero.

b) −6
To represent −18 on the number line, count 18 points to the left of the zero.

c) + 12
To represent +12 on the number line, count 12 points to the right of the zero.

d) −23
To represent −23 on the number line, count 23 points to the left of the zero.



e) +13

To represent +12 on the number line, count 12 points to the right of the zero.

5. Write all the positive integers between −1 to 5.

Solution:

Positive integers are the whole numbers that are greater than zero. The given range is from -1 to 5.

The positive integers between -1 to 5 would be all the whole numbers in that range that are greater than 0.

Therefore, the positive integers between -1 to 5 are:

1, 2, 3, 4, 5.

6. Write the integers between the given integers.

a) +3 and +9 b) −7 and +1 c) −11 and −4 d) −27 and −15

Solution:

a) +3 and +9

To find the integers between +3 and +9, we need to list all integers that are greater than +3 but less than +9. These are: +4, +5, +6, +7, +8.

b) −7 and +1

To find the integers between −7 and +1, we need to list all integers that are greater than −7 but less than +1. These are: -6, -5, -4, -3, -2, -1, 0.

c) −11 and −4

To find the integers between −11 and −4, we need to list all integers that are greater than −11 but less than −4. These are: -10, -9, -8, -7, -6, -5.

d) −27 and −15

To find the integers between −27 and −15, we need to list all integers that are greater than −27 but less than −15. These are:

-26, -25, -24, -23, -22, -21, -20, -19, -18, -17, -16.

7. Write the next 4 integers for each number sequence.

a) −3, −5, −7 b) +1, +6, +11 c) −13, −3, +7 d) −10, −5, 0

Solution:

a) −3, −5, −7

This sequence is decreasing by 2 each time. So, to get the next number, we subtract 2 from the previous number. Hence, the next 4 integers are:

$-7 - 2 = -9$, $-9 - 2 = -11$, $-11 - 2 = -13$, $-13 - 2 = -15$.

b) +1, +6, +11

This sequence is increasing by 5 each time. So, to get the next number, we add 5 to the previous number. Hence, the next 4 integers are:

$+11 + 5 = 16$, $+16 + 5 = +21$, $+21 + 5 = +26$, $+26 + 5 = +31$

c) −13, −3, +7

This sequence is increasing by 10 each time. So, to get the next number, we add 10 to the previous number. Hence, the next 4 integers are:

$+7 + 10 = +17$, $+17 + 10 = +27$, $+27 + 10 = +37$, $+37 + 10 = +47$

d) $-10, -5, 0$
This sequence is increasing by 5 each time. So, to get the next number, we add 5 to the previous number. Hence, the next 4 integers are:
$0 + 5 = +5$, $+ 5 + 5 = +10$, $+ 10 + 5 = + 15$, $+ 15 + 5 = + 20$

8. Write the opposite integers for the following.

a) -8 b) $+7$ c) -38 d) -71 e) $+55$

Solution:

a) The opposite of $- 8$ is $+ 8$. This is because the opposite of a negative integer is a positive integer.
b) The opposite of $+ 7$ is -7. This is because the opposite of a positive integer is a negative integer.
c) The opposite of $- 38$ is $+ 38$. This is because the opposite of a negative integer is a positive integer.
d) The opposite of $- 71$ is $+ 71$. This is because the opposite of a negative integer is a positive integer.
e) The opposite of $+ 55$ is -55. This is because the opposite of a positive integer is a negative integer.

9. Compare and write the smallest and the greatest integer.

a) $-7, +5, -4, +11$ b) $-15, -17, +3, -3$ c) $-24, +21, -34, -20$

Solution:

a) $-7, +5, -4, +11$
On a number line, 11 lies to the right of the integers $+ 5$, $- 4$ and -7. So, $+11$ is the greatest integer.
Similarly, -4 lies to the right of -7. So, -7 is smaller than -4. Thus, 11 is the greatest and -7 is the smallest among these integers.
Symbolically $-7 < -4 < 5 < 11$.

b) $-15, -17, +3, -3$
On a number line, 11 lies to the right of the integers $+ 5$, $- 4$ and -7. So, $+11$ is the greatest integer.
Similarly, -4 lies to the right of -7. So, -7 is smaller than -4. Thus, 11 is the greatest and -7 is the smallest among these integers.
Symbolically $-7 < -4 < 5 < 11$.

c) $-24, +21, -34, -20$
On a number line, 11 lies to the right of the integers $+ 5$, $- 4$ and -7. So, $+11$ is the greatest integer.
Similarly, -4 lies to the right of -7. So, -7 is smaller than -4. Thus, 11 is the greatest and -7 is the smallest among these integers.
Symbolically $-7 < -4 < 5 < 11$.

10. Arrange the given integers in ascending and descending order.

a) −2, −6, −11, −14 b) −5, −4, +3, +1 +5 c) −11, −23, +25, +13

Solution:

a) −2, −6, −11, −14

Compare the negative numbers separately.

−14 < −11 < − 6 < −2 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

-5, -4, -2, +6

Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:

+ 6, −2, −4, −5

b) −5, −4, +3, +1 +5

Compare the negative numbers separately.

−5 < −4 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Similarly, compare the positive integers separately.

5 is the greatest positive integer here and +1 is the smallest positive integer.

So, 5 > 3 > 1.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

−5, −4, +1, +3, +5

Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:

+5, +3, +1, −4, −5

c) −11, −23, +25, +13

Compare the negative numbers separately.

−23 < −11 (A greater number with a negative sign is always smaller than every smaller number with a negative sign.)

Similarly, compare the positive integers separately.

25 is the greatest positive integer here and 13 is the smallest positive integer.

So, 25 < 13.

0 is smaller than the positive integers and greater than the negative integers. So, it will be between them.

Arranging the integers from the greatest to the smallest, i.e., in ascending order, we get:

−23, −11, 13, 25

Arranging the integers from the greatest to the smallest, i.e., in descending order, we get:

25, 13, −11, −23

11. Find the absolute value of the given integers.

a) −15 b) −44 c) −32 d) −56 e) +77

Solution:

a) The absolute value of -15 is |-15| = 15. This is because the absolute value of a number is its distance from zero on the number line, and |-15| represents the distance between -15 and 0, which is 15.

b) The absolute value of -44 is |-44| = 44. This is because the absolute value of a number is its distance from zero on the number line, and |-44| represents the distance between -44 and 0, which is 44.

c) The absolute value of -32 is |-32| = 32. This is because the absolute value of a number is its distance from zero on the number line, and |-32| represents the distance between -32 and 0, which is 32.

d) The absolute value of -56 is |-56| = 56. This is because the absolute value of a number is its distance from zero on the number line, and |-56| represents the distance between -56 and 0, which is 56.

e) The absolute value of +77 is |+77| = 77. This is because the absolute value of a number is its distance from zero on the number line, and |+77| represents the distance between +77 and 0, which is 77.

12. 12. Arrange the absolute values of the given integers in ascending and descending order.

a) −25, −28, +31, −39, −18 b) −9, +8, −5, −19 c) −1, +3, +11, −13, −16

Solution:

a) −25, −28, +31, −39, −18

In absolute notation, we can write as:

|−25| = 25, |−28| = 28, |+31| = 31, |−39| = 39, |−18| = 18

Arranging the absolute values from the greatest to the smallest, i.e., in ascending order, we get:

18, 25, 28, 31, 39

Arranging the absolute values from the greatest to the smallest, i.e., in descending order, we get:

39, 31, 28, 25, 18

b) −9, +8, −5, −19

In absolute notation, we can write as:

|−19| = 19, |−9| = 9, |−5| = 5, |+8| = 8

Arranging the absolute values from the greatest to the smallest, i.e., in ascending order, we get:

5, 8, 9, 19

Arranging the absolute values from the greatest to the smallest, i.e., in descending order, we get:

19, 9, 8, 5

c) −1, +3, +11, −13, −16

In absolute notation, we can write as:

|−1| = 1, |+3| = 3, |+11| = 11, |−13| = 13, |−16| = 16

Arranging the absolute values from the greatest to the smallest, i.e., in ascending order, we get:
1, 3, 11, 13, 16
Arranging the absolute values from the greatest to the smallest, i.e., in descending order, we get:
16, 13, 11, 3, 1

13. Solve the following to complete the tables.

a)

+	−7	−13	+15
−3	−10	−16	+12
+16	+9	+3	+31
−47	+40	+34	−32

b)

×	−2	−11	+25
+24	−48	−264	+600
+14	−28	−154	+350
−3	+6	+33	−75

14. Subtract the following.

a) −18 from +28 b) +17 from −78 c) −56 from −45

Solution:

a) −18 from +28

The first number + 28 remains unchanged. Change the operation from subtraction to addition and change the sign of the number – 18 that is to be subtracted. It will become + 28. Then add according to the rules for adding integers.

$(+28) - (-18) = (+28) + (+18)$

Take the absolute values.

$|+28| = 28$ and $|- 18| = 18$

Add the absolute values.

$28 + 18 = 46$

Put the common sign with the answer.

So, $(28) - (-18) = + 46$.

b) +17 from −78

The first number + 17 remains unchanged. Change the operation from subtraction to addition and change the sign of the number – 78 that is to be subtracted. It will become + 78. Then add according to the rules for adding integers.

$(+17) - (-78) = (+17) + (+78)$

Take the absolute values.

$|+17| = 17$ and $|- 78| = 78$

Add the absolute values.

$17 + 78 = 95$

Put the common sign with the answer.

So, (17) − (−78) = + 95.

c) −56 from −45

The first number – 56 remains unchanged. Change the operation from subtraction to addition and change the sign of the number −45 that is to be subtracted. It will become +45. Then add according to the rules for adding integers.

(-56) − (−45) = (-56) + (+45)

Take the absolute values.

|-56| = 56 and |-45| = 45

Subtract the absolute values.

56 – 45 = 11

Put the common sign with the answer.

So, (-56) − (−45) = −11.

15. Divide the given integers.

a) −16 by −4 b) +44 by −11 c) −86 by +2 d) +72 by +9

Solution:

a) −16 by −4

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.

$(-16) \div (-4) = |-16| \div |-4| = 16 \div 4 = 4$

$(-16) \div (-4) = +4$

b) +44 by −11

When we divide two integers with opposite sign, simply divide their absolute value and then put the negative sign with the answer.

$(+44) \div (-11) = |+44| \div |-11| = 44 \div 11 = 4$

$(+44) \div (-11) = -4$

c) −86 by +2

When we divide two integers with opposite sign, simply divide their absolute value and then put the negative sign with the answer.

$(-86) \div (+2) = |-86| \div |+2| = 86 \div 2 = 43$

$(-86) \div (+2) = -43$

d) +72 by +9

When we divide the integers with the same sign, simply divide the absolute value of the integers and then divide and put the positive sign with the result.

$(+72) \div (+9) = |+72| \div |+9| = 72 \div 9 = 8$

$(+72) \div (+9) = +8$

16. Verify the commutative property of addition and multiplication.

a) −6, −4 b) −11, +17 c) −26, −27 d) +15, +17

Solution:

a) −6 and −4

Let’s add −6 and −4.

$-6 + (-4) = -6 - 4 = -10$

$-4 + (-6) = -4 - 6 = -10$

Observe that the change in the order of −6 and

−4 does not affect the result.
Let's multiply −6 and −4.
$-6 \times (-4) = -6 \times (-4) = +24$
$-4 \times (-6) = -4 \times (-6) = +24$
Observe that the change in the order of −6 and +4 does not affect the result.

b) −11 and +17
Let's add −11 and +17.
$-11 + (+17) = -11 + 17 = +6$
$-11 + (+17) = -11 + 17 = +6$
Observe that the change in the order of −11 and +17 does not affect the result.
Let's multiply −11 and +17.
$-11 \times (+17) = +17 \times (-11) = -187$
$+17 \times (-11) = +17 \times (-11) = -187$
Observe that the change in the order of −11 and +17 does not affect the result.

c) −26 and −27
Let's add −26 and −27.
$-26 + (-27) = -26 - 27 = -53$
$-27 + (-26) = -27 - 26 = -53$
Observe that the change in the order of −26 and −27 does not affect the result.
Let's multiply −26 and −27.
$-26 \times (-27) = -26 \times (-27) = +702$
$-27 \times (-26) = -27 \times (-26) = +702$
Observe that the change in the order of −26 and −27 does not affect the result.

d) +15 and +17
Let's add +15and +17.
$+15 + (+17) = +15 + 17 = +32$
$+17 + (+15) = +17 + 15 = +32$
Observe that the change in the order of +15 and +17 does not affect the result.
Let's multiply +15 and +17.
$+15 \times (+17) = +15 \times (+17) = +255$
$+17 \times (+15) = +17 \times (+15) = +255$
Observe that the change in the order of +15 and +17 does not affect the result.

17. Verify the associative property of addition and multiplication.

a) −23, −14, −10 b) −2, +7, +12 c) −3, −15, −20 d) 14, −13, −21

Solution:

a) −23, −14, −10

For Addition:

$[(-23) + (-14)] + (-10) = -23 + [(-14) + (-10)]$
$L.H.S = [(-23) + (-14)] + (-10)$
$= [-23 - 14] + (-10)$
$= -37 + (-10)$

$= -37 - 10$
$= -47$
$R.H.S = -23 + [(-14) + (-10)]$

$= -23 + [(-14) + (-10)]$
$= -23 + [-14 - 10]$
$= -23 - 24$
$= -47$
So, L.H.S = R.H.S.
$[(-23) + (-14)] + (-10) = -23 + [(-14) + (-10)]$
$-47 = -47$
For Multiplication:
$[(-23) \times (-14)] \times (-10) = -23 \times [(-14) \times (-10)]$
$L.H.S = [(-23) + (-14)] + (-10)$
$= [23 \times 14] \times (-10)$
$= 322 \times (-10)$
$= -3220$
$R.H.S = -23 \times [(-14) \times (-10)]$
$= -23 \times [(-14) \times (-10)]$
$= -23 \times [14 \times 10]$
$= -23 \times 140$
$= -3220$
So, L.H.S = R.H.S.
$[(-23) \times (-14)] \times (-10) = -23 \times [(-14) \times (-10)]$
$3220 = -3220$

b) $-2, +7, +12$
For Addition:
$[(-2) + (+7)] + (+12) = -2 + [(+7) + (+12)]$
$L.H.S = [(-2) + (+7)] + (+12)$
$= [(-2) + (+7)] + (+12)$
$= [-2 + 7] + (+12)$
$= 5 + 12 \quad = 17$
$R.H.S = -2 + [(+7) + (+12)]$
$= -2 + [(+7) + (+12)]$
$= -2 + [7 + 12]$
$= -2 + 19 \quad = 17$
So, L.H.S = R.H.S.
$[(-2) + (+7)] + (+12) = -2 + [(+7) + (+12)]$
$17 = 17$
For Multiplication:
$[(-2) \times (+7)] \times (+12) = -2 \times [(+7) \times (+12)]$
$L.H.S = [(-2) \times (+7)] \times (+12)$
$= [(-2) \times (+7)] \times (+12)$
$= [-2 \times 7] \times (12)$

$= -14 \times 12 = -168$
$R.H.S = -2 \times [(+7) \times (+12)]$
$= -2 \times [7 \times 12]$
$= -2 \times 84 = -168$
So, L.H.S = R.H.S.
$[(-2) \times (+7)] \times (+12) = -2 \times [(+7) \times (+12)]$
$168 = -168$

c) −3, −15, −20
For Addition:
$[(-3) + (-15)] + (-20) = -3 + [(-15) + (-20)]$
$L.H.S = [(-3) + (-15)] + (-20)$
$= [-3 - 15] + (-20)$
$= -18 - 20$
$= -38$
$R.H.S = -3 + [(-15) + (-20)]$
$= -3 + [-15 - 20]$
$= -3 + (-35)$
$= -3 - 35$
$= -38$
So, L.H.S = R.H.S.
$[(-3) + (-15)] + (-20) = -3 + [(-15) + (-20)]$
$-38 = -38$
For Multiplication:
$[(-3) \times (-15)] \times (-20) = -3 \times [(-15) \times (-20)]$
$L.H.S = [(-3) \times (-15)] \times (-20)$
$= [-3 \times (-15)] \times (-20)$
$= 45 \times (-20)$
$= -900$
$R.H.S = -3 \times [(-15) \times (-20)]$
$= -3 \times [-15 \times (-20)]$
$= -3 \times 300$
$= -900$
So, L.H.S = R.H.S.
$[(-3) \times (-15)] \times (-20) = -3 \times [(-15) \times (-20)]$
$900 = -900$

d) $14, -13, -21$
For Addition:
$[(+14) + (-13)] + (-21) = +14 + [(-13) + (-21)]$
$L.H.S = [(+14) + (-13)] + (-21)$
$= [+14 - 13] + (-21)$
$= +1 + (-21)$
$= +1 - 21$
$= -20$
$R.H.S = +14 + [(-13) + (-21)]$

$= +14 + [-13 - 21]$
$= +14 + (-34)$
$= +14 - 34$
$= -20$
So, L.H.S = R.H.S.
$[(+14) + (-13)] + (-21) = +14 + [(-13) + (-21)]$
$-20 = -20$
For Multiplication:
$[(+14) \times (-13)] \times (-21) = +14 \times [(-13) \times (-21)]$
$L.H.S = [(+14) \times (-13)] \times (-21)$
$= [+14 \times (-13)] \times (-21)$
$= -182 \times (-21)$
$= +3822$
$R.H.S = +14 \times [(-13) \times (-21)]$
$= +14 \times [(-13) \times (-21)]$
$= +14 \times [13 \times 21]$
$= +14 \times 273$
$= +3822$
So, L.H.S = R.H.S.
$[(+14) \times (-13)] \times (-21) = +14 \times [(-13) \times (-21)]$
$+3822 = +3822$

18. Write the additive and multiplicative inverse for the given integers.

a) −4 b) −7 c) +16 d) −23 e) −19 f) +45

Solution:

a) −4
For additive inverse:
Add −4 and + 4.
$-4 + (+4) = -4 + 4 = 0$
Here, 4 and −4 are called the additive inverse of each other.
For multiplicative inverse:
Multiply −4 and $\frac{1}{-4}$
$-4 \times (\frac{1}{-4}) = 1$
Here, −4 and $\frac{1}{-4}$ are called the multiplicative inverse of each other.

b) −7
For additive inverse:
Add −7 and + 7.
$-7 + (+7) = -7 + 7 = 0$
Here, 7 and −7 are called the additive inverse of each other.
For multiplicative inverse:
Multiply −7and $\frac{1}{-7}$
$-7 \times (\frac{1}{-7}) = 1$

Here, −7 and $\frac{1}{-7}$ are called the multiplicative inverse of each other.

c) +16

For additive inverse:

Add +16 and −16.

$+16 + (-16) = +16 - 16 = 0$

Here, 16 and −16 are called the additive inverse of each other.

For multiplicative inverse:

Multiply + 16and $\frac{1}{+16}$

$+16 \times (\frac{1}{+16}) = 1$

Here, + 16 and $\frac{1}{+16}$ are called the multiplicative inverse of each other.

d) −23

For additive inverse:

Add −23 and + 23.

$-23 + (+23) = -23 + 23 = 0$

Here, 23 and −23 are called the additive inverse of each other.

For multiplicative inverse:

Multiply −23 and $\frac{1}{-23}$

$-23 \times (\frac{1}{-23}) = 1$

Here, −23 and $\frac{1}{-23}$ are called the multiplicative inverse of each other.

e) −19

For additive inverse:

Add −19 and + 19.

$-19 + (+19) = -19 + 19 = 0$

Here, 19 and −19 are called the additive inverse of each other.

For multiplicative inverse:

Multiply −19 and $\frac{1}{-19}$

$-19 \times (\frac{1}{-19}) = 1$

Here, −19 and $\frac{1}{-19}$ are called the multiplicative inverse of each other.

f) +45

For additive inverse:

Add +45 and − 45.

$-45 + (+45) = -45 + 45 = 0$

Here, 45 and −45 are called the additive inverse of each other.

For multiplicative inverse:

Multiply +27and $\frac{1}{+27}$

$+45 \times (\frac{1}{+45}) = 1$

Here, +45 and $\frac{1}{+45}$ are called the multiplicative inverse of each other.

19. Solve the following expressions.

$a)\ 5[6.4 - \{5.5 + 2\,(2.8 + 3.6 - 2.1)\}] \div 4$

$b)\ 184.75 - 4[26.24 - 3\,\{12.84 - (6.4 + 3.5)\}]$

$c)\ 25\frac{3}{4} + 6[\,7\frac{1}{2} - 8\,\{\frac{7}{15} \div (1\frac{2}{15} - \frac{2}{15})\}]$ $\qquad d)\ \frac{11}{12} - [\frac{3}{4} \times \{\frac{5}{6} - (\frac{1}{2} + \frac{1}{6})\}]$

20. Arham went grocery shopping. The grocery list below shows the items he bought along with the quantity and rates.

Sr. No	Item	Quantity	Rate
1	Sugar	3.5 kg	Rs. 160 per kg
2	Flour	5.2 kg	Rs. 70 per kg
3	Milk	3 litre	Rs. 240 per litre
4	Meat	2.5 kg	Rs. 825 per kg
5	Onion	4.25 kg	Rs. 220 per kg
6	Tomato	1.5 kg	Rs. 155 per kg

a) Determine the total cost of the purchase.
b) How much money is left if Arham had Rs 10000?
c) To confirm the total bill, write the single expression for Arham's shopping and solve it.

Solution:

a) Price of sugar per kg = Rs. 160
Price of sugar of 3.5 kg = 3.5 × Rs. 160
Price of flour per kg = Rs. 70
Price of flour of 5.2 kg = 5.2 × Rs. 70
Price of milk per kg = Rs. 240
Price of milk of 3 litre = 3 × Rs. 240
Price of meat per kg = Rs. 825
Price of meat of 2.5 kg = 2.5 × Rs. 825
Price of onion per kg = Rs. 220
Price of onion of 4.25 kg = 4.25 × Rs. 220
Price of tomato per kg = Rs. 155
Price of tomato of 1.5 kg = 1.5 × Rs. 155
Total cost = (3.5 × 160) + (5.2 × 70) + (2.5 × 825) + (4.25 × 220) + (1.5 × 155)
= 560 + 364 + 2062.5 + 935 + 232.5
= 4,154
Therefore, the total amount that Arham paid to the shopkeeper is Rs 4,154.

b) Arham have total amount = Rs. 10,000
Arham spent on grocery = Rs. 4,154
Amount left = Rs.10,000 – Rs.4,154
= Rs.5,846

c) The single expression for calculating Arham's total bill is:
Total amount = Amount spent on grocery + Amount remaining

Total bill = Rs. 4,154 + Rs. (10,000 - 4,154) Total bill = Rs. 4,154 + Rs. 5,846
Total bill = Rs. 10,000

Therefore, Arham's total bill is Rs. 10,000, which is the amount he had initially.

21. The sum of two integers is −133. If one integer is −67, find the other.

Solution:
The sum of two integers = −133
First integer = −67
Another integer = ?
Let x be another integer, then:
$x + (-67) = -133$
$x - 67 = -133$
Adding +67 on both sides, we get
$x - 67 + 67 = -133 + 67$
$x = -66$
Thus, another integer is − 67.

22. The product of two integers is 75. If one integer is −15, find the other.

Solution:
Product of two integers = + 75
First integer = −15
Another integer = ?
Let x be another integer, then:
$x \times (-15) = +75$
$x = (+75) \div (-15)$
$x = -5$
When we divide the integers with the opposite sign, then put the negative sign with the result.
$x = -7$
Thus, another integer is −7.

Learning Check 3.1

1. Find the value of the following.

a) 20% of 50 b) 10% of 150 c) 2.5% of 25

d) 14% of 200 e) $23\frac{1}{7}$ % of 980 f) 12.5% of 400

g) 28% of 520 m h) 7.8% of 20 kg i) 1.75% of Rs 450

j) 23.2% of 420 k) 90% marks out of 850 l) 1.25% of 75

Solution:

a) 20% of 50

First, write 20% as $\frac{20}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

20% of 50 = $\frac{20}{100} \times 50$

= 10

So, 20% of 50 is 10.

b) 10% of 150

First, write 10% as $\frac{10}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

10% of 150 = $\frac{10}{100} \times 150$

= 15

So, 10% of 150 is 15.

c) 2.5 % of 25

First, write 2.5 % as $\frac{2.5}{100}$ To find the fraction of a number is the same as multiplying the fraction with the number we have:

2.5 % of 25 = $\frac{2.5}{100} \times 25$

= $\frac{5}{8}$

So, 2.5 % of 25 is $\frac{5}{8}$.

d) 14 % of 200

First, write 14 % as $\frac{14}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

14 % of 200 = $\frac{14}{100} \times 200$

= 28

So, 14 % of 200 is 28.

e) $23\frac{1}{7}$ % of 980

First, write $23\frac{1}{7}$% as $\frac{162/7}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

$23\frac{1}{7}$ % of 980 = $\frac{162/7}{100} \times 980$

= $\frac{162}{7 \times 100}$

$= 226\frac{4}{5}$

So, $23\frac{1}{7}$% of 980 is $226\frac{4}{5}$.

f) 12.5 % of 400

First, write 12.5 % as $\frac{12.5}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

12.5 % of 400 = $\frac{12.5}{100}$ × 400

= 50

So, 12.5 % of 400 is 50.

g) 28% of 520 m

First, write 28 % as $\frac{28}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

28 % of 520 = $\frac{28}{100}$ × 520

= $145\frac{3}{5}$

So, 28 % of 520 is $145\frac{3}{5}$.

h) 7.8 % of 20 kg

First, write 7.8 % as $\frac{7.8}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

7.8 % of 20 = $\frac{7.8}{100}$ × 20

= 1.56

So, 7.8 % of 20 is 1.56.

i) 1.75 % of Rs 450

First, write 1.75 % as $\frac{1.75}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

1.75 % of 450 = $\frac{1.75}{100}$ × 450

= 7.875

So, 1.75 % of 450 is 7.875.

j) 23.2 % of 420

First, write 23.2 % as $\frac{23.2}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

23.2 % of 420 = $\frac{23.2}{100}$ × 420

= 97.44

So, 23.2 % of 420 is 97.44.

k) 90 % marks out of 850

First, write 90 % as $\frac{90}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

90 % of 850 = $\frac{90}{100}$ × 850

= 765

So, 90 % of 850 is 765.

l) 1.25 % of 75

First, write 1.25 % as $\frac{1.25}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

1.25 % of 75 = $\frac{1.25}{100} \times 75 = 0.9375$

So, 1.25 % of 75 is 0.9375.

2. Express:

a) 220 as a percentage of 500
b) 655 as a percentage of 900
c) 2.25 as a percentage of 120
d) 42 as a percentage of 100
e) 150 cm as a percentage of 800 cm
f) 25 as a percentage of 120
g) 90 as a percentage of 170 marks
h) 5 mg as a percentage of 35 mg
i) 78 m as a percentage of 140 m
j) 145 as a percentage of 340
k) 3 days as a percentage of 1-week
l) 70 g as a percentage of 750 g

Solution:

a) 220 as a percentage of 500

220 as a fraction of 500 = $\frac{220}{500}$

Therefore, 220 as a percentage of 500 = $\frac{220}{500} \times 100 = 44$ %

So, 220 is 44 % of 500.

b) 655 as a percentage of 900

655 as a fraction of 900 = $\frac{665}{900}$

Therefore, 655 as a percentage of 900 = $\frac{665}{900} \times 100 = 72.78$ %

So, 655 is 72.78 % of 900.

c) 2.25 as a percentage of 120

2.25 as a fraction of 120 = $\frac{2.25}{120}$

Therefore, 2.25 as a percentage of 120 = $\frac{2.25}{120} \times 100 = 1.875$ %

So, 2.25 is 1.875 % of 120.

d) 42 as a percentage of 100

42 as a fraction of 100 = $\frac{42}{100}$

Therefore, 42 as a percentage of 100 = $\frac{42}{100} \times 100 = 42$ %

So, 42 is 42 % of 100.

e) 150 cm as a percentage of 800 cm

150 as a fraction of 800 = $\frac{150}{800}$

Therefore, 150 as a percentage of 800 = $\frac{150}{800} \times 100 = 18.75$ %

So, 150 cm is 18.75 % of 800 cm.

f) 25 as a percentage of 120

25 as a fraction of 120 = $\frac{25}{120}$

Therefore, 25 as a percentage of 120 = $\frac{25}{120} \times 100$ = 20.83 %
So, 25 is 20.83 % of 120.

g) 90 as a percentage of 170 marks
90 as a fraction of 170 = $\frac{90}{170}$
Therefore, 90 as a percentage of 170 = $\frac{90}{170} \times 100$ = 52.94 %
So, 90 marks is 52.94 % of 170 marks.

h) 5 mg as a percentage of 35 mg
5 as a fraction of 35 = $\frac{5}{35}$
Therefore, 5 as a percentage of 35 = $\frac{5}{35} \times 100$ = 14.29 %
So, 5 mg is 14.29 % of 35 mg.

i) 78 m as a percentage of 140 m
78 as a fraction of 140 = $\frac{78}{140}$
Therefore, 78 as a percentage of 140 = $\frac{78}{140} \times 100$ = 55.71 %
So, 78 m is 55.71 % of 140 m.

j) 145 as a percentage of 340
145 as a fraction of 340 = $\frac{145}{340}$
Therefore, 145as a percentage of 340 = $\frac{145}{340} \times 100$ = 42.65 %
So, 145 is 42.65 % of 340.

k) 3 days as a percentage of 1-week
3 as a fraction of 7 = $\frac{3}{7}$
Therefore, 3 as a percentage of 7 = $\frac{3}{7} \times 100$ = 42.86 %
So, 3 days is 42.86 % of 7 days.

l) 70 g as a percentage of 750 g
70 as a fraction of 750= $\frac{70}{750}$
Therefore, 70 as a percentage of 750 = $\frac{70}{750} \times 100$ = 9.3 %
So, 70 g is 9.3 % of 750 g.

3. 78% of the 900 students in school are present. Find:
a) How many students are present?
b) How many students are absent?

Solution:

a) Total students = 900
Percentage of present students = 78 %

Number of student's present = $\frac{78}{100} \times 900 = 702$

Therefore, 702 students are present.

b) Number of students absent = total students – present students

= 900 – 702

= 198

Therefore, 198 students are absent.

4. In a housing society there are 500 houses, of which 150 houses are under construction. What percentage of houses are under construction?

Solution:

Total number of houses = 500

Number of houses under construction = 150

Percentage of houses under construction = $\frac{150}{500} \times 100$

= 30 %

Therefore, 30% of the houses in the housing society are under construction.

5. Nida spent 5% of the day reciting Holy Quran. How much time did she spend reciting Holy Quran?

Solution:

Since, there are 24 hours in a day. So,

The time Nida spent reciting Holy Quran = 5% of 24 hours

= $\frac{5}{100} \times 24$ hours

= 1 hour 12 minutes

Therefore, Nida spent 1 hour 12 minutes reciting Holy Quran.

6. A shopkeeper has a stock of 150 kg of dates in Ramadan. He sold 45 kg of dates in one day. What percentage of dates did he sell?

Solution:

Total amount of dates in stock = 150 kg

Amount of dates sold = 45 kg

Percentage of dates sold = $\frac{sold\ amount\ of\ dates}{total\ amount\ of\ dates} \times 100$

$= \frac{45}{150} \times 100$

= 30 %

Therefore, the shopkeeper sold 30% of the dates he had in stock.

7. Out of 140 pages of a storybook, 35% of the pages have been read by Ibrahim. How many pages of the storybook did he read?

Solution:

Total number of pages in the book = 140

Percentage of pages read by Ibrahim = 35%

Number of pages read by Ibrahim = Percentage of pages read pages x Total number of pages in the book

$= \frac{35}{100} \times 140$

= 49 pages

Therefore, Ibrahim read 49 pages of the storybook.

8. Maria bought 900 g of butter. She used 150 g of butter. What percentage of butter is left?

Solution:

Total amount of butter bought = 900 g

Amount of butter used = 150 g

Amount of butter left = Total amount of butter bought - Amount of butter used

= 900 g - 150 g = 750 g

Percentage of butter left = $\frac{Amount\ of\ butter\ left}{Total\ amount\ of\ butter\ bought} \times 100$

$= \frac{750}{900} \times 100$

= 83.33%

Therefore, Maria has 83.33% of the butter left.

9. In a mathematics exam, Ahmad solved 65% of the questions. If there were a total of 10 questions, how many did he solve in all?

Solution:

Total number of questions = 10

Percentage of questions solved by Ahmad = 65%

Number of questions solved by Ahmad = Percentage of questions solved × Total number of questions

$= \frac{65}{100} \times 10$

= 6.5 questions

Therefore, Ahmad solved 6.5 questions in the mathematics exam.

Learning Check 3.2

1. Compare the quantities by using percentage.

a) 10 out of 15 and 15 out of 90
b) 440 out of 550 and 520 out of 710
c) 60 out of 110 and 35 out of 70
d) 200 out of 400 and 150 out of 350
e) 67 out of 100 and 44 out of 80
f) 30 out of 50 and 140 out of 180
g) 85 out of 130 and 12 out of 25
h) 76 out of 120 and 80 out of 140

Solution:

a) 10 out of 15 and 15 out of 90

10 out of 15 = $\frac{10}{15} \times 100\ \% = 66.67\ \%$

15 out of 90 = $\frac{15}{90} \times 100\ \% = 16.67\ \%$

Since 66.67 % is greater than 16.67 %, therefore 10 out of 15 is greater than 15 out of 90.

b) 440 out of 550 and 520 out of 710

440 out of 550 = $\frac{440}{550} \times 100\ \% = 80\ \%$

520 out of 710 = $\frac{520}{710} \times 100\ \% = 73.24\ \%$

Since 80 %is greater than 73.24 %, therefore 440 out of 550 is greater than 520 out of 710.

c) 60 out of 110 and 35 out of 70
60 out of 110 = $\frac{60}{110}$ × 100 % = 54.55 %
35 out of 70 = $\frac{35}{70}$ × 100 % = 50 %
Since 54.55 % is greater than 50 %, therefore 60 out of 110 is greater than 35 out of 70.

d) 200 out of 400 and 150 out of 350
200 out of 400 = $\frac{200}{400}$ × 100 % = 50 %
150 out of 350 = $\frac{150}{350}$ × 100 % = 42.86 %
Since 50 % is greater than 42.86 %, therefore 200 out of 400 is greater than 150 out of 350.

e) 67 out of 100 and 44 out of 80
67 out of 100 = $\frac{67}{100}$ × 100 % = 67 %
44 out of 80 = $\frac{44}{80}$ × 100 % = 55 %
Since 67 % is greater than 55 %, therefore 67 out of 100 is greater than 44 out of 80.

f) 30 out of 50 and 140 out of 180
30 out of 50 = $\frac{30}{50}$ × 100 % = 60 %
140 out of 180 = $\frac{140}{180}$ × 100 % = 77.78 %
Since 60 % is less than 77.78 %, therefore 30 out of 50 is less than 140 out of 180.

g) 85 out of 130 and 12 out of 25
85 out of 130 = $\frac{85}{130}$ × 100 % = 65.38 %
12 out of 25 = $\frac{12}{25}$ × 100 % = 48 %
Since 65.38 % is greater than 48 %, therefore 85 out of 130 is greater than 12 out of 25.

h) 76 out of 120 and 80 out of 140
76 out of 120 = $\frac{76}{120}$ × 100 % = 63.33 %
80 out of 140 = $\frac{80}{140}$ × 100 % = 57.14 %
Since 63.33 % is greater than 57.14 %, therefore 76 out of 120 is greater than 80 out of 140.

2. Calculate the increase and decrease in quantity.

a) 20 increased by 5%
b) 650 decreased by 12.5%
c) 55 decreased by 12%
d) 298 decreased by 32%
e) 400 increased by 16%
f) 320 increased by 54%
g) 100 increased by 98%
h) 640 decreased by 62%

Solution:

a) 20 increased by 5%
First, work out 5 % of 20:

5 % of 20 = $\frac{5}{100}$ × 20 = 1
This is the value that you need to add to 20.
20 + 1 = 21
So, 20 increased by 5 % is 21.

b) 650 decreased by 12.5%
First, work out 12.5 % of 650:
12.5 % of 650 = $\frac{12.5}{100}$ × 650 = 81.25
This is the value that you need to subtract from 650.
650 – 81.25 = 568.75
So, 650 decreased by 12.5 % is 568.75.

c) 55 decreased by 12%
First, work out 12% of 55:
12 % of 55 = $\frac{12}{100}$ × 55 = 6.6
This is the value that you need to subtract from 55.
55 – 6.6 = 48.4
So, 55 decreased by 12 % is 48.4.

d) 298 decreased by 32%
First, work out 32% of 298:
32 % of 298= $\frac{32}{100}$ × 298 = 95.36
This is the value that you need to subtract from 298.
298 – 95.36 = 202.64
So, 298 decreased by 32 % is 202.64.

e) 400 increased by 16%
First, work out 16 % of 400:
16 % of 400 = $\frac{16}{100}$ × 400 = 64
This is the value that you need to add to 464.
400 + 64 = 464
So, 400 increased by 16 % is 60.

f) 320 increased by 54%
First, work out 54 % of 320:
54 % of 320 = $\frac{54}{100}$ × 320 = 172.8
This is the value that you need to add to 320.
320 + 172.8 = 492.8
So, 320 increased by 54 % is 492.8.

g) 100 increased by 98 %
First, work out 98 % of 100:
98 % of 100 = $\frac{98}{100}$ × 100 = 98
This is the value that you need to add to 100.
100 + 98 = 198
So, 100 increased by 98 % is 198.

h) 640 decreased by 62%
First, work out 62 % of 640:

62 % of 640 = $\frac{62}{100}$ × 640 = 396.8
This is the value that you need to subtract from 650.
640 – 396.8 = 243.2
So, 640 decreased by 62 % is 243.2.

3. **In a quiz program, one team gave 7 correct answers out of 15 and the second team gave 8 correct answers out of 12. Compare the number of correct answers both teams gave using percentages.**
Solution:
For the first team:
Number of correct answers = 7
Total number of questions = 15
Percentage of correct answers = $\frac{7}{15}$ × 100 % = 46.67%
For the second team:
Number of correct answers = 8
Total number of questions = 12
Percentage of correct answers = $\frac{8}{12}$ × 100% = 66.67%
Therefore, the second team gave a higher percentage of correct answers (66.67%) compared to the first team (46.67%).

4. **Madiha gave Rs 6000 as a donation from her salary of Rs 40000 and Umar gave Rs 8000 from his salary of Rs 48000. Compare who gave a larger donation by percentage.**
Solution;
For Madiha:
Donation amount = Rs 6000
Salary = Rs 40000
Percentage of donation = ?
Percentage of donation = $\frac{6000}{40000}$ × 100% = 15%
For Umar:
Donation amount = Rs 8000
Salary = Rs 48000
Percentage of donation = ?
Percentage of donation = $\frac{8000}{48000}$ × 100% = 16.67%
Therefore, Umar gave a larger donation by percentage (16.67%) compared to Madiha (15%).

5. **Sufyan bought 5 kg of sugar for Rs 1050. If the price of sugar increased by 3.6% per kg, what is the increased price of sugar per kg?**
Solution:
Cost price of 5 kg sugar = Rs 1050
Increased % = 3.6 % per kg
New increased price of sugar = ?
We know that:

Price per kg = $\frac{Total\ price}{Qusntity}$
Price per kg = $\frac{1050}{5}$ = 210
If the price of sugar increased by 3.6% per kg, the new price per kg can be calculated as:
New price per kg = Old price per kg + (Increase percentage x Old price per kg) New price per kg = Rs 210 + (0.036 x Rs 210)
New price per kg = Rs 210 + Rs 7.56
New price per kg = Rs 217.56
Therefore, the increased price of sugar per kg is Rs 217.56, rounded to two decimal places.

6. **The price of the shirt is Rs 2500. If the price is decreased by 12% during a sale, what is the price of the shirt after the discount?**
Solution:
Price of a shirt = Rs 2500
Discount % = 12 %
Price after discount = ?
Discount amount = 12% x Rs 2500
= $\frac{12}{100} \times 2500$
Discount amount = Rs 300
The price of the shirt after the discount can be calculated as:
Price after discount = Original price - Discount amount Price after discount
= Rs 2500 - Rs 300
Price after discount = Rs 2200
Therefore, the price of the shirt after the discount is Rs 2200.

7. **Sadia solved 18 questions of mathematics in 35 minutes and Dawood solved 15 questions in 30 minutes. Compare using percentages to see who took more time to solve the questions.**
Solution:
For Sadia:
Number of questions solved = 18
Time taken = 35 minutes
Percentage of questions solved per minute = $\frac{18}{35} \times 100\%$ = 51.43%
For Dawood:
Number of questions solved = 15
Time taken = 30 minutes
Percentage of questions solved per minute = $\frac{15}{30} \times 100\%$ = 50%
Therefore, Sadia took more time to solve the questions as she solved a lower percentage of questions per minute compared to Dawood.

8. **Usman's salary is Rs 48000. What is his salary after an increment of 15%?**
   Solution:
   Usman's salary = Rs 48000
   Percentage increment = 15 %
   Increment amount = 15% of Rs 48000
   Increment amount = $\frac{15}{100} \times 48000$
   Increment amount = Rs 7200
   The new salary after the increment can be calculated as:
   New salary = Old salary + Increment amount
   New salary = Rs 48000 + Rs 7200 New salary = Rs 55200
   Therefore, Usman's new salary after an increment of 15% is Rs 55200.
9. **Saman bought a motorcycle for Rs 98000.**
   a) Find the price of the motorcycle if the price increased by 4%.
   b) Find the price of the motorcycle if the price decreased by 3%.
   Solution:

a) Cost price of motorcycle = Rs 98000
   Increase percentage = 4 %
   New price of motorcycle = ?
   New price = Old price + (Increase percentage x Old price)
   New price = Rs 98000 + (4% x Rs 98000)
   $= \text{Rs } 98000 + \frac{4}{100} \times 98000$
   New price = Rs 101920
   Therefore, the price of the motorcycle would be Rs 101920 if the price increased by 4%.

b) Cost price of motorcycle = Rs 98000
   Decrease percentage = 4 %
   New price of motorcycle = ?
   New price = Old price - (Decrease percentage x Old price)
   New price = Rs 98000 - (3% x Rs 98000)
   $= \text{Rs } 98000 - \frac{3}{100} \times 98000$
   New price = Rs 95060
   Therefore, the price of the motorcycle would be Rs 95060 if the price decreased by 3%.

Learning Check 3.3

1. **Find the ratio of:**
   a) 30 to 80 b) 12 kg to 40 kg c) 90 cm to 12 m d) Rs 300 to Rs 450
   e) 40 to 80 f) 400 g to 2 kg g) 550 *l* to 220 l h) 7 h to 40 min

Solution:

a) 30 to 80

To find the ratio of 30 to 80, we simply write it as 30:80
Since the HCF of 30 and 80 is 10, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 10.

$\frac{30}{10} : \frac{80}{10}$

3 : 8

Therefore, the ratio of 30 to 80 is 3:8.

b) 12 kg to 40 kg

To find the ratio of 12 kg to 40 kg, we write it as 12:40.
Since the HCF of 12 and 40 is 4, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 4.

$\frac{12}{4} : \frac{40}{4}$

3 : 10

Therefore, the ratio of 12 kg to 40 kg is 3:10.

c) 90 cm to 12 m

To find the ratio of 90 cm to 12 m, we need to convert one of the measurements so that they are in the same units.
1 m = 100 cm
12 m = 1200 cm and 90 cm
Since the HCF of 90 and 1200 is 30, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 30.

$\frac{90}{30} : \frac{1200}{30}$

3 : 40

Therefore, the ratio of 90 cm to 12 m is 3:40.

d) Rs 300 to Rs 450

To find the ratio of Rs 300 to Rs 450, we write it as 300:450.
Since the HCF of 300 and 450 is 150, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 150.

$\frac{300}{150} : \frac{450}{150}$

2 : 3

Therefore, the ratio of Rs 300 to Rs 450 is 2:3.

e) 40 to 80

To find the ratio of 40 to 80, we simply write it as 40:80
Since the HCF of 40 and 80 is 40, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 40.

$\frac{40}{40} : \frac{80}{40}$

1 : 2

Therefore, the ratio of 40 to 80 is 1:2.

f) 400 g to 2 kg

To find the ratio of 400 g to 2 kg, we need to convert one of the measurements so that they are in the same units.
400 g
Since, 1 kg = 1000 g, so 2 kg = 2000 g

Since the HCF of 400 and 2000 is 400, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 400.
$\frac{400}{400}:\frac{2000}{400}$
1 : 5
Therefore, the ratio of 400 g to 2 kg is 1:5.

g) $550\ l$ to $220\ l$
To find the ratio of 550 to 220, we simply write it as 550 : 220
Since the HCF of 550 and 220 is 110, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 110.
$\frac{550}{110}:\frac{220}{110}$
5 : 2
Therefore, the ratio of $550\ l$ to $220\ l$ is 5 : 2.

h) 7 h to 40 min
To express 7 hours and 40 minutes as a ratio, we need to convert both quantities to the same unit of time.
1 hour = 60 minutes
7 hours = 420 minutes
To find the ratio of 420 min to 40 min, we simply write it as 420 : 20
Since the HCF of 30 and 420 is 20, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 20.
$\frac{420}{20}:\frac{40}{20}$
21 : 2
Therefore, the ratio of 7 hours to 40 minutes is 21 : 2.

2. Simplify the following ratios.

a) 6 : 12	b) 20 : 65	c) 40 : 64	d) 42 : 14
e) 17 : 51	f) 56 : 96	g) 5 : 120	h) 13 : 39

Solution:

a) 6 : 12
To find the ratio of 6 to 12, we simply write it as 6 : 12
Since the HCF of 6 and 12 is 6, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 6.
$\frac{6}{6}:\frac{12}{6}$
1 : 2
Therefore, the ratio of 6 and 12 is 1 : 2.

b) 20 : 65
To find the ratio of 20 to 65, we simply write it as 20 : 65
Since the HCF of 20 and 65 is 5, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 5.
$\frac{20}{5}:\frac{65}{5}$
4 : 13
Therefore, the ratio of 20 and 65 is 4 : 13.

c) 40 : 64
To find the ratio of 40 to 64, we simply write it as 40 : 64
Since the HCF of 40 and 64 is 8, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 8.

$\frac{40}{8} : \frac{64}{8}$

5 : 8

Therefore, the ratio of 40 and 64 is 5 : 8.

d) 42 : 14

To find the ratio of 42 to 14, we simply write it as 42 : 14

Since the HCF of 42 and 14 is 14, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 14.

$\frac{42}{14} : \frac{14}{14}$

3 : 1

Therefore, the ratio of 42 and 14 is 3 : 1.

e) 17 : 51

To find the ratio of 17 to 51, we simply write it as 17 : 51

Since the HCF of 17 and 51 is 17, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 17.

$\frac{17}{17} : \frac{51}{17}$

1 : 3

Therefore, the ratio of 17 and 51 is 1 : 3.

f) 56 : 96

To find the ratio of 56 to 96, we simply write it as 56 : 96

Since the HCF of 56 and 96 is 8, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 8.

$\frac{56}{8} : \frac{96}{8}$

7 : 12

Therefore, the ratio of 56 and 96 is 7 : 12.

g) 5 : 120

To find the ratio of 5 to 120, we simply write it as 5 : 120

Since the HCF of 5 and 120 is 5, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 5.

$\frac{5}{5} : \frac{120}{5}$

1 : 24

Therefore, the ratio of 5 and 120 is 1 : 24.

h) 13 : 39

To find the ratio of 13 to 39, we simply write it as 13 : 39

Since the HCF of 13 and 39 is 13, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 13.

$\frac{13}{13} : \frac{39}{13}$

1 : 3

Therefore, the ratio of 13 and 39 is 1 : 3.

3. Find the continued ratio for the following values.

a) If $a : b = 1 : 3$ and $b : c = 2 : 4$ then $a : b : c$
b) If $x : y = 9 : 2$ and $y : z = 5 : 11$ then $x : y : z$
c) If $d : e = 7 : 9$ and $e : f = 3 : 13$ then $d : e : f$
d) If $a : b = 3 : 8$ and $b : c = 5 : 12$ then $a : b : c$
e) If $p : q = 4 : 9$ and $q : r = 1 : 5$ then $p : q : r$
f) If $x : y = 11 : 12$ and $y : z = 13 : 14$ then $x : y : z$
g) If $a : b = 2 : 8$ and $b : c = 5 : 7$ then $a : b : c$
h) If $m : n = 7 : 5$ and $n : o = 6 : 11$ then $m : n : o$

Solution:

a) If a : b = 1 : 3 and b : c = 2 : 4 then a : b : c

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

a	:	b	:	c
1	:	3		
		2	:	4

a : b : c = 2 : 6 : 12

Therefore, the continued ratio is a : b : c = 2 : 6 : 12.

b) If *x* : y = 9 : 2 and y : z = 5 : 11 then *x* : y : z

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

x	:	y	:	z
9	:	2		
		5	:	11

x : y : z = 45 : 10 : 22

Therefore, the continued ratio is x : y : z = 45 : 10 : 22 .

c) If d : e = 7 : 9 and e : f = 3 : 13 then d : e : f

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

d	:	e	:	f
7	:	9		
		3	:	13

d : e : f = 21 : 27 : 117 = 7 : 9 : 39

Therefore, the continued ratio is d : e : f = 7 : 9 : 39

d) If a : b = 3 : 8 and b : c = 5 : 12 then a : b : c
To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

a	:	b	:	c
3	:	8		
		5	:	12

a : b : c = 15 : 40 : 96
Therefore, the continued ratio is a : b : c = 15 : 40 : 96 .

e) If p : q = 4 : 9 and q : r = 1 : 5 then p : q : r
To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

p	:	q	:	r
4	:	9		
		1	:	5

p : q : r = 4 : 9 : 45
Therefore, the continued ratio is p : q : r = 4 : 9 : 45.

f) If *x* : y = 11 : 12 and y : z = 13 : 14 then *x* : y : z
To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

x	:	y	:	z
11	:	12		
		13	:	14

x : y : z = 143 : 156 : 168
Therefore, the continued ratio is x : y : z = 143 : 156 : 168.

g) If a : b = 2 : 8 and b : c = 5 : 7 then a : b : c
To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

a	:	b	:	c
2	:	8		
		5	:	7

a : b : c = 10 : 40 : 56 = 5 : 20 : 28
Therefore, the continued ratio is a : b : c = 5 : 20 : 28 .

h) If m : n = 7 : 5 and n : o = 6 : 11 then m : n : o

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

m	:	n	:	o
7	:	5		
		6	:	11

m : n : o = 42 : 30 : 55

Therefore, the continued ratio is m : n : o = 42 : 30 : 55.

4. There are 25 students in a Hifz class. Out of these, 20 students were present on Monday. Find the ratio between the total students and the students that are present. Simplify if possible.

Solution:

Total number of students = 25

Number of student's present = 20

Ratio of total students to present students can be written as:

Total number of students : Number of students present

25 : 20

Since the HCF of 25 and 20 is 5, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 5.

$\frac{25}{5} : \frac{20}{5}$

5 : 4

Therefore, the ratio of total students to present students is 5:4.

5. Salma and Shazia bought some gifts for their friend on Eid. Salma contributed Rs 760 and Shazia contributed Rs 916. Find the ratio between the amounts contributed by Salma and Shazia. Simplify if possible.

Solution:

Salma contribution = Rs 760

Shazia contribution = Rs 916

So, the ratio is:

Salma's contribution : Shazia's contribution

760 : 916

Since the HCF of 760 and 916 is 4, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 4.

$\frac{760}{4} : \frac{916}{4}$

190 : 229

Therefore, the ratio between the amounts contributed by Salma and Shazia is 190 : 229.

6. Wali's marks in the first term in mathematics are 78 and in the second term they are 85. Find the ratio between his first- and second-term marks. Simplify if possible.

Solution:

First term marks in mathematics = 78

Second term marks in mathematics = 85

So, the ratio is:

First term marks in mathematics : Second term marks in mathematics

78 : 85

Since the HCF of 78 and 85 is 1, we can further simplify the ratio.

So, we not simplify further the given ratio

Therefore, the ratio between first term and second term in mathematics is 78 : 85.

7. The ratio among the capacity of the jug and the bucket is 1 : 5 and the ratio among the capacity of the bucket and the tub is 3 : 4. Find the continued ratio among the capacities of the three objects. Simplify if possible.

Solution: Given that

Jug : Bucket = 1 : 5

Bucket : Tube = 3 : 4

Let's denote the capacity of the jug, bucket, and tub as J, B, and T, respectively.

Then, we have:

J : B = 1 : 5 an B : T = 3 : 4

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

J	:	B	:	T
1	:	5		
		3	:	4

J : B : T = 3 : 15 : 20

Therefore, the continued ratio is J : B : T = 3 : 15 : 20.

8. The length of one piece of rope is 660 m and the length another piece of rope is 780 m. Find the ratio between the lengths of the two ropes and express in simplified form if possible.

Solution:

Length of the first rope = 660 m

Length of the second rope = 780 m

So, the ratio is:

Length of the first rope : Length of the second rope

660 : 780

Since the HCF of 660 and 780 is 60, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 60.

$$\frac{660}{60} : \frac{780}{60}$$

11 : 13

Therefore, the ratio between the length of first and second rope is 11 : 13.

9. The ratio between Marwa's height and Sadia's height is 2 : 3 and the ratio of Sadia's height to Sidra's height is 7 : 9. Find the continued ratio among their heights. Simplify if possible.

Solution:

Marwa's height : Sadia's height =2 : 3

Sadia's height : Sidra's height = 7 : 9

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

Marwa	:	Sadia	:	Sidra
2	:	3		
		7	:	9

Marwa's height : Sadia's height : Sidra's height = 14 : 21 : 27

Therefore, the continued ratio is Marwa's height : Sadia's height : Sidra's height = 14 : 21 : 27.

Learning Check 3.4

1. Express the unit rate for the following.

a) Rs 150 for 3 packs of biscuits b) 10 pages read in 45 minutes
c) 4 kg bag of apples for Rs 920 d) Rs 2560 for 8 books
e) Rs 1150 for 2.5 metres of fabric

Solution:

a) Rs 150 for 3 packs of biscuits

To find the per pack rate of the biscuits, we need to divide the price of biscuits by the number of packs.

Let's write it as a ratio by removing the units:

$\frac{150}{3}$ = 150 : 3

To reduce it to unit rate, divide both terms by 3.

150 : 3 = $\frac{150 \div 3}{3 \div 3}$ = $\frac{50}{1}$

So, the unit rate (per pack rate) of the biscuit is Rs 50.

b) 10 pages read in 45 minutes

To find the per minute rate of the pages, we need to divide the pages by the minutes.

Let's write it as a ratio by removing the units:

$\frac{10}{45}$ = 10 : 45

To reduce it to unit rate, divide both terms by 45.

10: 45 = $\frac{10 \div 45}{45 \div 45}$ = $\frac{2 \div 9}{1}$ = $\frac{2}{9}$

So, the unit rate (per minute rate) of the pages is $\frac{2}{9}$.

c) 4 kg bag of apples for Rs 920
To find the per kilogram rate of the apples, we need to divide the price of apples by the kilogram.
Let's write it as a ratio by removing the units:
$\frac{920}{4}$ = 920 : 4
To reduce it to unit rate, divide both terms by 4.
920 : 4 = $\frac{920 \div 4}{4 \div 4} = \frac{230}{1}$
So, the unit rate (per kilogram rate) of the apples is Rs 230.

d) Rs 2560 for 8 books
To find the price rate per book, we need to divide the price by the number of books.
Let's write it as a ratio by removing the units:
$\frac{2560}{8}$ = 2560 : 8
To reduce it to unit rate, divide both terms by 8.
2560 : 8 = $\frac{2560 \div 8}{8 \div 8} = \frac{320}{1}$
So, the unit rate (per book rate) is Rs 320.

e) Rs 1150 for 2.5 metres of fabric
To find the per metre rate of the fabric, we need to divide the price of fabric by the metre.
Let's write it as a ratio by removing the units:
$\frac{1150}{2.5}$ = 1150 : 2.5
To reduce it to unit rate, divide both terms by 3.
1150 : 2.5 = $\frac{1150 \div 2.5}{2.5 \div 2.5} = \frac{460}{1}$
So, the unit rate (per metre rate) of the fabric is Rs 460.

2. Fahad's annual salary is Rs 948000. What is his salary per month?
Solution:
Annual salary = Rs 948000
Salary per month = ?
To find the per month salary of the Fahad, we need to divide the annual salary of the Fahad by the 12 months.
Let's write it as a ratio by removing the units:
$\frac{948000}{12}$ = 948000 : 12
To reduce it to unit rate, divide both terms by 12.
948000 : 12 = $\frac{948000 \div 12}{12 \div 12} = \frac{79000}{1}$
Therefore, Fahad's monthly salary is Rs 79000.

3. Zoo entry tickets for 15 people costs Rs 3900. What is the rate per ticket?
Solution:
To find the rate per ticket, we need to divide the total cost by the number of tickets:
Rate per ticket = $\frac{Total\ cost}{Number\ of\ tickets\ Rate\ per\ ticket} = \frac{3900}{15}$
Rate per ticket = Rs 260

Therefore, the rate per ticket is Rs 260.

4. **A sack of 50 kg of rice costs Rs 12000. Find the rate of rice per kg.**

   Solution:

   To find the rate of rice per kg, we need to divide the total cost by the weight of the rice:

   Rate per kg = $\frac{Total\ cost}{Weight\ of\ rice}$

   Rate per kg = $\frac{12000}{50}$= Rs 240

   Therefore, the rate of rice per kg is Rs 240.

5. **Saba covered 5 kilometers in 2 hours during her morning walk. What is her speed per hour?**

   Solution:

   To find Saba's speed per hour, we need to divide the distance she covered by the time she took:

   Speed per hour = $\frac{Distance}{Time\ speed\ per\ hour}$= $\frac{5}{2}$

   Speed per hour = 2.5 km/hour

   Therefore, Saba's speed per hour is 2.5 km/hour.

6. **Oil costs Rs 23940 for 76 litres. What is the price per litre of oil?**

   Solution:

   To find the price per litre of oil, we need to divide the total cost by the number of litres:

   Price per litre = $\frac{Total\ cost}{Number\ of\ litres}$

   Price per litre = $\frac{23940}{76}$ = Rs 315

   Therefore, the price per litre of oil is Rs 315.

7. **A 12 metre length of fabric costs Rs 6720. Find the unit rate (or per metre rate) of fabric.**

   Solution:

   To find the unit rate (or per metre rate) of fabric, we need to divide the total cost by the length of the fabric:

   Unit rate of fabric = $\frac{Total\ cost}{Length\ of\ fabric}$

   Unit rate of fabric = $\frac{6720}{12}$

   Unit rate of fabric = Rs 560/metre

   Therefore, the unit rate (or per metre rate) of fabric is Rs 560/metre.

Unit Check 3

1. **Encircle the correct answer.**

   a) The term that is used to compare two or more quantities of the same kind is called:

   i) percentage   ii) ratio   iii) rate   iv) fraction

b) There are 300 trees in a farm. If 35% of trees are mango trees, then how many other trees are there?
   i) 105 ii) 195 iii) 200 iv) 155

c) If a : b = 4 : 5 and b : c = 5 : 3 then a : b : c equals:
   i) 4 : 3 : 5 ii) 5 : 3 : 4 iii) 4 : 5 : 3 iv) 3 : 4 : 5

d) 15% increase in 7800 equals:
   i) 1175 ii) 1170 iii) 8790 iv) 8970

e) The price of 5 litres of petrol is Rs 1075. The unit rate of petrol is:
   i) Rs 215 ii) Rs 205 iii) Rs 190 iv) Rs 200

2. Fill in the blanks.

a) A percentage shows the quantity out of **100**.
b) **rate** is the comparison of quantities of different kinds having different units.
c) **ratio** is used to compare two or more quantities of the same kind.
d) 56 out of 112 in percentage form is **50 %**.
e) 5% of 80 is **4**.

3. Express the following as percentage.

a) 52 m out of 96 m b) 420 out of 550 c) 21 out of 84
d) 78 out of 80 e) 12 kg out of 50 kg f) 15 min out of 4 hours

Solution:

a) 52 m out of 96 m
52 out of 96 = $\frac{52}{96}$ × 100 % = 54.17 %
So, 52 meters out of 96 meters is equal to 54.17%.

b) 420 out of 550
420 out of 550 = $\frac{420}{550}$ × 100 % = 80 %
So, 420 out of 550 is equal to 76.36%.

c) 21 out of 84
21 out of 84 = $\frac{21}{84}$ × 100 % = 23.81 %
So, 21 out of 84 is equal to 25%.

d) 78 out of 80
78 out of 80 = $\frac{78}{80}$ × 100 % = 97.50 %
So, 78 out of 80 is equal to 97.5%.

e) 12 kg out of 50 kg
12 out of 50 = $\frac{12}{50}$ × 100 % = 24 %
So 12 kg out of 50 kg is equal to 24%.

f) 15 min out of 4 hours
We first convert hours into minutes, As
1 hour = 60 minutes
4 hours = 4 × 60 = 240 minutes
15 out of 240 = $\frac{15}{240}$ × 100 % = 6.25 %
So, 15 minutes out of 4 hours is equal to 6.25%.

4. Calculate the value of the following.

a) 6% of 540 b) 24% of 64 c) 80% of 55 d) 2% of 12

Solution:

a) 6 % of 540

First, write 6 % as $\frac{6}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

6 % of 540 = $\frac{6}{100} \times 540$

= 32.4 %

So, 6 % of 540 is 32.4 %.

b) 24 % of 64

First, write 24 % as $\frac{24}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

24% of 64 = $\frac{24}{100} \times 64$

= 15.36

So, 24 % of 64 is 15.36.

c) 80 % of 55

First, write 80 % as $\frac{80}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

80% of 55 = $\frac{80}{100} \times 55$

= 44

So, 80 % of 55 is 44.

d) 2 % of 12

First, write 2 % as $\frac{2}{100}$. To find the fraction of a number is the same as multiplying the fraction with the number we have:

2 % of 12 = $\frac{2}{100} \times 12$

= 0.24

So, 2 % of 12 is 0.24.

5. Calculate and compare the value of the following using percentage.

a) 2 out of 10 and 3 out of 21 b) 14 out of 28 and 2 out of 4

c) 62 out of 90 and 55 out of 80 d) 18 out of 54 and 6 out of 54

Solution:

a) 2 out of 10 and 3 out of 21

2 out of 10 = $\frac{2}{10}$ × 100 % = 20 %

3 out of 21 = $\frac{3}{21}$ × 100 % = 14.29 %

Since 20 % is greater than 14.29 %, therefore 2 out of 10 is greater than 3 out of 21.

b) 14 out of 28 and 2 out of 4

14 out of 28 = $\frac{14}{28}$ × 100 % = 50 %

2 out of 4 = $\frac{2}{4}$ × 100 % = 50 %

Since 50 % is equal to 50 %, therefore 14 out of 28 is equal to 2 out of 4.

c) 62 out of 90 and 55 out of 80

62 out of 90 = $\frac{62}{90}$ × 100 % = 68.89 %
55 out of 80 = $\frac{55}{80}$ × 100 % = 68.75 %
Since 68.89 % is greater than 68.75 %, therefore 62 out of 90 is greater than 55 out of 80.
d) 18 out of 54 and 6 out of 54
18 out of 54 = $\frac{18}{54}$ × 100 % = 33.33 %
6 out of 54 = $\frac{6}{54}$ × 100 % = 11.11 %
Since 33.33 % is greater than 11.11 %, therefore 18 out of 54 is greater than 6 out of 54.

6. Calculate the following.

a) 40 increased by 13% b) 106 decreased by 7%
c) 15 increased by 95% d) 12 decreased by 4%

Solution:

a) 40 increased by 13%
First, work out 13 % of 40:
13 % of 40 = $\frac{13}{100}$ × 40 = 5.2
This is the value that you need to add to 40.
40 + 5.2 = 45.2
So, 40 increased by 13 % is 45.2.
b) 106 decreased by 7%
First, work out 7 % of 106:
7 % of 106 = $\frac{7}{100}$ × 106 = 7.42
This is the value that you need to subtract to 106.
106 – 7.42 = 98.58
So, 106 increased by 7 % is 98.58.
c) 15 increased by 95%
First, work out 95 % of 15:
95 % of 15 = $\frac{95}{100}$ × 15 = 14.25
This is the value that you need to add to 15.
15 + 14.25 = 29.25
So, 15 increased by 95 % is 29.25.
d) 12 decreased by 4%
First, work out 4 % of 12:
4 % of 12 = $\frac{4}{100}$ × 12 = 0.48
This is the value that you need to subtract to 12.
12 – 0.48 = 11.52
So, 12 increased by 4 % is 11.52.

7. Find the ratio of the following quantities.

a) 48 to 60 b) 94 to 98 c) 35 to 60 d) 54 to 27 e) 34 to 70

Solution:

a) 48 to 60

To find the ratio of 48 to 60, we simply write it as 48 : 60
Since the HCF of 48 and 60 is 12, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 12.

$\frac{48}{12} : \frac{60}{12}$

3 : 5

Therefore, the ratio of 48 to 60 is 3 : 5.

b) 94 to 98

To find the ratio of 94 to 98, we simply write it as 94 : 98
Since the HCF of 94 and 98 is 2, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 2.

$\frac{94}{2} : \frac{98}{2}$

47 : 49

Therefore, the ratio of 94 to 98 is 47 : 49.

c) 35 to 60

To find the ratio of 35 to 60, we simply write it as 30:80
Since the HCF of 35 and 60 is 5, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 5.

$\frac{35}{5} : \frac{60}{5}$

7 : 12

Therefore, the ratio of 35 to 60 is 7 : 12.

d) 54 to 27

To find the ratio of 54 to 27, we simply write it as 30:80
Since the HCF of 54 and 27 is 27, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 27.

$\frac{54}{27} : \frac{27}{27}$

2 : 1

Therefore, the ratio of 54 to 27 is 2 : 1.

e) 34 to 70

To find the ratio of 34 to 70, we simply write it as 30:80
Since the HCF of 34 and 70 is 2, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 2.

$\frac{34}{2} : \frac{70}{2}$

17 : 35

Therefore, the ratio of 34 to 70is 17 : 35.

8. Simplify the given ratios.

a) 5 : 50 b) 216 : 748 c) 60 : 40 d) 13 : 17 e) 99 : 121

Solution:

a) 5 : 50

To find the ratio of 5 to 50, we simply write it as 5 : 50
Since the HCF of 5 and 50 is 5, we can further simplify the ratio.
Divide both terms of the ratio by the HCF 5.

$\frac{5}{5} : \frac{50}{5}$

1 : 10

Therefore, the ratio of 5 to 50 is 1 : 10.

b) 216 : 748

To find the ratio of 216 to 748, we simply write it as 216 : 748

Since the HCF of 216 and 748 is 4, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 4.

$\frac{216}{4} : \frac{748}{4}$

54 : 187

Therefore, the ratio of 216 to 748 is 54 : 187.

c) 60 : 40

To find the ratio of 60 to 40, we simply write it as 60 : 40

Since the HCF of 60 and 40 is 20, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 20.

$\frac{60}{20} : \frac{40}{20}$

3 : 2

Therefore, the ratio of 60 to 40 is 3 : 2.

d) 13 : 17

To find the ratio of 13 to 17, we simply write it as 13 : 17

Since the HCF of 13 and 17 is 1, so we cannot further divide the given ratio.

$$13 : 17$$

Therefore, the ratio of 13 to 17 is 13 : 17.

e) 99 : 121

To find the ratio of 99 to 121, we simply write it as 99 : 121

Since the HCF of 99 and 121 is 11, we can further simplify the ratio.

Divide both terms of the ratio by the HCF 11.

$\frac{99}{11} : \frac{121}{11}$

9 : 11

Therefore, the ratio of 99 to 121 is 9 : 11.

9. Solve the following.

a) If a : b = 2 : 5 and b : c = 4 : 7 then find a : b : c

b) If s : t = 6 : 13 and t : u = 1 : 12 then find s : t : u

c) If x : y = 3 : 9 and y : z = 7 : 11 then find x : y : z

Solution:

a) If a : b = 2 : 5 and b : c = 4 : 7 then find a : b : c

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

a	:	b	:	c
2	:	5		
		4	:	7

a : b : c = 8 : 20 : 35

Therefore, the continued ratio is a : b : c = 8 : 20 : 35.

b) If s : t = 6 : 13 and t : u = 1 : 12 then find s : t : u

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

s	:	t	:	u
6	:	13		
		1	:	12

s : t : u = 6 : 13 : 156

Therefore, the continued ratio is s : t : u = 6 : 13 : 156.

c) If x : y = 3 : 9 and y : z = 7 : 11 then find x : y : z

To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

x	:	y	:	z
3	:	9		
		7	:	11

x : y : z = 21 : 63 : 99 = 7 : 21 : 33

Therefore, the continued ratio is x : y : z= 7 : 21 : 33.

10. Express the unit rates for the following quantities.

a) 12 litres of milk for Rs 1440 b) 105 pencils in 5 boxes

c) 1 dozen oranges for Rs 120 d) Rs 552 distributed among 4 children

Solution:

a) Rs 1440 for 12 litre of milk.

To find the per litre rate of the milk, we need to divide the price of milk by the litre of milk.

Let's write it as a ratio by removing the units:

$\frac{1440}{12}$ = 1440 : 12

To reduce it to unit rate, divide both terms by 12.

1440 : 12 = $\frac{1440 \div 12}{12 \div 12} = \frac{120}{1}$

So, the unit rate (per litre rate) of the milk is Rs 120.

b) 105 pencils in 5 boxes.

To find the per box rate of the pencils, we need to divide the 105 pencils by the number of 5 boxes.

Let's write it as a ratio by removing the units:

$\frac{105}{5}$ = 105 : 5

To reduce it to unit rate, divide both terms by 5.

105 : 5 = $\frac{105 \div 5}{5 \div 5} = \frac{21}{1}$

So, the unit rate (per box rate) of the pencils is 21.

c) 1 dozen oranges for Rs 120
Since, 1 dozen = 12 oranges
To find the per price per orange rate, we need to divide the price of oranges by the number of oranges.
Let's write it as a ratio by removing the units:
$\frac{120}{12}$ = 120 : 12
To reduce it to unit rate, divide both terms by 12.
120 : 12 = $\frac{120 \div 12}{12 \div 12} = \frac{10}{1}$
So, the unit rate (per orange rate) of the orange is Rs 10.

d) Rs 552 distributed among 4 children.
To find the per child rate of the rupees, we need to divide the 552 rupees among the 4 children.
Let's write it as a ratio by removing the units:
$\frac{552}{4}$ = 552 : 4
To reduce it to unit rate, divide both terms by 4.
552 : 4 = $\frac{552 \div 4}{4 \div 4} = \frac{138}{1}$
So, the unit rate (per child rate) of the rupees is Rs 138.

11. Zeemal has Rs 12000 as savings. If she donated Rs 2000 to a needy person, what percentage of her savings did she give?
Solution:
Zeemal savings amount = Rs 12000
Zeemal donated amount = Rs 2000
Zeemal gave her saving = ?
We know that
Zeemal gave her saving %= $\frac{donated\ amount}{saving\ amount} \times 100$
$= \frac{2000}{12000} \times 100 = 16.67\ \%$
So, Zeemal gave 16.67% of her savings.

12. A storybook has 184 pages. If Wali read 56% pages, how many pages are left unread?
Solution:
Total pages in a storybook = 184
Wali read pages = 56 %
Left unread pages = ?
We know that
Wali read pages = 56 % × 184
$= \frac{56}{100} \times 184$
= 103 pages (approx)
Wali unread pages = 184 – 103 = 81 (approximately)

13. Out of 600 students in a school, 415 are boys. Find the simplified form of the ratio of the number of girls to the number of boys.

Solution:
Total number of students = 600
Number of boys = 415
Number of girls = Total number of students - Number of boys
= 600 – 415 = 185
So, the ratio of the number of girls to the number of boys can be expressed as:
Number of girls : Number of boys
185 : 415
To simplify this ratio, we can divide both numbers by their greatest common factor, which is 5:
185 : 415 = $\frac{185}{5}$: $\frac{415}{5}$ = 37 : 83
Therefore, the simplified form of the ratio of the number of girls to the number of boys is 37 : 83.

14. In a zoo, 50 out of 240 birds are sparrows and 20 out of 240 birds are parrots. Compare the number of sparrows and parrots using percentage.
Solution:
Number of sparrows = 50
Number of parrots = 20
Total number of birds = 240
Percentage of sparrows = $\frac{Number\ of\ sparrows}{Total\ number\ of\ sparrows}$ × 100
Percentage of sparrows = $\frac{50}{240}$ × 100
= 20.83%
Percentage of parrots = $\frac{Number\ of\ parrots}{Total\ number\ of\ parrots}$ × 100
Percentage of parrots = $\frac{20}{240}$ × 100
Percentage of parrots = 8.33%
So, the percentage of sparrows is 20.83% and the percentage of parrots is 8.33%. Therefore, there are a higher percentage of sparrows compared to parrots in the zoo.

15. Kamal bought a table for Rs 5500. Find the price of the table if:
a) There is a 16% increase in the price of the table.
b) There is a 7% decrease in the price of the table.
Solution:
a) To find the price of the table after a 16% increase, we can use the formula:
Price after increase = Original price + (Original price x Percentage increase)
Percentage increase = 16%
Original price = Rs 5500
Price after increase = 5500 + (5500 x 0.16) = 5500 + 880 = Rs 6380
So, the price of the table after a 16% increase is Rs 6380.
b) To find the price of the table after a 7% decrease, we can use the formula:
Price after decrease = Original price - (Original price x Percentage decrease)
Percentage decrease = 7%

Original price = Rs 5500
Plugging in the values, we get:
Price after decrease = 5500 - (5500 x 0.07) = 5500 - 385 = Rs 5115
So, the price of the table after a 7% decrease is Rs 5115.

16. Ali, Waqar and Zaeem contributed some amount to help the poor. The ratio of Ali and Waqar's contribution is 3 : 4 and the ratio of Waqar and Zaeem's contribution is 5 : 6. Find the continued ratio between their contributions.

Solution:

Ali : Waqar = 3 : 4 and Waqar : Zaeem = 5 : 6
To find the continued ratio, first we need to find the common term. The product of the first terms of both ratios will become the first term, the product of the last terms of two ratios will become the last term and the product of the first term of the second ratio and the second term of the first ratio will become the middle (common) term of the continued ratio. As

Ali	:	Waqar	:	Zaeem
3	:	4		
		5	:	6

Ali : Waqar : Zaeem = 15 : 20 : 24
Therefore, the continued ratio is Ali : Waqar : Zaeem = 15 : 20 : 24.

17. The cost of almonds is Rs 2500 per kg. What is the reduced price of almonds if the cost decreased by 1.4%?

Solution:

To find the reduced price of almonds after a 1.4% decrease in cost, we can use the formula:
Reduced price = Original price - (Original price x Percentage decrease)
Percentage decrease = 1.4%
Original price = Rs 2500 per kg
Reduced price = 2500 - (2500 x 0.014) = 2500 - 35 = Rs 2465
So, the reduced price of almonds after a 1.4% decrease in cost is Rs 2465 per kg.

18. The cost of 21 kg of mangoes is Rs 3150. What is the rate per kilogram of mangoes?

Solution:

To find the rate per kilogram of mangoes, we can divide the total cost of 21 kg of mangoes by the weight of mangoes:
Cost of 21 kg of mangoes = Rs 3150
Weight of mangoes = 21 kg
Rate per kilogram of mangoes = $\frac{Cost\ of\ mangoes}{Weight\ of\ mangoes}$
Rate per kilogram of mangoes = $\frac{3150}{21}$ = Rs 150
So, the rate per kilogram of mangoes is Rs 150.

Learning Check 4.1

1. Identify if the collection of given objects is a set.

a) The set of talented students of grade 6.

b) The set of the first 4 prime numbers. c) The set of prayers in a day.

d) {4, 5, 6, 7, 7} e) The set of whole numbers less than 20.

f) The set of students in a school. g) A collection of grade 6 books

h) The set of beautiful cities of Pakistan. i) {s, t, u, d, e, n, t} j) {+,-, =, +}

k) The set of tasty food in a restaurant. l) {6, 7, 8, 9, 10}

Solution:

a) The set of talented students of grade 6
No it is not a set. According to the definition the collection of well-defined and distinct objects is called a set. this collection is not well defined as for some person it is talented but for others may be not talented.

b) The set of the first 4 prime numbers
Yes, this is a set. It is a collection of distinct objects (prime numbers) with no specific order. It satisfies the definition of a set.

c) The set of prayers in a day
Yes, this is a set. It is a collection of distinct objects (prayers) that occur in a day. It satisfies the definition of a set.

d) {4, 5, 6, 7, 7}
No, this is not a set. Sets cannot have duplicate elements, and this collection contains the element '7' twice. If the duplicate element is removed, then it would be a set {4, 5, 6, 7}.

e) The set of whole numbers less than 20
Yes, this is a set. It is a collection of distinct objects (whole numbers) with no specific order, which are less than 20. It satisfies the definition of a set.

f) The set of students in a school
Yes, this is a set. It is a collection of distinct objects (students) with no specific order, which belong to a school. It satisfies the definition of a set.

g) A collection of grade 6 books
Yes it is a set because the collection of books of grade 6 are distinct and well defined.

h) The set of beautiful cities of Pakistan
Yes, this is a set. It is a collection of distinct objects (beautiful cities) with no specific order, which belong to Pakistan. It satisfies the definition of a set.

i) {s, t, u, d, e, n, t}
Replace (No it is not a set as t is repeated in this collection so the element of collection of objects in not distinct.)

j) {+,-, =, +}
Replace (No it is not a set as the symbol + is repeated in this collection so the element of collection of objects in not distinct.)

k) The set of tasty food in a restaurant
No, this is not necessarily a set. Tasty food is subjective and can vary from person to person, so the collection of tasty food in a restaurant may not have distinct objects and may not satisfy the definition of a set.

l) {6, 7, 8, 9, 10}
Yes, this is a set. It is a collection of distinct objects (numbers) with no specific order.

2. If X = {4, 6, 8, 9, 10, 12, 14, 15, 16}, then write true or false for the following statements.

a) $6 \in X$ b) $9 \notin X$ c) $15 \in X$ d) $12 \notin X$ e) $11 \in X$

Solution:

a) True: The number 6 is an element of set X.
b) False: The number 9 is an element of set X.
c) True: The number 15 is an element of set X.
d) False: The number 12 is an element of set X.
e) False: The number 11 is not an element of set X.

3. If A = {1, 3, 5, 7, 9}, then fill in the blanks with the appropriate symbol of $\in$ or $\notin$.

a) 7____________ A b) 13 ____________A c) 3____________A
d) 2 ____________A e) 5____________A f) 6____________A

Solution:

a) $7 \in A$ b) $13 \notin A$ c) $3 \in A$ d) $2 \notin A$ e) $5 \in A$ f) $6 \notin A$

4. Write the cardinality of the following sets.

a) B = {0, 1, 2, 3, 4,…15} b) Q = {red, blue, orange}
c) N = { } d) K = {Tahir, Asim, Asad, Ali, Umar}
e) R = ø f) S = {5, 10, 15, 20}
g) P = {4, 6, 8, 9, 10, 12, 14, 15} h) E = {a, e, i, o, u}

Solution:

a) The set B contains the numbers 0 to 15. Therefore, the cardinality of set B is 16 because there are 16 elements in the set.
b) The set Q contains three elements, which are "red", "blue", and "orange". Therefore, the cardinality of set Q is 3.
c) The set N is an empty set, which means it has no elements. Therefore, the cardinality of set N is 0.
d) The set K contains the names Tahir, Asim, Asad, Ali, and Umar. Therefore, the cardinality of set K is 5 because there are 5 elements in the set.
e) The set R is an empty set, which means it has no elements. Therefore, the cardinality of set R is 0.
f) The set S contains the numbers 5, 10, 15, and 20. Therefore, the cardinality of set S is 4 because there are 4 elements in the set.

g) The set P contains the numbers 4, 6, 8, 9, 10, 12, 14, and 15. Therefore, the cardinality of set P is 8 because there are 8 elements in the set.

h) The set E contains the vowels a, e, i, o, and u. Therefore, the cardinality of set E is 5 because there are 5 elements in the set.

Learning Check 4.2

1. Separate the finite and infinite sets from the following.

a) N = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} b) P = {2, 3, 5, 7, …}

c) O = {1, 3, 5, 7, 9, …} d) C = {4, 6, 8, 9, 10}

e) The set of multiples of 5 f) The set of countries of the world

g) The set of whole numbers h) The set of even numbers greater than 100

i) The set of factors of 120 j) X = {103, 105, 107, 109, …}

Solution:

a) Set N is a finite set since it has a fixed number of elements, as set of natural numbers from 1 upto 10.

b) Set P is an infinite set since it contains all prime numbers and there is no end to the prime numbers.

c) Set O is an infinite set since it contains all odd numbers and there is no end to odd numbers.

d) Set C is a finite set since it has a fixed number of elements, as 4, 6, 8, 9, 10.

e) The set of multiples of 5 is an infinite set since there is no limit to the number of multiples of 5.

f) The set of the countries of the world are countable so it is a finite set.

g) The set of whole numbers is an infinite set since there is no limit to the number of whole numbers.

h) The set of even numbers greater than 100 is an infinite set because there is no upper limit to the number of even numbers greater than 100.

i) The set of factors of 120 is a finite set since it has a fixed number of elements. Therefore, the set of factors of 120 is:
{1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120}

j) Set X is an infinite set since it contains all odd prime numbers greater than or equal to 103, and there is no end to the odd prime numbers.

2. Identify and separate the empty sets.

a) A = {10, 15, 20, 25, 30} b) The set of odd multiples of 4

c) B = set of numbers less than 20

d) The set of prime numbers that have at least 5 factors

e) The set of multiples of 8 between 30 and 40

f) The set of even numbers between 7 and 10

g) The set of triangles with four sides

h) The set of days of the week that start with the letter J

i) The set of months of the year

j) The set of prime numbers between 8 and 10

Solution:

a) A = {10, 15, 20, 25, 30} is not an empty set. It contains 5 elements.

b) The set of odd multiples of 4 is an empty set.
Multiples of 4 are numbers that can be expressed in the form 4n, where n is an integer. Odd numbers are numbers that cannot be expressed in the form 2n, where n is an integer. Therefore, the set of odd multiples of 4 is empty because there are no numbers that satisfy both conditions.

c) B = set of numbers less than 20 is not an empty set.
It contains the elements
{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19}.

d) The set of prime numbers that have at least 5 factors is an empty set.
A prime number has exactly 2 factors (1 and itself). Therefore, no prime number can have 5 or more factors, and the set of prime numbers that have at least 5 factors is empty.

e) The set of multiples of 8 between 30 and 40 is a not an empty. As the 32 is the multiple of 8 that is between 30 and 40.

f) The set of even numbers between 7 and 10 is not an empty set as the 8 is the even number that lies between 7 and 10.

g) The set of triangles with four sides is an empty set.
A triangle is a polygon with three sides, so there cannot be a triangle with four sides.

h) The set of days of the week that start with the letter J is an empty set.
The days of the week are Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday. None of these days start with the letter J.

i) The set of months of the year is not an empty set.
It contains the elements
{"January", "February", "March", "April", "May", "June", "July", "August", "September", "October", "November", "December"}.

j) The set of prime numbers between 8 and 10 is an empty set.
The only prime numbers less than 10 are 2, 3, 5, and 7. None of these numbers are between 8 and 10. Therefore, the set of prime numbers between 8 and 10 is empty.

3. Identify the singleton sets.

a) The set of the greatest two 2-digit number
b) The set of prime numbers between 1 and 3
c) The set of whole numbers less than 1
d) The set of provinces in Pakistan
e) The set of months of the year that start with the letter M
f) The set of common factors of 12, 40, 55
g) The set of numbers that divides both 8 and 15
h) The set of multiples of 9 between 10 and 20

Solution:

a) The set of the greatest two 2-digit numbers is a singleton set as it contains only one element, which is 99.
b) The set of prime numbers between 1 and 3 is a singleton set as there is only one number that is 2 lies between 1 and 3 which is prime number.
c) The set of whole numbers less than 0 is the singleton set as 0 is the only whole number that is less than 1.
d) The set of provinces in Pakistan is not a singleton set as it contains more than one element.
e) The set of months of the year that start with the letter M is a singleton set as it contains only one element, which is {May}.
f) The set of common factors of 12, 40, 55 is a singleton set as it contains only one element, which is {1}.
g) The set of numbers that divide both 8 and 15 is a singleton set as it contains only one element, which is {1}.
h) The set of multiples of 9 between 10 and 20 is the singleton set as there is only one multiple of 9 that is 18 lies between 10 and 20.

4. Identify the universal set from the given sets and then show those sets by using a Venn diagram.

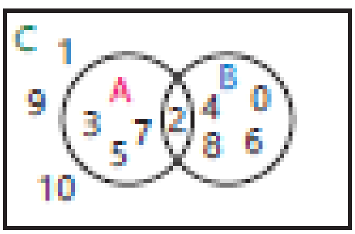
a) A = set of prime numbers less than 10, B = set of even numbers less than 10 and C = set of whole numbers up to 10

b) E = {a, b, c, d}, F = {a, e, i, o, u} and G = {a, b, c, d, e, …, Z}

c) T = {1, 2, 3, 4, 5, …, 10}, U = {1, 3, 5, 7} and V = {2, 3, 4}

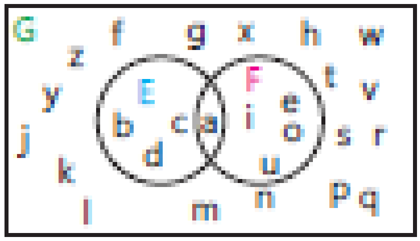
Solution:

a) The universal set in this case would be the set of whole numbers up to 10:
C = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
To create the Venn diagram for this, we draw a rectangle to represent the universal set, and then draw circles to represent the subsets A, and B, where 2 is common element in the set A and B.
Venn Diagram:

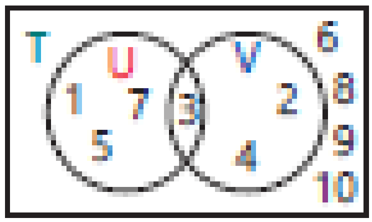


b) The universal set in this case would be the set of all alphabets:
G = {a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z}.
To create the Venn diagram for this, we draw a rectangle to represent the universal set, and then draw circles to represent the subsets E, F, where a is common element in the set E and F.

Venn Diagram:



c) The universal set in this case would be the set of natural numbers up to 10:
N = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
To create the Venn diagram for this, we draw a rectangle to represent the universal set, and then draw circles to represent the subsets U and V, where 3 is common element in the set U and V.
Venn Diagram:



Unit Check 4

1. Encircle the correct answer.

a) A collection of objects that are distinct and well-defined is called:
   i) group ii) things iii) set iv) elements

b) The symbol used for the elements of a set is:
   i) ∈ ii) φ iii) ∉ iv) ≈

c) The cardinality of the set of factors of 18 is:
   i) 5 ii) 3 iii) 2 iv) 6

d) The empty set is:
   i) The set of whole numbers between 1 and 5
   ii) The set of days of the week
   iii) The set of integers between −1 and 1
   iv) The set of odd multiples of 2

e) Which one is not a set?
   i) The set of months of the year
   ii) The set of Islamic months
   iii) The set of prayers in a day
   iv) The set of tasty food items

f) Which one is the singleton set?
   i) The set of provinces in Pakistan
   ii) The set of vowels
   iii) The set of solar months that have 3 letters
   iv) The set of integers less than 1

2. **Define the following:**

   a) set b) cardinality of a set c) finite set
   d) empty set e) universal set f) infinite set g) Venn diagram

   Solution:

   a) The collection of such distinct and well-defined objects is called a **set**.
   b) The total number of elements that are present in a set is called the **cardinality** of the set. For example, the number of elements in a day's prayers is 5.
   c) A set that consists of a limited number of elements is called a **finite set**.
   d) A set that has no element is called an **empty set.**
   e) A set that contains all the elements of the sets that are under consideration in a particular context is called a **universal set**
   f) A set that consists of an unlimited or uncountable number of elements is called an **infinite set**.
   g) A diagram that represents mathematical sets and their relationships is called a **Venn diagram**.

3. **Identify which of the following are sets.**

   a) A collection of good players of cricket
   b) A collection of months of the solar year
   c) A collection of Muslim countries in the world
   d) A collection of countries in Europe
   e) A collection of seven colours in a rainbow
   f) A collection of the first ten natural numbers
   g) A collection of interesting storybooks
   h) A collection of the English alphabets
   i) A collection of whole numbers
   j) A collection of even numbers

   Solution:

   a) A collection of good players of cricket:
   This is not a set since it is not a collection of distinct objects. It may contain duplicate players, and it is unclear what criteria define a "good" player.
   b) A collection of months of the solar year:
   This is a set since it is a collection of distinct objects, namely the twelve months of the year.
   c) A collection of Muslim countries in the world:
   This is a set since it is a collection of distinct objects, namely countries with a significant Muslim population or with an Islamic government.
   d) A collection of countries in Europe:
   This is a set since it is a collection of distinct objects, namely the countries located on the European continent.
   e) A collection of seven colors in a rainbow:
   This is a set since it is a collection of distinct objects, namely the seven colors of the spectrum.
   f) A collection of the first ten natural numbers:
   This is a set since it is a collection of distinct objects, namely the numbers 1 through 10.

g) A collection of interesting storybooks:
This is not a set since it is not a collection of distinct objects. It is unclear what criteria define an "interesting" storybook.
h) A collection of the English alphabets:
This is a set since it is a collection of distinct objects, namely the 26 letters of the English alphabet.
i) A collection of whole numbers:
This is a set since it is a collection of distinct objects, namely the set of numbers that includes all positive integers, zero, and negative integers.
j) A collection of even numbers:
This is a set since it is a collection of distinct objects, namely the set of numbers that are divisible by 2.

4. Sort out finite or infinite sets from the given sets.

a) P ={3, 6, 9, 12, 15, …}
b) A ={ 1, 3, 5, 7,…21}
c) E = {2, 4, 6, 8, …}
d) The set of factors of 100
e) The set of multiples of 20 less than 200

Solution:

a) P = {3, 6, 9, 12, 15, …}
This is an infinite set since it has an infinite number of elements, generated by adding 3 to each preceding element.
b) A = {1, 3, 5, 7, …21}
This is a finite set since it has a specific number of elements, namely 11.
c) E = {2, 4, 6, 8, …}
This is an infinite set since it has an infinite number of elements, generated by adding 2 to each preceding element.
d) The set of factors of 100
This is a finite set since it has a specific number of elements, namely 9. The factors of 100 are 1, 2, 4, 5, 10, 20, 25, 50, and 100.
e) The set of multiples of 20 less than 200
This is a finite set since it has a specific number of elements, namely 10. The multiples of 20 less than 200 are 20, 40, 60, 80, 100, 120, 140, 160, 180.

5. Which of the following are empty sets?

a) The set of even numbers between 2 and 4
b) The set of odd numbers between 2 and 4
c) The set of fractions with a denominator of 0
d) The set of multiples of 15 less than 10
e) The set of whole numbers other than natural numbers
f) The set of even prime numbers greater than 2
g) The set of circles with two corners

Solution:

a) The set of even numbers between 2 and 4
This is an empty set since there are no even numbers between 2 and 4.
b) The set of odd numbers between 2 and 4
This set of odd numbers between 2 and 4 is not an empty set as 3 is an odd number lies between 2 and 4.

c) The set of fractions with a denominator of 0
   This is an empty set since a fraction with a denominator of 0 is undefined.
d) The set of multiples of 15 less than 10
   This is an empty set since there are no multiples of 15 less than 10.
e) The set of whole numbers other than natural numbers
   The set of whole numbers other than natural number is not an empty set as 0 is the whole number that is not a natural number.
f) The set of even prime numbers greater than 2
   This is an empty set since the only even prime number is 2.
g) The set of circles with two corners
   This is an empty set since a circle does not have any corners.

6. Which of the following sets are singleton sets?

a) The set of natural numbers less than 2
b) The set of natural numbers between 35 and 37
c) The set of whole numbers between 2 and 4
d) The set of even numbers between 2 and 4
e) The set of odd numbers between 5 and 9
f) The set of the capital city of Pakistan
g) The set of provincial capitals of Pakistan

Solution:

a) The set of natural numbers less than 2
   This is a singleton set since it only contains the element 1.
b) The set of natural numbers between 35 and 37
   This is a singleton set since it only contains the element 36.
c) The set of whole numbers between 2 and 4
   The set of whole numbers between 2 and 3 is a singleton set as only 3 lies between 2 and 4 which is a whole number.
d) The set of even numbers between 2 and 4
   This is not a singleton set as there is only one element 3 that lies between 2and 4 that is not even number.
e) The set of odd numbers between 5 and 9
   This is the singleton set as only 7 lies between 5 and 9 that is odd number.
f) The set of the capital city of Pakistan
   This is a singleton set since Islamabad is the only capital city of Pakistan.
g) The set of provincial capitals of Pakistan
   This is not a singleton set since Pakistan has multiple provincial capitals.

7. Write the number of elements of the given sets.

a) The set of whole numbers less than 5
b) The set of natural numbers between 5 and 6
c) The set of divisors of 10 that are greater than 5
d) The set of multiples of 3 that are greater than 9 and less than 50
e) The set of multiples of 2 that are less than 20
f) The set of factors of 8

Solution:

a) The set of whole numbers less than 5

This set contains the elements 0, 1, 2, 3, and 4. Therefore, it has 5 elements.

b) The set of natural numbers between 5 and 6
This set only contains the element 5. Therefore, it has 1 element.

c) The set of divisors of 10 that are greater than 5
This set contains the elements 6 and 10, since these are the only divisors of 10 that are greater than 5. Therefore, it has 2 elements.

d) The set of multiples of 3 that are greater than 9 and less than 50
This set contains the elements 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, and 48. Therefore, it has 13 elements.

e) The set of multiples of 2 that are less than 20
This set contains the elements 2, 4, 6, 8, 10, 12, 14, 16, and 18. Therefore, it has 9 elements.

f) The set of factors of 8
This set contains the elements 1, 2, 4, and 8. Therefore, it has 4 elements.

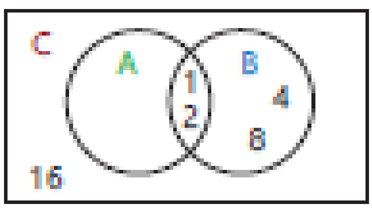
8. Which one of the given sets is the universal set? Show it using a Venn diagram.

a) A = set of factors of 2, B = set of factors of 8, C = set of factors of 16

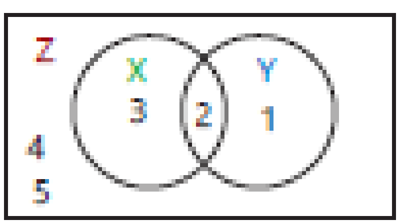
b) X = {2, 3}, Y = {1, 2}, Z = {1, 2, 3, 4, 5}

Solution:

a) The universal set in this case would be the set C: C = set of factors of 16.
To draw Venn diagram, we first draw a rectangle to represent the universal set, which is set C. Then we draw two circles inside the rectangle to represent the sets A and B. We make sure that each element in each set is represented by at least one point inside the corresponding circle. If two sets have elements in common, we draw overlapping regions to represent those elements.
Venn Diagram:



b) The universal set in this case would be the set Z, as Z = {1, 2, 3, 4, 5}.
To draw Venn diagram, we first draw a rectangle to represent the universal set, which is set Z. Then we draw two circles inside the rectangle to represent the sets X and Y. We place the elements of each set inside the corresponding circle. If two sets have elements in common, we draw overlapping regions to represent those elements.
Venn diagram:



Learning Check 5.1

1. **Identify the term-to-term rules for the following patterns and write the next three terms.**

   Solution:

   a) The term-to-term rule for this pattern is to add 4 to the previous term. The next three terms are: 56, 60, 64.

   b) The term-to-term rule for this pattern is to subtract 4 from the previous term. The next three terms are: 73, 69, 65.

   c) The term-to-term rule for this pattern is to divide the previous term by 2. The next three terms are: 400, 200, 100.

   d) The term-to-term rule for this pattern is to divide the previous term by 5. The next three terms are: 625, 125, 25.

2. **By using the position-to-term rule, find the 7th, 20th, 55th and 90th terms of the following patterns.**

   Solution:

   a) The position-to-term rule for this pattern is to add the position number to 4. Therefore, the 7th term is 11, the 20th term is 24, the 55th term is 59, and the 90th term is 94.

   b) The position-to-term rule for this pattern is to multiply the position number by 4. Therefore, the 7th term is 28, the 20th term is 80, the 55th term is 220, and the 90th term is 360.

   c) The position-to-term rule for this pattern is to add the position number to 19. Therefore, the 7th term is 26, the 20th term is 39, the 55th term is 74, and the 90th term is 109.

   d) The position-to-term rule for this pattern is to multiply the position number by 5. Therefore, the 7th term is 35, the 20th term is 100, the 55th term is 275, and the 90th term is 450.

   e) The position-to-term rule for this pattern is to multiply the position number by 12. Therefore, the 7th term is 84, the 20th term is 240, the 55th term is 660, and the 90th term is 1080.

   f) The position-to-term rule for this pattern is to simply add 1 to the position number. Therefore, the 7th term is 8, the 20th term is 21, the 55th term is 56, and the 90th term is 91.

3. Ibrahim recited 2 pages of the Holy Quran on Monday, 5 pages on Tuesday and 8 pages on Wednesday. If he keeps reciting the pages of the Holy Quran each day, how many pages will he recite on Thursday, Friday and Saturday, respectively?

Solution:

Since Ibrahim recited 2 pages on Monday, 5 pages on Tuesday, and 8 pages on Wednesday, we can assume that he is reciting an arithmetic sequence where the first term is 2, the second term is 5, and the third term is 8.

To find the common difference, we can subtract the second term from the first term, and then subtract the third term from the second term:

Common difference = 5 - 2 = 3 Common difference = 8 - 5 = 3

So the common difference is 3. Therefore, the fourth, fifth, and sixth terms of the sequence will be:

Fourth term = 8 + 3 = 11 Fifth term = 11 + 3 = 14 Sixth term = 14 + 3 = 17

So Ibrahim will recite 11 pages on Thursday, 14 pages on Friday, and 17 pages on Saturday.

4. An author is writing a storybook. He wrote 10 pages in the first week, 15 pages in the second week, 20 pages in the third week and 25 pages in the fourth week. If he keeps writing the pages in the same pattern, how many pages will he write in the fifth and sixth week?

Solution:

To find out how many pages the author will write in the fifth week, we need to add the common difference to the fourth term of the sequence, since the pattern is arithmetic with a common difference of 5.

The fourth term of the sequence is 25, and the common difference is 5. Therefore, the fifth term of the sequence is:

25 + 5 = 30

So the author will write 30 pages in the fifth week.

To find out how many pages he will write in the sixth week, we need to add the common difference to the fifth term of the sequence, which is 30. Therefore, the sixth term of the sequence is:

30 + 5 = 35

So the author will write 35 pages in the sixth week.

Learning Check 5.2

1. Identify and separate the variables in each of the following.

a) $2x + 5$	b) $7x^2 + 2y + c$	c) $9a + 4$	d) $2u^2 + 2v$
e) $5w + 7x – 2$	f) $7x + y – 4$	g) $1 + 4a + 2b$	h) $-4x + 7y + z$

Solution:

a) The variable in this expression is "x".

b) The variables in this expression are "x" and "y".

c) The variable in this expression is "a".

d) The variables in this expression are "u" and "v".

e) The variables in this expression are "w" and "x".

f) The variables in this expression are "x" and "y".

g) The variables in this expression are "a" and "b".

h) The variables in this expression are "x", "y", and "z".

2. Enlist the terms of the algebraic expressions.

a) $-4x^2 + 2x + 1$	b) $5ab - 7$	c) $2x - 3yx + 7$	d) $5y + 2yz - 11$
e) $6y + 2z - 3y^2$	f) $2w^2 - 5w - 7$	g) $x^2 - 2xy + y^2$	h) $7a^2 + 5ab + c^2$

Solution:

a) The terms of the expression $-4x^2 + 2x + 1$ are:

$-4x^2$ (a term with coefficient -4 and variable x^2)

2x (a term with coefficient 2 and variable x)

1 (a constant term)

b) The terms of the expression 5ab − 7 are:

5ab (a term with coefficient 5 and variables a and b)

−7 (a constant term)

c) The terms of the expression 2x − 3yx + 7 are:

2x (a term with coefficient 2 and variable x)

−3yx (a term with coefficient −3y and variable x)

7 (a constant term)

d) The terms of the expression 5y + 2yz − 11 are:

5y (a term with coefficient 5 and variable y)

2yz (a term with coefficient 2y and variable z)

−11 (a constant term)

e) The terms of the expression $6y + 2z - 3y^2$ are:

$6y$ (a term with coefficient 6 and variable y)

$2z$ (a term with coefficient 2 and variable z)

$-3y^2$ (a term with coefficient -3 and variable y^2)

f) The terms of the expression $2w^2 - 5w - 7$ are:

$2w^2$ (a term with coefficient 2 and variable w^2)

$-5w$ (a term with coefficient -5 and variable w)

-7 (a constant term)

g) The terms of the expression $x^2 - 2xy + y^2$ are:

x^2 (a term with coefficient 1 and variable x^2)

$-2xy$ (a term with coefficient $-2y$ and variables x and y)

y^2 (a term with coefficient 1 and variable y^2)

h) The terms of the expression $7a^2 + 5ab + c^2$ are:

$7a^2$ (a term with coefficient 7 and variable a^2)

$5ab$ (a term with coefficient 5 and variables a and b)

c^2 (a term with coefficient 1 and variable c^2)

3. Identify and write down the coefficients of the terms in the following algebraic expressions.

a) $31x + 4$	b) $2xy + y^2$	c) $7y^2 + 9x^4 - 9zy$	d) $23x^2 + 9y$
e) $4xy + 2yz$	f) $xy + yz + xyz$	g) $15t^3 - 4x^2y^2$	h) $2b^3 + 4$

Solution:

a) The coefficients are 31 and 4.

b) The coefficients are 2 and 1.

c) The coefficients are 7, 9, and -9.

d) The coefficients are 23 and 9.

e) The coefficients are 4 and 2.

f) The coefficients are 1, 1, and 1.

g) The coefficients are 15, -4, x^2, and y^2.

h) The coefficients are 2 and 4.

4. Write down the exponents of the variables in the following.

a) $-4x^5$ b) $7b^3$ c) y^7 d) t^3 e) $7h^7$ f) $2z^4$

Solution:

a) The exponent of x is 5.

b) The exponent of b is 3.

c) The exponent of y is 7.

d) The exponent of t is 3.

e) The exponent of h is 7.

f) The exponent of z is 4.

5. Write down the algebraic expressions by using the given terms.

a) $7a, -3b^2, -4ab$ b) $2x^3, -4y, -7$ c) $3u^4, 6v^2, -2uv$

Solution:

a) The algebraic expression is: $7a - 3b^2 - 4ab$.

b) The algebraic expression is: $2x^3 - 4y - 7$.

c) The algebraic expression is: $3u^4 + 6v^2 - 2uv$.

Learning Check 5.3

1. Substitute the given values for the variables to evaluate each expression.

a) $xy - (y + z) + x^2$ when $x = -2$, $y= 4$ and $z = 1$

b) $4 (a^3 + b^2 + 4) + 3abc^4$ when $a = -3$, $b = 2$ and $c = -5$

c) $2p^3q^2 + 4p^2q^3 + 7q^2p^3 - 2p^3q^2$ when $p = -7$, and $q = 4$

d) $4x^3 + 5y^3 + 4$ when $x = -1$ and $y = 0$

e) $a^3 - 4ab^2 + 5ab - 6$ when $a = 4$ and $b = -6$

f) $x^3 + y^3$ when $x = -5$ and $y = 5$

Solution:

a) $xy - (y + z) + x^2 = (-2)(4) - (4 + 1) + (-2)^2 = -8 -5 + 4 = -9$

b) $4(a^3 + b^2 + 4) + 3abc^4 = 4((-3)^3 + 2^2 + 4) + 3(-3)(2)(-5)^4 = 4(-27 + 4 + 4) + 3(-6)(625) = -76 - 11250 = -11326$

c) $2p^3q^2 + 4p^2q^3 + 7q^2p^3 - 2p^3q^2$, we get:

$2(-7)^3(4)^2 + 4(-7)^2(4)^3 + 7(4)^2(-7)^3 - 2(-7)^3(4)^2$

$-10976 + 12544 - 38416 + 10976 = -25872$

d) $4x^3 + 5y^3 + 4 = 4(-1)^3 + 5(0)^3 + 4 = -4 + 0 + 4 = 0$

e) $a^3 - 4ab^2 + 5ab - 6 = 4^3 - 4(4)(-6)^2 + 5(4)(-6) - 6 = 64 - 576 - 120 - 6 = -638$

f) $x^3 + y^3 = (-5)^3 + 5^3 = -125 + 125 = 0$

2. Evaluate the algebraic expressions when w = 3, x = −6 , y = −1 and z = 2.

a) xy/z

b) $2x + y + 2z - w^2$

c) $7x^2 + 2y3 + z$

d) $4w^5 + 2x^2y^4$

e) $(x + y)(x - y) / x^2 + 2xy + y^2$

f) $3x + 4y + (2w/7z^2)$

g) $w + z/x + y$

h) $w^2 + 2y^3z^2 + 4/x^2 - y^2$

Solution:

a) $xy/z = (-6)(-1)/2 = 6/2 = 3$

b) $2x + y + 2z - w^2 = 2(-6) + (-1) + 2(2) - 3^2 = -12 - 1 + 4 - 9 = -18$

c) $7x^2 + 2y^3 + z = 7(-6)^2 + 2(-1)^3 + 2 = 252 - 2 + 2 = 252$

d) $4w^5 + 2x^2y^4 = 4(3)^5 + 2(-6)^2(-1)^4 = 4(243) + 2(36)(1) = 972 + 72 = 1044$

e) $(x + y)(x - y)/x^2 + 2xy + y^2 = (-6 + (-1))(-6 - (-1))/(-6)^2 + 2(-6)(-1) + (-1)^2$

$= \frac{(-7)(-5)}{36} + 12 + 1 = \frac{35}{36} + 13 = \frac{35+468}{36} = \frac{503}{36} = 13.97$

f) $3(-6) + 4(-1) + (2(3)/(7(2)^2)) = -18 - 4 + (6/28) = -22 + 3/14$

$= -308 + 3/14 = -305/14 = -21\frac{11}{14}$

g) $w + z/x + y = 3 + 2/(-6) + (-1) = 3 - 1/3 - 1 = 9 - 1 - 3/3 = 5/3$

h) $w^2 + 2y^3z^2 + 4/x^2 - y^2 = 3^2 + 2(-1)^3(2)^2 + 4/(-6)^2 - (-1)^2$

$= 9 - 8 + 4/36 - 1 = 4/36 = 1/9$

3. If $x = 5$ and $y = -7$, then evaluate $5x^2 - 2xy + x$.

Solution:

$5x^2 - 2xy + x = 5(5)^2 - 2(5)(-7) + 5 = 5(25) + 70 + 5 = 125 + 70 + 5 = 200$

Learning Check 5.4

1. Add the following algebraic expressions horizontally.

a) xy, xy b) $3x^3, 5x^3$ c) $uv, 5uv, 4uv$ d) $13x, 9x, x$

e) $5y, -4y, y$ f) $7p, 17q, 27pq$ g) $-8y, 3y$ h) $-11xz, xz$

Solution:

a) $xy + xy = (1 + 1)xy = 2xy$

b) $3x^3 + 5x^3 = (3 + 5)x^3 = 8x^3$

c) $uv + 5uv + 4uv = = (1 + 5 + 4)uv = 10uv$

d) $13x + 9x + x = (13 + 9 + 1)x = 23x$

e) $5y - 4y + y = (5 - 4 + 1)\ y = 2y$

f) $7p + 17q + 27pq = 27pq + 7p + 17q$

g) $-8y + 3y = -5y$

h) $-11xz + xz = -10xz$

2. Add the following algebraic expressions vertically.

a) $11x + y, x - 11y$

b) $7u + 9v, 5u + 12v$

c) $-2m^3 + 5n^2, m^3 + 6n^2, 2m^3 - 2n^2$

d) $x + 3y - 2z, x - y + z$

e) $2x^2 + 9y^3 - 7, x^2 - y^3 + 2$

f) $5x^4 + 5x^3y^3 + 6, - x^4 - 3x^3y^2 - 2y$

g) $2x + 3y - 4, -2x + y + 4, x - y + 11$

h) $x^2 + 5x + 6, 3x^2 + 4x, 2x^2 - x - 3$

Solution:

a) $11x + y, x - 11y$

$11x + y$ $x - 11y$
$12x - 10y$

b) $7u + 9v, 5u + 12v$

$7u + 9v$ $5u + 12v$
$12u + 21v$

c) $-2m^3 + 5n^2$, $m^3 + 6n^2$, $2m^3 - 2n^2$

$-2m^3 + 5n^2$
$m^3 + 6n^2$
$2m^3 - 2n^2$
$m^3 + 9n^2$

d) $x + 3y - 2z$, $x - y + z$

$x + 3y - 2z$
$x - y + z$
$2x + 2y - z$

e) $2x^2 + 9y^3 - 7$, $x^2 - y^3 + 2$

$2x^2 + 9y^3 - 7$
$x^2 - y^3 + 2$
$3x^2 + 8y^3 - 5$

f) $5x^4 + 5x^3y^3 + 6$, $- x^4 - 3x^3y^2 - 2y$

$5x^4 \quad + 5x^3y^3 + \quad 6$
$-x^4 - 3x^3y^2 \quad - 2y$
$4x^4 - 3x^3y^2 + 5x^3y^3 - 2y + 6$

g) $2x + 3y - 4$, $-2x + y + 4$, $x - y + 11$

$2x + 3y - 4$
$-2x + y + 4$
$x - y + 11$
$x + 4y + 11$

h) $x^2 + 5x + 6$, $3x^2 + 4x$, $2x^2 - x - 3$

$x^2 + 5x + 6$
$3x^2 + 4x$
$2x^2 \quad - x - 3$
$6x^2 + 8x + 3$

3. Subtract the first algebraic expression from the second one horizontally.

a) $2x + 5y - 3z$ from $4x - 9y + 11z$

b) $3x^2 - 6x - 4$ from $5 + x - 2x^2$

c) $3a^2 + b^2 - 3abc$ from $9a^2 - 5b^2 + ab$

d) $5x^3 + 2y^2 - 1$ from $x^3 - y^2 + 5$

e) $-7x^4 + 5yx - 4y^2$ from $x^2 - 5xy + 2y^2$

f) $+11x^2 - 5$ from $-5x^2 - 12$

Solution:

a) $2x + 5y - 3z$ from $4x - 9y + 11z$

Let us subtract $2x + 5y - 3z$ from $4x - 9y + 11z$

$4x - 9y + 11z - (2x + 5y - 3z)$

$= 4x - 9y + 11z - 2x - 5y + 3z$

$= 2x - 14y + 14z$

b) $3x^2 - 6x - 4$ from $5 + x - 2x^2$

Let us subtract $3x^2 - 6x - 4$ from $5 + x - 2x^2$

$5 + x - 2x^2 - (3x^2 - 6x - 4)$

$= 5 + x - 2x^2 - 3x^2 + 6x + 4$

$= - 2x^2 - 3x^2 + x + 6x + 4 + 5$

$= - 5x^2 - 3x^2 + 7x + 9$

c) $3a^2 + b^2 - 3abc$ from $9a^2 - 5b^2 + ab$

$9a^2 - 5b^2 + ab - (3a^2 + b^2 - 3abc)$

$= 9a^2 - 5b^2 + ab - 3a^2 - b^2 + 3abc$

$= 9a^2 - 3a^2 - 5b^2 - b^2 + ab + 3abc$

$= 6a^2 - 6b^2 + ab + 3abc$

d) $5x^3 + 2y^2 - 1$ from $x^3 - y^2 + 5$

$= x^3 - y^2 + 5 - (5x^3 + 2y^2 -1)$

$= x^3 - y^2 + 5 - 5x^3 - 2y^2 +1$

$= x^3 - 5x^3 - y^2 - 2y^2 + 5 + 1$

$= - 4x^3 - 3y^2 + 6$

e) $-7x^4 + 5yx - 4y^2$ from $x^2 - 5xy + 2y^2$

$= x^2 - 5xy + 2y^2 - (-7x^4 + 5yx - 4y^2)$

$= x^2 - 5xy + 2y^2 + 7x^4 - 5yx + 4y^2$

$= x^2 + 7x^4 + 2y^2 + 4y^2 - 5yx - 5xy$

$= x^2 + 7x^4 + 6y^2 - 10yx$

f) $+11x^2 - 5$ from $-5x^2 -12$

$= -5x^2 -12 - (+11x^2 - 5)$

$= -5x^2 -12 - 11x^2 + 5$

$= -5x^2 - 11x^2 -12 + 5$

$= - 16x^2 -7$

4. Subtract the first algebraic expression from the second one vertically.

a) $9x^2 - 5x + 6$ from $-3x^2 - 4x - 3$

b) $4a + 7b - 7c$ from $9b - c - 3a$

c) $15x^2 - 2xy + 6z^2$ from $4z^2 + 3xy - 11x^2$

d) $7y^2 + 5x^2 - w^2 - 3xyw$ from $4y^2 - w^2 + 6x^2 - 7wxy$

e) $13a^3 + 11a^2b - 17ab^2 - 6ab$ from $20a^3 - 17ab^2 - 6ab - 11b^3$

f) $18x^4 + 10x^3y + 14x^2y^2 - 25xy^3$ from $15x^4 + 14x^3y + 14x^2y^2 - 25xy - 30y^4$

Solution:

a) $9x^2 - 5x + 6$ from $-3x^2 - 4x - 3$

Let us subtract $9x^2 - 5x + 6$ from $-3x^2 - 4x - 3$ vertically.

$$\begin{array}{c} -3x^2 - 4x - 3 \\ \pm 9x^2 \mp 5x \pm 6 \\ \hline -12x^2 + x - 9 \end{array}$$

b) $4a + 7b - 7c$ from $9b - c - 3a$

Let us subtract $4a + 7b - 7c$ from $9b - c - 3a$ vertically.

$$\begin{array}{l} \quad 4a + 7b - 7c \\ \mp 3a \pm 9b \mp c \\ \hline \quad 7a - 2b \end{array}$$

c) $15x^2 - 2xy + 6z^2$ from $4z^2 + 3xy - 11x^2$

Let us subtract $15x^2 - 2xy + 6z^2$ from $4z^2 + 3xy - 11x^2$ vertically.

$$\begin{array}{l} \quad 15x^2 - 2xy + 6z^2 \\ \mp 11x^2 \pm 3xy \pm 4z^2 \\ \hline \quad 26x^2 - 5xy + 2z^2 \end{array}$$

d) $7y^2 + 5x^2 - w^2 - 3xyw$ from $4y^2 - w^2 + 6x^2 - 7wxy$

Let us subtract $7y^2 + 5x^2 - w^2 - 3xyw$ from $4y^2 - w^2 + 6x^2 - 7wxy$ vertically.

$$\begin{array}{c} 4y^2 - w^2 + 6x^2 - 7wxy \\ \pm 7y^2 \mp w^2 \pm 5x^2 \mp 3xyw \\ \hline -3y^2 \qquad + x^2 - 4wxy \end{array}$$

e) $13a^3 + 11a^2b - 17ab^2 - 6ab$ from $20a^3 - 17ab^2 - 6ab - 11b^3$

Let us subtract $13a^3 + 11a^2b - 17ab^2 - 6ab$ from $20a^3 - 17ab^2 - 6ab - 11b^3$ vertically.

$$\begin{array}{llllll}
 & 20a^3 & - 17ab^2 & - 6ab & - 11b^3 & \\
 & \pm 13a^3 & \mp 17ab^2 & \mp 6ab & & \pm 11a^2b \\
\hline
 & 7a^3 & & & 11b^3 & - 11a^2b
\end{array}$$

f) $18x^4 + 10x^3y + 14x^2y^2 - 25xy^3$ from $15x^4 + 14x^3y + 14x^2y^2 - 25xy - 30y^4$

Let us subtract $18x^4 + 10x^3y + 14x^2y^2 - 25xy^3$ from $15x^4 + 14x^3y + 14x^2y^2 - 25xy - 30y^4$ vertically.

$$\begin{array}{lllllll}
15x^4 & + 14x^3y & + 14x^2y^2 & - 25xy & - 30y^4 & \\
\pm 18x^4 & \pm 10x^3y & \pm 14x^2y^2 & & & \mp 25xy^3 \\
\hline
- 3x^4 & + 4x^3y & + 14x^2y^2 & - 25xy & - 30y^4 & + 25xy^3
\end{array}$$

5. Subtract $4x^2y + 2yx + z$ from $5yx^2 - 7xy + 2y^2x - 6z$.

Solution:

$= 5yx^2 - 7xy + 2y^2x - 6z - (4x^2y + 2yx + z)$

$= 5yx^2 - 7xy + 2y^2x - 6z - 4x^2y - 2yx - z$

$= 5yx^2 - 4x^2y - 7xy - 2yx + 2y^2x - 6z - z$

$= \ yx^2 - 9xy + 2y^2x - 7z$

6. What should be added to $5ab^2 - 3a^2b + 7$ to get $11ab^2 + 3a^2b + 11$?

Solution:

To get what should be added to $5ab^2 - 3a^2b + 7$ to $11ab^2 + 3a^2b + 11$, we have to subtract $5ab^2 - 3a^2b + 7$ from $11ab^2 + 3a^2b + 11$.

$= 11ab^2 + 3a^2b + 11 - (5ab^2 - 3a^2b + 7)$

$= 11ab^2 + 3a^2b + 11 - 5ab^2 + 3a^2b - 7$

$= 11ab^2 - 5ab^2 + 3a^2b + 3a^2b + 11 - 7$

$= 6ab^2 + 6a^2b + 4$

7. Subtract the sum of $2p^2q^3 + 4pq^2 - 7pq$ and $8pq^2 + 11pq$ from $5p^2q^3 - 11q^2p - 12pq + 2qp^2 + 3pq + 7$.

Solution:

First we find the sum of $2p^2q^3 + 4pq^2 - 7pq$ and $8pq^2 + 11pq$

$2p^2q^3 + 4pq^2 - 7pq + 8pq^2 + 11pq = 2p^2q^3 + 12pq^2 + 4pq$

Now subtract $2p^2q^3 + 12pq^2 + 4pq$ from $5p^2q^3 - 11q^2p - 12pq + 2qp^2 + 3pq + 7$

$= 5p^2q^3 - 11q^2p - 12pq + 2qp^2 + 3pq + 7 - (2p^2q^3 + 12pq^2 + 4pq)$

$= 5p^2q^3 - 11q^2p - 12pq + 2qp^2 + 3pq + 7 - 2p^2q^3 - 12pq^2 - 4pq$

$= 5p^2q^3 - 2p^2q^3 - 11q^2p - 12pq^2 + 2qp^2 + 7 - 4pq + 3pq - 12pq$

$= 3p^2q^3 - 23q^2p + 2qp^2 + 7 - 13pq$

Learning Check 5.5

Simplify the following algebraic expressions.

a) $2y(z - x)$

b) $15 - 2x (x + y)$

c) $5u(u + v + w) - uv - vw - 3$

d) $3y^2 - 1 (x - y) + 2$

e) $2(u + v - 12) + uv - 2w$

f) $u^2 + v(u - v) + uv$

g) $\{(u - v) - (u + v) - 3v^2 \} + u^2$

h) $(x + y)(x - y)$

i) $25\{xy - x (2 - (x - y))\}$

j) $3 - 2 \{u^2 + v^2(u + v) - 3uv\}$

k) $40a - 3[15a - 2\{(1 - 2a) - (3a - (2a + 5))\}]$

Solution:

a) $2y(z - x) = 2yz - 2yx$

b) $15 - 2x (x + y) = 15 - 2x^2 - 2xy$

c) $5u(u + v + w) - uv - vw - 3 = 5u^2 + 5uv + 5uw - uv - vw - 3 = 5u^2 + 4uv + 5uw - vw - 3$

d) $3y^2 - 1 (x - y) + 2 = 3y^3 - x + y + 2$

e) $2(u + v - 12) + uv - 2w = 2u + 2v - 24 + uv - 2w$

f) $u^2 + v(u - v) + uv = u^2 + uv - v^2 + uv = u^2 + 2uv - v^2$

g) $\{(u - v) - (u + v) - 3v^2\} + u^2 = u - v - u - v - 3v^2 + u^2 = u^2 - 2v - 3v^2$

h) $(x + y)(x - y) = x^2 - y^2$

i) $25\{xy - x(2 - (x - y))\}$

$= 25(xy - x(2 - x + y))$

$= 25(xy - 2x + x^2 - xy)$

$= 25x^2 - 50x$

$= 25x(x - 2)$

j) $3 - 2\{u^2 + v^2(u + v) - 3uv\}$

$= 3 - 2(u^2 + 2uv^2 + v^3 - 3uv)$

$= 3 - 2u^2 - 4uv^2 - 2v^3 + 6uv$

$= -2v^3 - 4uv^2 + 6uv - 2u^2 + 3$

k) $40a - 3[15a - 2\{(1 - 2a) - (3a - (2a + 5))\}]$

$= 40a - 3[15a - 2\{1 - 2a - (3a - 2a - 5)\}]$

$= 40a - 3[15a - 2\{1 - 2a - 3a + 2a + 5\}]$

$= 40a - 3[15a - 2\{-3a + 6\}]$

$= 40a - 3[15a + 6a - 12]$

$= 40a - 3[21a - 12]$

$= 40a - 63a + 36$

$= -23a + 36$

2. Simplify and then evaluate the following algebraic expressions for the given values of the variables.

a) $4p + [3q - 2r(p - q + r)]$, $p = 1$, $q = 2$, $r = -1$

b) $(2 + 5x)(11 + 12x)$, $x = -2$

c) $xy(x + y - 2) - 2x + z^2$, $x = 3$, $y = 0$, $z = -4$

d) $2/5r - 5(2 - st)$, $r = 15$, $s = 5$, $t = 1$

e) $5x + x(2y + 3 + y/x)$, $x = 2$, $y = -2$

Solution:

a) $4p + [3q - 2r(p - q + r)] = 4(1) + [3(2) - 2(-1)(1 - 2 + (-1))]$

$= 4 + (6 - 4) = 4 + 2 = 6$

b) $(2 + 5x)(11 + 12x) = (2 + 5(-2))(11 + 12(-2)) = (-8)(-13) = 104$

c) $xy(x + y - 2) - 2x + z^2 = 3(0)(3 + 0 - 2) - 2(3) + (-4)^2 = 0 - 6 + 16 = 10$

d) $2/5r - 5(2 - st) = 2/5(15) - 5(2 - 5(1)) = 6 - 5(2-5) = 6 -5(-3) = 6 + 15 = 21$

e) $5x + x(2y + 3 + y/x)$

$= 5(2) +2\{(2(-2) + 3 + -2/2)\}$

$= 10 + 2\{-4 + 3-1\}$

$= 10 + 2\{-2\}$

$= 10 - 6$

$= 4$

Unit Check 5

1. Encircle the correct answer.

a) The branch of mathematics in which we use different letters to represent unknown values is called:

i) numbers ii) algebra iii) measurement iv) geometry

b) The next term of the pattern by term-to-term rule is: 16, 32, 48, ___

i) 768 ii) 60 iii) 64 iv) 56

c) The power of any number or variable is called:

i) constant ii) coefficient iii) exponent iv) term

d) The value of the algebraic expression $6a - 3b - 7c$ when $a = 10$, $b = 7$ and $c = 2$ is:

i) 20 ii) 23 iii) 27 iv) 25

e) When we subtract $12x - 6y - 10z$ from $19x + 11y - 15z$, we get:

i) $31x - 5y + 25z$ ii) $7x -17y + 5z$

iii) $7x + 17y - 5z$ iv) $31x + 5y - 25z$

2. Fill in the blanks.

a) A number **pattern** is a sequence of numbers that increases, decreases or repeats by following a specific rule.

b) A/An **expression** is a combination of variables, constants, coefficients and exponents.

c) A quantity that is not constant and may take different values is called **variable**.

d) The terms that have the same variables and exponents are called **like** terms.

e) The sum of $x^4 + 5$ and $4x^4 + 2xy + 9$ is **$5x^4 + 2xy + 14$**

3. Find the next terms by using the term-to-term rule.

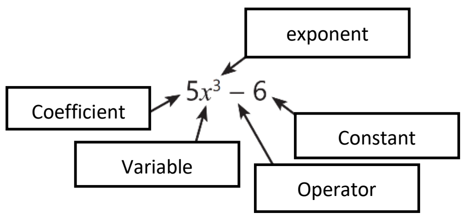
a) 17, 21, 25, **29, 33, 37** b) 98, 90, 82, **74, 66, 58**

c) 4, 8, 12, 16, **20, 24, 28** d) 262144, 32768, 4096, **512, 64, 8**

4. By using the position-to-term rule, find the 10th, 15th and 19th terms of the following:

a) 7, 8, 9, **16, 21, 25** b) 6, 12, 18, 24, **60, 90, 114** c) 25, 26, 27, **34, 39, 43**

5. Label the parts of the following algebraic expression.



6. Write the coefficient and exponent of the following algebraic expressions:

a) $3x^4 + y^5$ b) $7x^3$ c) $11x + 4$ d) $9x^2 + 1$ e) $9y^3$

Solution:

a) 3,1 and 4, 5 b) 7, 3 c) 11 and 1 d) 9 and 2 e) 9 and 3

7. If $x = -4$, $y = 5$ and $z = -2$, then evaluate the following algebraic expressions.

a) $4x^2 + 3xy^2 - 2yz^2$ b) $7xy + 9yz\ x^2 - z^2$ c) $x(x + z) / y(x - z)$

d) $x/x\ (y - z)$ e) $4y^2 - x^2/4x^2 + 2y + 4$ f) $9x^3 + 2xy^2 + 2yz^3 + 9$

Solution:

a) $4x^2 + 3xy^2 - 2yz^2 = 4(-4)^2 + 3(-4)(5)^2 - 2(5)(-2)^2$

[Substitute $x = -4$, $y = 5$ and $z = -2$] $= 4(16) + 3(-4)(25) - 2(5)(4) = 64 - 300 + (-40) = -276$

b) $7xy + 9yz / x^2 - z^2 = 7(-4)(5) + 9(5)(-2) / (-4)^2 - (-2)^2$ [Substitute $x = -4$, $y = 5$ and $z = -2$] $= -140 - 90 / 16 - 4$

$$= -144 - 45/8 = -1152 - \frac{45}{8} = -1197/8 = -149.625$$

c) $x(x + z)/y(x - z) = (-4)(-4 + (-2)) / 5(-4 - (-2))$

[Substitute $x = -4$, $y = 5$ and $z = -2$] $= (-4)(-6) / (5)(-2) = 24 / -10 = -12 / 5$

d) $\frac{x}{x(y-z)} = \frac{-4}{(-4)(5-(-2))}$ [Substitute x = -4, y = 5 and z = -2] = 1 / (-4)(7) = $-\frac{1}{28}$

e) We can simplify the second term by canceling the common factor of $4y^2 - (x^2 / 4x^2) + (2y + 4) = 4y^2 - (1 / 4) + (2y + 4)$ [Cancel the common factor of x^2] = $4y^2 + 2y + (15 / 4)$

If x = −4 and y = 5, then we substitute the values of x and y into the expression and simplify: $4y^2 + 2y + (15 / 4) = 4(5)^2 + 2(5) + (15 / 4)$ [Substitute x = −4 and y = 5] = 100 + 10 + (15 / 4)

= 400 + 40 + 15/4 = 455/4 = 113 3/4

f) $9x^3 + 2xy^2 + 2yz^3 + 9 = 9(-4)^3 + 2(-4)(5)^2 + 2(5)(-2)^3 + 9$

[Substitute x = −4 and y = 5] = $-576 - 200 - 80 + 9 = -856 + 9 = -847$

8. Add the following algebraic expressions vertically and horizontally.

a) $2m^3 - 6m^2n + 7mn^2$, $5m^3 + 3mn^2 + 7m^2n - 21$

b) $25p^3 + 40p^2q + 20pq^2$, $20p^2q + 30pq^2 - 40p^2q^2 + 50pq$

c) $12z^5 + 17z^4 - z^2 - 18$, $11z^5 - 12z^4 + 2z^2 + 21$, $5z^4 + 4z^2 - 35$

d) $x^5 + 6x^4 - 15x^3 + 34x$, $14x^4 - 10x^3 + 16x - 18$, $10x^5 + 12x^2 - 50x - 13$

e) $14y^4 - 19y^3 + 16y^2 - 21$, $20y^3 + 15y^2 - 11y - 15$, $18y^3 - 31y^2 + 11y + 36$

Solution:

a) $2m^3 - 6m^2n + 7mn^2$, $5m^3 + 3mn^2 + 7m^2n - 21$

First add horizontally:

$= 2m^3 - 6m^2n + 7mn^2 + 5m^3 + 3mn^2 + 7m^2n - 21$

$= 2m^3 + 5m^3 - 6m^2n + 7m^2n + 7mn^2 + 3mn^2 - 21$

$= 7m^3 + m^2n + 10mn^2 - 21$

$2m^3 - 6m^2n + 7mn^2$
$5m^3 + 7m^2n + 3mn^2 - 21$
$7m^3 + m^2n + 10mn^2 - 21$

b) $25p^3 + 40p^2q + 20pq^2$, $20p^2q + 30pq^2 - 40p^2q^2 + 50pq$

First add horizontally:

$= 25p^3 + 40p^2q + 20p^2q + 20pq^2 + 30pq^2 - 40p^2q^2 + 50pq$

$= 25p^3 + 60p^2q + 50pq^2 - 40p^2q^2 + 50pq$

$$\begin{array}{l} 25p^3 + 40p^2q + 20pq^2 \\ \qquad\quad 20p^2q + 30pq^2 - 40p^2q^2 + 50pq \\ \hline 25p^3 + 60p^2q + 50pq^2 - 40p^2q^2 + 50pq \end{array}$$

c) $12z^5 + 17z^4 - z^2 - 18,\ 11z^5 - 12z^4 + 2z^2 + 21,\ 5z^4 + 4z^2 - 35$

First add horizontally:

$= 12z^5 + 17z^4 - z^2 - 18 + 11z^5 - 12z^4 + 2z^2 + 21 + 5z^4 + 4z^2 - 35$

$= 12z^5 + 11z^5 + 17z^4 - 12z^4 + 5z^4 - z^2 + 2z^2 + 4z^2 - 18 + 21 - 35$

$= 23z^5 + 10z^4 + 5z^2 - 32$

$$\begin{array}{l} 12z^5 + 17z^4 - z^2 - 18 \\ 11z^5 - 12z^4 + 2z^2 + 21 \\ \qquad\quad 5z^4 + 4z^2 - 35 \\ \hline 23z^5 + 10z^4 + 5z^2 - 32 \end{array}$$

d) $x^5 + 6x^4 - 15x^3 + 34x,\ 14x^4 - 10x^3 + 16x - 18,\ 10x^5 + 12x^2 - 50x - 13$

First add horizontally:

$= x^5 + 6x^4 - 15x^3 + 34x + 14x^4 - 10x^3 + 16x - 18 + 10x^5 + 12x^2 - 50x - 13$

$= x^5 + 10x^5 + 6x^4 + 14x^4 - 15x^3 - 10x^3 + 12x^2 + 34x + 16x - 50x - 13 - 18$

$= 11x^5 + 20x^4 - 25x^3 + 12x^2 - 31$

$$\begin{array}{l} \quad x^5 + 6x^4 - 15x^3 \qquad\quad + 34x \\ \qquad\qquad 14x^4 - 10x^3 \qquad + 16x - 18 \\ 10x^5 \qquad\qquad\qquad + 12x^2 - 50x \qquad - 13 \\ \hline 11x^5 + 20x^4 - 25x^3 + 12x^2 - 31 \end{array}$$

e) $14y^4 - 19y^3 + 16y^2 - 21,\ 20y^3 + 15y^2 - 11y - 15,\ 18y^3 - 31y^2 + 11y + 36$

First add horizontally:

$= 14y^4 - 19y^3 + 16y^2 - 21 + 20y^3 + 15y^2 - 11y - 15 + 18y^3 - 31y^2 + 11y + 36$

$= 14y^4 - 19y^3 + 20y^3 + 18y^3 + 16y^2 + 15y^2 - 31y^2 + 11y - 11y + 36 - 21 - 15$

$= 14y^4 - 19y^3$

$14y^4 - 19y^3 + 16y^2 \quad - 21$
$20y^3 + 15y^2 - 11y - 15$
$18y^3 - 31y^2 + 11y + 36$
$14y^4 - 19y^3$

9. Subtract the first expressions from the second one in each of the following.

a) $60a^4 + 50a^3 - 40a^2 - 30a$, $50a^3 - 35a^2 + 30a - 70$

b) $15p^3 - 12p^2q + 18pq^2 + 21q^3$, $2p^3 - 12p^2q + 20q^3 - 6$

c) $70x^3 - 40x^2y^2 + 90y^3$, $40x^3 - 30x^2y^2 + 50y^3$

d) $14x^4 - 8x^3y - 18x^2y^2 + 6xy^3$, $10x^4 - 18x^2y^2 - 4xy^3 - 5y^4$

e) $90x^5 - 60x^4 + 40x^3 - 15$, $-10x^5 - 60x^4 - 30x^2 + 25$

Solution:

a) $60a^4 + 50a^3 - 40a^2 - 30a$, $50a^3 - 35a^2 + 30a - 70$

First subtract horizontally:

$= 50a^3 - 35a^2 + 30a - 70 - (60a^4 + 50a^3 - 40a^2 - 30a)$

$= 50a^3 - 35a^2 + 30a - 70 - 60a^4 - 50a^3 + 40a^2 + 30a$

$= - 60a^4 + 50a^3 - 50a^3 - 35a^2 + 40a^2 + 30a + 30a - 70$

$= - 60a^4 + 5a^2 + 60a - 70$

$50a^3 - 35a^2 + 30a - 70$
$\pm 60a^4 \pm 50a^3 \mp 40a^2 \mp 30a$
$- 60a^4 \quad + 5a^2 + 60a - 70$

b) $15p^3 - 12p^2q + 18pq^2 + 21q^3$, $2p^3 - 12p^2q + 20q^3 - 6$

First subtract horizontally:

$= 2p^3 - 12p^2q + 20q^3 - 6 - (15p^3 - 12p^2q + 18pq^2 + 21q^3)$

$= 2p^3 - 12p^2q + 20q^3 - 6 - 15p^3 + 12p^2q - 18pq^2 - 21q^3$

$= 2p^3 - 15p^3 - 12p^2q + 12p^2q + 20q^3 - 21q^3 - 18pq^2 - 6$

$= - 13p^3 + q^3 - 18pq^2 - 6$

$$\begin{array}{|l|}
\hline
\quad 2p^3 + 20q^3 - 12p^2q \qquad\qquad - 6 \\
\pm 15p^3 \pm 21q^3 \mp 12p^2q \pm 18pq^2 \\
\hline
-13p^3 + q^3 \qquad\qquad - 18pq^2 - 6 \\
\hline
\end{array}$$

c) $70x^3 - 40x^2y^2 + 90y^3$, $40x^3 - 30x^2y^2 + 50y^3$

First subtract horizontally:

$= 40x^3 - 30x^2y^2 + 50y^3 - (70x^3 - 40x^2y^2 + 90y^3)$

$= 40x^3 - 30x^2y^2 + 50y^3 - 70x^3 + 40x^2y^2 - 90y^3$

$= 40x^3 - 70x^3 - 30x^2y^2 + 40x^2y^2 + 50y^3 - 90y^3$

$= -30x^3 + 10x^2y^2 - 40y^3$

$$\begin{array}{|l|}
\hline
\quad 40x^3 - 30x^2y^2 + 50y^3 \\
\pm 70x^3 \mp 40x^2y^2 \pm 90y^3 \\
\hline
-30x^3 + 10x^2y^2 - 40y^3 \\
\hline
\end{array}$$

d) $14x^4 - 8x^3y - 18x^2y^2 + 6xy^3$, $10x^4 - 18x^2y^2 - 4xy^3 - 5y^4$

First subtract horizontally

$= 10x^4 - 18x^2y^2 - 4xy^3 - 5y^4 - (14x^4 - 8x^3y - 18x^2y^2 + 6xy^3)$

$= 10x^4 - 18x^2y^2 - 4xy^3 - 5y^4 - 14x^4 + 8x^3y + 18x^2y^2 - 6xy^3$

$= 10x^4 - 14x^4 - 18x^2y^2 + 18x^2y^2 - 4xy^3 - 6xy^3 + 8x^3y - 5y^4$

$= -4x^4 - 10xy^3 + 8x^3y - 5y^4$

Let us subtract vertically:

$$\begin{array}{|l|}
\hline
\quad 10x^4 - 18x^2y^2 - 4xy^3 \qquad - 5y^4 \\
\pm 14x^4 \mp 18x^2y^2 \pm 6xy^3 \mp 8x^3y \\
\hline
-4x^4 \qquad\qquad - 10xy^3 + 8x^3y - 5y^4 \\
\hline
\end{array}$$

e) $90x^5 - 60x^4 + 40x^3 - 15$, $-10x^5 - 60x^4 - 30x^2 + 25$

First subtract horizontally

$= -10x^5 - 60x^4 - 30x^2 + 25 - (90x^5 - 60x^4 + 40x^3 - 15)$

$= -10x^5 - 60x^4 - 30x^2 + 25 - 90x^5 + 60x^4 - 40x^3 + 15$

$= -10x^5 - 90x^5 - 60x^4 + 60x^4 - 40x^3 - 30x^2 + 25 + 15$

$= -100x^5 - 40x^3 - 30x^2 + 40$

Now subtract vertically:

$$\begin{array}{rrrrr} -10x^5 & -60x^4 & & -30x^2 & +25 \\ \pm 90x^5 & \mp 60x^4 & \pm 40x^3 & & \mp 15 \\ \hline -100x^5 & & -40x^3 & -30x^2 & +40 \end{array}$$

10. Simplify the following expressions.

a) 3[6x – 2/5 {5x − 10(2x + 7)}]

b) 40[2a − 5{16a − (7a − 25 + 5a)}]

c) −8q [1/2 (4m − 6n) – 2/3 {15m − 3(2m − 6n)}]

d) 13[12a − 3 {6b − (12a − 5 + 6b)}

Solution:

a) First, we simplify the expression inside the curly brackets:

5x - 10(2x + 7) = 5x - 20x - 70 = -15x - 70

Next, we simplify the expression inside the square brackets using the result above:

6x – 2/5(–15x – 70) = 6x + (–2/5)(–15x) + (–2/5)(-70)= 6x + 6x + 28 = 12x + 28

Finally, we multiply the result by 3:

3(12x + 28) = 36x + 84

Therefore, 3[6x – 2/5{5x – 10(2x + 7)}] simplifies to 36x + 84

b) First, we need to simplify the expression inside the curly brackets:

7a - 25 + 5a = 12a - 25

So, now we can substitute that expression back into the original expression:

40[2a – 5{16a – (7a - 25 + 5a)}] = 40[2a – 5(16a – 12a + 25)]

= 40[2a – 5(4a + 25)]

= 40[2a – 20a - 125]

$= 40[-18a - 125]$

$= -720a - 5000$

Therefore, the simplified expression is $-720a - 5000$

c) Let's simplify the expression step by step:

i. Start by simplifying the terms inside the brackets:

$1/2(4m - 6n) = 2m - 3n$

$15m - 3(2m - 6n) = 15m - 6m + 18n = 9m + 18n$

So the expression inside the brackets simplifies to:

$2m - 3n - 2/3(9m + 18n) = 2m - 3n - 6m - 12n = -4m - 15n$

ii. Now substitute this simplified expression back into the original expression:

$-8q\ [-4m - 15n] = 32qm + 120qn$

Therefore, the simplified expression is $32qm + 120qn$.

d) Let's simplify the given expression step by step:

First, we need to simplify the expression inside the round brackets: $6b - (12a - 5 + 6b) = 6b - 12a + 5 - 6b = -12a + 5$

Substituting this value in the original expression, we get: $13[12a - 3(-12a + 5)] = 13[12a + 36a - 15] = 13[48a - 15] = 624a - 195$

Therefore, the simplified expression is $624a - 195$.

Learning Check 6.1

1. **Identify and separate out the algebraic equations.**

a).$4x^2+5=1$

Solution:

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

b):$2+x=4$

Solution:

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

c): $5a^3+4b+c$

Solution:

It is an algebraic expression that contains three terms, each separated by an addition symbol (+.).It does not have an equal (=) sign. And there is only one side of the equation.

d): $9y^3+z^2-2=7$

Solution:

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

e): $5a+b$

Solution:

The expression $5a+b$ is an algebraic expression because it does not have an equal (=) sign. And there is only one side of the equation.

f): $20x+6=20$

Solution:

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

g): $6b^4+2b^2-5$

Solution:

The expression $6b^4+2b^2-5$ is an algebraic expression because it does not have an equal (=) sign. And there is only one side of the equation.

h):$3t^2+5=-1$

Solution:

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

i): $24+x^2=15$

Solution:

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

2. **Identify and separate out the Linear equations from the given Algebraic equations.**

a): $2x+1=-1$

Solution:

It is the linear equation with variable x has 1 as an exponent, $2x+1= -5$ is a linear equation.

b): $4x^2-5=2$

Solution:

Since the variable x has 2 as an exponent, so the equation is not linear.

c): $7y^3 + 9= -20$

Solution:

Since the variable y has 3 as an exponent, so the equation is not linear.

d): $7c^2-5a^3=10$

Solution:

Since the variable c and a have 2 and 3 as an exponent, so the equation is not linear.

e): $10+5x= -3$

Solution:

Since the variable x has 1 as an exponent, so the equation is a linear equation.

f):$2t+5=9$

Solution:

Since the variable t has 1 as an exponent, so the equation is a linear equation.

g): $t+3=1$

Solution:

Since the variable t has 1 as an exponent, so the equation is a linear equation.

h): $5x^2+2x-1= 9$

Solution:

Since the variable x has 2 as an exponent, so the equation is not linear.

i): $5z^3+9z^2= -11$

Solution:

Since the variable z have 3 and 2 as an exponent, so the equation is not linear.

3. **Define a linear equation in one variable and give five examples of a linear equation in one variable.**

Linear equation in one variable:

The linear equation that contains only one variable is called a linear equation in one variable.

Examples

$2x-4=8$, $3y-1=0$, $9+2x=8$, $x+3=1$, $10+5x= -3$

Each of these equations has only one variable, so these are linear equations in one variable.

Learning Check 6.2

1. **Write the algebraic expressions for the following phrases.**

a) 12 added to half of a number

b) A number is subtracted from 17

c) Four times a number subtracted from 9

d) Five times a number increases by 6

e) Sum of a number and 4 is divided by 5

f) Twice the number decreases by 10

g) Four times a number subtracted from 4

a) 12 added to half of a number

Solution:

$12+x/2$

b) A number is subtracted from 17

Solution:

$17-x$

c) Four times a number subtracted from 9

Solution:

$9-4x$

d) Five times a number increases by 6

Solution:

5x+6

e) Sum of a number and 4 is divided by 5

Solution:

(x+4)/5

f) Twice the number decreases by 10

Solution:

2x-1

g) Four times a number subtracted from 4

Solution:

4-4x

2. **Write the linear equations for the following.**

a) The sum of a number and 7 is 15.

b) The sum is 25 when 10 is added to twice a number.

c) Five times the sum of a number and 4 is 25.

d) The sum of 1.1 and half of a number is equal to 2.3

e) Four times a number decreased by one-fourth equals 5.

f) Thrice a number divided by 5 added to 2.4 is 6.5.

g) The result of subtraction of twice a number from 30 is 24.

h) Four times the result of the subtraction of 3 from a number is 4 times the subtraction of 1 from thrice the number.

i) Six times a number plus 2 equals −19.

j) Twice a number divided by 4 is equal to one-fourth of the number.

Solution:

a) The sum of a number and 7 is 15.

Solution:

x+7=15

b) The sum is 25 when 10 is added to twice a number.

Solution:

10+2x=25

c) Five times the sum of a number and 4 is 25.

Solution:

5x+4=25

d) The sum of 1.1 and half of a number is equal to 2.3

Solution:

1.1+x/2=2.3

e) Four times a number decreased by one-fourth equals 5.

Solution:

4x=5

f) Thrice a number divided by 5 added to 2.4 is 6.5.

Solution:

3x/5+2.4=6.5

g) The result of subtraction of twice a number from 30 is 24.

Solution:

13-2x=24

h) Four times the result of the subtraction of 3 from a number is 4 times the subtraction of 1 from thrice the number.

Solution:

4(x-3) =4(3x-1)

i) Six times a number plus 2 equals −19.

Solution:

6x+2= -19

j) Twice a number divided by 4 is equal to one-fourth of the number.

Solution:

x/2=x/4

3. Write the given algebraic equations as statements.

a) $12x + 6 = 14$	b) $5a - 2 = 3$	c) $7m + 5 = -11$
b) $\frac{2}{3}x + 7 = 15$	e) $7y + 1 = 12$	f) $2(5x + \frac{7}{2}) = -1$
g) $3r + 5 = -13$	h) $4t + 0.4 = 6.2$	I) $\frac{3}{5}t + 7 = 15$
j) $2x + 7 = \frac{1}{4}$	k) $1.3x + 5 = 2$	l) $5r + \frac{3}{4} = \frac{2}{3}$

Solution:

a) $12x + 6 = 14$

Twelve times a number added to six is equal to fourteen.

b) $5a - 2 = 3$

Five times a number subtracted by two equals to three.

c) $7m + 5 = -11$

Seven times a number added to five equals to minus eleven.

d) $\frac{2}{3}x + 7 = 15$

Two third of a number added to seven equals to fifteen.

e) 7y + 1 = 12

Seven times a number added to one equal to twelve.

f) $2(5x + \frac{7}{2}) = -1$

Five times a number added to 7/2 then multiply the entire expression by 2 equals to minus

g) $3r + 5 = -13$

Three times a number added to five equals to minus thirteen.

h) 4t + 0.4 = 6.2

Four times a number added to 0.4 equal to 6.2.

i) $\frac{3}{5}t + 7 = 15$

3/5 times a number added to seven equals to fifteen.

j) $2x + 7 = \frac{1}{4}$

Two times a number added to a seven equal to one fourth.

k) $1.3x + 5 = 2$

1.3 times a number added to a five equal to two.

l) $5r + \frac{3}{4} = \frac{2}{3}$

Five times a number added to ¾ equal to 2/3.

4. Solve the following linear equations.

a) 3/5x=12

Solution:

To solve this eq. we first isolate variable x

By multiplying 5 on both sides of the eq.

We get,

3x=60

Dividing 3 on both sides

3/3x=60/3

x=20

b) 1+4x=4/5

Solution:

To solve this eq. we first isolate variable x

4x = 4/5 - 1

$4x = (4-5)/5$

Dividing 4 on both sides

$4x = -1/5$

$x = -1/20$

c) 2/7(14m-12)=20

Solution:

Multiplying both sides by 7/2 in order to isolate the variable m,

$7/2 \times 2/7(14m - 12) = 20 \times 7/2$

By simplifying we get,

$14m - 12 = 70$

Adding 12 on both sides of the eq.

$14m - 12 + 12 = 70 + 12$

We get,

$14m = 82$

Dividing both sides by 14

$m = 82/14$

$m = 41/7$

d) 2b + 3 = 4

Solution:

By isolating 'b' we have eq,

$2b = 4 – 3$

$2b = 1$

Dividing 2 on both sides

$b = 1/2$

e) 6+5c= -11

Solution:

Isolating the variable 'c'

$5c = -11 - 6$

We get,

$5c = - 17$

Dividing 5 on both sides

$c = -17/5$

f) 4(1/3p+7/4) = 1/4

Solution:

Multiplying 4 we get

4/3p+7 =1/4

By isolating variable ‘p’

4/3p= 1/4 – 7

4/3p = (1 – 28)/4

4/3p = -27/4

Multiplying both sides by ¾,

p = -27/4 X 3/4

p = - 81/16

g) 4/5u + 1/2 = 6

Solution:

By Isolating ‘u’

4/5u = 6 – ½

4/5u = (12 – 1)/6

4/5u = 11/6

Multiplying both sides by $\frac{5}{4}$, we get

U= 11/6 x 5/4

U = 55/8

h) -3a+7=9

Solution:

By simplifying we get,

-3a = 9 – 7

- 3a = 2

Dividing by - 3 on both sides

a = - 2/3

Solution:

i) 6x-4-2(4+x)=3x+7

Solution:

By solving this eq,

6x – 4 – 8 – 2x = 3x+ 7

6x -12 -2x =3x+7

4x – 12 =3x +7

Taking variables on one side

4x – 3x = 7+12

x = 19

j) 4(0.4x+0.2) =1.2x+9

Solution:

1.6x + 0.8 = 1.2x + 9

1.6x -1.2x = 9 – 0.8

0.4x = 8.2

Dividing 0.4 on both sides

x= 8.2/0.4

x = 20.5

l): 3(x+1) + 7(x- 1)=2(2-x)+9(4+x)

Solution:

3x +3 +7x – 7 =4 – 2x +36+9x

10x – 4 = 7x + 40

10x=7x +40+4

10x -7x = 44

Dividing 3 on both sides

3x = 44

x = 44/3

5. **The price of a geometry box is twice the price of a notebook. The price of 3 geometry boxes and 5 notebooks is Rs 1375. Find the price of one geometry box.**

Solution:

Let the price of one notebook = x

Then the price of one geometry box = 2x

Price of three geometry boxes = 3(2x) = 6x

The price of 5 notebooks = 5x

So, the total cost of 3 geometry boxes and 5 notebooks = 6x + 5x = 11x

Total cost = 1375 Rs

We can write the eq,

11x = 1375

Dividing 11 on both sides we get,

x = 125

So the price of 1 notebook is Rs.125

And the price of 1 geometry box = 2x = 2(125) = Rs 250

Therefore,

The price of 1 geometry box = Rs 250

6. **Find the number if thrice the number subtracted from 50 is 5.**
   Solution:
   Let thrice the number = x
   According to the statement,
   50 – 3x = 5
   Isolate 'x'
   50 – 5 = 3x
   45 = 3x
   Dividing 3 on both sides
   x= 45/3
   x = 15
   So the number we are looking is 15
7. **Ahad's age is one-fourth of his father's age. If the sum of their ages after 5 years is 55, then find their present ages.**
   Solution:
   Suppose Ahad age = x
   So, Ahad's father age = 4x
   According to the statement
   After 5 years, Ahad's age would be x + 5 and his father's age would be 4x + 5
   Therefore,
   (x + 5) + (4x + 5) = 55
   5x + 10 = 55
   Subtracting 10 from both sides we get,
   5x = 45
   Dividing by 5 on both sides
   x = 9
   so Ahad's current age is 9 years and his father age is 4x = 36 years
8. **The cost of petrol has risen by Rs 20. Find the old price of petrol if the increased price is Rs 215.**
   Solution:
   Let the price of the petrol = x
   Price of the petrol rises by =x + 20
   Increased price = Rs 215
   Now set up the equation
   New price = Rs 215

(x+20) =215

Subtracting 20 on both sides

X + 20 – 20 = 215 – 20

x = 195

Therefore, the old price of petrol was Rs 195.

9. **The total length of a red and a green ribbon is 96 m. If the length of the green ribbon is three times the length of the red ribbon, find the length of both ribbons**.

Solution:

Let length of red ribbon = x

Length of green ribbon =3x

total length of a red and a green ribbon is 96 m.

so, write the equation

length of red ribbon + length of green ribbon = total length

x + 3x = 96

by solving for x

4x = 96

Dividing 4 on both sides

x = 96/4

x = 24

Therefore, the length of red ribbon is 24 m while length of green ribbon is 3(24) = 72 m.

10. **The sum of the marks of Sara and Amara is 54. If Sara's marks are 12 less than the marks obtained by Amara, then find the marks of Amara and Sara.**

Solution:

Let's assume Sara's marks = x – 12

Sum of Sara and Amara marks = 54

Set up the equation

x+ (x - 12) = 54

2x – 12 = 54

Adding 12 on both sides we get,

2x = 54 + 12

2x = 66

Dividing by 2 on both sides we get,

x = 66/2

$x = 33$

Therefore, Amara's marks are 33 and Sara marks are $(33 - 12) = 21$

11. Irum thought of a number. When she multiplied that number by 2, she got 4/5. Find the number.

Solution:

Let's assume the number the Irum thought = x

When multiplied that number with 2x she got = 4/5

By simplifying for x we get,

$2x = 4/5$

Dividing 2 on both sides

$x = (4/5)/2$

$x = 4/10$

$x = 2/5$

so, the number thought by Irum is 2/5

Unit Check 6

1. Encircle the correct answer.

a) Which of the following is true for all the values of the variables?

i) **expression** ii) pattern iii) coefficient iv) equation

b) An algebraic equation is a mathematical statement that has two equals:

i**) expressions** ii) equations iii) numbers iv) variables

c) The linear equation is:

i) $2x^3 +1 = 5$ ii) $2x^2 + 7= 1$ iii) $\mathbf{1 + 3x = 9}$ iv) $3x^2 + 9 = 5$

d) The equation of the statement "twice of a number subtracted from the sum of the number and 7 is 12" is:

i) $2x - (x - 7) = 12$ ii) $2x - 7 = 12 + x$

iii) $12 - 2x = x +7$ iv) $\mathbf{(x +7) - 2x = 12}$

e) The solution of the equation $23(3x +6) = 21$ is:

i) 17 ii) **17** iii) 21 iv) 17

2. Fill in the blanks.

a) A **linear** equation has degree equal to 1.

b) The equal sign between two expressions means that both sides of the equation are **equal.**

c) Expressions are **true** for all values of the variables.

d) The equation for "a number decreased by 6" is **x – 6**.

e) The linear equation in one variable has only one **degree**.

3. **Identify and separate out the Algebraic equations.**

a) $6y^4 + 2 = 4$ b) $7u + 5$ c) $9b^2 + 1 = 5$

d) $10t^3 + 4t + 2$ e) $y^3 + 2y^2 + 1 = -2$ f) $7x + 1$

Solution:

a) $6y^4 + 2 = 4$

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

b) $7u + 5$

No, it is not an algebraic equation. The expression $7u+5$ is an algebraic expression because it does not have an equal (=) sign. And there is only one side of the equation.

c) $9b^2 + 1 = 5$

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

d) $10t^3 + 4t + 2$:

No, it is not an equation. The expression $10t^3 +4t +2$ is an algebraic expression because it does not have an equal (=) sign. And there is only one side of the equation.

e) $y^3 + 2y^2 + 1 = -2$

It is an algebraic equation in which two expressions are set equal to each other. It usually consists of a variable, coefficients and constants. And there are left and right sides of an equation.

f) $7x + 1$

The expression $7x + 1$ is an algebraic expression because it does not have an equal (=) sign. And there is only one side of the equation.

4. **Identify and separate out the Linear equations.**

Solution:

a) $4x + 4$

It is the linear equation with variable ‘x’ has 1 as an exponent, $4x + 4$ is a linear equation.

b) $7a^2 + b =19$

Since the variable ‘a’ has 2 as an exponent, so the equation is not linear.

c)- 2c + 4 = -1

It is the linear equation with variable 'c' has 1 as an exponent, - 2c + 4 = -1, is a linear equation.

d)b^2 +- 2c^3 = 5

Since the variable 'b' has 2 as an exponent, so the equation is not linear.

e)5 – 5t = -6

It is the linear equation with variable 't' has 1 as an exponent, 5 – 5t = -6, is a linear equation.

f)2a^3 + 5b +c =0

Since the variable 'a' has 3 as an exponent, so the equation is not linear.

5. **Write the Algebraic Expression for the following phrases.**

   Solution:

   a) One quarter of a number added to 14.
      14 + x/4
   b) Three times a number decreased by one - half
      3x – 1/2
   c) A number is subtracted from 7.
      7 – x
   d) 15 is added to one fifth of a number.
      15 + 1/5x
   e) 6 is subtracted from three times a number
      6 – 3 x

6. **Write the Linear equations for the following.**

   Solution:

   a) Twice a number plus 5 is 12.
      2x + 5=12
   b) Five subtracted from thrice a number is equal to four times that number.
      5 – 3x = 4x
   c) Three times a number is equal to one half.
      3x = ½
   d) One third of a number equals 16.
      1/3x = 16
   e) One – Seventh of a number subtracted from 9 is 12.
      9 – 1/7x = 12

7. **Solve the following Linear equations.**

a) $\mathbf{4x + 5 = 20}$

Solution:

$4x = 20 - 5$

$4x = 15$

Dividing both sides by 4 we get,

$x = 15/4$

b) $\mathbf{9 - 3(2 + x) = 1 + 4x}$

Solution:

$9 - 6 - 3x = 1 + 4x$

$3 - 3x = 1 + 4x$

$4x + 3x = 3 - 1$

$7x = 2$

Dividing both sides by 7 we get,

$x = 2/7$

c) $\mathbf{2x + 1 = 7x + 4}$

Solution:

$2x - 7x = 4 - 1$

$-5x = 3$

Dividing both sides by 5 we get,

$x = -3/5$

d) $\mathbf{2 - \frac{3}{4}(4x + 20) = 15}$

Solution:

$2 - 12/4x + 60/4 = 15$

$2 + 60/4 - 15 = 12/4x$

$60/4 - 13 = 12/4x$

$(60 - 52)/4 = 12/4x$

$8/4 = 12/4x$

$2 = 3x$

Dividing both sides by 3 we get,

$x = 2/3$

e) $\mathbf{8 + x = 2x + 4}$

Solution:

$8 - 4 = 2x - x$

$4 = x$

f) 29 + 1 = 1/3(3a + 6)

Solution:

30 = a + 2

a = 30 – 2

By subtracting 2 from both sides we get,

a = 28

g) 2(x + 5) + 7(x - 1) + 15 =21

Solution:

2x 10 + 7x – 7 + 15= 2

9x+ 3 + 15 = 21

9x + 18 = 21

9x= 21 - 18

x = 3/9

By simplifying

x = 1/3

h) - 4(x + 5) = - 2(7x + 4)

Solution:

-4x – 20 = -14x – 8

- 4x + 14x = - 8 + 20

10x = 12

Dividing both sides by 10 we get,

x = 12/10

x = 6/5

i) 3/4b + 1/4b = 9

Solution:

(3b+1b)/4 = 9

4b/4 = 9

b = 9

8. When a number is subtracted from 10 and the result is added to twice that number, the answer is 24. Find the number.

Solution:

Let's suppose number = x

When we subtract 10 from this number = 10 – x

Added twice the number to this result = (10 – x) + 2x

This expression equals to 24 so,

$(10 - x) + 2x = 24$

Simplifying this equation we get

$10 + x = 24$

Subtracting both sides by 10 we get,

$x = 24 – 10$

$x = 14$

So, the required number is 14.

9. **Tahir and Usman weigh 84 kg together. Tahir weighs three times as much as Usman. Find Usman's and Tahir's weights.**

   Solution:

   Sum of Tahir and Usman weight = 84 kg

   We write Usman weight $=x$

   Tahir weight $= 3x$

   Therefore, according to the statement,

   $x + 3x = 84$

   $4x = 84$

   Dividing by 4 on both sides we get,

   $x = 84/4$

   $x = 21$ kgs

   So, Tahir weight is $3x = 3 \times 21 = 63$ Kgs and Usman weight is 21 Kgs

10. **The length of two ropes is 36 m. The length of one rope is 3 times the length of the other rope. Find the length of each piece of rope.**

    Solution:

    The length of shorter piece of rope = x

    Other piece of rope $=3x$

    Given condition $= x+ 3x= 36$

    $4x = 36$

    $x = 9$

    Therefore, the length of shorter rope = 9m and the length of longer rope is 3x is $3 \times 9 = 27$m.

11. **The length of the rectangular hall is three times its width. If the perimeter of the hall is 26 m, find the length and width of the hall.**

    Solution:

    Length of Rectangular hall, $L =3x$

    Width of Rectangular hall, $W = x$

Perimeter of hall = 26m

2(L + W) = Perimeter

By putting values in this equation

2(3x +x) = 26

6x +2x =26

8x = 26

Dividing both sides by 8 we get,

x = 26/8 = 3.25m

Length = 3 x =3 X 3.25 = 9.75m

Width = 3.25m

12. Harris' age and the age of his younger brother together equal 18. Calculate Harris' and his brother's ages if Harris is twice as old as his brother.

Solution:

Suppose Brother Age =x

Harris age =2x

Given condition:

2x + x = 18

Put Harris age in given condition we get,

3x = 18

Dividing both sides by 3 we get,

x = 18/3

x = 6 years

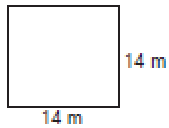
Now we know that Harris brother age is 6 years old. we can get the Harris age which is 2x = 2 x 6 = 12 years

Learning check 7.1

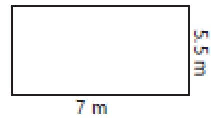
1. Find the perimeter of the following figures.

a)

b)



c)



Solution:

a) Length of rectangle = 3.8 cm

Width of rectangle = 12 cm

Perimeter of rectangle = 2(L+W)

= 2(3.8+1.2)

= 2 x 5= 10 m

b) Length of square = 14 cm

Perimeter of square = L+L+L+L

= 14+14+14+14

= 56 m

c) Length of rectangle = 7 m

Width of rectangle = 5.5 m

Perimeter of Rectangle = 2(7+ 5.5)

= 2(12.5) = 25m

2. Find the Area and Perimeter of a square of the given lengths.

a) 4cm

Area = L x L

= 4 x 4 = 16 cm^2

Perimeter = L + L + L+ L

= 4 + 4 + 4 + 4 = 16 cm

b) 21 cm

Area = L x L

= 21 x 21 = 441 cm^2

Perimeter = L + L + L+ L

= 21 + 21 +21 + 21= 84 cm

c) 6.3 m

Area = L x L

= 6.3 x 6.3 = 39.69 m^2

Perimeter = L + L + L+ L

= 6.3 + 6.3+ 6.3+ 6.3

= 25.2 m

d) 11 m

Area = L x L

= 11 x 11

= 121 m^2

Perimeter = L + L + L+ L

= 11 +11 + 11 + 11

= 44 m

e) 5.5 cm

Area = L x L

= 5.5 x 5.5

= 30.25 cm^2

Perimeter = L + L + L+ L

=5.5 + 5.5 + 5.5 +5.5

= 22 cm

3. Find the Area and Perimeter of a rectangle of the given lengths.

a) L=4.5 m W= 3.7 m

Solution:

Area = L x W

Area = 4.5 x 3.7

= 16.65 m^2

Perimeter= 2(L+W)

= 2(4.5+3.7)

=2 x 8.2

= 16.4 m

b) L = 11 cm, W =8.2 cm

Solution:

Area = L x W

=11 x 8.2

=90.2 cm^2

Perimeter= 2(L+ W)

= 2(11 + 8.2)

=2 x 19.2

=38.4 cm

c) L = 17 cm, W =12 cm

Solution:

Area = L x W

= 17 x 12 = 204cm^2

Perimeter= 2(L+W)
= 2(17 + 12)
= 2x 29
= 58 cm

d) L = 7.8m W = 5m

Solution:

Area = Lx W
= 7.8 x 5
= 39 m^2

Perimeter= 2(L+W)
= 2(7.8 + 5)
=2 x 12.8
= 25.6 m

4. **Find the area of the surface of a square-shaped table if its length is 107 cm.**

Solution:

Length of a square-shaped table = 107 m
Area of a square-shaped table = ?
We know that
Area of a square-shaped table = $(\text{length})^2$
= $(107)^2$
= 11,449 m^2

Therefore, the area of the surface of the square-shaped table is 11,449 cm^2.

5. **Find the area of a square shaped table if its length is 107 cm**.

Solution:

Length of square shaped table = 107 cm
Area =?

As we know
Area of a square table = L x L
= 107 x 107
= 11,449 m^2

6. **Find the length of a square if its perimeter is 65 m.**

Solution:

Length of a square (L) =?
Perimeter of a square (P) = 65 m
As we know,

Perimeter of a square= 4 × L

P = 4L
65 = 4L
Dividing 4 on both sides
L = 65/4
L = 16.25 m

7. Find the area of a square shaped wall whose length is 15 m. Also find the cost of cementing wall at the rate of rupees 290 per square metre.

Solution:

Length of a wall = 15 m
Rate of cementing the wall = 290/m^2
Area of a square shaped wall =?
Cost of cementing the wall =?
Area = $(\text{length})^2$
= $(15)^2$
= 225 m^2
Cost of cementing the wall = area x cost/ square meter
= 225 x 290
= Rs 65,250

8. The length of a square garden is 52 m. Find the cost of fencing the square garden at the rate of Rs.40 per meter.

Solution:

Length of square garden = 52 m
Cost of fencing per metre = Rs.40
Perimeter = 4 × L
= 4 x 52
=208 m
Cost= Perimeter x Cost/ meter
= 208 x 40
= Rs 8,320

9. Find the area of a rectangular frame if its width is 8 cm and its length is 25 cm.

Solution:

Length of frame = 25 cm
Width of frame = 8 cm
Area of a rectangular frame =?
Area of a rectangular frame = L x W
= 25 x 8
= 200 cm^2

10. The perimeter of a rectangular floor of the hall in a masjid is 210 m. If the hall's width is 40 m, calculate its length.

Solution:

Perimeter of rectangular floor = 210m
Hall Width = 40m
Hall Length =?
As we know that,
P = 2(L x W)
210m =2 (L + 40)
Dividing 2 on both sides
L + 40 = 210/2
L+ 40 = 105m
Subtracting 40 on both sides
L + 40 – 40 = 105 – 40
L = 65m
So, the length of the rectangular floor of the hall in Masjid is 65m.

11. Find the length of the rectangular floor of a room if its area is 375 m^2 and its width is 15 m. Also find the cost of flooring the room at the rate of Rs 180 per m^2.

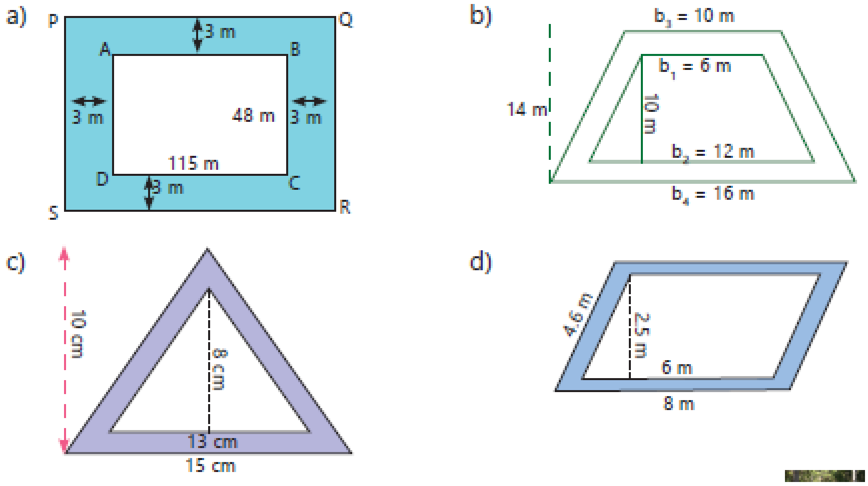
Solution:

Area of the rectangular floor of a room = $375m^2$
Width of the rectangular floor of a room =15m
Length of the rectangular floor of a room =?

Area of a rectangular floor =L x W
375 =L x 15
Dividing 15 on both sides
L =375/15
L = 25m
As we know,
Cost of flooring the room = area x cost/meter
= 375 x 180
= Rs 67,500

Learning Check 7.2

1. Find the area of the path for the following figures.



a) Solution:

Area of the rectangular path = L × W

Area of the inner rectangular path = 115 × 48

= 5520m^2

Area of the outer rectangular path =?

Length of the outer shape = length of the inner shape + 2 × (width of the path or border)

Width of the outer shape = width of the inner shape + 2 × (width of the path or border)

Length of the outer shape = 115 + 2 x 3 = 121

Width of the outer shape = 48 + 2 x 3 = 54

Area of the outer rectangular path = L x W

= 121 x 54 = 6534

Area of a path around a shape = Area of the outer shape – Area of the inner shape

Area of a path around a shape = 6534 – 5520 = 1014m^2

b) Solution:

First, we find the area of the inner trapezium.

Area of the inner trapezium = altitude × $(b_1 + b_2)/2$

= 10cm × (6 cm +12 cm)/2

= 10 cm × (18 cm)/2

= 5 cm × 18 cm = 90 cm^2

Area of the outer trapezium = altitude × $(b_3 + b_4)/2$

= 14cm × (10cm + 16 cm)/2

$= 14 \text{ cm} \times (26 \text{ cm})/2$
$= 14 \text{ cm} \times 13 \text{ cm}$
$= 182 \text{ cm}^2$

Area of a path around a trapezium = Area of the outer trapezium − Area of the inner
trapezium

Area of the path around the trapezium = $182 \text{ cm}^2 - 90 \text{ cm}^2 = 92 \text{ cm}^2$

c) Solution:

Base of outer triangle = 15 cm
Base of inner triangle = 13 cm
Altitude of outer triangle =10 cm
Altitude of inner triangle = 8 cm
Area of outer triangle = ½ x (altitude × base)
$= 1/2 (10\text{cm} \times 15\text{cm}) = 75 \text{ cm}^2$
Area of inner triangle = 1/2 (altitude × base)
$= 1/2 (8\text{cm} \times 13 \text{ cm})$
$= 52 \text{ cm}^2$
Area of the path around the triangle = $75 \text{ cm}^2 – 52 \text{ cm}^2 = 23 \text{ cm}^2$

d) Solution:

Area of the inner parallelogram = base × altitude
$= 6 \text{ m} \times 2.5 \text{ m}$
$= 15 \text{ m}^2$
Area of the outer parallelogram = base × altitude
$= 8 \text{ m} \times 4.6 \text{ m}$
$= 36.8 \text{ m}^2$
Area of a path around a parallelogram = Area of the outer parallelogram − Area of the inner parallelogram
Area of the path around the parallelogram = $36.8 \text{ m}^2 – 15 \text{ m}^2 = 21.8 \text{ m}^2$

2. Find the area of a triangle-shaped front wall of a house if the length of its base is 18.2 m and the length of its altitude is 16.3 m.

Solution:

Length of the base = 18.2m
Length of its Altitude = 16.3m
As we know that
Area = (1/2) x Base x Altitude
= (1/2) x 18.2 x 16.3
=1/2 x 296.66 = 148.33m^2

3. **The area of a triangle-shaped flower bed is 7 m2 and its altitude is 4 m. find the length of the base of the flower bed.**

   Solution:

   Area of triangle shaped flower bed = $7m^2$

   Altitude = 4m

   Length or base = ?

   As we know that

   Area = (1/2) x Base x Altitude

   Base = 2(Area)/ Altitude

   By putting values

   Base = (2 x7)/4

   Base = 3.5m

   Therefore, the length of the base of the flower bed is 3.5m

4. **Find the length of the altitude of a trapezium-shaped window if the lengths of two parallel sides are 1.2 m and 3.8 m, respectively. The area of the trapezium is 7.5 m^2.**

   Solution:

   Area of the trapezium = $7.5m^2$

   Length of parallel side, b1 = 1.2m

   Length of parallel side, b2 = 3.8m

   Altitude of Trapezium =?

   As we know that

   Area = Altitude x (b1+b2)/2

   7.5 = Altitude x (1.2 + 3.8)/2

   7.5 = Altitude x 5/2

   7.5 = Altitude x 2.5

   Altitude = 7.5/2.5

   Altitude = 3 m

5. **Find the area of a trapezium if the lengths of two parallel sides are 45 cm and 30 cm, respectively. The perpendicular distance between two parallel sides is 18 cm.**

   Solution:

   Area of the trapezium = ?

   Length of parallel side, b1 = 45 cm

   Length of parallel side, b2 = 30 cm

   Perpendicular distance between two sides = 18 cm

   Area = Perpendicular distance x (b1+b2)/2

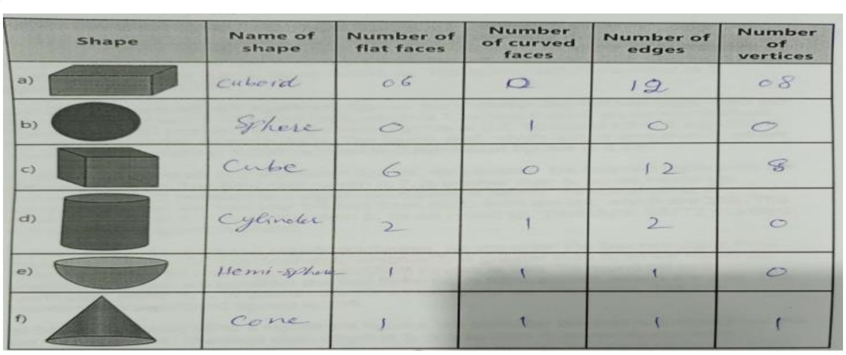
= 18 x (45 + 30)/ 2
= 18 x 75/2
= 675 cm^2

6. **The base of a parallelogram-shaped wooden sheet is 14 cm, and its area is 126 cm^2. Find the altitude of the sheet.**
Solution:
Altitude of the sheet =?
Base of a parallelogram-shaped wooden sheet = 14cm
Area of a parallelogram-shaped wooden sheet = 126cm^2
Area of a parallelogram is
Area = Base x altitude
Re arranging the formula,
Altitude = Area/Base
= 126/14
= 9 cm

7. **The base and the altitude of a parallelogram are 48 m and 35 m, respectively. Find the area of the parallelogram.**
Solution:
The base of a parallelogram = 48m
The altitude of a parallelogram = 35m
Area of the parallelogram =?
Area of a parallelogram is
Area = Base x altitude
= 48 x 35
= 1680m^2
Therefore, the area of the parallelogram is 1680m^2

Learning Check 7.3

Write the name of the following shapes and the number of faces, edges, and vertices for each.

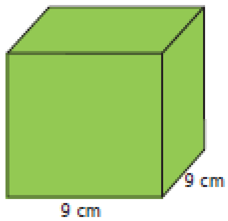


Shape	Name of shape	Number of flat faces	Number of curved faces	Number of edges	Number of vertices
a)	Cuboid	06	0	12	08
b)	Sphere	0	1	0	0
c)	Cube	6	0	12	8
d)	Cylinder	2	1	2	0
e)	Hemi-sphere	1	1	1	0
f)	Cone	1	1	1	1

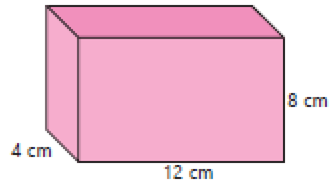
Learning check 7.4

1. Find the volume and surface area of the following cubes and cuboids.

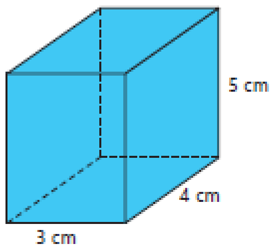
a)



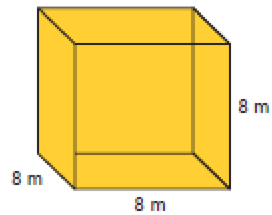
b)



c)



d)



a) Solution:

As we know that

Volume of a cube = l × l × l

By putting given values

Volume of a cube = 9 x 9 x 9 = 729cm^3

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(9)^2$ =6 x 81 = 486cm^2

b) Solution:

As we know that

Volume of a cuboid = l × w × h

Volume of a cuboid = 12 x 8 x 4 = 384cm^3

Total Surface area of a cuboid = 2 × [(l × w) + (l × h) + (w × h)]

Total Surface area of a cuboid = 2 x [(12 x 8) + (12 x 4) + (8 x 4)]

= 2 x [96 + 48 +32]

= 2 x 176

= 352cm^2

c) Solution:

As we know that

Volume of a cuboid = l × w × h

Volume of a cuboid = 4 x 3 x 5

= 60cm^3

Total Surface area of a cuboid = 2 × [(l × w) + (l × h) + (w × h)]

Total Surface area of a cuboid = 2 x [(4 x 3) + (4 x 5) + (3 x 5)]

= 2 x [12 + 20 + 15]

$= 2 \times 47$
$= 94cm^2$

d) Solution:

As we know that
Volume of a cube = l × l × l
By putting given values
Volume of a cube = 8 x 8 x 8 = $512cm^3$
Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$
Surface area of a cube = $6(8)^2 = 6 \times 64 = 384cm^2$

2. 2.Find the area and volume of the cubes of the given length by using the formula.

Solution:

a) 6 cm

As we know that
Volume of a cube = l × l × l
By putting given values
Volume of a cube = 6 x 6 x 6 = $216cm^3$
Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$
Surface area of a cube = $6(6)^2 = 6 \times 36 = 216cm^2$

b) 10 m

As we know that
Volume of a cube = l × l × l
By putting given values
Volume of a cube = 10 x 10 x 10 = $1000m^3$
Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$
Surface area of a cube = $6(10)^2 = 6 \times 100 = 600m^2$

c)15cm

As we know that
Volume of a cube = l × l × l
By putting given values
Volume of a cube = 15 x 15 x 15 = $3375cm^3$
Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$
Surface area of a cube = $6(15)^2 = 6 \times 225 = 1350cm^2$

d) 7.5 cm

As we know that
Volume of a cube = l × l × l
By putting given values
Volume of a cube = 7.5 x 7.5 x 7.5 = $421.875cm^3$

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(7.5)^2 = 6 \times 56.25 = 337.5cm^2$

e) 13.8 m

As we know that

Volume of a cube = $l \times l \times l$

By putting given values

Volume of a cube = $13.8 \times 13.8 \times 13.8 = 2628.072m^3$

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(13.8)^2 = 6 \times 190.44 = 1142.64m^2$

f) 24 m

As we know that

Volume of a cube = $l \times l \times l$

By putting given values

Volume of a cube = $24 \times 24 \times 24 = 13,824m^3$

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(24)^2 = 6 \times 576 = 3,456m^2$

g) 3.4 m

As we know that

Volume of a cube = $l \times l \times l$

By putting given values

Volume of a cube = $3.4 \times 3.4 \times 3.4 = 39.30m^3$

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(3.4)^2 = 6 \times 11.56 = 69.36m^2$

h) 9 cm

As we know that

Volume of a cube = $l \times l \times l$

By putting given values

Volume of a cube = $9 \times 9 \times 9 = 729cm^3$

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(9)^2 = 6 \times 81 = 486cm^2$

i)17 m

As we know that

Volume of a cube = $l \times l \times l$

By putting given values

Volume of a cube = $17 \times 17 \times 17 = 1,749\ m^3$

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(17)^2 = 6 \times 289 = 1734\ m^2$

j) 24 cm

As we know that

Volume of a cube = $l \times l \times l$

By putting given values

Volume of a cube = 24 x 24 x 24 = 13,824cm^3

Surface area of a cube = $l^2 + l^2 + l^2 + l^2 + l^2 + l^2 = 6 \times l^2 = 6l^2$

Surface area of a cube = $6(24)^2$ =6 x 576 = 3,456cm^2

3. Find the area and volume of the cubes of the given length by using the Formula.

a) l = 4.2 cm w = 1.4 cm h = 3.7 cm

Solution:

As we know that

Volume of a cuboid = $l \times w \times h$

Volume of a cuboid = 4.2 x1.4 x 3.7 = 21.76cm^3

Total Surface area of a cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

Total Surface area of a cuboid = 2 x [(4.2 x 1.4) + (4.2 x 3.7) + (1.4 x 3.7)]

= 2 x [5.88 +15.54 + 5.18]

= 11.76+31.08+10.36

= 53.2cm^2

b) L = 8.9 m w = 3.6 m h = 5.3 m

Solution:

As we know that

Volume of a cuboid = $l \times w \times h$

Volume of a cuboid = 8.9 x 3.6 x 5.3 = 169.812m^3

Total Surface area of a cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

Total Surface area of a cuboid = 2 x [(8.9 x 3.6) + (8.9 x 5.3) + (3.6 x 5.3)]

= 2 x [32.04 + 8.953 +19.08]

= 64.08+17.906+38.16

= 196.58m^2

c) L = 15 cm w = 5 cm h = 10 cm

Solution:

As we know that

Volume of a cuboid = $l \times w \times h$

Volume of a cuboid = 15 x 5 x 10 = 750cm^3

Total Surface area of a cuboid = $2 \times [(l \times w) + (l \times h) + (w \times h)]$

Total Surface area of a cuboid = 2 x [(15 x 5) + (15 x10) + (5 x 10)]

= 2 x [75 + 150 +50]

= 150+300+100

= 550cm^2

d) L = 24 m w = 8 m h = 16 m

Solution:

As we know that

Volume of a cuboid = l × w × h

Volume of a cuboid = 24 x 8 x 16 = 3072m^3

Total Surface area of a cuboid = 2 × [(l × w) + (l × h) + (w × h)]

Total Surface area of a cuboid = 2 x [(24 x 8) + (24 x 16) + (8 x 16)]

= 2 x [192 +384 +128]

= 384+768+256

= 1408m^2

e) L = 4.7 m w = 3.6 m h = 4.1 m

Solution:

As we know that

Volume of a cuboid = l × w × h

Volume of a cuboid = 4.7 x 3.6 x 4.1 = 69.372m^3

Total Surface area of a cuboid = 2 × [(l × w) + (l × h) + (w × h)]

Total Surface area of a cuboid = 2 x [(4.7 x 3.6) + (4.7 x 4.1) + (3.6 x 4.1)]

= 2 x [16.92 + 19.27 +14.76]

= 33.84+38.54+29.52

= 101.9m^2

f) l = 16 cm, w = 10 cm, h = 14 cm

As we know that

Volume of a cuboid = l × w × h

Volume of a cuboid =16 x 10 x 14 = 2240cm^3

Total Surface area of a cuboid = 2 × [(l × w) + (l × h) + (w × h)]

Total Surface area of a cuboid = 2 x [(16 x 10) + (16 x 14) + (10 x 14)]

= 2 x [160+224+140]

= 2 x 160+ 2 x224+2 x 140

= 1048cm^2

g) L = 7.5 m w = 4.5 m h = 8.2 m

As we know that

Volume of a cuboid = l × w × h

Volume of a cuboid =7.5 x 4.5 x 8.2 = 276.75cm^3

Total Surface area of a cuboid = 2 × [(l × w) + (l × h) + (w × h)]

Total Surface area of a cuboid = 2 x [(7.5 x 4.5) + (7.5 x 8.2) + (4.5 x 8.2)]

= 2 x [33.75+61.5+36.9]

= 2 x 33.75+ 2 x 61.5+2 x 36.9

= 264.3m^2

4. **Ali measures the length, width and height of his classroom, which is 12 m, 7 m and 8 m, respectively. Find the area of the classroom.**

Solution:

As we know the surface area of a room **=**2[(l x w) + (l x h) + (w x h)]

Putting values we get

surface area of a room **=**2[(12 x 7) +(12 x 8) +(7 x 8)]

= 2 x 84 +2 x96 +2 x56

= 168 +192+112 =472m^2

5. **Find the capacity of a water tank whose length is 75 m, width is 40 m, and its height is 60 m. Find the capacity of the water tank in litres.**

Solution:

Capacity of water tank = Length x width x Height

= 75 x 40 x 60

= 180,000 m^3

Now convert cubic metre into litres

Since, 1 cubic metre = 1000 litres

So, 180,000 m^3 = 1000 ×180000 litres = 180000000 litres

So, capacity of a water tank is 180000000 litres.

6. **A cuboid oil container is 10 m long, 6 m wide and 8 m high. Calculate the volume of the container.**

Solution:

Volume of the cuboid = Length x width x Height

=10 x 6 x 8 =480m^3

So, the volume of a container is 480 m^3.

7. **A cube-shaped hall of a masjid is 8 m long. Find the cost of painting four of its walls at the rate of Rs 350 per m$^{2.}$**

Solution:

Since Hall is cube shaped its walls are identical squares. Therefore the area of each wall can be calculated as:

Area of each wall =Length x Height

= 8 x 8 = 64 m^2

Total area of 4 walls =4 x 64sq.m

= 256 m^2

Cost of painting = total area x cost per m^2

$= 256\ m^2 \times 350\ Rs / m^2$

$= Rs\ 89,600$

So, cost of painting the four walls is Rs 89,600.

8. The total surface area of a cuboid gift box is 992 cm2. If its length is 20 cm and its width is 12 cm, respectively, find the height of the gift box.

Solution:

Let's assume height of gift box = h

Total surface area of cuboid = 2[(l x w) + (l x h) + (w x h)]

Substituting the given values,

$992 = 2 \times (20cm \times 12cm + 20cm \times h + 12cm \times h)$

$992 = 2 \times (240cm^2 + 32cm \times h)$

$992 = 480 + 64cm \times h$

$992 – 480 = 32\ cm \times h$

$512cm^2 = 64cm \times h$

$h = 512/64 = 8cm$

Thus, height of a gift box is 8 cm.

9. A wooden cupboard has a length of 214 cm, a width of 80 cm, and a height of 256 cm. Find:

a) The volume of the cupboard.

b) The surface area of the cupboard.

Solution:

First we convert the dimensions to meters

Length = 214 cm

Width = 80 cm

Height = 256 cm

a) Volume of the cupboard = Length x width x height

$= 214m \times 80m \times 256m$

$= 4,382,720\ cm^3$

b) Surface area of the cupboard = 2 x (length x width + length x height + width x height)

$= 2 \times (214 \times 80 + 214 \times 256 + 80 \times 256)$

$= 2 \times (17120 + 54784 + 20480)$

$= 2(92384)$

$=184,768\ m^2$

10. An auditorium is 20 m long, 14 m wide and 12 m high. Find:

a) The total surface area of the room including the roof and floor.

b) The cost of painting four of its walls at the rate of Rs 400 per m^2.

c) The cost of painting the ceiling at the rate of Rs 320 per m^2

Solution:

a) Area of the floor= product of the length and width of auditorium

=20 x 14= 280m^2

Similarly,

Area of the ceiling= Product of the length and width of auditorium

= 20 x 14= 280m^2

Total surface area of the room= area of the four walls

Area of the wall1= H x L= 12 x 20= 240m^2

Area of the wall 2= Hx L= 12 x 20= 240m^2

Area of wall 3= H x W= 12 x 14= 168m^2

Area of wall 4= H x W= 12 x 14 = 168 m^2

Total area of four walls= 816

Total surface area = area of floor+ area of ceiling+ total area of four walls

= 280 + 280+ 816

=1376m^2

b) Solution:

The total surface area of the four walls=H x L1+H x L2+H x W1+H x W2

=12x 20+12x20+12x14+12x14

=816m^2

Cost of painting= Total surface area x cost per square meter

=816 x 400

=Rs 326400

c) Solution:

Area of the ceiling = Product of the length and width of auditorium

= 20 x 14= 280m^2

Cost of painting= Total surface area x cost per square meter

=280 x 320

= 89600 R s

Check – 7

1. Encircle the correct answer.

a) The formula to find the perimeter of a rectangle is:

i) $l \times w$ ii) l^2 iii) $4 \times l$ iv**) $2(l + w)$**

b) If the length of a square-shaped clock is 8 cm, then the perimeter of the clock is:

i) 32 cm ii) **32 cm^2** iii) 64 cm iv) 64 cm^2

c) If a triangle's base is 6 cm long and its height is 9 cm, then the area of the triangle is:

i) **27 cm^2** ii) 28 cm^2 iii) 54 cm^2 iv) 120 cm^2

d) The surface area of a cube of length 9 m is:

i) 496 m^2 ii) 406 m^2 iii) **486 m^2** iv) 586 m^2

e) If the length of a cube is 20 cm, then the volume is:

i) 120 cm^2 ii) 400 cm^2 iii) **8000 cm^3** iv) 120 cm^3

2. Fill in the blanks.

a) The area of any shape is the space occupied by it.

b) The formula to find the surface area of a cuboid is 2(l x w) +2(w x h) +2(h x l).

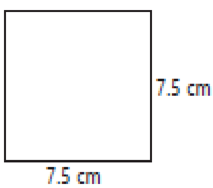
c) The number of edges of a cylinder is 2.

d) The formula to find the area of a triangle is 1/2(B x H).

e) The three dimensions of a solid are its length, width and height.

3. Find the Area and Perimeter of the following.

a)



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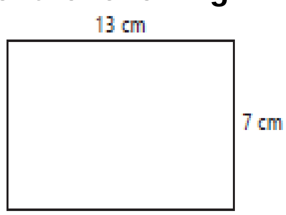


Solution:

a) Area = L X L

= 7.5 x 7.5

= 56.25cm^2

Perimeter = 4L

= 4 x 7.5

= 30cm

b) Area = L x B

= 13 x 7

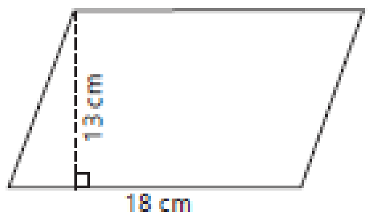
= 91 cm^2

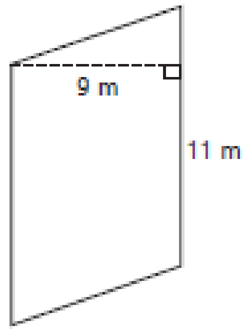
Perimeter = 2(L+ B)

= 2(13 + 7)

= 2 x 20
= 40cm

4. Find the area of the following figures.

a)


b)


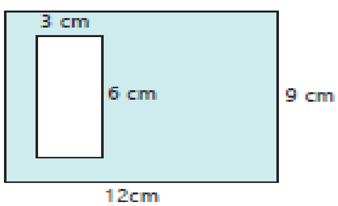
a) Solution:

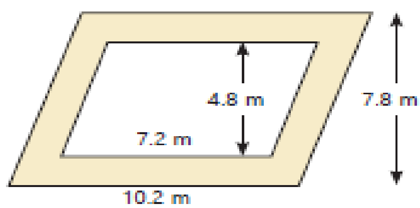
Area of a parallelogram= L x H
= 18 x 13
= $234cm^2$

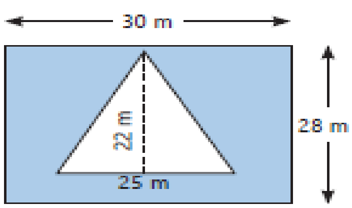
b) Solution:

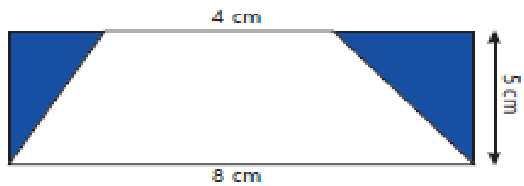
Area of parallelogram= L x H
= 9 x 11
= $99\ m^2$

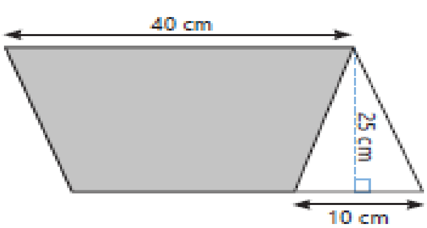
5. Find the area of the shaded parts following figures.

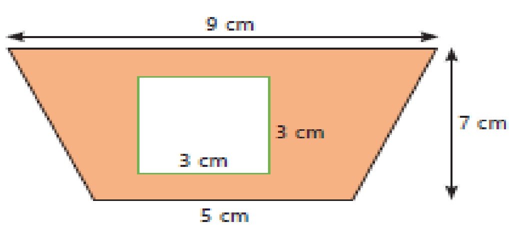
a)


b)


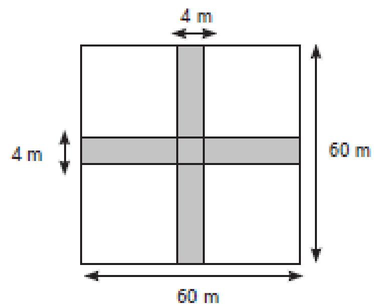
c)


d)


e)


f)


g)



a) Solution:

Area of the Outer Rectangle = L x W

= 12 x 9 = 108cm^2

Area of the Inner Rectangle = L x W

= 6 x 3 = 18cm^2

Area of the shaded part = Area of the Outer Rectangle - Area of the Inner Rectangle

= 108 cm^2 – 18 cm^2 = 90cm^2

b) Solution:

Area of the Outer Parallelogram = B x A = 10.2 x 7.8 = 79.56 m^2

Area of the Inner Parallelogram = B x A = 7.2 x 4.8 = 34.56 m^2

Area of the Shaded Parallelogram =Area of the Outer Parallelogram - Area of the Inner Parallelogram

= 79.56 m^2 – 34.56 m^2 = 45m^2

c) Solution:

Area of the triangle = ½(B x H)

= (22 x 25)/2

= 275 m^2

Area of the rectangle = L X W

= 30 x 28

= 840 m^2

Area of the shaded region = Area of the rectangle - Area of the triangle

= 840 m^2 – 275 m^2 = 565 m^2

d) Solution:

Area of the Trapezium = ½ x Altitude x (b1 + b2)

= ½ x 5 x (8+ 4)

= ½ x 5 x 12 = 30cm

Area of the Rectangle = L x W

= 8 x 5 = 40cm

Area of the shaded part = Area of the Rectangle - Area of the Trapezium

$= 40 – 30 = 10\ cm^2$

e) Solution:

Area of the Parallelogram = B x A = 40 x 25 = 1000 m^2

Area of the Trapezium = ½ x Altitude x (b1 + b2)

= ½ x 25 x (40 + 30)

= ½ x 25 x 70 = ½ x 1750 =875 m^2

Area of the shaded part = Area of the Parallelogram - Area of the Trapezium

=1000 – 875 = 125 m^2

f) Solution:

Area of the Trapezium = ½ x 7(9 + 5)

= ½ x 7(14) = 98/2 = 49 cm^2

Area of the Square = L x L = 3 x 3 =9 cm^2

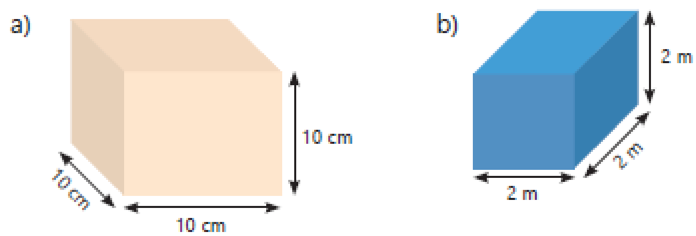
Area of the shaded part = Area of the Trapezium - Area of the Square

= 49 cm^2 – 9 cm^2 = 40 cm^2

f) Solution:

Area of the Square = (60 × 4) + (4 × 60) – (4 × 4) =464 m^2

6. Find the surface area and volume of the given cubes.



Solution:

a) Surface area of a cube = $6L^2$

$= 6(10)^2 = 6 \times 100 = 600cm^2$

Volume of a cube = L X L X L

= 10 x 10 x10 = 1000cm^3

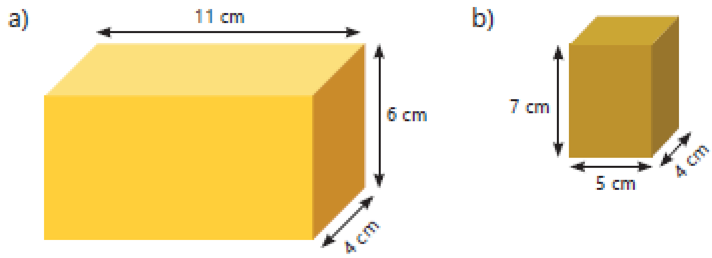
b) Surface area of a cube = $6L^2$

$= 6(2)^2 = 6 \times 4 = 24m^2$

Volume of a cube = L X L X L

= 2 x 2 x 2 = 8m^3

7. Find the surface area and volume of the cuboids.



Solution:

a) Area of a cuboid = L x W X H

=11 x 4 x 6 = 264 cm^3

Surface area of a cuboid = 2 x [(l x w) + (l x h) + (w x h)]

=2 x (11 x 4 + 11 x 6 + 4 x 6)

= 2 x (44 + 66+ 24) = 268 cm^2

b) Volume of a cuboid = L x W X H

=5 x 4 x7 = 140 cm^3

Surface area of a cuboid = 2 x [(l x w) + (l x h) + (w x h)]

=2 x (5 x 4 + 5 x 7 + 4 x 7)

= 2 x (20 + 35 + 28) = 166 cm^2

8. Complete the given table:

a) 1680cm^3 ,856cm^2

b) 6m, 126 m^2

c) 8 m ,360 m^3

d) 4.9m, 450.8m^3

e) 72cm^3,138cm^2

9. Find the area of a square-shaped whiteboard whose length is 25 cm.

Solution:

Area of a square-shaped whiteboard =?

Length of a square-shaped whiteboard = 25cm

Area of a square = L x L =25 x 25 = 625cm^2

10. Determine the perimeter of a rectangular garden with a length of 85 m and a width of 68 m.

Solution:

Length = 85m

Width = 68m

Perimeter of a rectangular garden =**?**

As we know that

Perimeter of a rectangle = 2(L + W)

= 2(85 + 68)

= 170 + 136 = 306m

11. A rectangle-shaped rose flower bed has a length of 9 m and a width of 7 m. If a 3.2 m wide flower border is around the flower bed find the total area of the border.

Solution:

Length = 9m

Width = 7m

Total area of the rectangular flower bed =L X W = 9 x 7 = 63m^2

To find the length of the flower border we add the area of the border on all the four sides of the flower bed.

Length with border = length + 2 x border width

= 9m + 2 x 3.2m

= 15.4m

Width with border = Width + 2 x 3.2m

=7+ 2 x 3.2m =13.4m

Area of the flower bed with border = Length with border x Width with border

= 15.4 x 13.4 = 206.36m^2

Total area of the flower border = Area of the flower bed with border - Area of the flower bed

= 206.36 – 63 = 143.36m^2

12. Find the altitude of a parallelogram if its base is 24 cm and its area is 360 cm^2.

Solution:

Altitude of a parallelogram= ?

Base = 24cm

Area = 360cm2

As we know that,

Area of a parallelogram = B x H

Substituting values we get,

$360 = 24 \times H$

$360/24 = H,$

$H = 15cm$

13. The lengths of two parallel sides of a trapezium are 12.6 cm and 7.8 cm, respectively. Find the area of the trapezium if the distance between its two parallel sides is 8 cm.

Solution:

Area of trapezium = ½ (sum of parallel sides x distance between them)

= ½ (12.6 + 7.8) x 8cm

= ½ (20.04) x 8

= 81.6 cm^2

14. A wheat field is 290 metres wide and 320 metres long. The outside of the field is surrounded by a path that is 4 metres wide. Find the area of the path.

Solution:

Length of inner rectangular field = 320 m

Width of inner rectangular field = 290 m

Wide path = 4 m

Length of outer rectangular field = 320m + 2(4m) = 328m

Width of outer rectangular field = 290m + 2(4m) = 298m

Area of the outer rectangular field = Length x Width

= 328m x 298m = 97,744m^2

Area of the inner rectangular field = Length x Width

= 320m x 290m Area = 92,800m^2

Area of path = Area of outer rectangle - Area of inner rectangle

= 97,744m^2 - 92,800m^2 = 4,944m^2

Therefore, the area of the path surrounding the wheat field is 4,944 square metres.

15. Find the width of the floor of a triangle-shaped hall if its area is 450 cm^2 and its base is 50 cm.

Solution:

Area of a triangle shaped hall = 450cm^2

Base = 50cm

Width =?

As we know that Area of a triangle = ½ (B x W)

So, 450 = ½ (50 x W)

450 x 2= 50 x W

900 = 50 x W

900/50 = W

Width = 18cm

16. Find the total surface area of a cuboid-shaped model whose length, width and height are 11.03 m, 12.86 m and 13.1 m, respectively.

Solution:

Surface area of Cuboid shaped model = 2 x [(L X W) + (L X H) + (W X H)]

Length = 11.03m

Width = 12.86m

Height = 13.1m

By putting values, we get,

Surface area of Cuboid shaped model =

= 2 x [(11.03 x 12.86) + (11.03 x 13.1) + (12.86 x 13.1)]

= 2 x (141.84 + 144.4 + 168.4)

= 283.69 + 288.8 + 336.8 = 909.6096m^2

17. Find the capacity of an aquarium whose length, width and height are 150 cm, 90 cm and 110 cm, respectively.

Solution:

Length = 150cm

Width = 90cm

Height =110cm

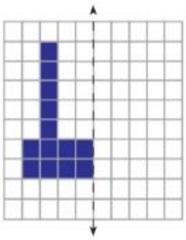
Capacity of the aquarium = Length x Width x Height

= 150 x 90 x 110 = 1485000cm^3

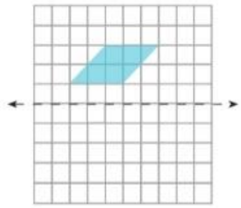
Learning Check 8.1

1. Reflect the images that are given on a square grid.

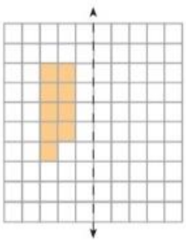
a)



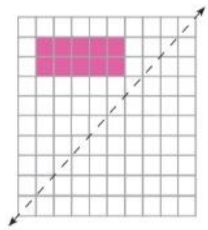
b)



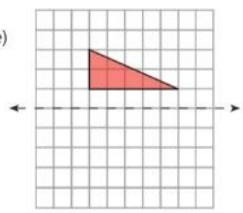
c)



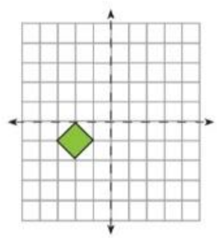
d)



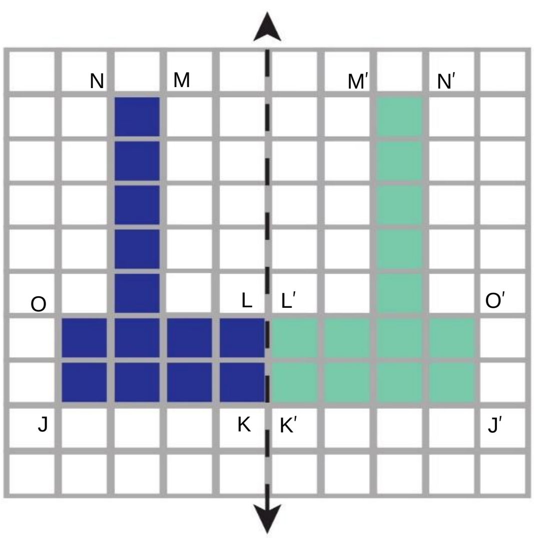
e)



f)



a) Solution:

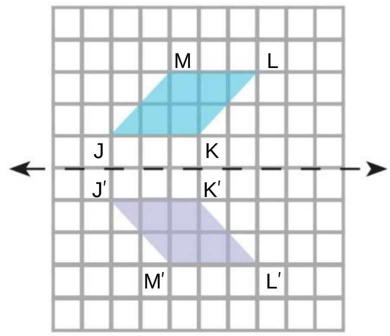


Step 1: Name the points (vertices) JKLMNO of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

b) Solution:

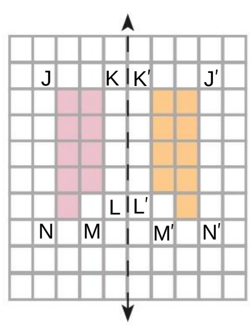


Step 1: Name the points (vertices) JKLM of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

c) Solution:

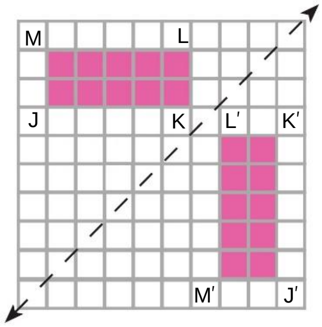


Step 1: Name the points (vertices) JKLMN of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

d) Solution:

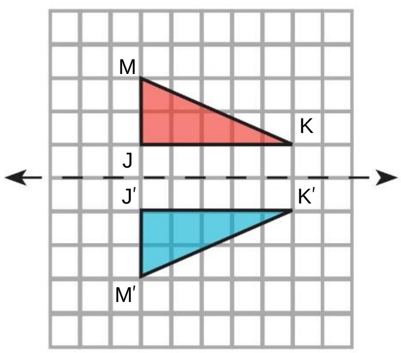


Step 1: Name the points (vertices) JKLM of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

e) Solution:

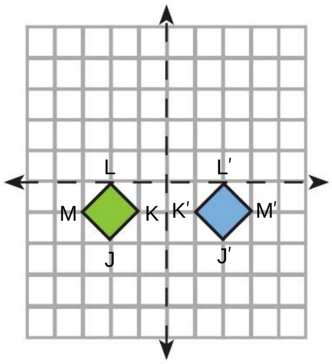


Step 1: Name the points (vertices) JKM of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

f) Solution:



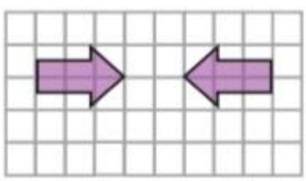
Step 1: Name the points (vertices) JKLM of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

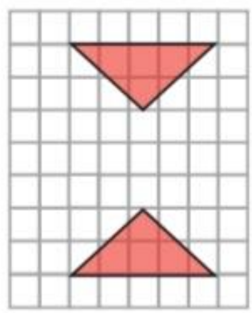
Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

2. **Find the line of reflection of the given images and draw it by using a compass**.

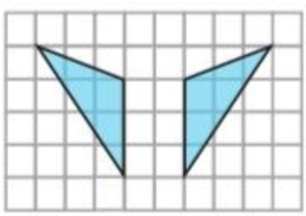
a)



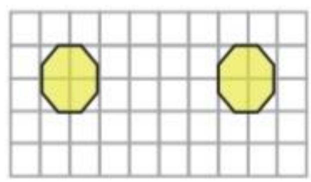
b)



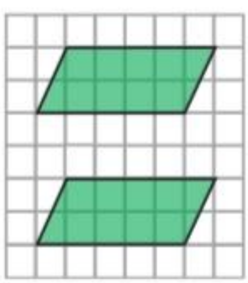
c)



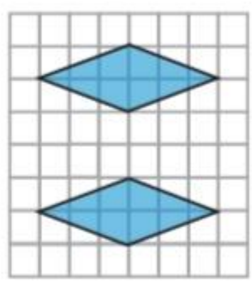
d)



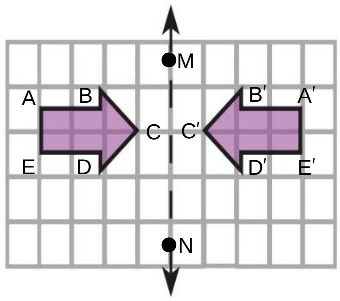
e)



f)



a) Solution:



Follow the steps to construct the line of reflection.

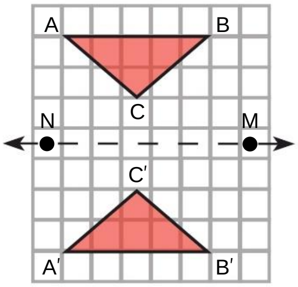
Step I: Using a straight line, connect any vertex of the given triangle ABCDE to the vertex of the reflected image A' B' C' D’ E’. C and C' are joined here.

Step II: Open the compass with more than half the measure of the line segment CC' and draw an arc on the top and bottom of CC’, placing the compass pointer at C.

Step III: Similarly, draw an arc on the top and bottom of CC’, placing the compass pointer at C', which intersects the previous arcs at points M and N, respectively.

Step IV: Join M and N. The perpendicular bisector MN to the line CC' is the required line of reflection.

b) SOLUTION:



Follow the steps to construct the line of reflection.

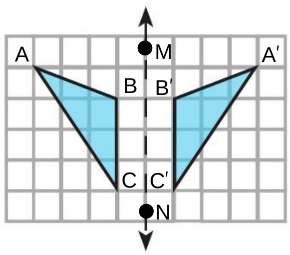
Step I: Using a straight line, connect any vertex of the given triangle ABC to the vertex of the reflected image A' B' C'. B and B' are joined here.

Step II: Open the compass with more than half the measure of the line segment BB' and draw an arc on the top and bottom of BB', placing the compass pointer at B.

Step III: Similarly, draw an arc on the top and bottom of BB, placing the compass pointer at B' , which intersects the previous arcs at points M and N, respectively.

Step IV: Join M and N. The perpendicular bisector MN to the line BB' is the required line of reflection.

c) Solution:



Follow the steps to construct the line of reflection.

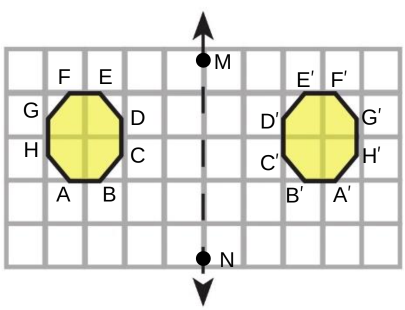
Step I: Using a straight line, connect any vertex of the given triangle ABC to the vertex of the reflected image A' B' C'. B and B' are joined here.

Step II: Open the compass with more than half the measure of the line segment BB' and draw an arc on the top and bottom of BB', placing the compass pointer at B

. **Step III:** Similarly, draw an arc on the top and bottom of BB’, placing the compass pointer at B’, which intersects the previous arcs at points M and N, respectively.

Step IV: Join M and N. The perpendicular bisector MN to the line BB' is the required line of reflection.

d) Solution:



Follow the steps to construct the line of reflection.

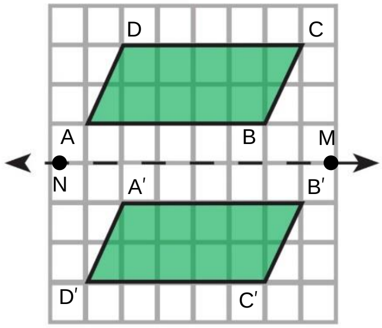
Step I: Using a straight line, connect any vertex of the given triangle ABCDEFGH to the vertex of the reflected image A' B' C' D’ E’ F’ G’ H. B and B' are joined here.

Step II: Open the compass with more than half the measure of the line segment BB' and draw an arc on the top and bottom of BB’, placing the compass pointer at B.

Step III: Similarly, draw an arc on the top and bottom of BB', placing the compass pointer at B', which intersects the previous arcs at points M and N, respectively.

Step IV: Join M and N. The perpendicular bisector MN to the line BB' is the required line of reflection.

e) Solution:



Follow the steps to construct the line of reflection.

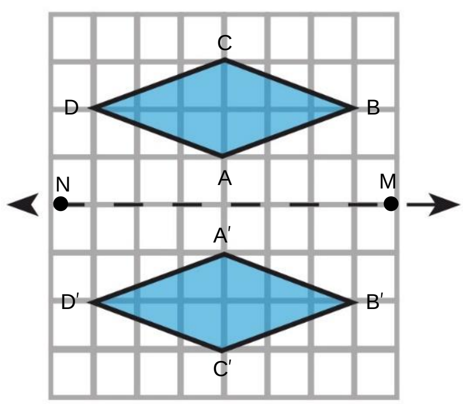
Step I: Using a straight line, connect any vertex of the given triangle ABCD to the vertex of the reflected image A' B' C' D'. A' and A' are joined here.

Step II: Open the compass with more than half the measure of the line segment AA' and draw an arc on the top and bottom of AA', placing the compass pointer at A.

Step III: Similarly, draw an arc on the top and bottom of AA, placing the compass pointer at A', which intersects the previous arcs at points M and N, respectively.

Step IV: Join M and N. The perpendicular bisector MN to the line AA' is the required line of reflection.

f) Solution:



Follow the steps to construct the line of reflection.

Step I: Using a straight line, connect any vertex of the given triangle ABCD to the vertex of the reflected image A' B' C' D’. D and D' are joined here.

Step II: Open the compass with more than half the measure of the line segment DD' and draw an arc on the top and bottom of DD', placing the compass pointer at D.

Step III: Similarly, draw an arc on the top and bottom of DD', placing the compass pointer at D' , which intersects the previous arcs at points M and N, respectively**.**

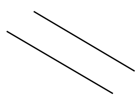
Step IV: Join M and N. The perpendicular bisector MN to the line DD' is the required line of reflection.

Learning check 8.2

1. Identify and tick ☑ the parallel lines.

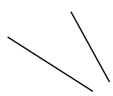
Solution:

a)



These are parallel lines because we know that parallel lines never cross each other. no matter how far these lines move in one direction.

b)

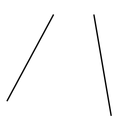


These lines are "non-parallel" because if they are extended in the same direction, they meet at point O

c)

These are parallel lines because we know that parallel lines never cross each other. no matter how far these lines move in one direction.

d)



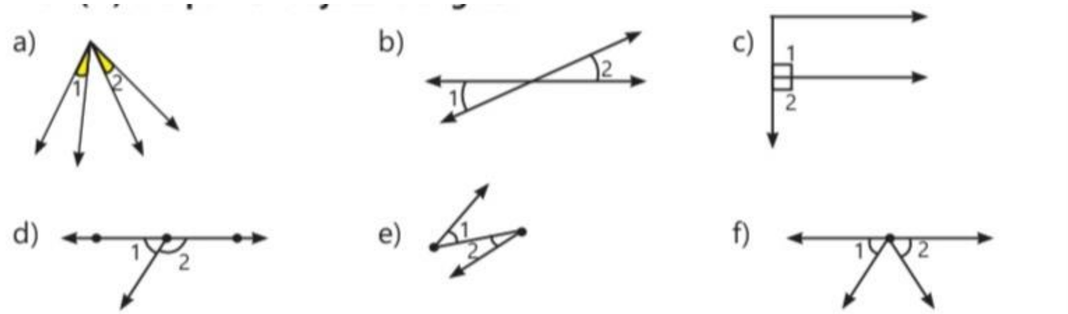
These lines are "non-parallel" because if they are extended in the same direction, they meet at point O

2. **Determine which pairs of lines are perpendicular.**

Solution:

b, c and d. As these lines are intersecting each other at 90 degrees.

3. **Tick ☑ the pair of adjacent angles.**



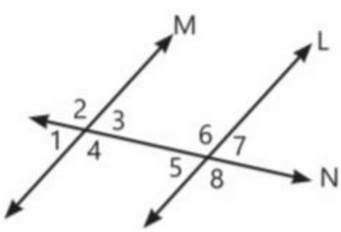
Solution:

c and d because they share the same vertex, same arm and they do not overlap.

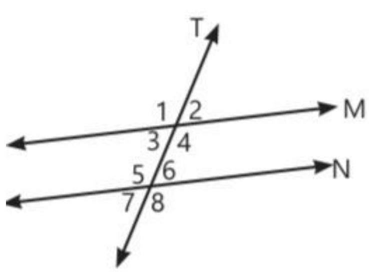
Learning check 8.3

1. **A transversal intersects two parallel lines, as depicted in the figures:**

i)



ii)



a) Write the names of parallel lines and a transversal.

b) Write the names of pairs of:

• Corresponding angles

• Alternate interior angles

• Vertically opposite angles

i) Solution:

a) i) Parallel lines: M and L Transversal: N

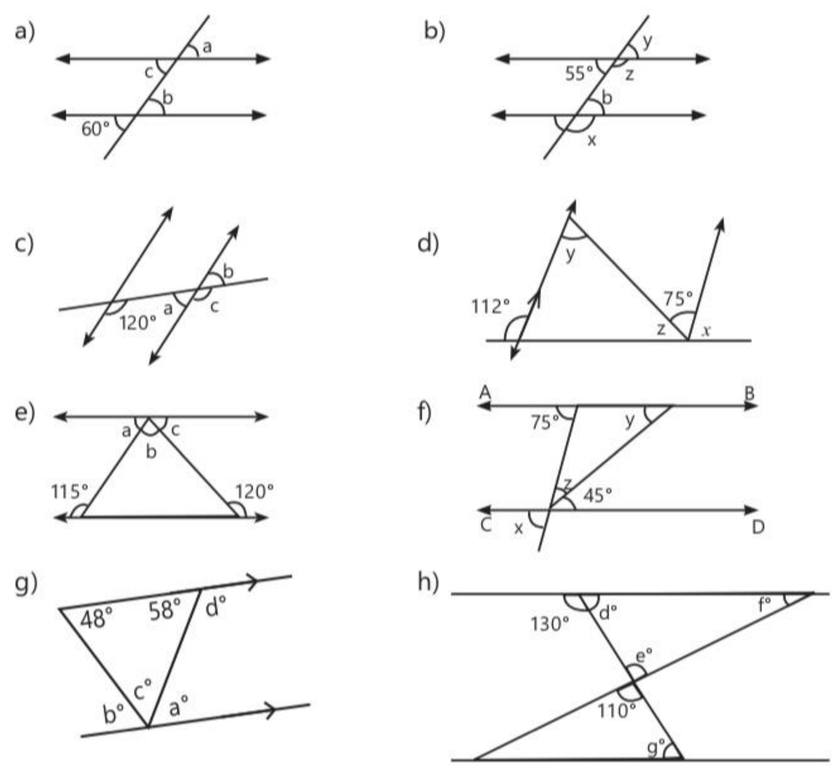
b) i) Corresponding angles: ∠1 and ∠5; ∠4 and ∠8, ∠2 and ∠6, ∠3 and ∠7. ii) Alternate interior angles ∠4 and ∠6, ∠3 and ∠5. iii) vertically opposite angles: ∠2 and ∠4; ∠1 and ∠3; ∠6and∠8; ∠5 and ∠7

ii) Solution:

a) i) Parallel lines: M and N, Transversal: T

b) i) Corresponding angles: ∠1 and ∠5; ∠4 and ∠8, ∠2 and ∠6, ∠3 and ∠7. ii) Alternate interior angles ∠3 and ∠6, ∠4 and ∠5. iii) vertically opposite angles: ∠1 and ∠4; ∠2 and ∠3; ∠5 and∠8; ∠6 and ∠7

2. **Find the unknown angles in the following figures by utilizing the properties of parallel lines and a transversal**.



a) Solution:

We know that vertically opposite angles are equal.

So, $\angle b = 60°$.

Now the corresponding angles are also equal.

So, $\angle c = 60°$.

Also, the alternate angles are equal.

So, $\angle a = 60°$

b) Solution:

$b = 55°$ (vertically opposite angles).

$b + z = 180°$ (interior angles on the same side of the transversal).

$55° + z = 180°$

$z = 180° - 55° = 125°$

as we know

corresponding angles are also equal.

Then $x = z = 125^0$

And $b = y = 55^0$

c) Solution:

$a + 120^0 = 180^0$ (Interior angles of the same side)

$m< a = 180^0 - 120^0 = 60^0$

$m< b = 60^0$ (Vertically opposite angles)

$m < c = 120^0$(corresponding angles)

d) Solution:

$x + 112^0 = 180^0$ (Interior angles of the same side)

$x = 180^0 - 112^0$

$x = 68^0$

$y = 75^0$ (corresponding angles)

$z = 112^0 - y$

putting the value of y we get,

$z = 112^0 - 75^0$

$z = 37^0$

e) Solution:

$a + 115^0 = 180^0$

$a = 180^0 - 115^0$

$a = 65^0$

$c + 120^0 = 180^0$

$c = 180^0 – 120^0$
$c = 60^0$
120^0 - < a = b
Putting the value of a we get
$120^0 – 65^0 = b$
$b = 55^0$

f) Solution:

$x = 75^0$(corresponding angles)
$y = 45^0$(vertically opposite angles)
$z = x – y$
$z = 75^0 – 45^0 = 30^0$

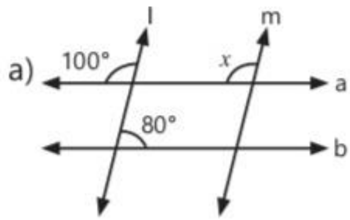
g) Solution:

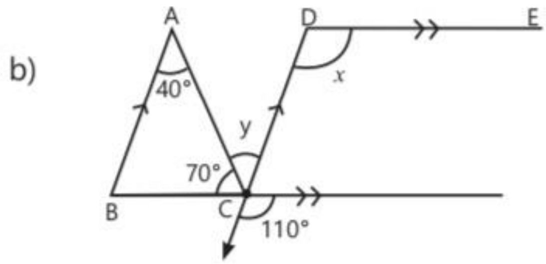
As we know, $a = 58^0$(Alternate interior angles)
$d + a = 180^0$
$d+ 58^0 = 180^0$
$d = 180^0 - 58^0 = 122^0$
$b = 48^0$(vertically opposite angles)
$c = d – b$
$c = 122^0 - 48^0$
$c = 74^0$

h) Solution:

$e = 110^0$(vertically opposite angles are same)
$d + 130^0 = 180^0$
$d = 50^0$
as we know $g^0 = d^0$(opposite angles are same)
so,
$g = 50^0$
$f = 130^0 - 110^0$
$f = 20^0$

3. Find the value of x.





a) Solution:

As we know that,

Corresponding angles are same

So,

$x = 100^0$

b) Solution:

As we know that,

Corresponding angles are same

So,

$X = 110^0$

Learning check 8.4

1. Determine which signs have rotational symmetry. Mark the point of rotation for the symmetric figures and write the order of rotation.

Solution:

a) No rotational symmetry

b) No rotational symmetry

c) No rotational symmetry

d) Rotational symmetry; order of rotational symmetry:2 times; point of rotation: Center point

e) Rotational symmetry; order of rotational symmetry:2 times; point of rotation: Center point

f) No Rotational symmetry

g) Rotational symmetry; order of rotational symmetry:2 times; point of rotation: Center point

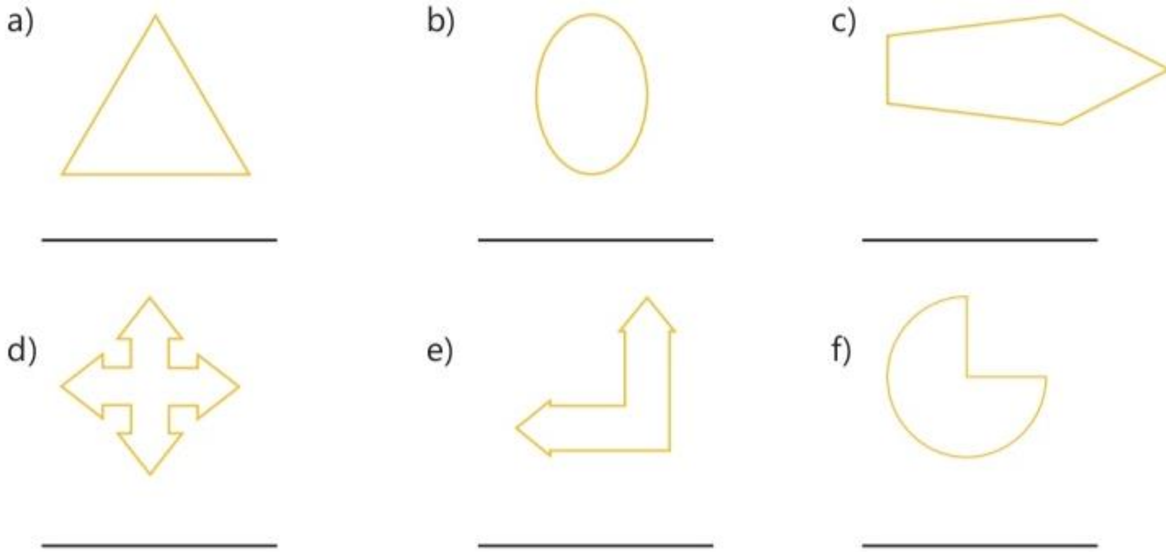
h) Rotational symmetry; order of rotational symmetry:2 times; point of rotation: Center point

i) No Rotational symmetry

j) No Rotational symmetry

k) No Rotational symmetry

2. **Identify the figures that have rotational symmetry. Write the order of rotation and mark the point of rotation for the symmetric figures.**
   Solution:



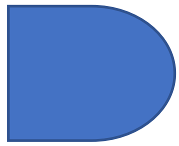
a) No rotational symmetry
b) No rotational symmetry
c) No rotational symmetry
d) Rotational symmetry;4 times; center Point
e) No rotational symmetry
f) No rotational symmetry

3. **Draw three shapes that have rotational symmetry and mention the order of their rotational symmetry as well as the point of rotation.**
   Solution:

Order of symmetry = 4 times
Point of rotation = center point.

4. Draw two shapes that have no rotational symmetry.

Solution:

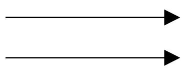
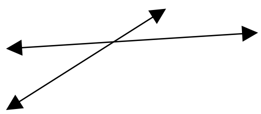
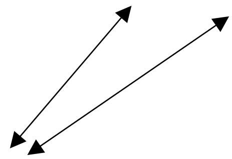
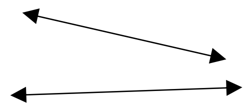
Unit Check 8

1. Encircle the correct answer.

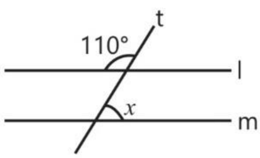
a) Perpendicular lines intersect each other at:

i) 60^o **ii) 90^o** iii) 180^o iv) 360^o

b) The pairs of lines that cannot intersect each other are:

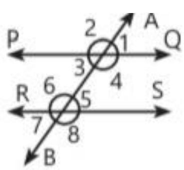
i)

ii)

iii)

iv)


c) The value of x in the given figure is:

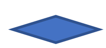


i) 70° ii) 110° iii) 180° iv) 120°

d) The alternate exterior angles are:



i) ∠1, ∠2, ∠5, ∠6 ii) ∠3, ∠4, ∠7, ∠8 **iii) ∠1, ∠2, ∠7, ∠8** iv) ∠3, ∠4, ∠5, ∠6

e) The order of rotation of the given figure is: 

i) 2 ii) 1 iii) 4 iv) 3 ii

2. Fill in the blanks.

a) Co-interior angles are interior angles on the same side of the Transversal.

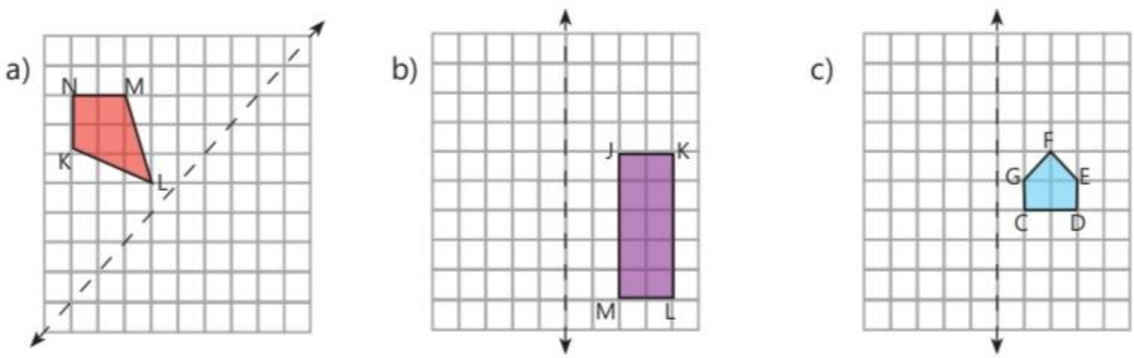
b) Parallel lines never meet each other.

c) The line that crosses two or more given lines is called transversal**.**

d) The sum of the measure of two or more angles on a straight line is equal to 180^o.

e) A shape that looks the same at least two times when rotated around a point in a full rotation is said to have rotational symmetry.

3. Reflect the image on the grid paper.

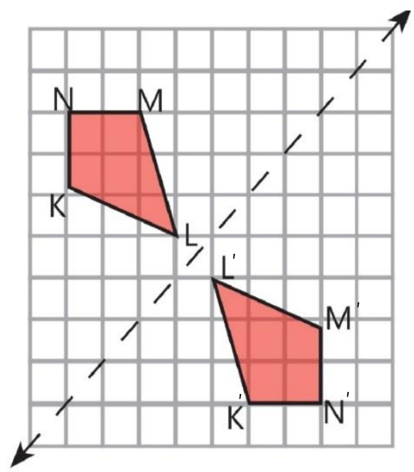


a) Solution:

Step 1: Name the points (vertices) KNML of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex L.

Step 3: For vertex L, mark L, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

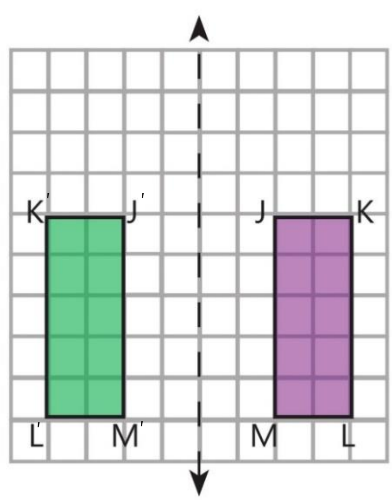


b) Solution:

Step 1: Name the points (vertices) KJML of the given shape for convenience.

Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex J.

Step 3: For vertex J, mark J, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

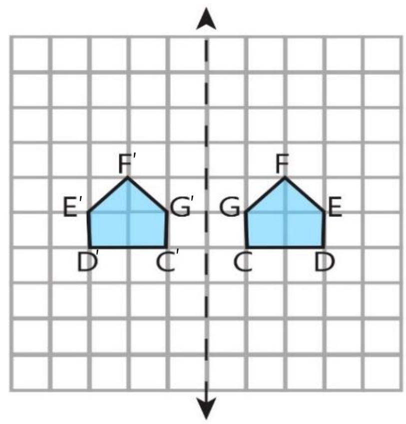


c) Solution:

Step 1: Name the points (vertices) FEDCG of the given shape for convenience.

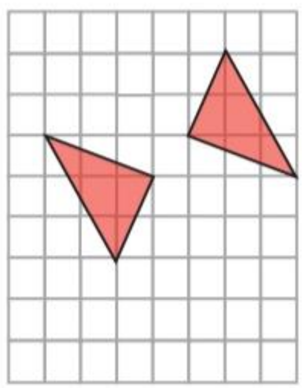
Step 2: Choose any of the closest point (from the line of reflection) of the given image and mark it on the opposite side of the line at the same distance as the original image. Here we take the vertex F.

Step 3: For vertex F, mark F, which is the same distance from the line of reflection. Similarly, mark all the other points on the other side.

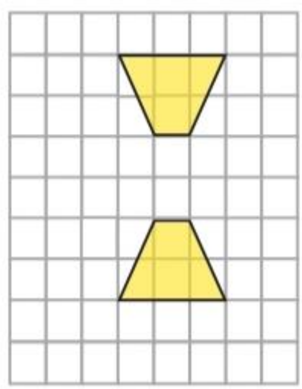


4. Find and draw the line of reflection for these images using a compass.

a)



b)

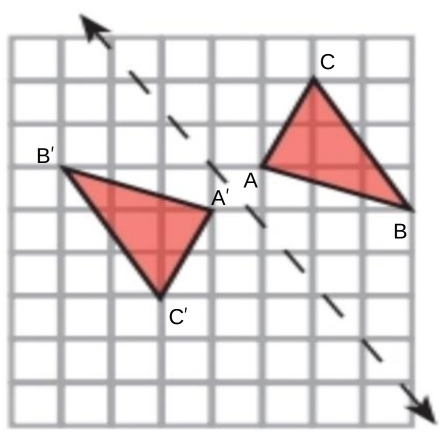


a) Solution:

Follow the steps to construct the line of reflection.

Step I: Using a straight line, connect any vertex of the given triangle ABC to the vertex of the reflected image A' B' C'. C and C' are joined here.

Step II: Open the compass with more than half the measure of the line segment CC' and draw an arc on the top and bottom of CC' , placing the compass pointer at C.

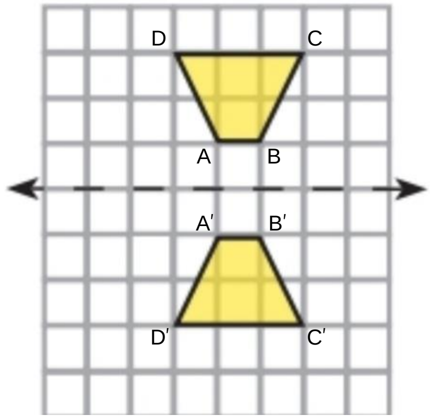


Step III: Similarly, draw an arc on the top and bottom of CC' , placing the compass pointer at C' , which intersects the previous arcs at points M and N, respectively.

Step IV: Join M and N. The perpendicular bisector MN to the line CC' is the required line of reflection.

b) Solution:

Follow the steps to construct the line of reflection.



Step I: Using a straight line, connect any vertex of the given triangle ABCD to the vertex of the reflected image A' B' C' D. C and C' are joined here.

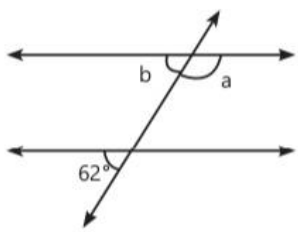
Step II: Open the compass with more than half the measure of the line segment CC' and draw an arc on the top and bottom of CC', placing the compass pointer at C.

Step III: Similarly, draw an arc on the top and bottom of CC', placing the compass pointer at C', which intersects the previous arcs at points M and N, respectively.

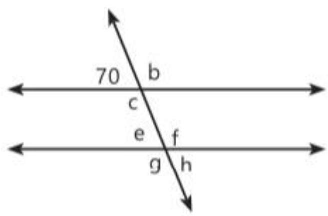
Step IV: Join M and N. The perpendicular bisector MN to the line CC' is the required line of reflection.

5. Find the unknown angles of the following figures.

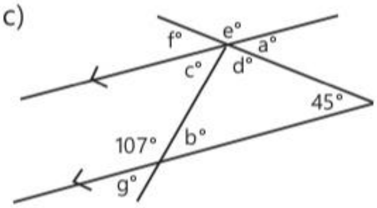
a)



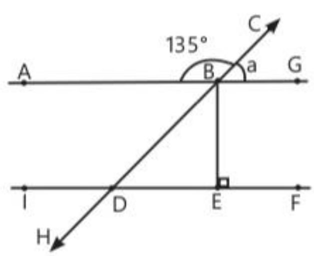
b)



c)



d)



a) Solution:

As we know corresponding angles are same

$b = 62^0$

$a + b = 180^0$

$a + 62^0 = 180^0$

$a = 180^0 - 62^0$

$a = 118^0$

b) Solution:

As we know vertically opposite angles are same

$D^0 = 70^0$

$E^0 = h^0$

$G^0 = f^0$

$B^0 = c^0$

So, $b + d = 180^0$

$B + 70^0 = 180^0$

$B = 180^0 - 70^0$

$B = 110^0$

c) Solution:

As we know $b^0 = g^0 = c^0$

$g + 107^0 = 180^0$

$g = 180^0 - 107^0$

$g = 73^0$

so,

$b = c = g = 73^0$

$a + 135^o = 180^o$

$a = 180^o – 135^o = 45^o$

$a = 45^o$

as, $a = f$

so, $a = f = 45^o$

$d + b + 45^o = 180^0$

$d + 73^o + 45^o = 180^o$

$d + 118^o = 180^0$

$d = 62^o$

$e + f = 180^o$ (corresponding angles)

putting values of ‘f’ we get ‘e’

$e + 45^o = 180^o$

$e = 180^o – 45^o = 135^o$

d) Solution:

Let two parallel line AG and IF intersect by a transversal Cat D and B.

<ABC = 135^0

<ABC = <DBA (are on the same line)

<ABC + <DBA = 180^0

135^0 + <DBA = 180^0

<DBA = 180^0 - 135^0

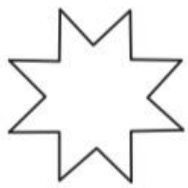
<DBA =45^o

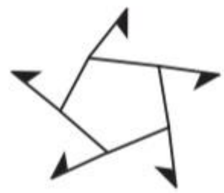
<DBA and <CBG are Vertically opposite angles so they must be equal. <DBA and <CBG =45^0

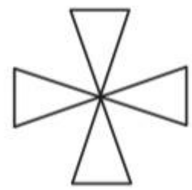
< ABC = <DBG = 135^0

6. Write the order of rotation of each figure and mark the point of rotation for the symmetric figures.

a) 

b) 

c) 

d) 

Solution:

a) 0, point of rotation: center point

b) 8, point of rotation: center point

c) 5, point of rotation: center point

d) 4, point of rotation: center point

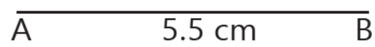
Learning Check 9.1

1. Draw the right bisectors of the following line segments using a pair of compasses.

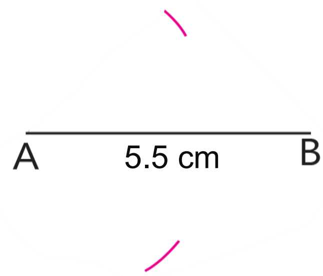
a)

Steps of Construction:

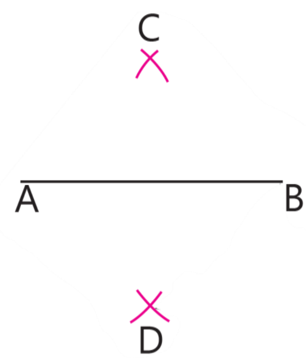
Step I: Draw a line segment AB=5.5 cm



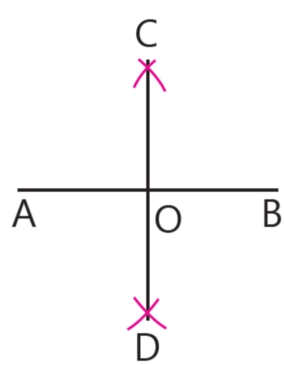
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.

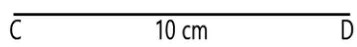


CD is the right bisector of AB and O is the mid point.

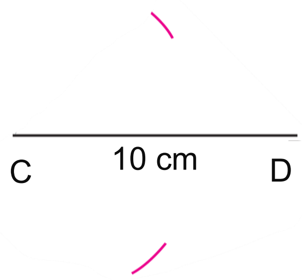
b)

Steps of Construction

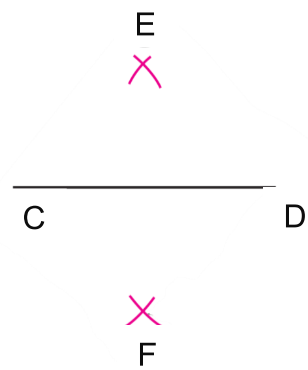
Step I: Draw a line segment CD=10 cm



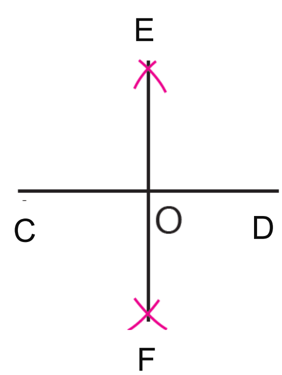
Step II: Place the pointer of the compass at point C. Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.



Step III: Similarly, using the same opening of the compass,
place the pointer of the compass at point D and draw two arcs on the top and bottom of CD which intersects the previous arcs at point E and F respectively.



Step IV: Join the point E and F by a line using a ruler that will cut the line segment CD at point O.

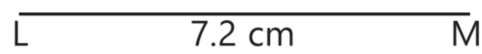


EF is the right bisector of CD and O is the mid point.

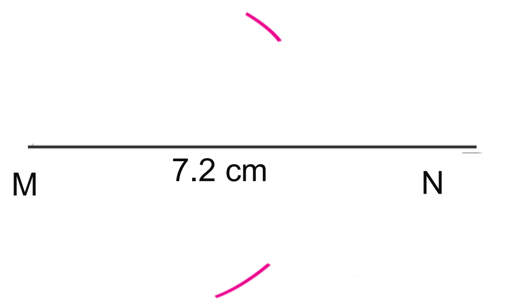
c)

Steps of Construction

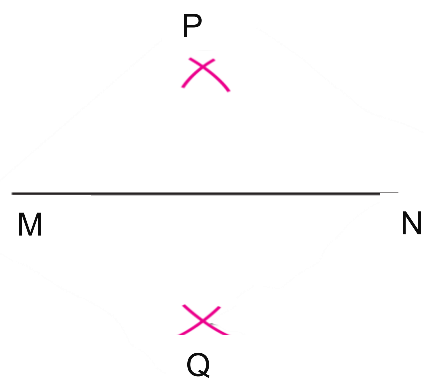
Step I: Draw a line segment LM=7.2 cm



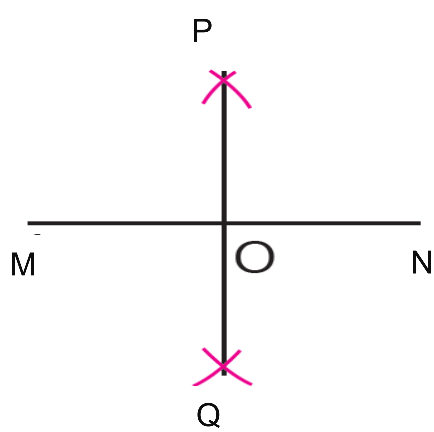
Step II: Place the pointer of the compass at point M. Open the compass with more than half of the measure of the line segment MN and draw an arc on the top and bottom of MN.



Step III: Similarly, using the same opening of the compass,
place the pointer of the compass at point N and draw two
arcs on the top and bottom of MN which intersects the
previous arcs at point P and Q respectively.



Step IV: Join the point P and Q by a line using a ruler that
will cut the line segment MN at point O.

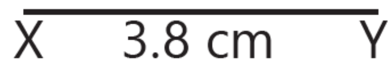


PQ is the right bisector of MN and O is the mid point.

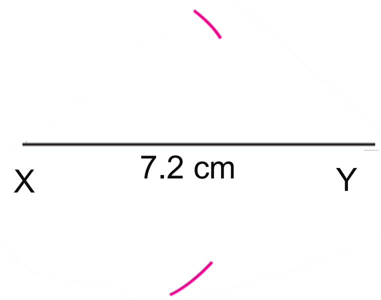
d)

Steps of Construction

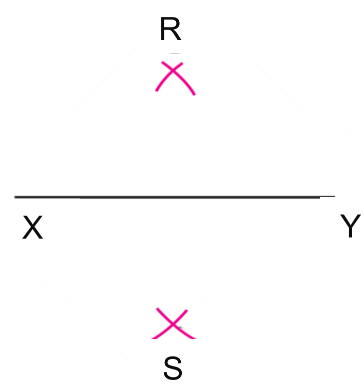
Step I: Draw a line segment XY=3.8 cm



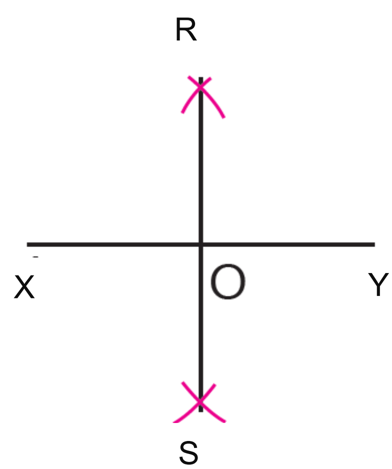
Step II: Place the pointer of the compass at point X. Open the compass with more than half of the measure of the line segment XY and draw an arc on the top and bottom of XY.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point Y and draw two arcs on the top and bottom of XY which intersects the previous arcs at point R and S respectively.



Step IV: Join the point R and S by a line using a ruler that will cut the line segment XY at point O.



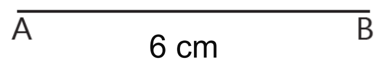
RS is the right bisector of XY and O is the mid point.

2. Draw the line segments of the following lengths and bisect them.

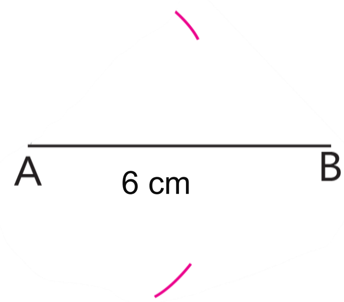
a) 6 cm

Steps of Construction:

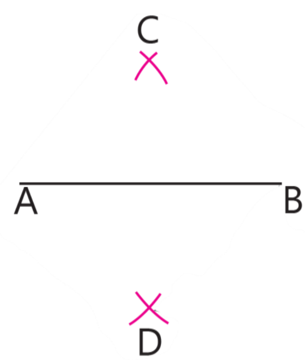
Step I: Draw a line segment AB=6 cm



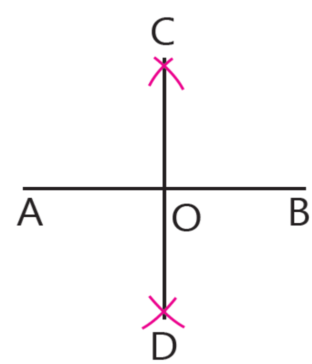
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass,
place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.

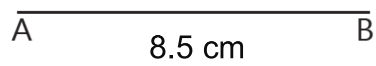


CD is the right bisector of AB and O is the mid point.

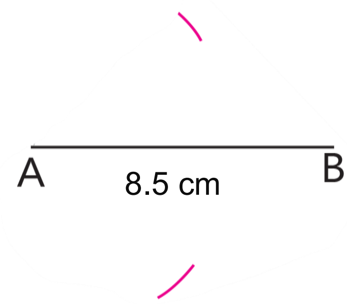
b) 8.5 cm

Steps of Construction:

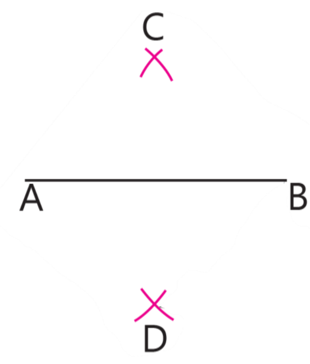
Step I: Draw a line segment AB=8.5 cm



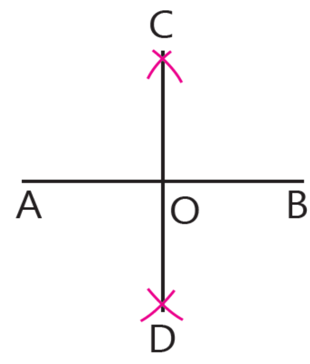
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.

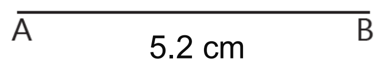


CD is the right bisector of AB and O is the mid point.

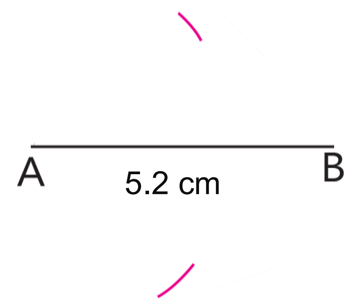
c) 5.2 cm

Steps of Construction:

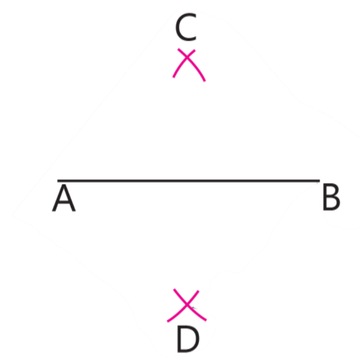
Step I: Draw a line segment AB=5.2 cm



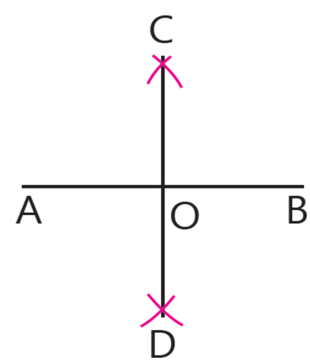
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.

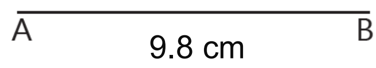


CD is the right bisector of AB and O is the mid point.

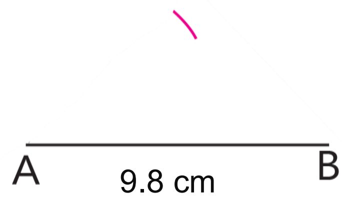
e) 9.8 cm

Steps of Construction:

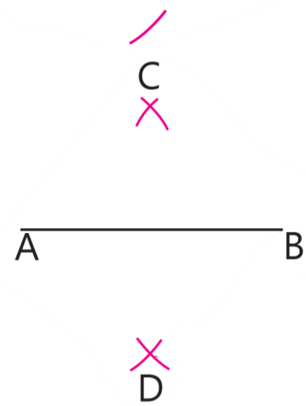
Step I: Draw a line segment AB=9.8 cm



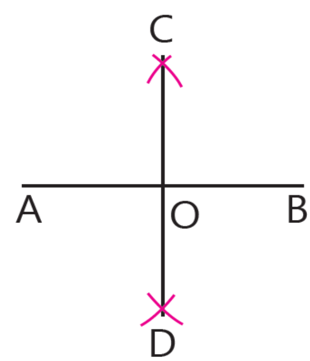
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.



CD is the right bisector of AB and O is the mid point.

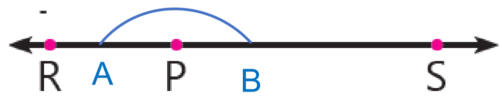
3. Draw perpendiculars to the lines from the given points on them using a compass.

a)

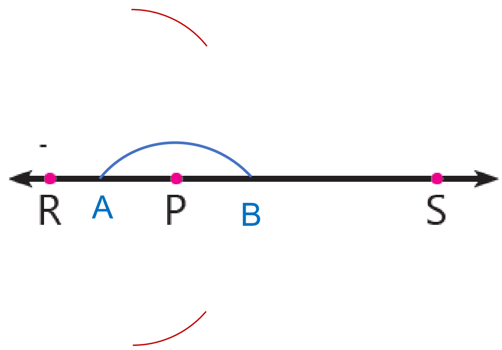
Steps of constructions:

RS is the given line and P is a point on it.

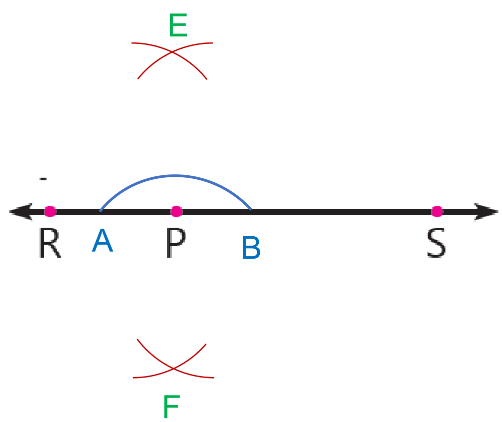
Step I: Place the pointer of the compass at point P and draw an arc of suitable radius that will cut the line RS at two points A and B respectively, such that PA = PB.

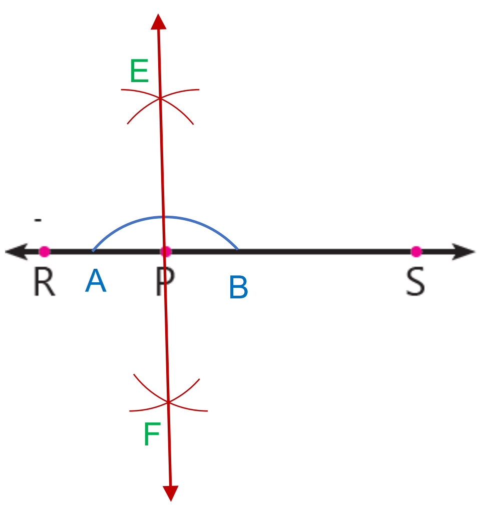


Step II: Place the pointer of the compass at point A and draw two arcs at top and bottom of radius greater than PA as shown in the figure.



Step III: With the same opening of the compass, place the pointer of the compass at point B and draw two arcs at top and bottom which cuts the previous arcs at point E and F as shown in the figure.





Step IV: Join the point E to P and extend it to F. So, EF is the required perpendicular to the given line RS at point P.

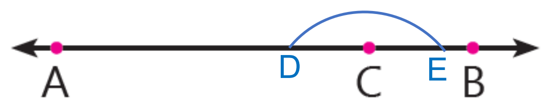
∴ EF ⊥ RS

b)

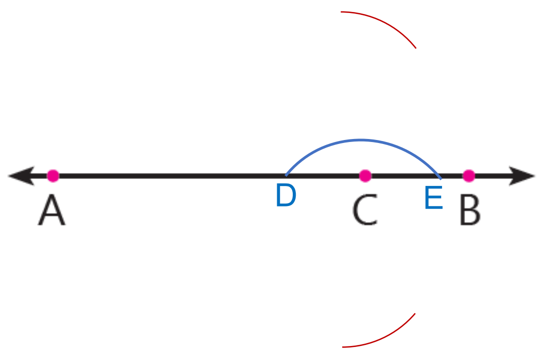
Steps of constructions:

AB is the given line and C is a point on it.

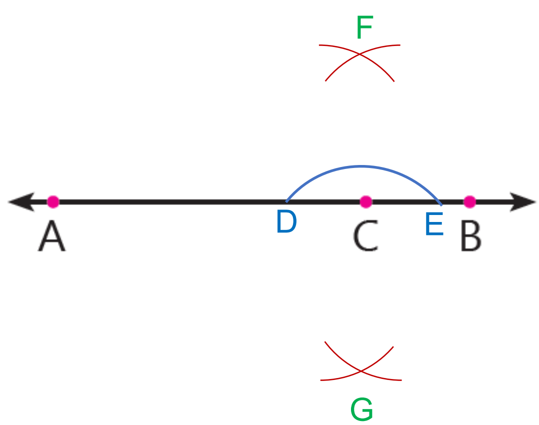
Step I: Place the pointer of the compass at point C and draw an arc of suitable radius that will cut the line AB at two points D and E respectively, such that CD = CE.

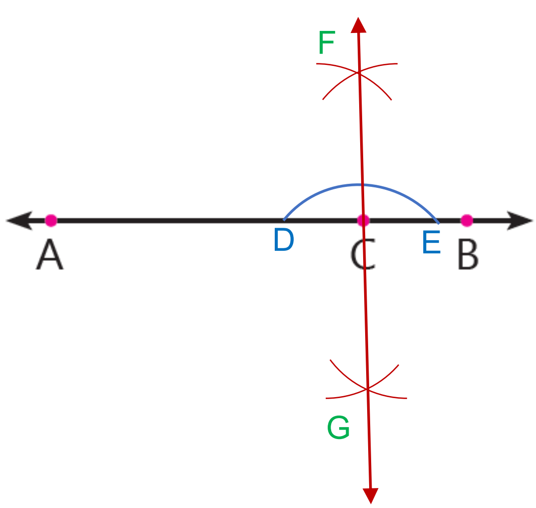


Step II: Place the pointer of the compass at point D and draw two arcs at top and bottom of radius greater than CD as shown in the figure.



Step III: With the same opening of the compass, place the pointer of the compass at point E and draw two arcs at top and bottom which cuts the previous arcs at point F and G as shown in the figure.





Step IV:

Join the point F to C and extend it to G. So, FG is the required perpendicular to the given line AB at point C.

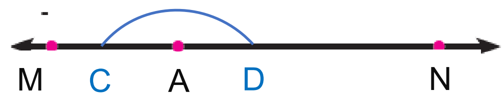
∴ FG ⊥ AB

c)

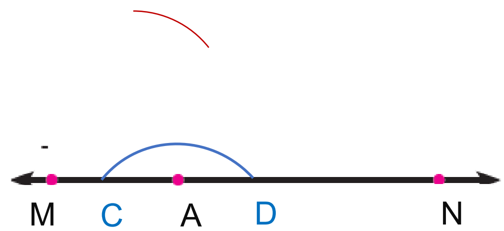
Steps of constructions:

MN is the given line and A is a point on it.

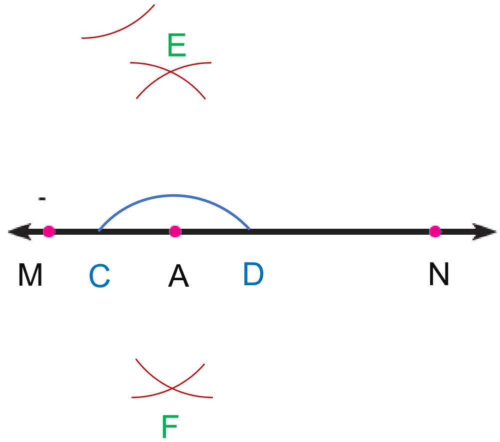
Step I: Place the pointer of the compass at point A and draw an arc of suitable radius that will cut the line MN at two points C and D respectively, such that AC = AD.

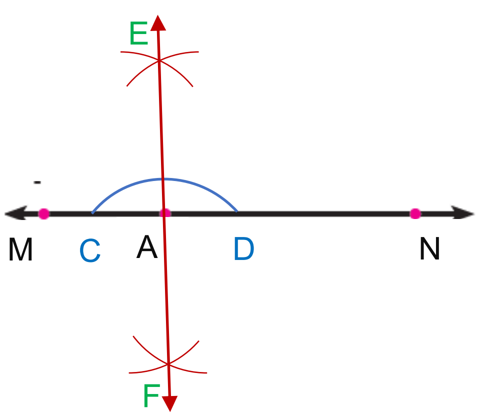


Step II: Place the pointer of the compass at point C and draw two arcs at top and bottom of radius greater than AC as shown in the figure.



Step III: With the same opening of the compass, place the pointer of the compass at point D and draw two arcs at top and bottom which cuts the previous arcs at point E and F as shown in the figure.





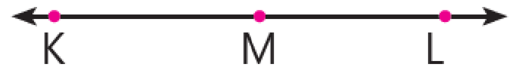
Step IV: Join the point E to A and extend it to F. So, EF is the required perpendicular to the given line MN at point A.

$\therefore EF \perp MN$

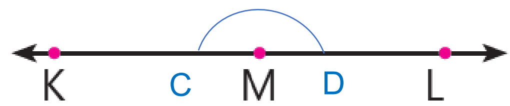
d)

Steps of constructions:

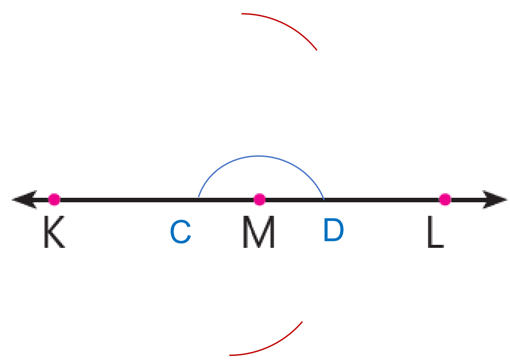
KL is the given line and M is a point on it.



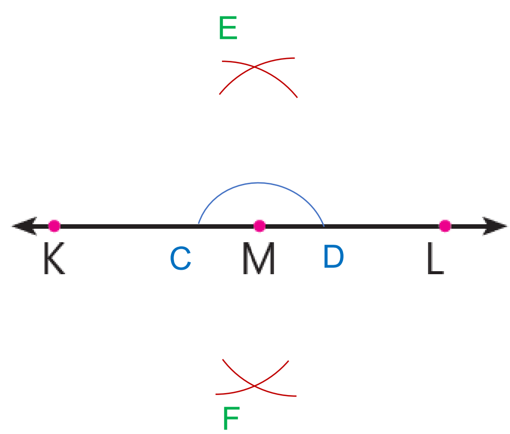
Step I: Place the pointer of the compass at point M and draw an arc of suitable radius that will cut the line KL at two points C and D respectively, such that MC = MD.

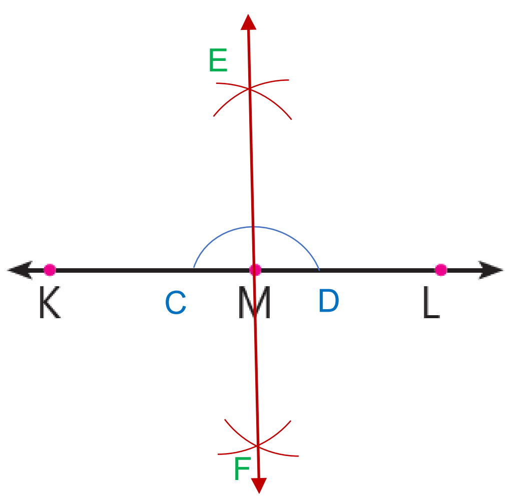


Step II: Place the pointer of the compass at point C and draw two arcs at top and bottom of radius greater than MC as shown in the figure.



Step III: With the same opening of the compass, place the pointer of the compass at point D and draw two arcs at top and bottom which cuts the previous arcs at point E and F as shown in the figure.





Step IV: Join the point E to M and extend it to F. So, EF is the required perpendicular to the given line KL at point M.

∴ EF ⊥ KL

4. Draw perpendiculars to the given lines from point P not on them using a compass.

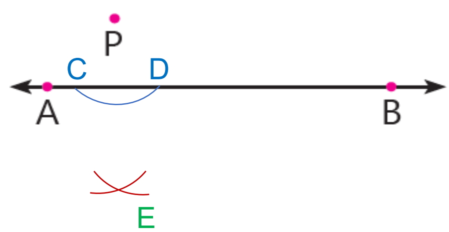
a)

AB is the given line. A point P (not on it) is also given.

Steps of Construction:

Step I: Place the pointer of the compass at point P and draw an arc that will cut the line AB at two points C and D respectively.

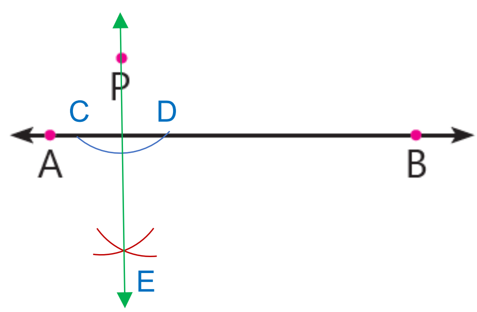
Step II: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of AB that cut each other at point E.



Step III: Join P to E.

PE is the required perpendicular to the given line AB through point P not on it.

∴ PE ⊥ AB

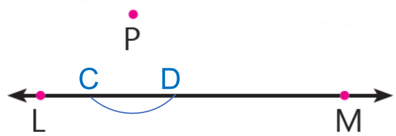


b)

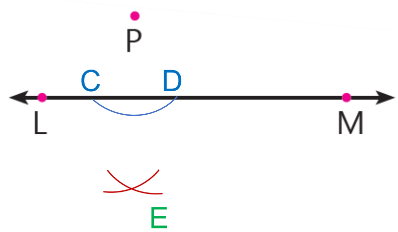
LM is the given line. A point P (not on it) is also given.

Steps of Construction:

Step I: Place the pointer of the compass at point P and draw an arc that will cut the line LM at two points C and D respectively.



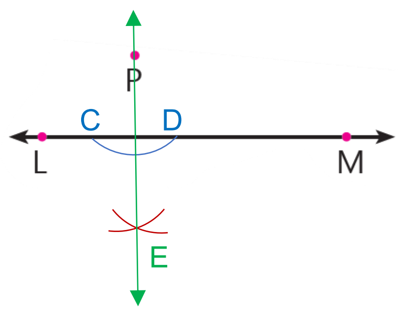
Step II: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of LM that cut each other at point E.



Step III: Join P to E.

PE is the required perpendicular to the given line LM through point P not on it.

$\therefore PE \perp LM$

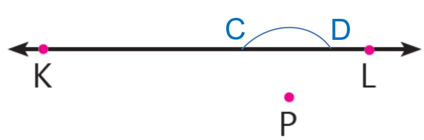


c)

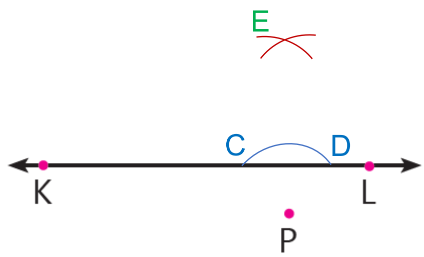
KL is the given line. A point P (not on it) is also given.

Steps of Construction:

Step I: Place the pointer of the compass at point P and draw an arc that will cut the line KL at two points C and D respectively.



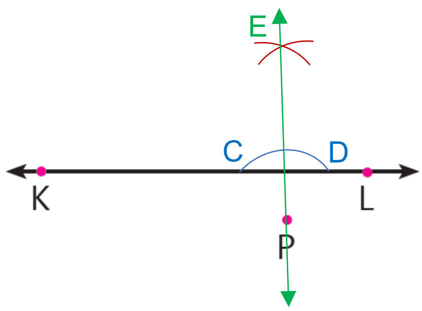
Step II: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of KL that cut each other at point E.



Step III: Join P to E.

PE is the required perpendicular to the given line KL through point P not on it.

∴ PE ⊥ KL

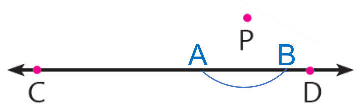


d)

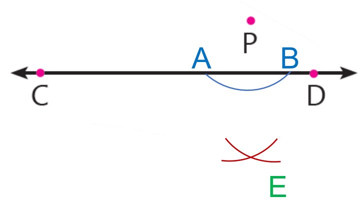
CD is the given line. A point P (not on it) is also given.

Steps of Construction:

Step I: Place the pointer of the compass at point P and draw an arc that will cut the line CD at two points A and B respectively.

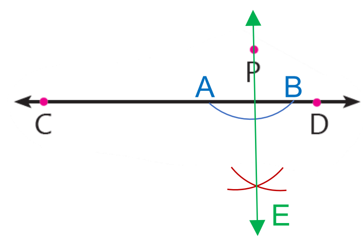


Step II: Using the same opening of the compass, place the pointer of the compass at point A and B and draw two arcs on the other side of CD that cut each other at point E.



Step III: Join P to E.

PE is the required perpendicular to the given line CD through point P not on it.



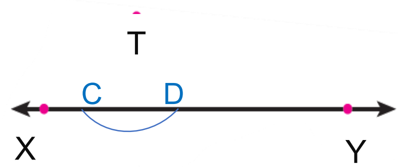
∴ PE ⊥ CD

5. Draw a line XY and draw a perpendicular on it from point T not on it.

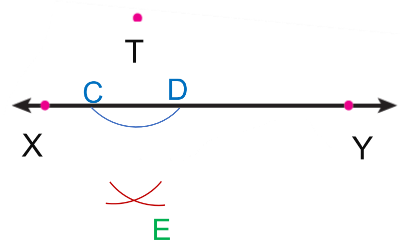
Steps of Construction:

Step I: Draw a line XY using ruler and put a point T not on it.

Step II: Place the pointer of the compass at point T and draw an arc that will cut the line XY at two points C and D respectively.

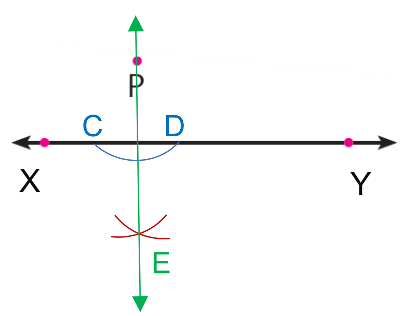


Step III: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of XY that cut each other at point E.



Step IV: Join T to E.

TE is the required perpendicular to the given line XY through point T not on it.

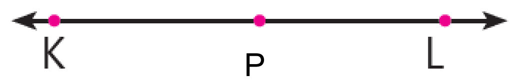


∴ TE ⊥ XY

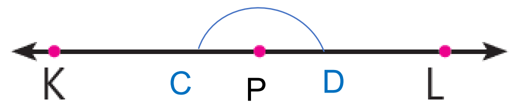
6. Draw a line KL and draw a perpendicular on it from point P on it.

Steps of constructions:

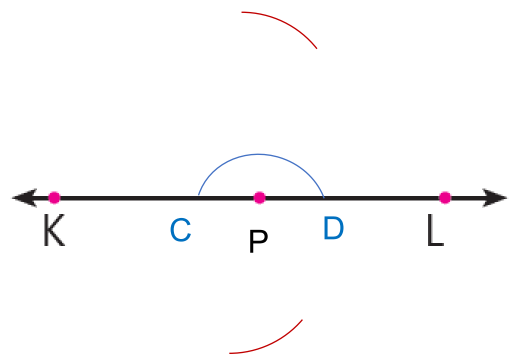
Step I: Draw a line KL using ruler and put a point M on it.



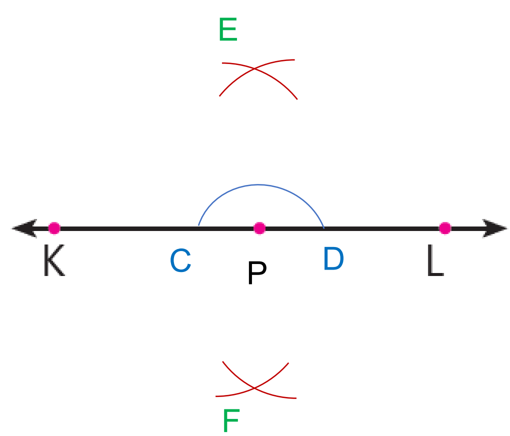
Step II: Place the pointer of the compass at point P and draw an arc of suitable radius that will cut the line KL at two points C and D respectively, such that PC = PD.

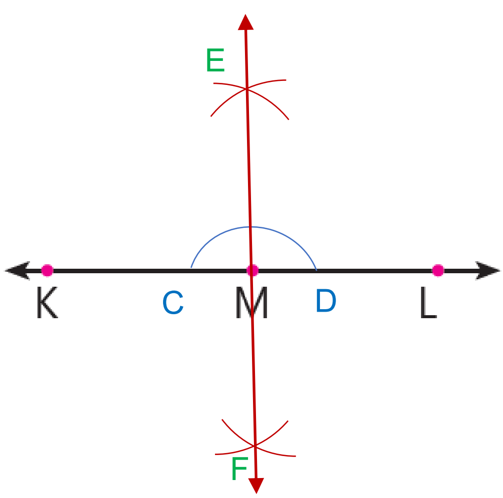


Step III: Place the pointer of the compass at point C and draw two arcs at top and bottom of radius greater than PC as shown in the figure.



Step IV: With the same opening of the compass, place the pointer of the compass at point D and draw two arcs at top and bottom which cuts the previous arcs at point E and F as shown in the figure.





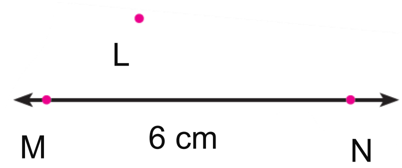
Step IV: Join the point E to M and extend it to F. So, EF is the required perpendicular to the given line KL at point P.

∴ EF ⊥ KL

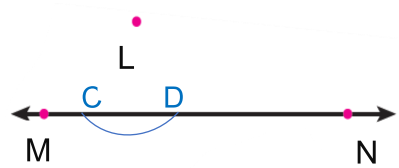
7. Draw a line segment mMN = 6 cm. Draw a perpendicular from point L not on it.

Steps of Construction:

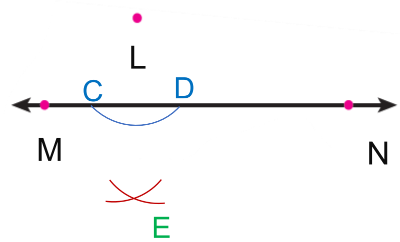
Step I: Draw a line MN=6 cm using ruler and put a point L not on it.



Step II: Place the pointer of the compass at point L and draw an arc that will cut the line MN at two points C and D respectively.

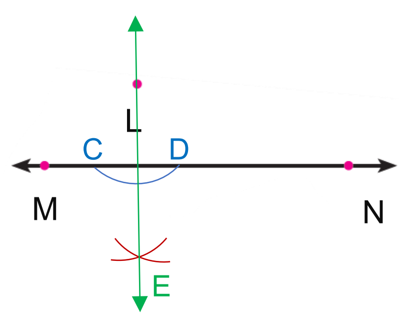


Step III: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of MN that cut each other at point E.



Step IV: Join L to E.

LE is the required perpendicular to the given line MN through point L not on it.



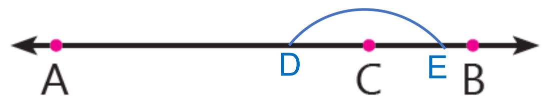
∴ LE ⊥ MN

8. Draw a line segment mAB = 7.2 cm. Construct a perpendicular at point C on it.

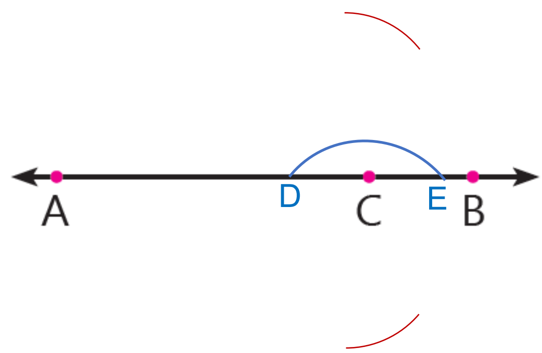
Steps of constructions:

Step I: Draw a line AB=7.2 cm and mark a point C on it.

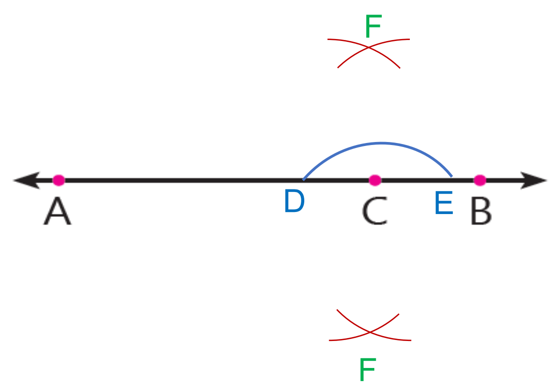
Step II: Place the pointer of the compass at point C and draw an arc of suitable radius that will cut the line AB at two points D and E respectively, such that CD = CE.

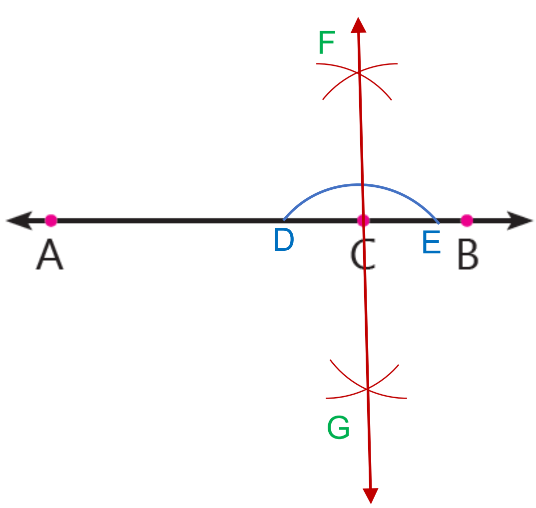


Step III: Place the pointer of the compass at point D and draw two arcs at top and bottom of radius greater than CD as shown in the figure.



Step IV: With the same opening of the compass, place the pointer of the compass at point E and draw two arcs at top and bottom which cuts the previous arcs at point F and G as shown in the figure.





Step IV: Join the point F to C and extend it to G. So, FG is the required perpendicular to the given line AB at point C.

$\therefore FG \perp AB$

Learning Check 9.2

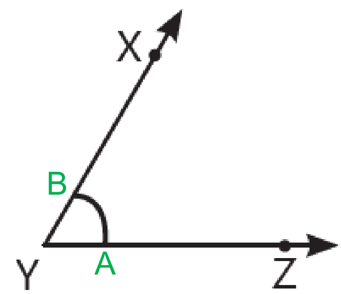
1. Copy the angles and then bisect them.

a)

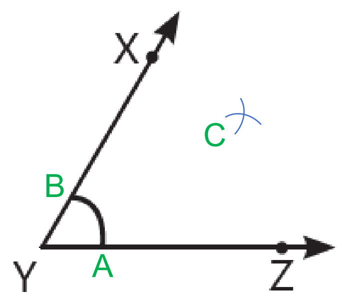
The given angle is XYZ.

Steps of Construction:

Step I: Name the points where the arc of angle cuts ray YZ and YX as A and B respectively.

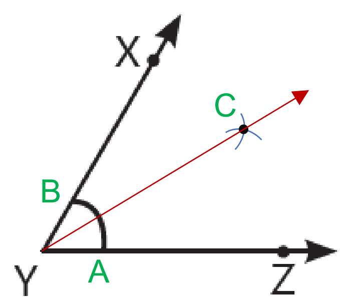


Step II: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step III: Using a ruler draw a ray YC passing through the point C. This line is bisecting the angle ∠XYZ into two equal parts.

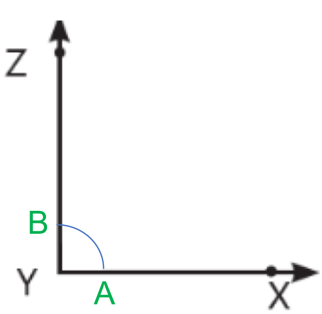
So, m∠XYC = m∠ZYC.



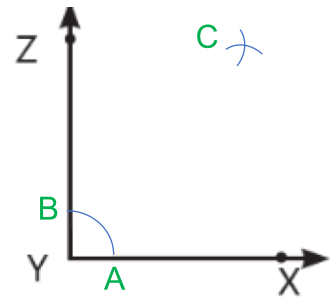
b)

Steps of Construction:

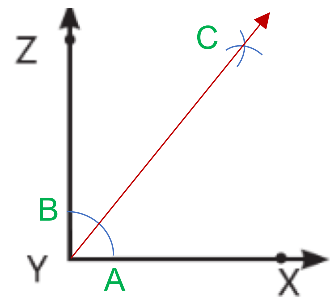
Step I: Place the pointer of the compass at point Y and draw an arc of suitable radius that will cut the ray YX at point A and cuts the ray YZ at point B.



Step II: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.

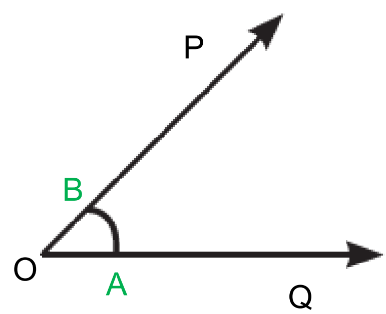


Step III: Using a ruler draw a ray YC passing through the point C. This line is bisecting the angle ∠ZYX into two equal parts.
So, m∠ZYC = m∠XYC.

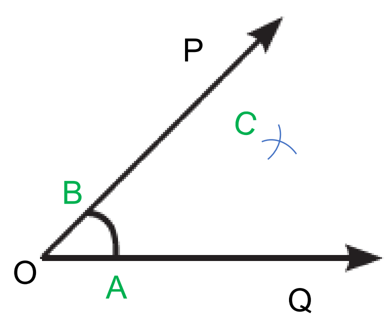


A

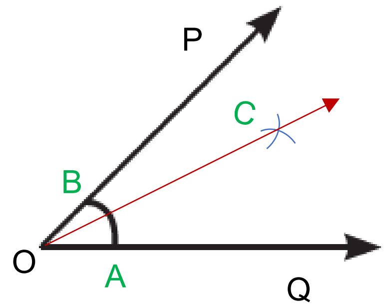
c)
Steps of Construction:
Step I: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively



Step II: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.

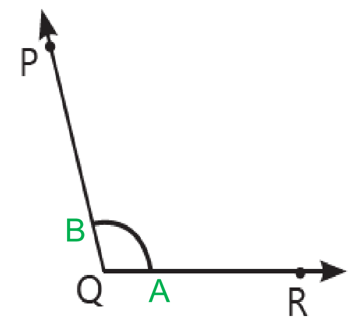


Step III: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.
So, m∠POC = m∠QOC.

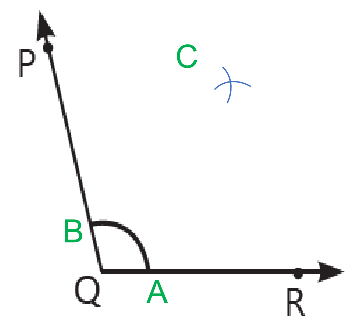


d)
The given angle is PQR.

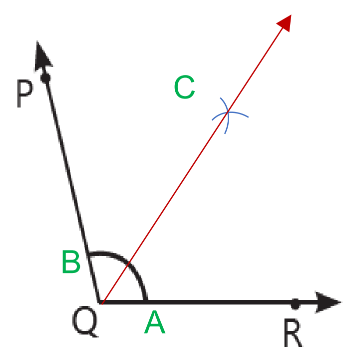
Steps of Construction:
Step I: Name the points where the arc of angle cuts ray QP and QR as A and B respectively.



Step II: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step III: Using a ruler draw a ray QC passing through the point C. This line is bisecting the angle ∠PQR into two equal parts.
So, m∠PQC = m∠RQC.



2. Construct the following angles by using a protractor and bisect them by using a pair of compasses.

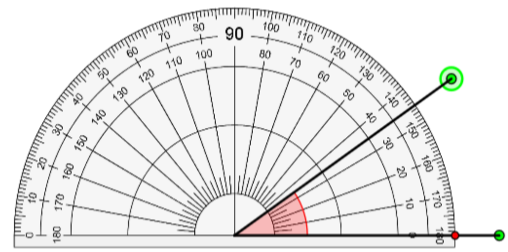
a) 35° b) 70° c) 85° d) 110° e) 150°

Solution:

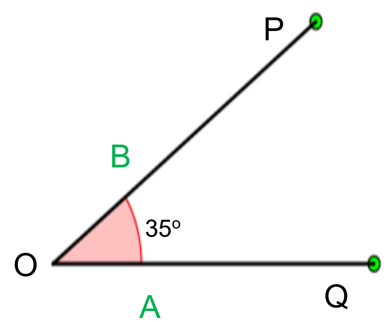
a) 35°

Steps of Construction:

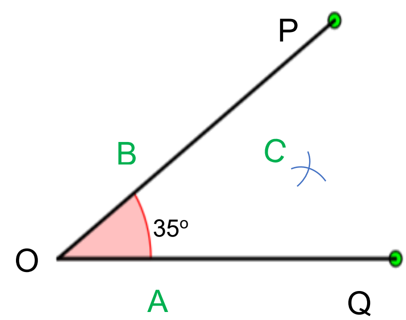
Step I: Use a protractor and draw an angle of 35º.



Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively

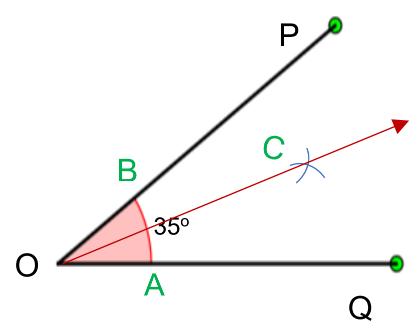


Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.

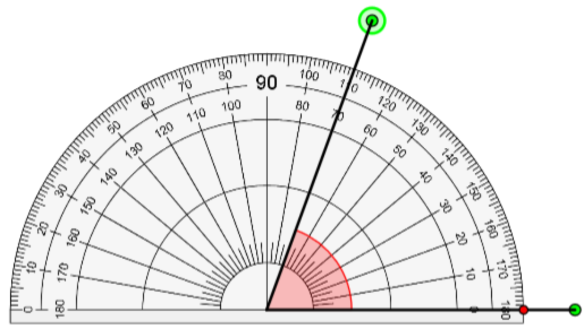
So, m∠POC = m∠QOC= 35º/2= 17.5º.



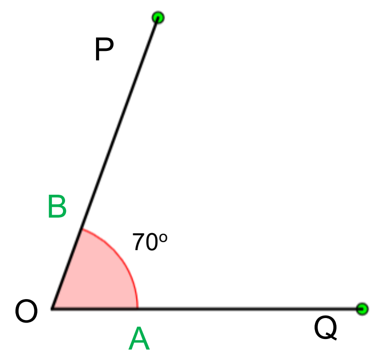
b) 70°

Steps of Construction:

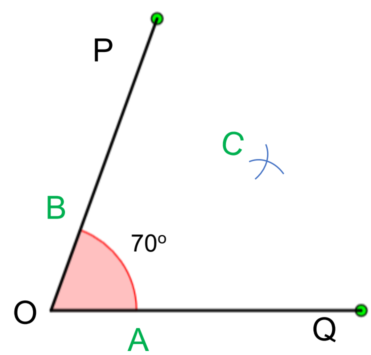
Step I: Use a protractor and draw an angle of 70°.



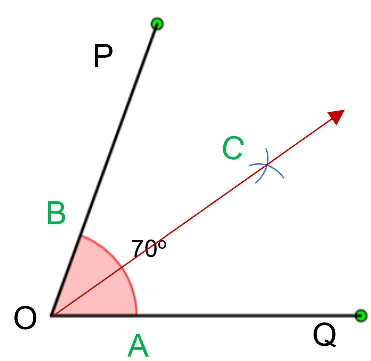
Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively



Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



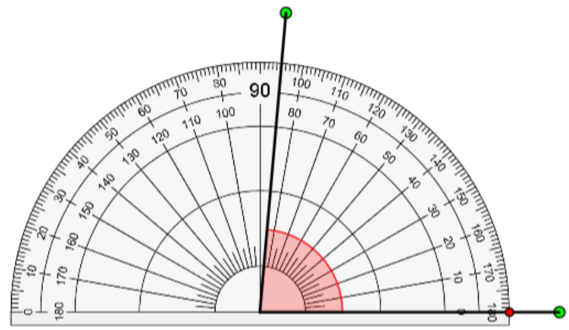
Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.
So, m∠POC = m∠QOC= 70°/2= 35°.



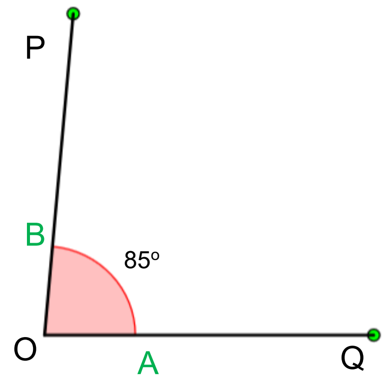
c) 85°

Steps of Construction:

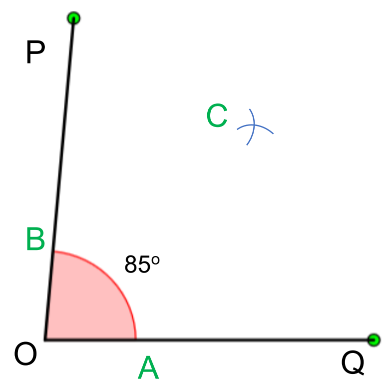
Step I: Use a protractor and draw an angle of 85°.



Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively.

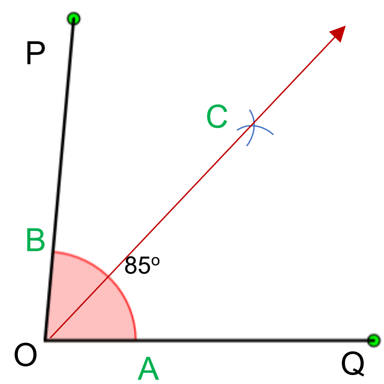


Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



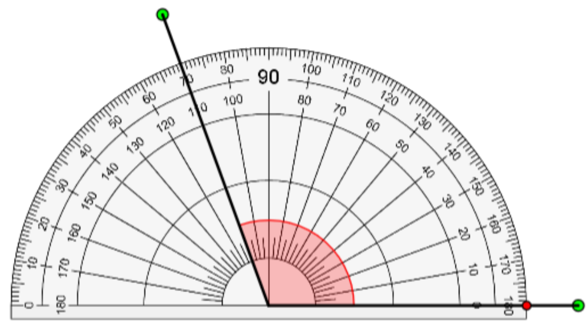
Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.

So, $m\angle POC = m\angle QOC = 85^\circ/2 = 42.5^\circ$.



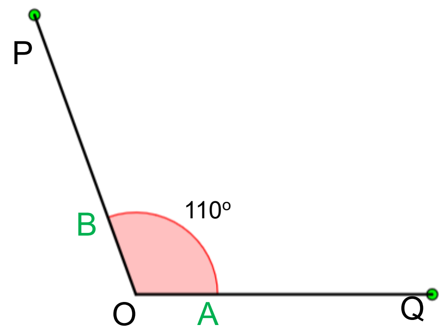
d) 110º

Steps of Construction:

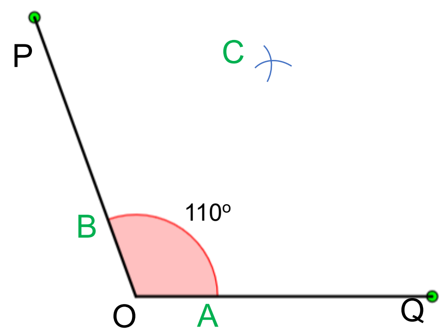


Step I: Use a protractor and draw an angle of 110º.

Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively.

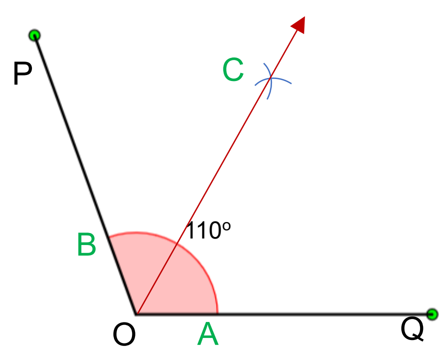


Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.

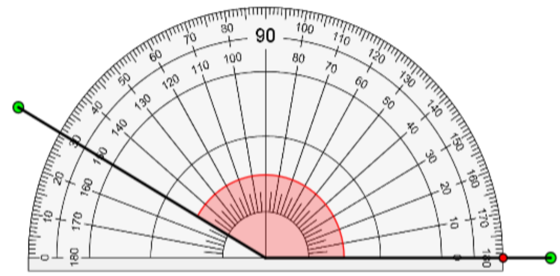
So, m∠POC = m∠QOC= 110º/2= 55º



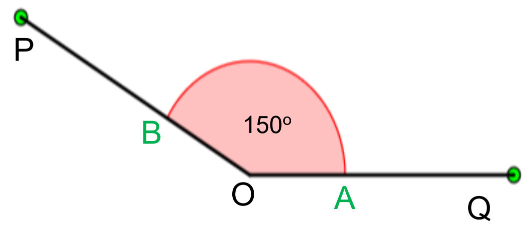
e) 150º

Steps of Construction:

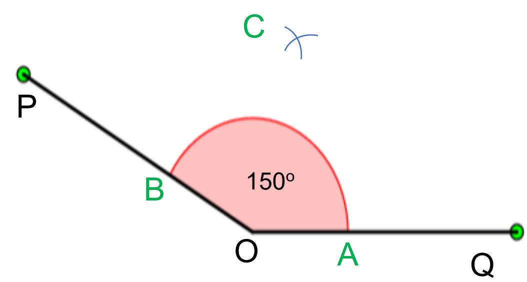
Step I: Use a protractor and draw an angle of 150º.



Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively.

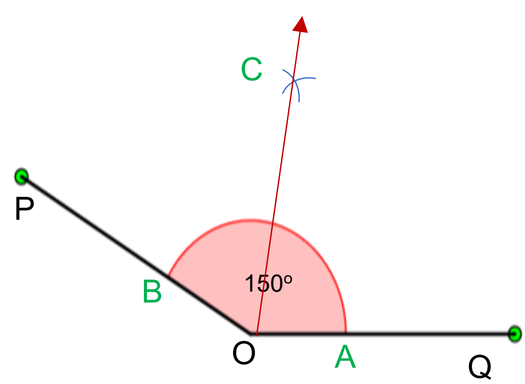


Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.

So, m∠POC = m∠QOC= 150º/2= 75º



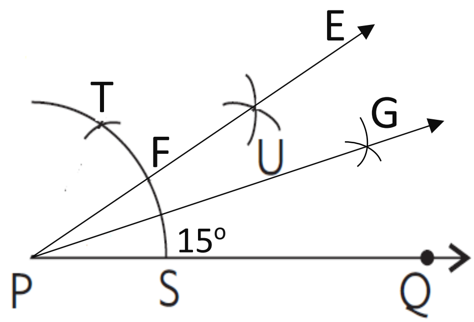
3. Construct the following angles by using a pair of compasses.

a) 15° b) 30° c) 45° d) 60° e) 75° f) 90° g) 120° h) 135°

Solution:

a) 15°

Steps of Construction:



Step I: Draw a ray PQ.

Step II: With P as centre, draw an arc of a suitable radius which cuts PQ at S.

Step III: Using the same opening of compasses, with S as centre, draw an arc which cuts the previous arc at point T.

Step IV: Using the same opening of compasses, with S and T as centres, draw arcs which cut each other at U.

Step V: Draw PE passing through U and cutting the arc $\widehat{ST}$ at F.

Step VI: with S and F as centres, draw arcs which cut each other at G.

Step I: Join P to G.

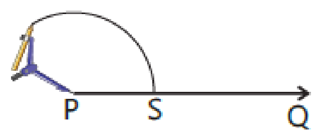
<GPQ= 15° is the required angle.

b) 30°

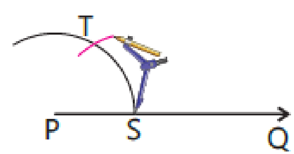
Steps of Construction:

Step I: Draw a ray $\overrightarrow{PQ}$ using a ruler.

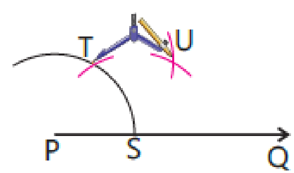
Step II: Place the pointer of the compass at point P and draw an arc of suitable radius that will cut the ray $\overrightarrow{PQ}$ at a point S.



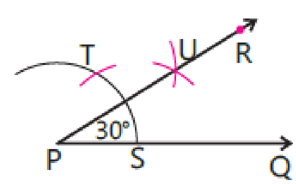
Step III: With the same opening of the compass, place the pointer of the compass at point S and draw an arc that will cut the previous arc at point T.



Step IV: Again with the same opening of the compass, place the pointer of the compass at point S and T and draw two arcs of same radius that will cut each other at point U.



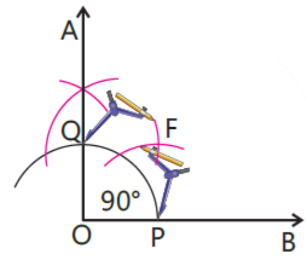
Step V: Using a ruler draw a ray $\overrightarrow{PR}$ passing through the point U. ∠QPR = 30° is the required angle.



c) 45°

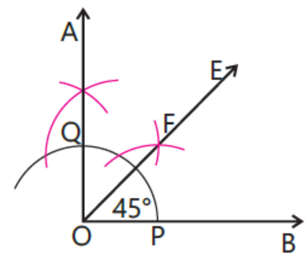
Steps of Construction:

Step I: Draw $\angle AOB = 90^\circ$.



Step II: Place the pointer of the compass at point P and Q and draw two arcs of same radius that will intersect each other at point F.

Step III: Using a ruler draw a ray $\overrightarrow{OE}$ passing through the point F. $\angle BOE = 45^\circ$ is the required angle.

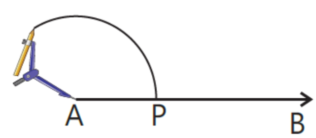


d)60°

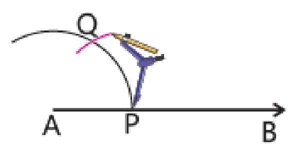
Steps of Construction:

Step I: Draw a ray $\overrightarrow{AB}$ using a ruler.

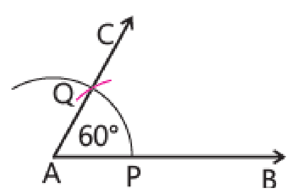
Step II: Place the pointer of the compass at point A and draw an arc of suitable radius that will cut the ray $\overrightarrow{AB}$ at a point P.



Step III: With the same opening of the compass, place the pointer of the compass at point P and draw an arc that will cut the previous arc at point Q.

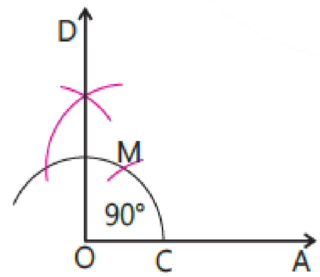


Step IV: Using a ruler draw a ray $\overrightarrow{AC}$ passing through the point Q. $\angle CAB = 60^\circ$ is the required angle.

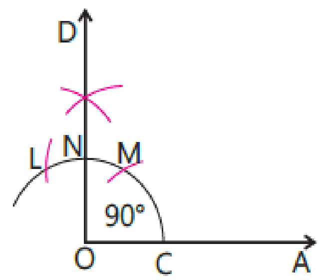


e) 75°

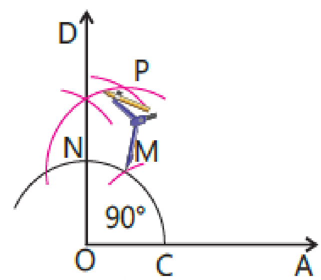
Step I: Draw an $\angle AOD$ equal to 90°.



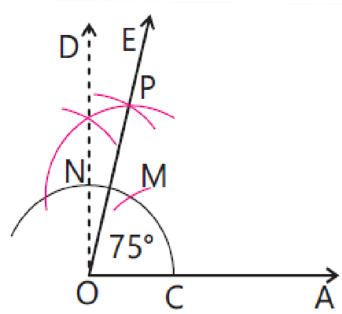
Step II: Mark a point N, where the arm $\overrightarrow{OD}$ of $\angle AOD$ is cut by the arc $\overset{\frown}{CL}$.



Step III: Place the pointer of the compass at point M and N and draw two arcs of same radius that will cut each other at point P.



Step IV: Using a ruler draw a ray $\overrightarrow{OE}$ passing through the point P. $\angle AOE = 75°$.

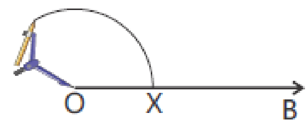


f) 90º

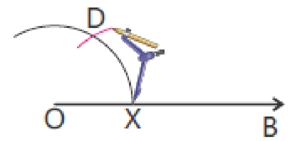
Steps of Construction:

Step I: Draw a ray $\overrightarrow{OB}$ using a ruler.

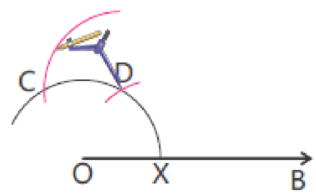
Step II: Place the pointer of the compass at point O and draw an arc of suitable radius that will cut the ray $\overrightarrow{OB}$ at a point X.



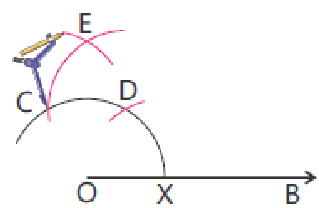
Step III: With the same opening of the compass, place the pointer of the compass at point X and draw an arc that will cut the previous arc at point D.



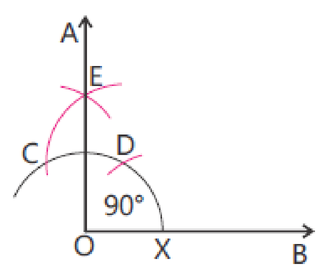
Step IV: Again using the same opening of the compass, place the pointer of the compass at point D and draw an arc that will cut the previous arc at point C.



Step V: Place the pointer of the compass at point D and C and draw two arcs of same radius that will cut each other at point E.



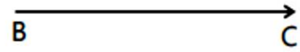
Step VI: Using a ruler draw a ray $\overrightarrow{OA}$ passing through the point E. $\angle AOB = 90°$ is the required angle.



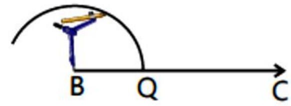
g) 120°

Steps of Construction:

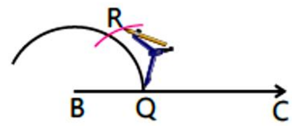
Step I: Draw a ray $\overrightarrow{BC}$ using a ruler.



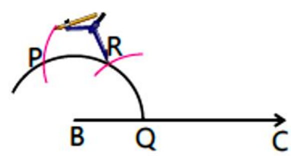
Step II: Place the pointer of the compass at point B and draw an arc of suitable radius that will cut the ray $\overrightarrow{BC}$ at any point Q.



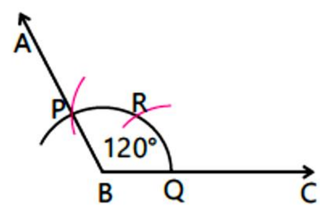
Step III: With the same opening of the compass, place the pointer of the compass at point Q and draw an arc that will cut the previous arc at point R.



Step IV: With the same opening of the compass, place the pointer of the compass at point R and draw an arc that will cut the initial arc at point P.

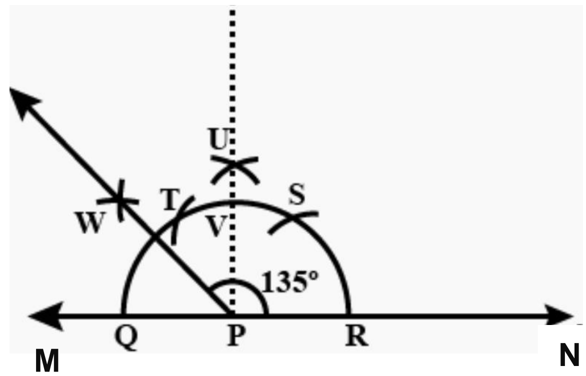


Step V: Using a ruler draw a ray $\overrightarrow{BA}$ passing through the point P. So, the $\angle ABC = 120°$.



h)135°

Steps of Construction:



Step I: Draw a line MN and mark a point P on it.

Step II: Taking P as centre and with a convenient radius, draw a semicircle which intersects line l at Q and R.

Step III: Taking R as centre and with the same radius as before, draw an arc intersecting the previously drawn arc to S.

Step IV: Taking S as centre and with the same radius as before, draw an arc intersecting the arc at T.

Step V: Taking S and T as centre, draw arcs of same radius to intersect each other at U.

Step VI: Join PU and let it intersect the arc at V.

Step VII: Taking Q and V as centres, draw arcs to intersect each other at W.

Join PW which is the required ray making 135° with line.

Unit Check

1. Encircle the correct answer.

a) The combination of points having no endpoint is called:
   i) point ii) segment iii) circle iv) line

b) The part of the line that has two endpoints is called:
   i) ray ii) point iii) line segment iv) square

c) The angle bisector bisects the angle exactly into:
   i) quarter ii) perpendicular iii) halves iv) any ratio

d) An angle can be constructed with the help of compass if it is a multiple of:
   i) 15 ii) 3 iii) 8 iv) 7

e) If we bisect a 180° angle, we get two angles of:
   i) 45° ii) 60° iii) 90° iv) 180°

2. Fill in the blanks.

a) A __line__ is a straight one dimensional figure which is a combination of points.

b) A __bisector__ is a line in geometry that divides a given line or angle into two parts.

c) A perpendicular bisector of a line segment always passes through the __center__ of the line segment.

d) The resulting angle of the bisection of 120° is __60°__.

e) Two lines or rays are said to be __perpendicular__ to each other if the angle between them is a right angle.

3. Draw the following line segments by using a ruler and then draw a perpendicular on it that bisects the lines.

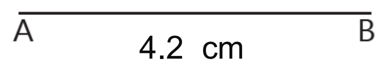
a) 4.2 cm b) 9.8 cm c) 12 cm d) 15.5 cm

Solution:

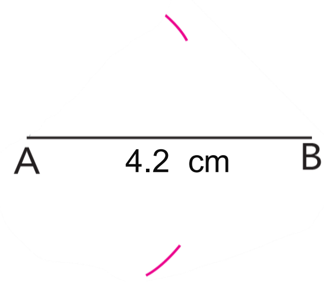
Steps of Construction:

a) 4.2 cm

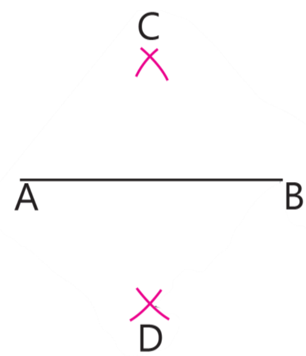
Step I: Draw a line segment AB=4.2 cm



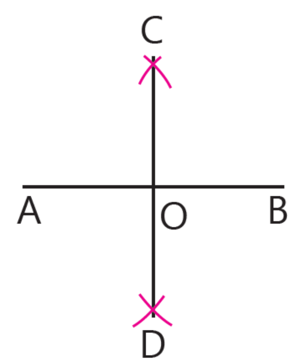
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.

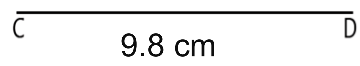


CD is the right bisector of AB and O is the mid point.

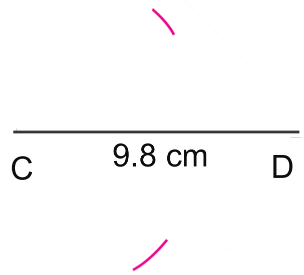
b) 9.8 cm

Steps of Construction

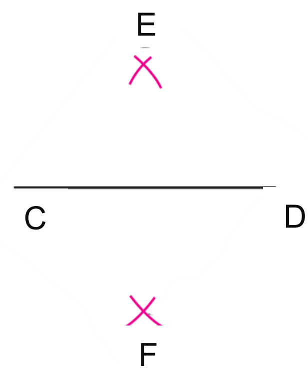
Step I: Draw a line segment CD=9.8 cm



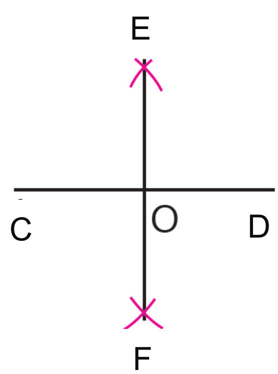
Step II: Place the pointer of the compass at point C. Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point D and draw two arcs on the top and bottom of CD which intersects the previous arcs at point E and F respectively.



Step IV: Join the point E and F by a line using a ruler that will cut the line segment CD at point O.

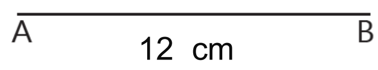


EF is the right bisector of CD and O is the mid point.

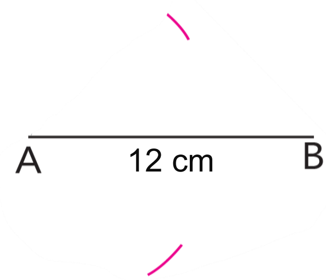
c) 12 cm

Steps of Construction:

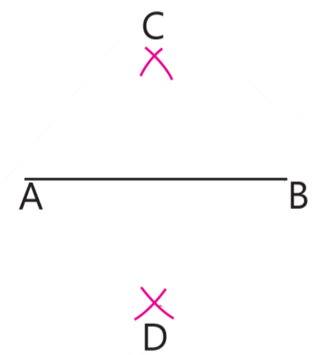
Step I: Draw a line segment AB=4.2 cm



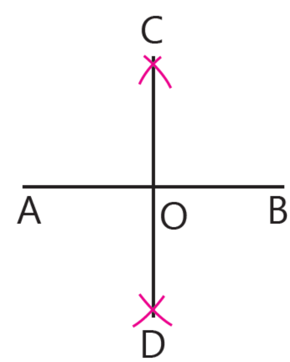
Step II: Place the pointer of the compass at point A. Open the compass with more than half of the measure of the line segment AB and draw an arc on the top and bottom of AB.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point B and draw two arcs on the top and bottom of AB which intersects the previous arcs at point C and D respectively.



Step IV: Join the point C and D by a line using a ruler that will cut the line segment AB at point O.

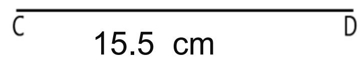


CD is the right bisector of AB and O is the mid point.

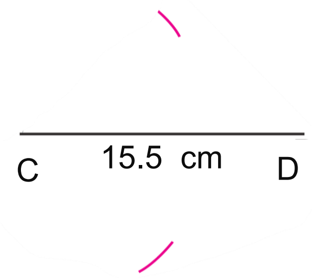
d) 15.5 cm

Steps of Construction

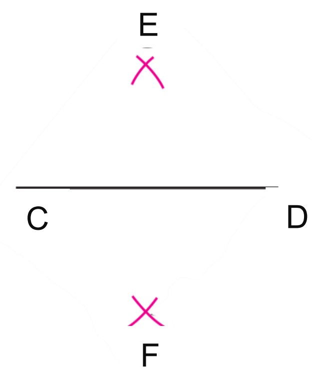
Step I: Draw a line segment CD=15.5 cm



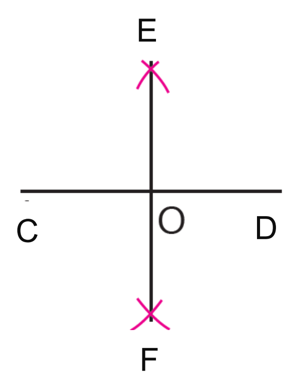
Step II: Place the pointer of the compass at point C. Open the compass with more than half of the measure of the line segment CD and draw an arc on the top and bottom of CD.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point D and draw two arcs on the top and bottom of CD which intersects the previous arcs at point E and F respectively.



Step IV: Join the point E and F by a line using a ruler that will cut the line segment CD at point O.



EF is the right bisector of CD and O is the mid point.

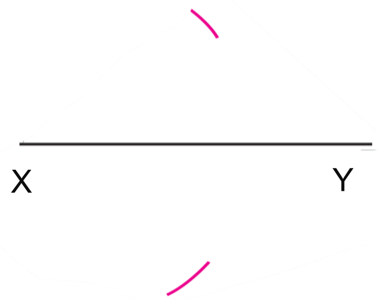
4. Draw the right bisectors of the following line segments by using a pair of compasses.

a)

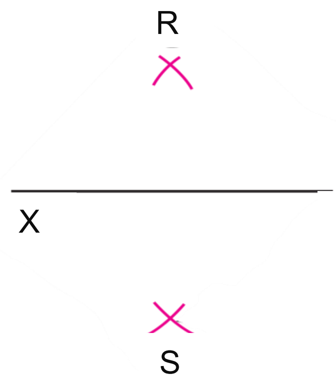
Steps of Construction

Step I: Draw a line segment XY.

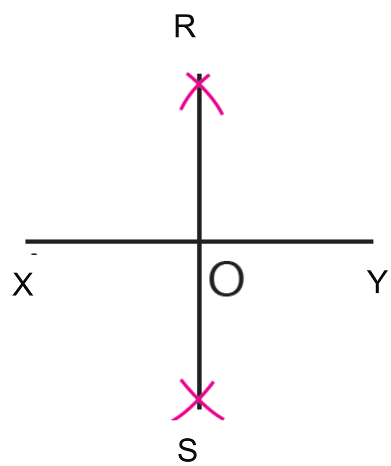
Step II: Place the pointer of the compass at point X. Open the compass with more than half of the measure of the line segment XY and draw an arc on the top and bottom of XY.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point Y and draw two arcs on the top and bottom of XY which intersects the previous arcs at point R and S respectively.



Step IV: Join the point R and S by a line using a ruler th will cut the line segment XY at point O.

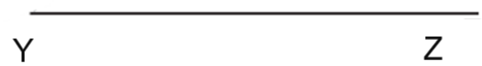


RS is the right bisector of XY and O is the mid point.

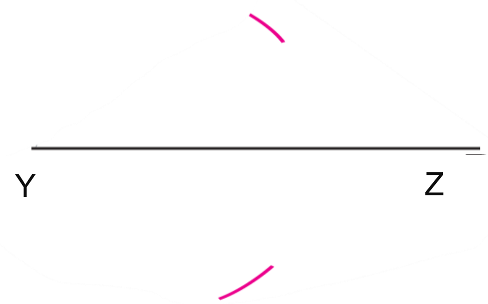
b)

Steps of Construction

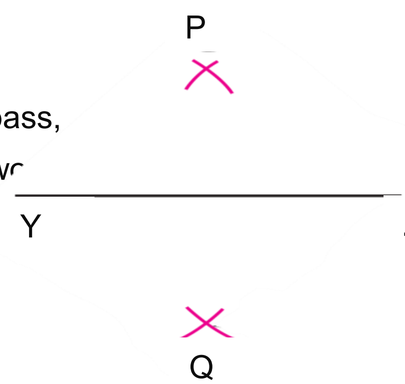
Step I: Draw a line segment YZ.



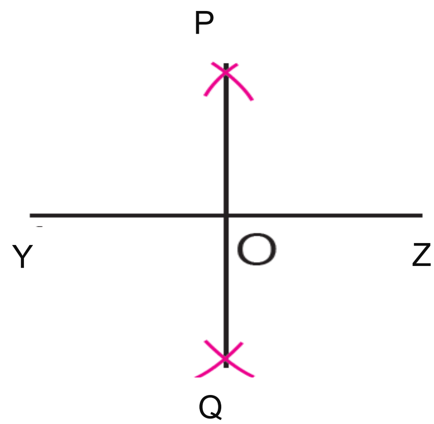
Step II: Place the pointer of the compass at point Y. Open the compass with more than half of the measure of the line segment YZ and draw an arc on the top and bottom of YZ.



Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point Z and draw two arcs on the top and bottom of YZ which intersects the previous arcs at point P and Q respectively.



Step IV: Join the point P and Q by a line using a ruler will cut the line segment YZ at point O.

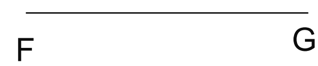


PQ is the right bisector of YZ and O is the mid point.

c)

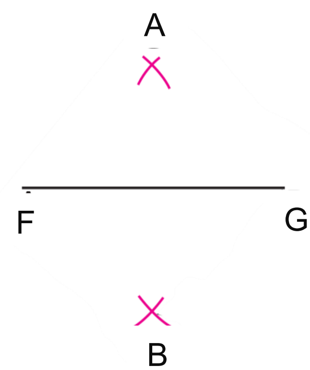
Steps of Construction

Step I: Draw a line segment FG.

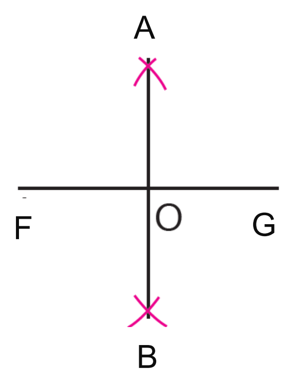


Step II: Place the pointer of the compass at point F. Open the compass with more than half of the measure of the line segment FG and draw an arc on the top and bottom of FG.

Step III: Similarly, using the same opening of the compass, place the pointer of the compass at point G and draw two arcs on the top and bottom of FG which intersects the previous arcs at point A and B respectively.



Step IV: Join the point A and B by a line using a ruler that will cut the line segment FG at point O.



AB is the right bisector of FG and O is the mid point.

5. Draw a perpendicular bisector on the given line segments from the points.

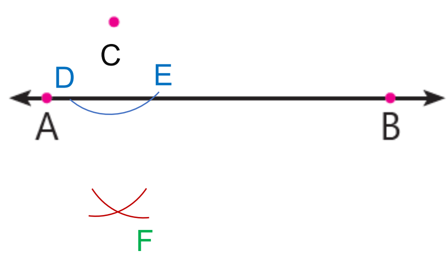
a)

AB is the given line. A point C (not on it) is also given.

Steps of Construction:

Step I: Place the pointer of the compass at point C and draw an arc that will cut the line AB at two points D and E respectively.

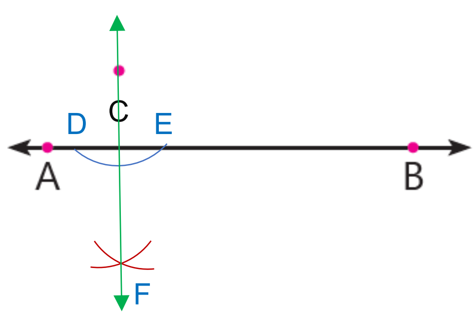
Step II: Using the same opening of the compass, place the pointer of the compass at point D and E and draw two arcs on the other side of AB that cut each other at point F.



Step III: Join C to F.

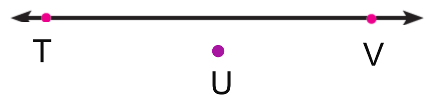
CF is the required perpendicular to the given line AB through point C not on it.

∴ CF ⊥ AB



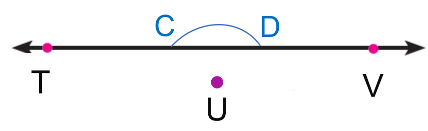
b)

TV is the given line. A point U (not on it) is also given.

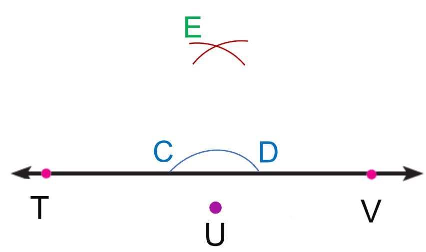


Steps of Construction:

Step I: Place the pointer of the compass at point U and draw an arc that will cut the line TV at two points C and D respectively.

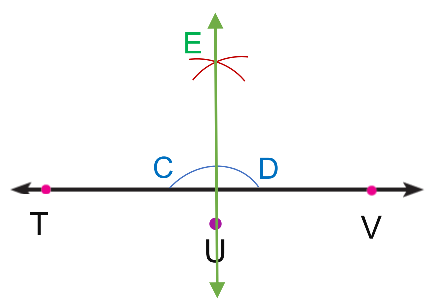


Step II: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of TV that cut each other at point E.



Step III: Join U to E.

UE is the required perpendicular to the given line TV through point U not on it.



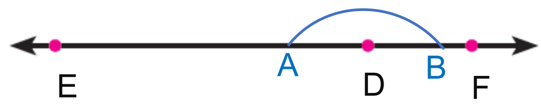
∴ UE ⊥ TV

c)

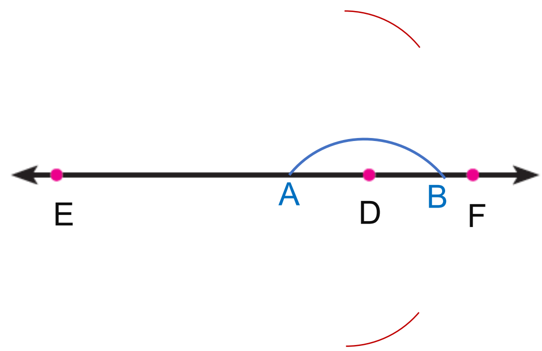
Steps of constructions:

EF is the given line and D is a point on it.

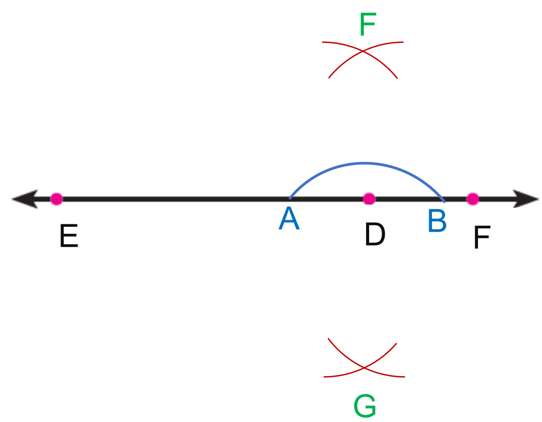
Step I: Place the pointer of the compass at point D and draw an arc of suitable radius that will cut the line EF at two points A and B respectively, such that AD = BD.

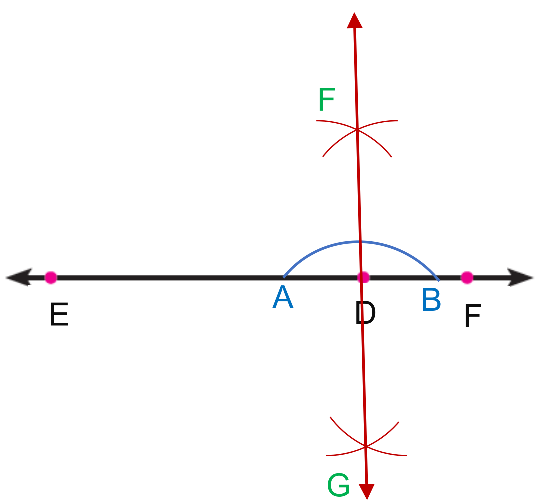


Step II: Place the pointer of the compass at point A and draw two arcs at top and bottom of radius greater than AD as shown in the figure.



Step III: With the same opening of the compass, place the pointer of the compass at point B and draw two arcs at top and bottom which cuts the previous arcs at point F and G as shown in the figure.





Step IV: Join the point F to D and extend it to G. So, FG is the required perpendicular to the given line EF at point D.

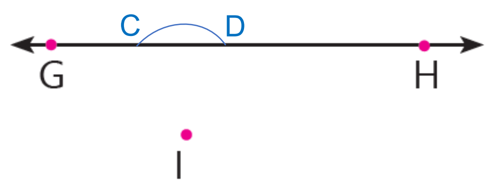
∴ $FG \perp EF$

d)

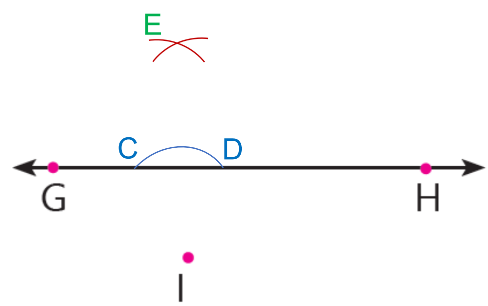
GH is the given line. A point I (not on it) is also given.

Steps of Construction:

Step I: Place the pointer of the compass at point I and draw an arc that will cut the line GH at two points C and D respectively.



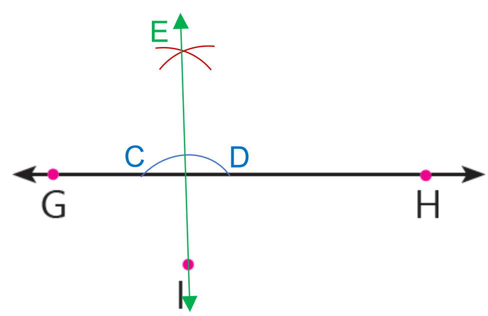
Step II: Using the same opening of the compass, place the pointer of the compass at point C and D and draw two arcs on the other side of GH that cut each other at point E.



Step III: Join I to E.

IE is the required perpendicular to the given line GH through point I not on it.

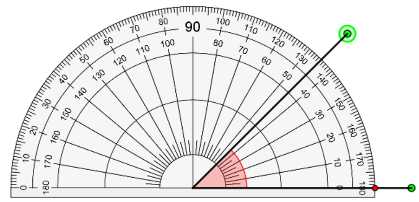
∴ EI ⊥ GH



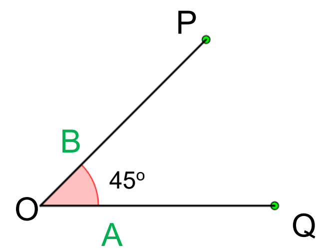
6. Construct the following angles by using a protractor and bisect them by using a pair of compasses.

a) 45°

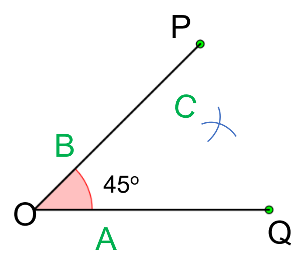
Step I: Use a protractor and draw an angle of 45º.



Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively

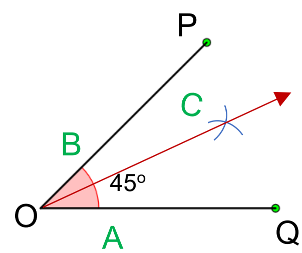


Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



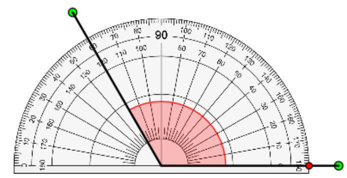
Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.

So, m∠POC = m∠QOC= 45º/2= 22.5º.

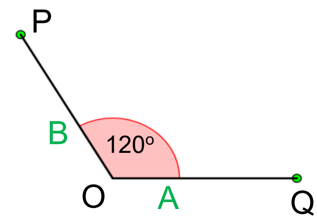


b) 120°

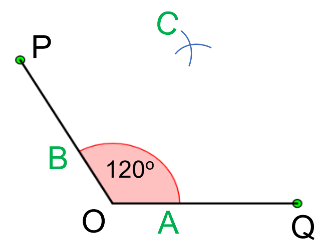
Step I: Use a protractor and draw an angle of 120°.



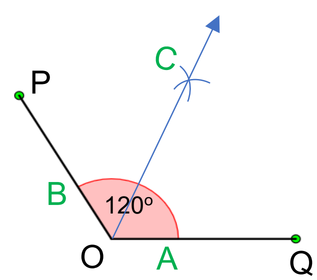
Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively



Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



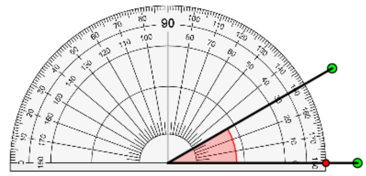
Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.



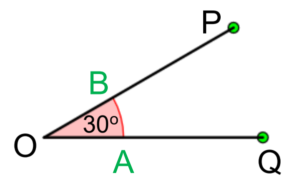
So, m∠POC = m∠QOC= 120°/2= 60°.

c) 30°

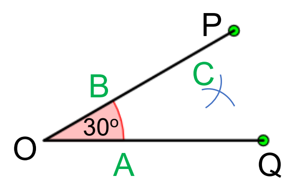
Step I: Use a protractor and draw an angle of 30°.



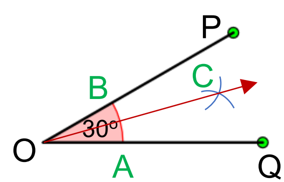
Step II: Name the vertex as O and arms of the given angles as OQ and OP respectively. Also name the points where the arc of angle cuts ray OQ and OP as A and B respectively



Step III: Place the pointer of the compass at point A and B and draw two arcs of suitable radius that will cut each other at point C.



Step IV: Using a ruler draw a ray OC passing through the point C. This line is bisecting the angle ∠POQ into two equal parts.



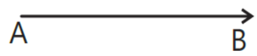
So, m∠POC = m∠QOC= 30°/2= 15°.

7. Construct the following angles by using a pair of compasses.

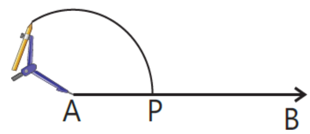
a) 60°

Steps of Construction:

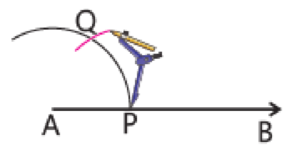
Step I: Draw a ray $\overrightarrow{AB}$ using a ruler.



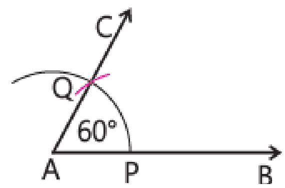
Step II: Place the pointer of the compass at point A and draw an arc of suitable radius that will cut the ray $\overrightarrow{AB}$ at a point P.



Step III: With the same opening of the compass, place the pointer of the compass at point P and draw an arc that will cut the previous arc at point Q.



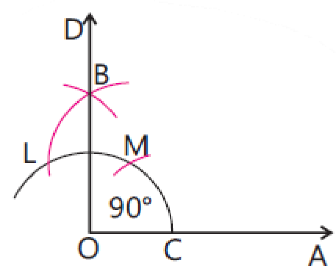
Step IV: Using a ruler draw a ray $\overrightarrow{AC}$ passing through the point Q. $\angle CAB = 60°$ is the required angle.



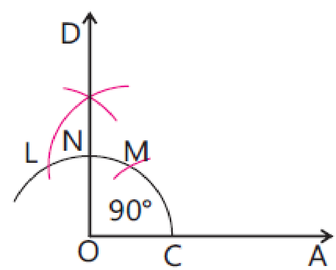
b) 105°

Steps of Construction:

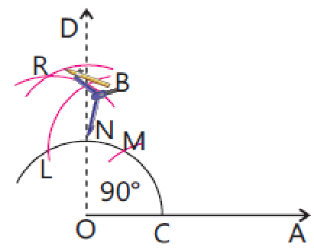
Step I: Draw an $\angle AOD = 90°$.



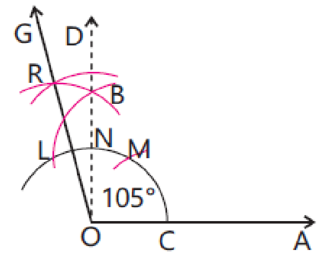
Step II: Mark a point N, where the arm $\overrightarrow{OD}$ of $\angle AOD$ is cut by the arc $\overset{\frown}{CL}$.



Step III: Place the point of the compass at point N and L and draw two arcs of same radius that will cut each other at point R.

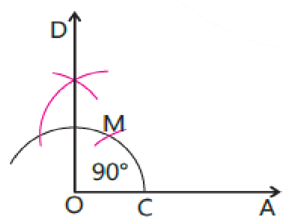


Step IV: Using a ruler draw a ray $\overrightarrow{OG}$ passing through the point R. So, $\angle AOG = 105°$

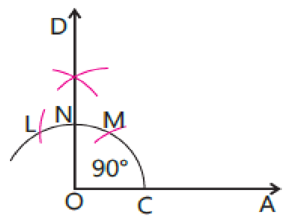


c) 75°

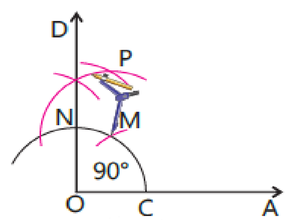
Step I: Draw an $\angle AOD$ equal to 90°.



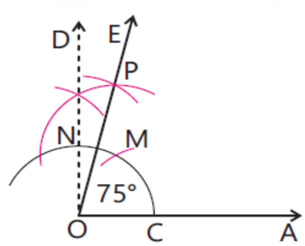
Step II: Mark a point N, where the arm $\overrightarrow{OD}$ of $\angle AOD$ is cut by the arc $\overset{\frown}{CL}$.



Step III: Place the pointer of the compass at point M and N and draw two arcs of same radius that will cut each other at point P.



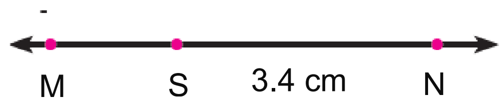
Step IV: Using a ruler draw a ray $\overrightarrow{OE}$ passing through the point P. $\angle AOE = 75°$.



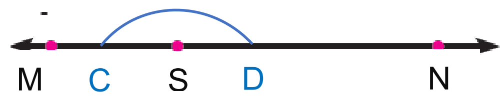
8. Draw a line segment MN of length 3.4 cm and then draw a perpendicular bisector on it from point S that is on it.

Steps of constructions:

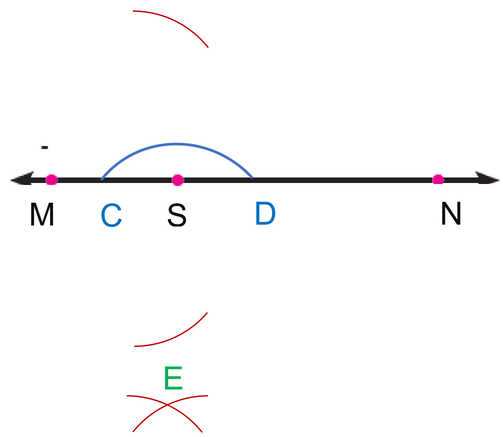
Step I: Draw a line segment MN= 3.4 cm and mark a point S on it.



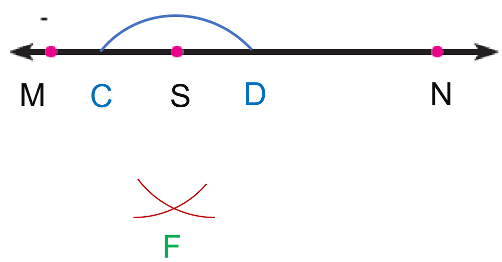
Step II: Place the pointer of the compass at point S and draw an arc of suitable radius that will cut the line MN at two points C and D respectively, such that SC = SD.

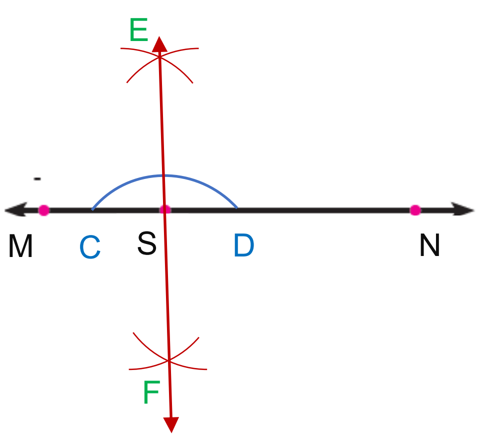


Step III: Place the pointer of the compass at point C and draw two arcs at top and bottom of radius greater than SC as shown in the figure.



Step IV: With the same opening of the compass, place the pointer of the compass at point D and draw two arcs at top and bottom which cuts the previous arcs at point E and F as shown in the figure.





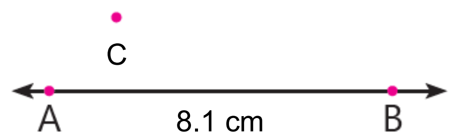
Step V: Join the point E to S and extend it to F. So, EF is the required perpendicular to the given line MN at point S.

∴ EF ⊥ MN

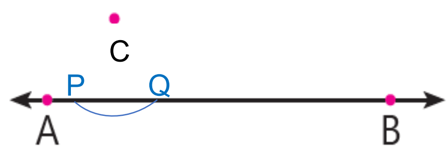
9. Draw a line segment AB of length 8.1 cm and then draw a perpendicular bisector on it from point C that is not on it.

Steps of Construction

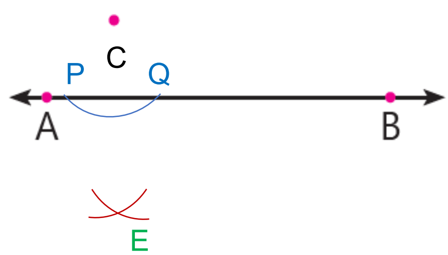
Step I: Draw a line segment AB=8.1 cm and mark a point C (not on it).



Step II: Place the pointer of the compass at point C and draw an arc that will cut the line AB at two points P and Q respectively.



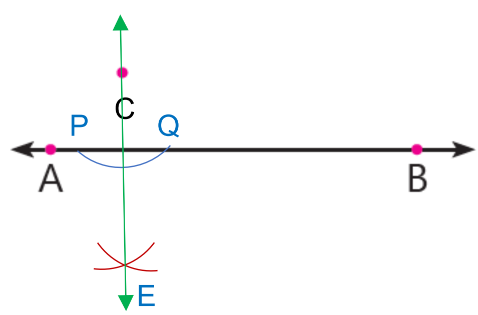
Step II: Using the same opening of the compass, place the pointer of the compass at point P and Q and draw two arcs on the other side of AB that cut each other at point E.



Step III: Join C to E.

CE is the required perpendicular to the given line AB through point C not on it.

$\therefore CE \perp AB$

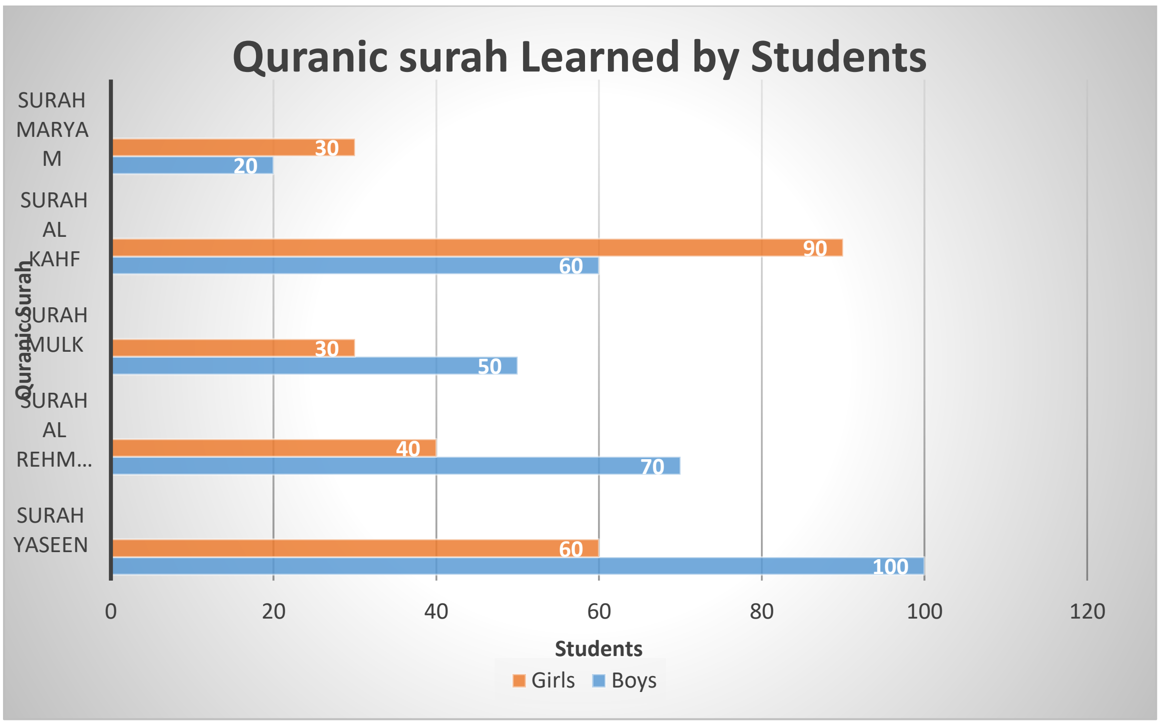


Learning Check 10.1

1. **The following data shows information about Hifz School students and surahs they have learned. Draw a horizontal multiple bar graph for this data using an appropriate scale.**

Students	Surah yaseen	Surah Al Reman	Surah Mulk	Surah Al Kahf	Surah Maryam
Boys	100	70	50	60	20
Girls	60	40	30	90	30

Solution: We use the following steps to draw the bar graph.



Step I: Draw an x-axis (horizontal line) and a y-axis (vertical line) perpendicular to each
other on graph paper.

Step II: Write the number of Surah's along the x-axis and the Students along the y-axis.Choose a color for each category (Surah Yaseen, Reman, Mulk, Alkahf,and Maryam).

Step III: Choose an appropriate scale. Here, one square represents one verse along the x-axis.

Step IV: Number of Boys and Girls Hifz all Surah mark on the graph paper on x-axis. So, on the y-axis we will mark the students (Boys & Girls)

Similarly, keep coloring up to the correct mark for each Surah's and draw the multiple bars.

The width of the bars must be the same throughout the multiple bar graph.

2. **The following data shows the marks in mathematics in the midterm and final term exams of five students in grade 6. Draw a vertical multiple bar graph for this data using and appropriate scale.**

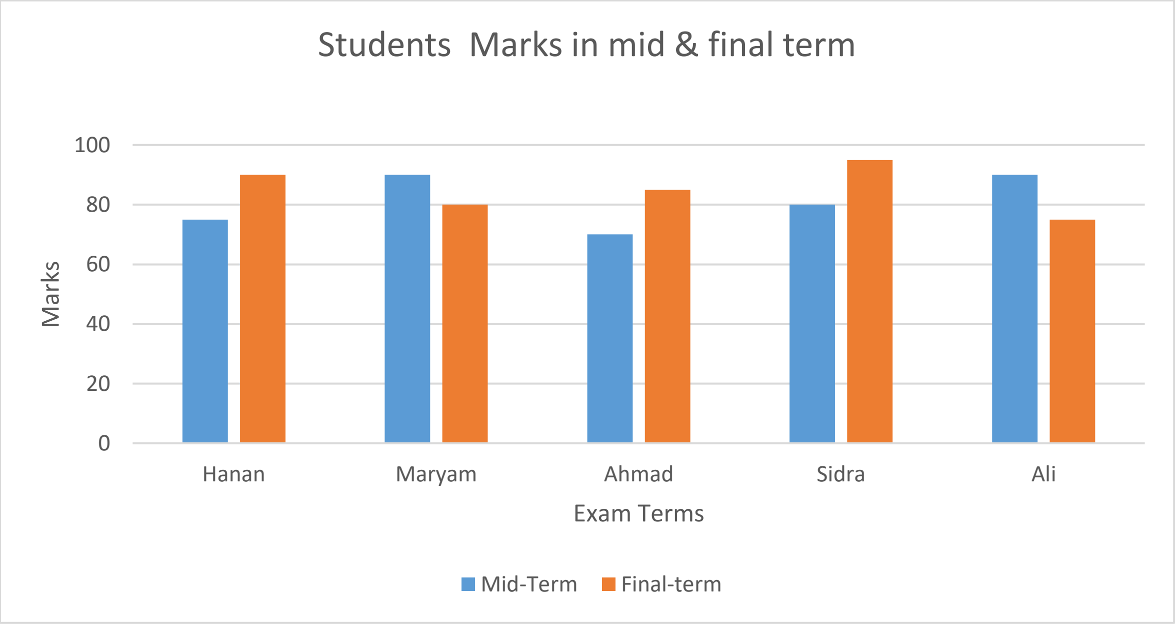
Examination	Hanan	Maryam	Ahmad	Sidra	Ali
Mid term	75	90	70	80	90
Final term	90	80	85	95	75

Solution: We use the following steps to draw a multiple bar graph

Step I: Draw horizontal and vertical lines that are perpendicular to each other on graph
paper.

Step II: Write the mid and final term along the x-axis and the number obtained by the students on the y-axis.

Choose a colour for each category (mid and final term).



Step III: Choose an appropriate scale. Here, 1 square represents 10 number along the y-axis.

Step IV: According to the given table, there are 5 students obtained the marks in the mid and final term. So, we will colour up to the mark of 5 students on the y-axis (using the pre-assigned colour)

Similarly, keep colouring up to the correct mark for each section and draw the multiple bars. The width of the bars must be the same throughout the multiple bar graph.

3. **The following data shows the results of a survey of 500 people about their favorite sports by age group. Draw a horizontal multiple bar graph for this data using an appropriate scale.**

Sports	**11-20**	**21-30**	**31-40**	**41-50**
Football	**90**	**40**	**60**	**50**
Cricket	**60**	**80**	**20**	**10**
Tennis	**30**	**20**	**30**	**10**

Solution: We use the following steps to draw the bar graph

Step I: Draw an x-axis (horizontal line) and a y-axis (vertical line) perpendicular to each

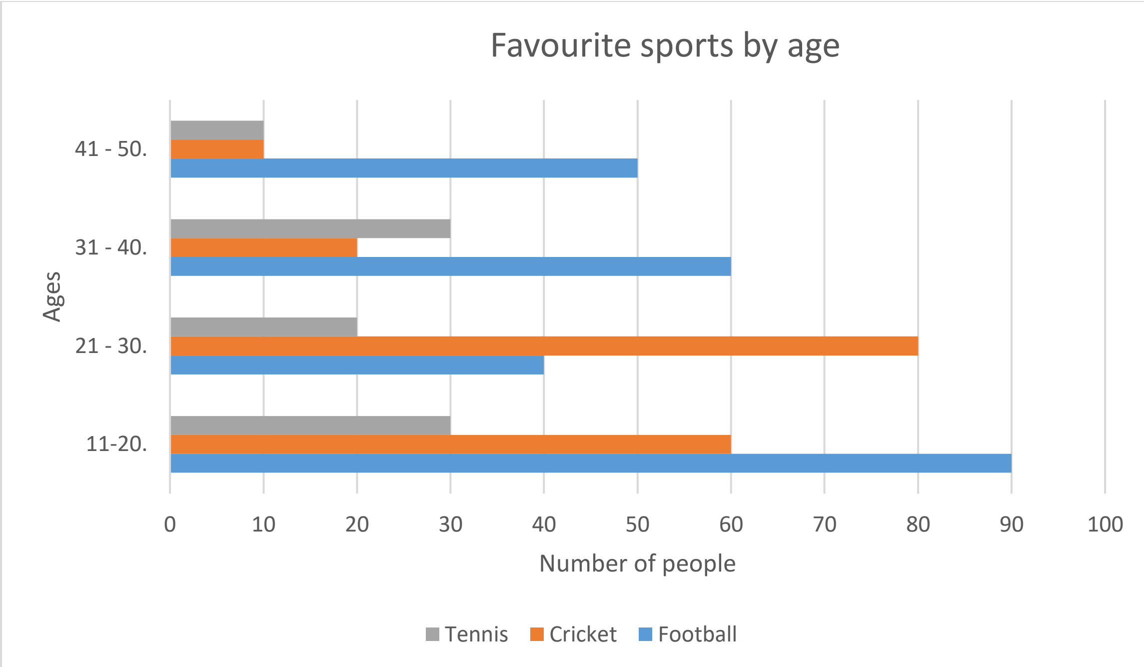
other on graph paper.

Step II: Write the number of people likes games by their age group along the x-axis and the games along the y-axis. Choose a color for each category.

Step III: Choose an appropriate scale. Here, one square represents no. of people along the x-axis.

Step IV: According to given data different age group of the people likes games (Football, Cricket, Tennis) are marked.

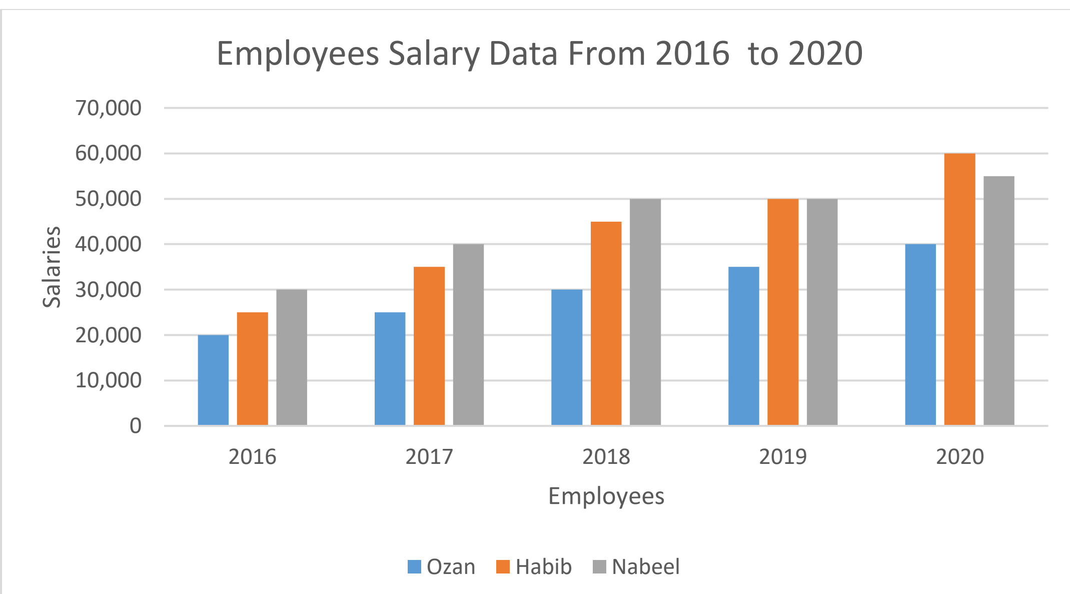
The width of the bars must be the same throughout the multiple bar graph.



4. **The following data shows the salaries of three different employees over a five-year period. Draw a vertical multiple bar graph for this data using an appropriate scale.**

Employees	2016	2017	2018	2019	2020
Ozan	20,000	25,000	30,000	35,000	40,000
Habib	25,000	35,000	45,000	50,000	60,000
Nabeel	30,000	40,000	50,000	50,000	55,000

Solution:



Step I: Draw horizontal and vertical lines that are perpendicular to each other on graph
paper.

Step II: Write the employees name along the x-axis and the salary of employees on the y-axis. Choose a colour for each employee salary.

Step III: Choose an appropriate scale. Here, 1 square represents 10,000 salary along the y-axis.

Step IV: According to the given table there is twenty thousand salary of Ozan in 2016. So, we will color up to mark of 20,000 on the y-axis for Ozan and up to the mark 25,000 on the y-axis for Habib salary. Similarly keep colouring up to the correct mark for each year and draw the multiple bars. The width of the bars must be the same throughout the multiple bar graph.

5. Observe the given multiple bar graph in horizontal form and answer the questions given.

Solution:

a) What is the title of the graph?
Solution: Temperature of Different Cities

b) What information is shown along the x-axis?
Solution: Temperature of different cities shown along the x-axis.

c) What information is shown along the y-axis?
Solution: Days of the week

d) What is the temperature on Monday in cities A and B?
Solution: City A= 10^0C & City B= 7.5°C

e) What is the temperature on Wednesday in city C?
Solution: 22^0C

f) Which city has the highest temperature and which city has the lowest temperature on Friday?
Solution: C and A

g) Which city has the lowest temperature on which day?
Solution: Monday, City B

h) What is the total temperature for Wednesday and Thursday together in city A?
Solution: 28°C

i) What is the difference between the temperature on Friday and Saturday in city B?
Solution: 6°C

6. Observe the given multiple bar graph in vertical form and answer the questions given.

a) What is the title of the graph?

Solution: Cups of coffee sold.

b) What information is shown along the x-axis?

Solution: Month of the year

c) What information is shown along the y-axis?

Solution: Number of cups.

d) How many cups were sold in March?

Solution: 30 cups

e) How many coffee cups were sold in February?

Solution: 105 cups

f) What is the difference between the coffee cups sold by the two malls in January?

Solution: 20

g) In which month did the least number of coffee cups get sold?

Solution: May

h) In which month were the greatest number of cups sold?

Solution: February

i) What is the total number of coffee cups sold from January to April in Mall 2?

Solution: 155 cups

j) What is the difference between the total number of cups sold in March and May?

Solution: 15

Learning check 10.2

1. Ali worked at a camping store and sold out of certain items in November. Draw a pie graph for the following data.

Item names	**Binoculars**	**Bags**	**Torches**	**Tents**
No. of Items	**16**	**24**	**32**	**8**

Solution:

Step 1: Find the total sum of items: 16 +24 + 32 + 8 = 80 Now, we have to find the angle for the sector of each category.

Step II: Find the measurement of the central angles by using the formula.

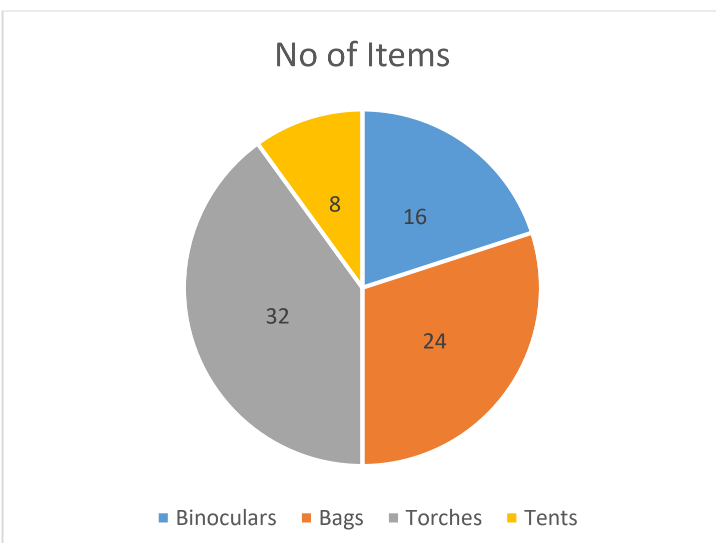
Item Names	**No. of items**	**Central angle for items**
Binoculars	16	$16/80 \times 360^o = 72^o$
Bags	24	$24/80 \times 360^o = 108^o$
Torches	32	$32/80 \times 360^o = 144^o$
Tents	8	$8/80 \times 360^o = 36^o$

Step III:

Draw a circle of any radius and draw the radial segment.

Step IV: Take a protractor and place it in the center of the circle. Draw an angle of 72^o for binoculars.

Step V: Again, place a protractor and align it on the center of the circle aligned with the radial segment (the arm of 72^o). Draw an angle of 108^o for bags. $144^{o,}$ 36^o Continue the procedure and make all the required angles for each item.



2. The data shows the number of people who say their prayers in the nearby masjid. Draw a pie graph for the following data.

Solution:

Prayers	**Fajr**	**Zuhr**	**Asar**	**Maghrib**	**Isha**
No. of people	**110**	**150**	**160**	**120**	**100**

Solution:

Step 1: Find the total sum of Prayers: 110 + 150 + 160 +120 +100 = 640

Now, we have to find the angle for the sector of each category.

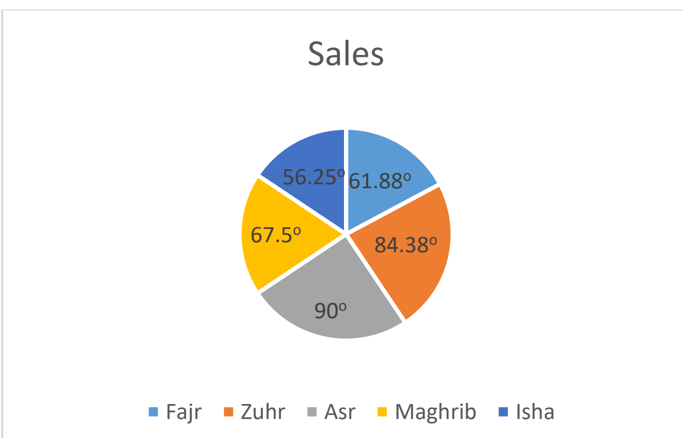
Step II: Find the measurement of the central angles by using the formula.

Prayers	**No.of people**	**Measure of the Central angle for Payers**
Fajr	110	$110/640 \times 360^o = 61.88^o$
Zuhr	150	$150/640 \times 360^o = 84.38^o$
Asar	160	$160/640 \times 360^o = 90^o$
Maghrib	120	$120/640 \times 360^o = 67.5^o$
Isha	100	$100/640 \times 360^o = 56.25^o$

Step II: Draw a circle of any radius and draw the radial segment AB.

Step III:

Take a protractor and place it in the center of the circle. Draw an angle of 61^o.for the first category Prayer, i.e., Fajar.



Step IV:

Again, place a protractor and align it on the center of the

circle aligned with the radial segment (the arm of 61°). Draw
an angle of 84° for the second category, i.e., Zuhr.

Step V:

Continue the procedure and make all the required angles
for each category.

This is the required pie graph for the given data.

3. **Asia and her friend went to the birdhouse and counted the number of birds of different types. Draw a pie graph for the following data.**

Birds	**Eagle**	**Goldfinch**	**Starling**	**Parrot**	**Pheasant**
No. of Birds	**80**	**60**	**120**	**90**	**30**

Solution:

Step 1: Find the total sum of Birds: 80 +60+120+90+ 30 = 380 Now, we have to find the angle for the sector of each category.

Step II: Find the measurement of the central angles by using the formula.

Birds	**No. of Birds**	**Measure of central angle of no. of birds**
Eagle	80	No. of Birds/Total sum x 360° = 80/380 x 360 = 75.79°
Goldfinch	60	60/380 x 360° = 56.84°
Starling	120	120/380 x 360° = 113.68°
Parrot	90	90/380 x 360° = 85.26°
Pheasant	30	30/380 x 360° = 28.42°

Step II: Draw a circle of any radius and draw the radial segment AB.

Step III:

Take a protractor and place it in the center of the circle. Draw an angle of 75.79°.for the first category Bird, i.e., Eagle.

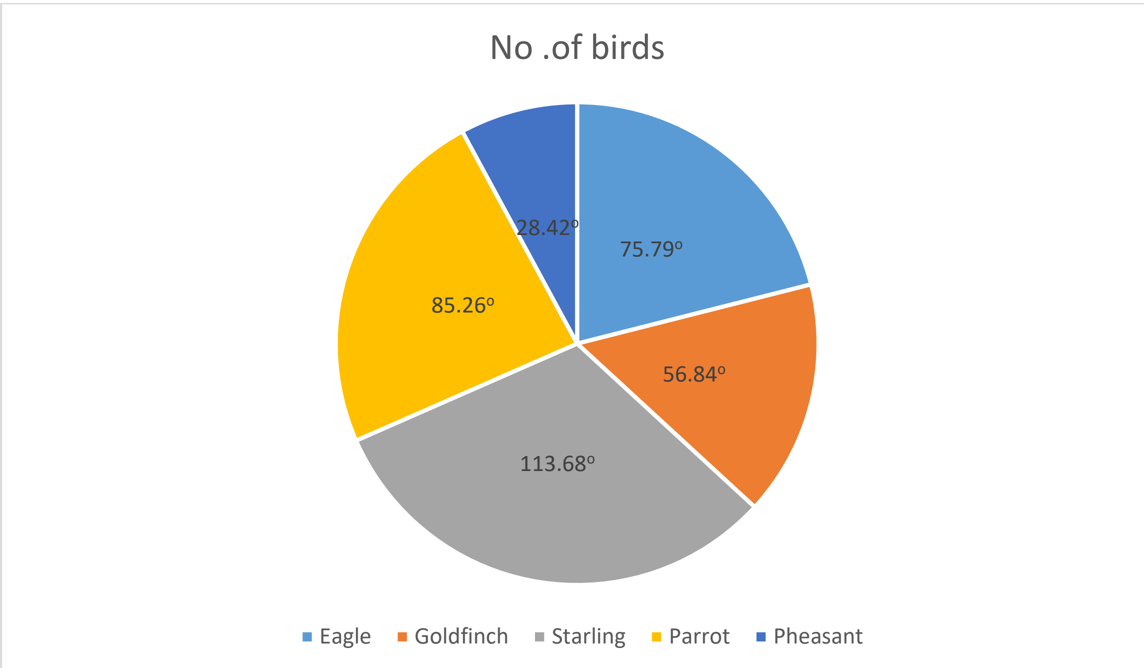
Step IV:

Again, place a protractor and align it on the center of the circle aligned with the radial segment (the arm of 56.84°). Draw an angle of 56.84° for the second category, i.e., Goldfinch.

Step V:

Continue the procedure and make all the required angles for each category.

This is the required pie graph for the given data.



4. Arham's activities for the whole day are shown by a pie graph. Read the graph and answer the following questions: (Hint: 1 day = 24 hours.)

a) How much time does he spend reciting the Quran?

Solution:

Total sum of Arham's activities = 360

Reciting Quran = 60^o

Hours for Reciting Quran = $60^o/360^o$ x 360 = 60

As we know

1 day = 24 hours

360^o/24hours =15^o/hour

So according to this knowledge we get

60/15 = 4 hours

b) How many hours does he sleep?

Solution:

Total sum of Arham's activities = 360

Sleeping = 120^o

Hours for Sleeping = $120^o/360^o$ x 360 = 120

As we know

1 day = 24 hours

360^o/24hours =15^o/hour

So according to this knowledge we get

120/15 = 8 hours

c) How many hours does he play?

Solution:

Total sum of Arham's activities = 360

Playing = 45^o

Hours for Playing = $45^o/360^o$ x 360 = 45

As we know

1 day = 24 hours

360^o/24hours =15^o/hour

So according to this knowledge we get

45/15 = 3 hours

d) In which activity does he spend the least amount of time and by how much?

Solution:

Arham spend the least amount of time in Studying and Playing i.e 45^o

Total sum of Arham's activities = 360

playing = 45^o

Hours for Playing = $45^o/360^o$ x 360 = 45

As we know

1 day = 24 hours

360^o/24hours =15^o/hour

So according to this knowledge we get

45/15 = 3 hours.

By calculation we come to know he spends 3 ,3 hours in Studying and Playing.

e) In which activity does he spend more time and by how much?

Solution:

Arham spend the more time in sleeping i.e. 120^o

Total sum of Arham's activities = 360

Sleeping = 120^o

Hours for Sleeping = $120^o/360^o$ x 360 = 120

As we know

1 day = 24 hours

$360^o/24$hours $=15^o$/hour

So according to this knowledge we get

120/15 = 8 hours.

By calculation we come to know he spends 8 hours in Sleeping.

f) How much time does he spend at school?

Solution:

Total sum of Arham's activities = 360

At school = 90^o

Hours at school = $90^o/360^o \times 360 = 90$

As we know

1 day = 24 hours

$360^o/24$hours $=15^o$/hour

So according to this knowledge we get

90/15 = 6 hours.

By calculation we come to know he spends 6 hours at school.

g) Tell the difference in time between playing and studying.

Solution:

From above answers we come to know that Arham spends time in playing = 3 hours

As studying and playing time is same that is 3 hours so

3 hours – 3 hours = 0 hours

Difference in time between playing and studying = 0

5. **Ehsan's salary is Rs 50,000. He made a monthly budget for his family. Read the graph and answer the following questions**.

   a) How much does he spend on house rent?

   Solution:

   Ehsan's salary = Rs 50,000

   he spends on house rent = 35^o

   Ehsan spent on house rent = $35^o/360^o \times 50{,}000$ = Rs 4861.11

   b) How much does he spend on education?

   Solution:

   Ehsan's salary = Rs 50,000

   he spends on Education = 70^o

   Ehsan spent on Education= $70^o/360^o \times 50{,}000$= Rs 9722.22

   c) Which item does he spend the most money on and by how much?

Solution:

Food**,** Ehsan's salary = Rs 50,000

he spends on Food = 140^o

Ehsan spent on Food= $140^o/360^o$ x 50,000= Rs 19444.44

d) What amount does he save?

Solution:

Ehsan's salary = Rs 50,000

He Saves = 25^o

Ehsan saves = $25^o/360^o$ x 50000= Rs 3472.22

e) On which things does he spend the same amount?

Solution:

Education, Clothing

f) How much does he spend on clothing?

Solution:

Ehsan's salary = Rs 50,000

He spends on Clothing = 70^o

Ehsan spent on Clothing= $70^o/360^o$ x 50000= Rs 9722.22

6. **The following pie graph shows the number of students in school from grades 4 to 8. There are 300 students in all from grades 4 to 8. Read the graph and answer the following questions:**

a) How many students are in grades 6 and 7 in all?

Solution:

Total number of students = 300

All Students in grades 6 and 7 = $72^o + 84^o = 156^o$

= $156^o/360^o$ x 300 = 130

b) How many less students are in grade 8 as compared to grade 4?

Solution:

Total number of students = 300

Difference between Grade 8 and Grade 4 students = $90^o – 60^o = 30^o$

= $30^o/360^o$ x 300 = 25

c) How many more students are in grade 6 than in grade 5?

Solution:

Total number of students = 300

Difference between Grade 6 and Grade 5 students = $72^o – 54^o = 18^o$

= $18^o/360^o$ x 300 = 15

d) How many students are in grade 7?

Solution:

Total number of students = 300

Grade 7 students = 84º

Grade 7 students = 84º/360º x 300 = 70

e) How many students are in grades 8 and 5 together?

Solution:

Total number of students = 300

Grade 8 and Grade 5 students altogether= 60º – 54º = 114º

= 114º/360º x 300 = 95

Learning Check 10.3

1. The following data shows the heights of 30 soldiers in the army. Show this in the form of grouped data and then answer the following questions: 180 cm, 181 cm, 179 cm, 180 cm, 182 cm, 182 cm, 183 cm, 182 cm, 179 cm, 178 cm, 180 cm, 183 cm, 184 cm, 182 cm, 178 cm, 179 cm, 180 cm, 183 cm, 177 cm, 178 cm, 176 cm, 177 cm, 183 cm, 184 cm, 182 cm, 180 cm, 181 cm, 182 cm, 179 cm, 178 cm

Solution:

Groups	Heights obtained	Tally marks	No. of soldiers
170- 180	180,179,180,179,178,180,178,179,180, 177,178,176,177,180,178,179	IIII IIII IIII I	16
180 -190	181,182,182,182,183,183,184,182,184, 182,181,182	IIII IIII II	12
			Total = 28

a) Find the highest frequency interval.

Solution:

16

b) Find the lowest frequency class interval.

Solution:

12

2. The following data shows the daily load-shedding duration in hours for 30 regions of the city. Show this in the form of grouped data and answer the following questions: 10, 2, 12, 5, 8, 7, 7, 6, 3, 8 10, 1, 7, 6, 11, 10, 12, 7, 6, 7, 9, 11, 7, 6, 9, 2, 7.

Groups	Load shedding duration	Tally marks	No. of hours
2 - 5	2,5,3,1,2	**~~IIII~~**	**5**
5-10	10,8,7,6,7,8,10,7,6,10,7,6,7,9,7,6,7	**~~IIII~~ ~~IIII~~ ~~IIII~~ II**	**17**
10 - 15	12,11	**II**	**2**
			Total = 24

a) Find the least load-shedding hours.

Solution:

2 hours

b) Find the most frequent load-shedding hours.

Solution:

17 hours

Learning check 10.4

1. Find the mean of the following data.

a) 1, 2, 2, 6, 7, 8, 9

b) 22, 35, 48, 59, 66

c) 152, 195, 225, 272, 348, 386.

Solution:

a) 1, 2, 2, 6, 7, 8, 9

Mean = $\sum x/ n$

= (1+2+2+6+7+8+9)/7

= 35/7 = 34.14

= (22 + 35 + 48 +59 +66)/ 5

$= \frac{35}{7}$

= 5

b) 22, 35, 48, 59, 66

Mean = $\sum x/ n$

= (22 + 35 + 48 +59 +66)/ 5

$= \frac{230}{5}$

= 46

c) 152, 195, 225, 272, 348, 386.

Mean = ∑ x/ n

= (152= 195 +225 + 272 + 348 + 380)/ 6

= $\frac{1578}{6}$

= 263

2. **The monthly income of 10 teachers in a school is given. Calculate the average of their monthly income. Rs 28000, Rs 25200, Rs 26200, Rs 25000, Rs 20500, Rs 19200, Rs 15400, Rs 20800, Rs 22000, Rs16500.**

Solution:

Average monthly income = (Rs28000 + Rs 25200 + Rs 26200+ Rs 25000+ Rs 20500+, Rs 19200+ Rs 15400+ Rs 20800+, Rs 22000+ Rs16500)/10

= 218800 / 10

Average monthly income = Rs. 21,880

3. **Find the median of the following sets of observations.**

a) 7, 2, 9, 2, 5

Solution:

First, we arrange in ascending order

2, 2, 5, 7, 9

Since there are an odd number of observations, the median is the middle value, which is 5.

Median = 9

b) 62, 45, 24, 97, 85, 39

Solution:

Arrange the data in ascending order we get

24,39,45,62,85,97

Since there are an even number of observations, the median is the average of the two middle values, which are 45 and 62. Such that

(45+62)/2

Median = 53.5

c) 657, 212, 361, 742, 535, 951, 897, 478.

Solution:

Arrange the data in ascending order we get

212,361,478,535,657,742,897,951

As the data is even

Then (535 + 657)/2

Median = 596

4. 4. Find the mode for each of the following:

a) 9, 6, 9, 12, 12, 16, 20

Solution:

As we know mode is that value which occur more than two times.

Mode =9, 12

b) 300, 200, 325, 687, 472, 200, 200, 648

Solution:

As we know mode is that value which occur more than two times.

Mode = 200

d)216, 720, 1024, 1215, 1648

Solution:

As we know mode is that value which occur more than two times.

Mode =0

Learning Check 10.5

1. Describe the following with examples:

a) Random experiment

A random experiment is an experiment that can be carried out numerous times under the same conditions. e.g. , before rolling a dice, we can't surely say what will be its outcome, but we know the possible outcomes as 1, 2, 3, 4, 5 and 6. Similarly, when tossing a coin, we cannot be sure whether it will be heads or tails. These are examples of a random experiment

b) Outcomes

Any potential result of a random experiment is considered to be an outcome. When tossing a coin, the possible outcomes are: heads or tails

c)Events

An outcome or a combination or collection of several outcomes is called an event. It is usually denoted by capital letters, e.g., A, B, C, etc. For example, the event (say A) of getting tails when a coin is tossed can be represented as: A = { T }. The probability of event A can be shown by P(A).

d)Probability

The likelihood of an event occurring is known as probability. For example, if we want to find the probability of getting heads when a coin is flipped, “getting heads” is the favorable or desired outcome.

2. Hadia and Mohid are tossing a coin to take the first turn in a game. Mohid chose heads. Find the probability of getting heads.

Solution:

Sample Space = {Head, Tail}

Number of possible outcomes = 2

Number of favorable outcomes = 1

Probability of getting heads = P (E) = Number of favorable (desired) outcome/Total no. of possible outcomes

Probability of getting heads = 1/2

3. A drawer consists of four red, five green, three black and eight white pairs of socks. Ali takes out one pair without looking at the socks. Find:

a) Sample space

b) P(red)

c) P(green)

d) P (not white)

e) P (not black)

Solution:

- **a)** Sample space = {Red, Green, Black, White} = (4,5,3,8) = 20
- **b)** P(Red) = Number of favorable (Red) outcome/Total no. of possible outcomes
  = 4/20 = 1/5
- **c)** P(Green) = Number of favorable (Green) outcome/Total no. of possible outcomes
  = 5/20 = 1/4
- **d)** P (not White) = Number of favorable (not White) outcome/Total no. of possible outcomes
  = P(Red) + P(Green) + P (Black)
  = 1/5 + 1/4 + 3/20 = 3/5
- **e)** P (not Black) = P(Red) + P(Green) + P (White)
  = 1/5 + 1/4 + 2/5 = 17/20

4. **Madiha has cards numbered from 1 to 30 and mixes them to put them in a box. She takes one card from the box. Find:**
a) Sample space b) P (10) c) P (15) d) P (not 5)
Solution:
a) Sample space = {1, 2, 3, ..., 29, 30}
b) P (10)
Number of favorable outcomes = 1 (there is only one card numbered 10)
Total number of possible outcomes = 30
P (10) = number of favorable outcomes / total number of possible outcomes = 1/30
So, the probability of drawing the card numbered 10 is 1/30.
c) P (15)
Number of favorable outcomes = 1 (there is only one card numbered 15)
Total number of possible outcomes = 30
P (15) = number of favorable outcomes / total number of possible outcomes = 1/30
So, the probability of drawing the card numbered 15 is also 1/30.
d) P (not 5)
Number of favorable outcomes (not 5) = 29 (there are 29 cards that are not 5)
Total number of possible outcomes = 30
P (not 5) = number of favorable outcomes / total number of possible outcomes = 29/30
So, the probability of not drawing the card numbered 5 is 29/30.

5. **Ayesha is playing cards with her friends when she draws a card from a deck of 30 cards numbered from 1 to 30. What is the probability that she draws a square number?**
Solution:
Sample space = 30
Square numbers from 1- 30 are 1,4,9,16,25
P (Square number) = number of favorable outcomes / total number of possible outcomes
P (Square number) = 5/30 = 1/6

6. **A box consists of 5 coins of Rs 1, 10 coins of Rs 2 and 15 coins of Rs 5. Ahad picked up a coin without looking. Find: a) Sample space b) P (Rs 2) c) P (Rs 5) d) P (not Rs 1).**
Solution:

a) Sample space = 5,10,15 = 5+10+15 = 30

b) P (Rs 2)

Number of favorable outcomes = 10

Total number of possible outcomes = 30

P (Rs 2) = number of favorable outcomes / total number of possible outcomes = 10/30

= 1/3

So, the probability of picking up a Rs 2 coin is 10/30 or 1/3.

c) P (Rs 5)

Number of favorable outcomes = 15

Total number of possible outcomes = 30

P (Rs 5) = number of favorable outcomes / total number of possible outcomes = 15/30

= 1/2

So, the probability of picking up a Rs 5 coin is 15/30 or 1/2.

d) P (not Rs 1)

Number of favorable outcomes (not Rs 1) = 25

Total number of possible outcomes = 30

P (not Rs 1) = number of favorable outcomes / total number of possible outcomes = 25/30 = 5/6

So, the probability of not picking up a Rs 1 coin is 25/30 or 5/6.

7. A bag contains two blue, five yellow, six white and seven black marbles. Alina picked up a marble from the bag without looking. Find: a) P(yellow)b) P(white)c) P (not blue) d) P (not black).

Solution:

Sample space = 2,5,6,7 =2+5+6+7= 20

a) P(yellow) = 5/20 =1/4

b) P(white) = 6/20 = 3/10

c) P (not blue) = 5/20 + 6/20 + 7/20 =18/20 = 9/10

d) P (not black) = P(yellow) + P(White) + P(blue)

= 1/4 + 3/10 + 2/20(1/10)

= 26/40 = 13/20

8. A dice is rolled. Find:

a) Sample space **b) P (even numbers)**

c) P(prime numbers) **d) P(odd numbers).**

Solution:

a) Sample space = {1, 2, 3, 4, 5, 6} =6

b) P (even numbers) = 3/6 = 1/2

c) P (prime numbers) = 3/6 = 1/2

d) P (odd numbers) = 3/6= ½

Unit Check 10

1. Encircle the correct answer.

a) 25% part of a pie graph is equal to:

i) $\frac{1}{2}$ ii) $\frac{1}{3}$ **iii)** $\mathbf{\frac{1}{4}}$ iv) 1

b) Which of these shows continuous data?

i) The students in a class ii) The number of chairs in the room

iii) The volume of liquid iv) The number of players in a hockey team

c) What is the missing number in 23, 35, ____, 56, 63, 78, 89 if its mean is 56?

i) 42 **ii) 48** iii) 56 iv) 92

d) The collection of every possible result of a random experiment is called:

i) event **ii) sample space** iii) trial iv) outcome

e) The mode of the data: 9, 10, 12, 13, 13, 13, 15, 15, 16, 16 is:

i) 9 ii) 15 iii) 16 **iv) 13**

2. Fill in the blanks.

a) The sum of the central angles of a pie graph must be equal to 360º.

b) Data collection is the process of gathering information about anything from any source.

c) Data in its raw form is known as ungrouped data.

d) The median of 5, 2, 7, 13, 8, 11 and 9 is 8.

e) The sum of a value divided by the number of values is called mean.

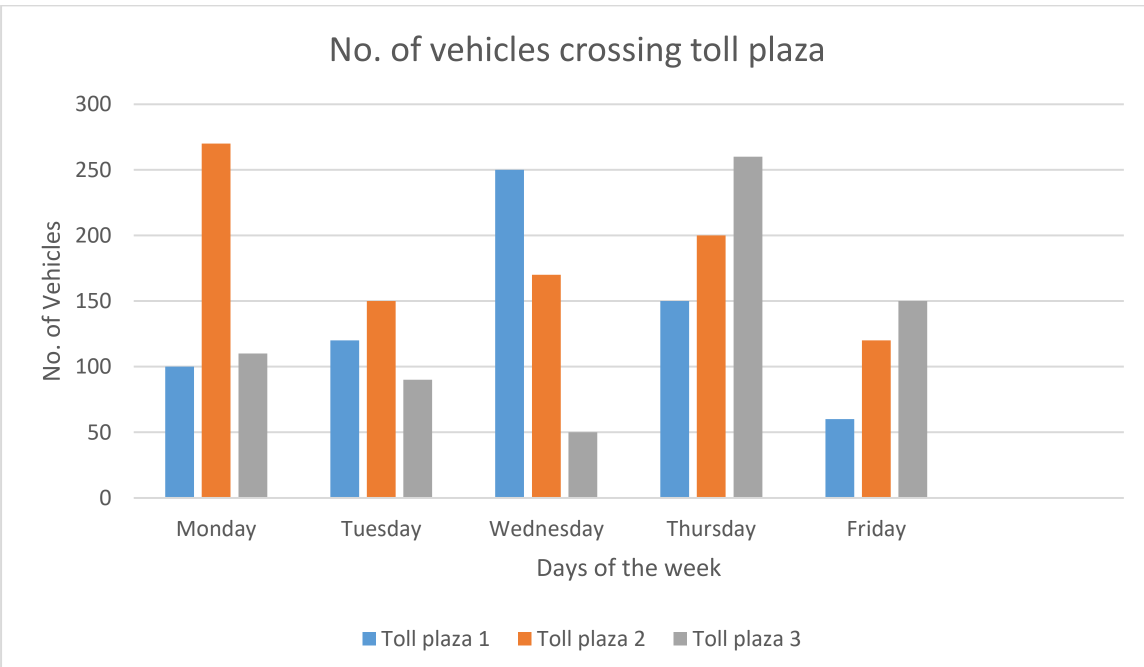
f) The probability or likelihood that a specific event will occur is defined as equally likely events.

3. The marks obtained by 40 students of class VI in a class test out of 25 marks are given below. Construct the frequency distribution table. 16, 17, 18, 3, 7, 23, 18, 13, 10, 21, 7, 1, 13, 21, 13, 15, 19, 24, 16, 2, 23, 15, 12, 18, 8, 12, 6, 8, 16, 5, 3, 5, 5, 7, 9, 12, 20, 10, 10, 23.

Solution:

Groups	Marks obtained	Tally marks	No. of Students
0 - 5	3,1,2,5,3,5,5	~~IIII~~ II	7
5 - 10	7,7,10,8,6,8,7,9,10,10	~~IIII~~ ~~IIII~~	10
10 - 15	13,13,13,15,15,12,12,12	~~IIII~~ ~~IIII~~	8
15 - 20	16,17,18,18,15,19,15,16,18,16,20	~~IIII~~ ~~IIII~~ I	11
20 - 25	23,21,21,23,24,23	~~IIII~~ I	6
			Total marks= 42

4. Solution:



5. Solution:

a) In how many days did Nida learn Surah Al Hadid?

30 days

b) Which two surahs did Hadia learn on the same day?

Surah Al Qalam and Surah Al Hadid

c) In how many days did Umar learn the three surahs?

80 days

d) In how many days did Ali learn Surah Qalam?

15 days

e) How many more days did Nida take to learn Surah Al Hadid than Hadia?

20 – 10 =10 days

f) Which surah did Ahmad learn in 10 days?

Surah Al Qalam

6. **The following data shows the favourite subjects of different grade 6 students. Draw a pie graph for this data**.

Solution:

Step I:

Total sum of Students = 8=10=15=4=9=6 =60

Step II: Find the measurement of the central angles by using the formula.

Subjects	No. of Students	Measure of the central angle
English	8	$8/60 \times 360^o = 48^o$
Science	10	$10/60 \times 360^o = 60^o$
Maths	15	$15/60 \times 360^o = 90^o$
Urdu	4	$4/60 \times 360^o = 24^o$
History	9	$9/60 \times 360^o = 54^o$
Islamiyat	6	$6/60 \times 360^o = 36^o$
Geography	8	$8/60 \times 360^o = 48^o$

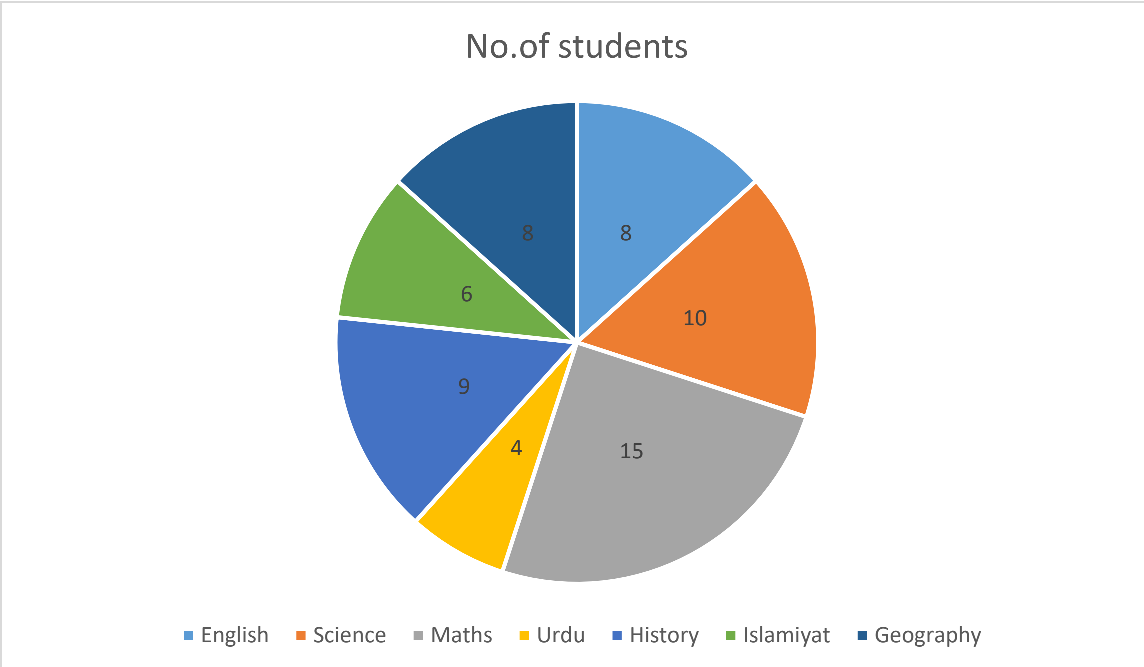
Step III: Draw a circle of any radius and draw the radial segment.

Step IV: Take a protractor and place it in the centre of the circle. Draw an angle of 48^o

for English.

Step V: Again, place a protractor and align it on the centre of the circle aligned with the radial segment (the arm of 48^o). Draw an angle of 60^o for Science.

Continue the procedure and make all the required angles for each day.



7. Solution:

Total number of students = 7200

a) How many students used the bus to come to school?

$150^o/360^o$ x 7200 = 3000 students

b) How many students use motorcycles and bicycles to come to school?

Students use motorcycles = $30^o/360^o$ x 7200 = 600

Students use Bicycles = $100^o/360^o$ x 7200 = 2000

= 2000 + 600 = 2600 students

C) Which mode of transportation do the majority of students use to get to school?

Bus

d) Which mode of transportation does the least number of students use to get to school?

Motorcycle

e) How many students come to school by car?

$80^o/360^o$ x 7200 = 1600 students

8. Find the mean, median and mode for the following observations:

Solution:

a) 43, 66, 75, 86, 66, 36

Mean: ∑ x/ n

= (43+ 66 +75 + 86 + 66 + 36)/ 6 = 372/6 = 62

Median: 36,43,66,66,75,86 (Arrange the data in ascending order then the average value is median if the total values are even)

= (66+ 66)/2 = 66

Mode: 66

b) 308, 204, 348, 135, 265

Mean: ∑ x/ n

= (308 + 204 + 348 + 135 + 265)/ 5 = 1260/5 = 252

Median: To find the median, we first need to put the observations in order:
135, 204, 265, 308, 348
The median is the middle value, which is 265.

Mode: 0

c) 11, 13, 13, 11, 11, 10

Mean: ∑ x/ n

= (11+ 13 +13 + 11 + 11 + 10)/ 6 = 69/6 = 11.5

Median: 10,11,11,11,13,13 (Arrange the data in ascending order then the average value is median if the total values are even)

= (11 + 11)/2 = 22/2 = 11

Mode: 11,13

d) 100, 120, 140, 140, 140

Mean: ∑ x/ n

= (100+ 120 +140 + 140 + 140 + 140)/ 5 = 640/5 = 128

Median: 100,120,140,140,140 (Arrange the data in ascending order then the mid value is median if the total values are odd)

= 140

Mode: 140

9. Look at the spinner and find:

a) What is the sample space?

b) Find the probability of: i) P(banana) ii) P(apple) iii) P(orange) iv) P(strawberry) v) P (not orange).

Solution:

Sample space = {Banana, Apple, Orange, Strawberry}

Number of possible outcomes = 8

Probability of an event = P (E) = Number of favourable (desired) outcomes/ Total number of possible

a) Number of favourable outcomes (Landing on banana) = 4
Probability of landing on banana = P (banana) = 4/8 = 1/2

b) Number of favourable outcomes (Landing on Apple) = 2
Probability of landing on Apple = P (Apple) = 2/8 = 1/4

c) Number of favourable outcomes (Landing on Orange) = 1
Probability of landing on Orange = P (Orange) = 1/8 = 1/8

d) Number of favourable outcomes (Landing on Strawberry) = 1
Probability of landing on Strawberry = P (Strawberry) = 1/8 = 1/8

e) Number of favourable outcomes (Landing on not orange) = 1/2 +1/4 +1/8
Probability of landing on not orange = P (not orange) = 7/8