بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

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AFAQ SUN SERIES

KEYBOOK

Mathematics

7

AFAQ

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Learning Check 1.1

1. Write all the integers between +2 and +9.

Solution:

To find the integers between two integers, you can start with the smaller integer and count up by one until you reach the larger integer. The integers between the two given integers are all the integers you counted, excluding the two given integers themselves.

To find the integers between +2 and +9, you can start with +3 and count upto +8:

+3, +4, +5, +6, +7 and +8.

So, the integers between +2 and +9 are:

+3, +4, +5, +6, +7 and +8.

2. Write all the integers between -8 and -16.

Solution:

To find the integers between -8 and -16, you can start with the larger integer (-8) and count down by one until you reach the smaller integer (-16). The integers between the two given integers are all the integers you counted, excluding the two given integers themselves.

To find the integers between -8 and -16, you can start with -9 and count down to -15:

-9, -10, -11, -12, -13, -14 and -15.

So, the integers between -8 and -16 are:

-15, -14, -13, -12, -11, -10 and -9.

3. Write the absolute value of the following integers.

a) +12 b) –9 c) +3 d) -15 e) +64 f) –72 g) +13

Solution:

Absolute value of all integers is always positive. So:

a) |+12| = 12

b) |-9| = 9

c) |+3| = 3

d) |-15| = 15

e) |+64| = 64

f) |-72| = 72

g) |+13| = 13

In each case, the vertical bars (| |) indicate the absolute value of the integer inside them.

4. Write the value of the underlined digit in the given numbers.

a) 1<u>6</u>7,238 b) 12,10<u>2</u>,865 c) 41.62<u>9</u> d) <u>1</u>374.283

e) <u>1</u>9,780 f) 8.14<u>9</u>4 g) 8<u>3</u>6.217 h) <u>1</u>0,310,100

Solution:

a) 1<u>6</u>7,238

Here the underlined digit is 6. To find the value of a digit in a number, you need to consider its place value. Here, the digit 6 is in the ten thousand's place. The value of a digit in the ten thousand's place is found by multiplying the digit by 10,000. So, the value of the digit 6 in this number is 6 x 10,000 = 6,000.

b) 12,10<u>2</u>,865

Here, the underlined digit 2 is in the thousands place. The value of a digit in the thousands place is found by multiplying the digit by 1000. So, the value of the digit 2 in this number is 2 x 1000 = 2000.

c) 41.62<u>9</u>

Here, the underlined digit 9 is in the thousandths place. The value of a digit in the thousandths place is found by multiplying the digit by 1/1000 (or divide by 1000). So, the value of the digit 9 in this number is 9 x 1/1000 = 9/1000= 0.009.

d) <u>1</u>374.283

Here, the underlined digit 1 is in the thousands place. The value of a digit in the thousands place is found by multiplying the digit by 1000. So, the value of the digit 1 in this number is 1 x 1000 = 1000.

e) <u>1</u>9,780

Here, the underlined digit 1 is in the ten thousands place. The value of a digit in the ten thousands place is found by multiplying the digit by 10000. So, the value of the digit 1 in this number is 1 x 10000 = 10000.

f) 8.14<u>9</u>4

Here, the underlined digit 9 is in the hundredth place. The value of a digit in the thousandths place is found by multiplying the digit by 1/100. So, the value of the digit 9 in this number is 9 x 1/100 = 9/100= 0.09.

g) 836.217

Here, the underlined digit 3 is in the tens place. The value of a digit in the ten tens place is found by multiplying the digit by 10. So, the value of the digit 3 in this number is 3 x 10 = 30.

h) 10,310,100

Here, the underlined digit 1 is in the ten million's place. The value of a digit in the ten million's place is found by multiplying the digit by 10,000,000. So, the value of the digit 1 in this number is 1 x 10,000,000 = 10,000,000.

5. **Show the number 2315816 in a place value table and then write it in words.**

Solution:

Here is the number 2,315,816 shown in a place value table:

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
2	3	1	5	8	1	6

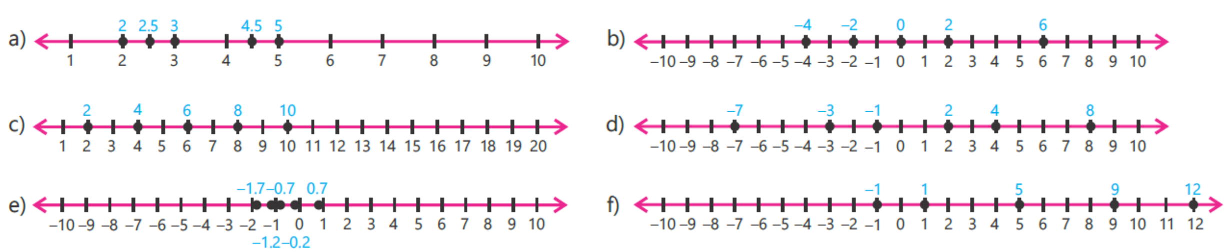
In words, the number 2,315,816 is written as "two million three hundred fifteen thousand eight hundred sixteen".

6. **Represent the given numbers on a number line.**

a) 2, 2.5, 3, 4.5, 5　　b) −4, −2, 0, 2, 6　　c) 2, 4, 6, 8, 10

d) −7, −3, −1, 2, 4, 8　　e) −1.7, −1.2, −0.7, −0.2, 0.7　　f) −1, 1, 5, 9, 12

Solution:



7. Represent the given fractions on a number line.

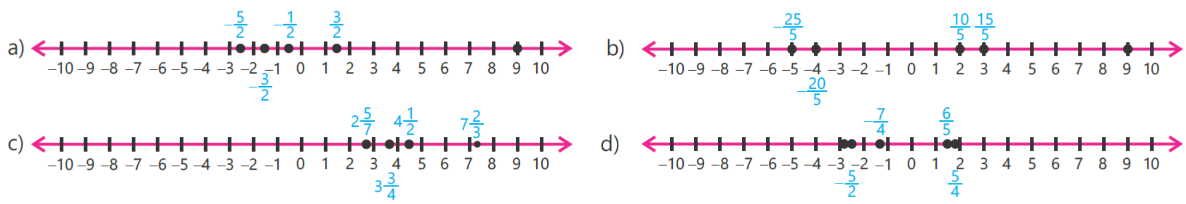
Solution:

a) $-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{3}{2}$

b) $-\frac{25}{5}, -\frac{20}{5}, \frac{15}{5}, \frac{10}{5}$

c) $2\frac{5}{7}, 3\frac{3}{4}, 4\frac{1}{2}, 7\frac{2}{3},$

d) $-\frac{7}{4}, -\frac{5}{2}, \frac{5}{4}, \frac{6}{5}$



8. Identify and separate the proper fractions, the improper fractions and the mixed numbers.

a) 6/11 **b) 3/8** **c) 3/2** **d) 6/5** **e) $5\frac{8}{9}$** **f) $10\frac{4}{5}$** **g) 25/17**

h) 11/16 **i) 18/13** **j) $1\frac{1}{2}$** **k) $4\frac{7}{9}$** **l) 8/13** **m) $12\frac{7}{12}$** **n) 23/18**

Solution:

a) 6/11 is a proper fraction because the numerator is less than the denominator.

b) 3/8 is a proper fraction because the numerator is less than the denominator.

c) 3/2 is an improper fraction because the numerator is greater than the denominator.

d) 6/5 is an improper fraction because the numerator is greater than the denominator.

e) 5 8/9 is a mixed number because there is a whole number (5) and a proper fraction (8/9).

f) $10\frac{4}{5}$ is a mixed number because there is a whole number (10) and a proper fraction (4/5). g) 25/17 is an improper fraction because the numerator is greater than the denominator.

g) 25/17 is an improper fraction because numerator is greater than the denominator.

h) 11/16 is a proper fraction because the numerator is less than the denominator.
i) 18/13 is an improper fraction because the numerator is greater than the denominator.
j) $1\frac{1}{2}$ is a mixed number because there is a whole number (1) and a proper fraction (1/2).
k) $4\frac{7}{9}$ is a mixed number because there is a whole number (4) and a proper fraction (7/9).
l) 8/13 is a proper fraction because the numerator is less than the denominator.
m) $12\frac{7}{12}$ is a mixed number because there is a whole number (12) and a proper fraction (7/12).
n) 23/18 is an improper fraction because the numerator is greater than the denominator.

9. Convert the following improper fractions to mixed numbers.

a) $\frac{3}{2}$ b) $\frac{6}{5}$ c) $\frac{8}{5}$ d) $\frac{12}{7}$ e) $\frac{24}{15}$

f) $\frac{20}{11}$ g) $\frac{15}{8}$ h) $\frac{14}{9}$

Solution:

To convert an improper fraction to a mixed number, we need to divide the numerator by the denominator. The whole number part of the result is the whole number in the mixed number, and the remainder is the numerator of the proper fraction.

a) 3/2 = $1\frac{1}{2}$ We divide 3 by 2 to get 1 with a remainder of 1. So the mixed number is $1\frac{1}{2}$.

b) 6/5 = $1\frac{1}{5}$ We divide 6 by 5 to get 1 with a remainder of 1. So the mixed number is 1 1/5.

c) 8/5 = $1\frac{3}{5}$ We divide 8 by 5 to get 1 with a remainder of 3. So the mixed number is $1\frac{3}{5}$.

d) 12/7 = $1\frac{5}{7}$ We divide 12 by 7 to get 1 with a remainder of 5. So the mixed number is $1\frac{5}{7}$.

e) 24/15 = $1\frac{9}{15}$ = $1\frac{3}{5}$ We divide 24 by 15 to get 1 with a remainder of 9. Then we simplify the fraction by dividing the numerator and denominator by their greatest common factor, which is 3. So the mixed number is $1\frac{3}{5}$.

f) 20/11 = $1\frac{9}{11}$ We divide 20 by 11 to get 1 with a remainder of 9. So the mixed number is $1\frac{9}{11}$.

g) 15/8 = $1\frac{7}{8}$ We divide 15 by 8 to get 1 with a remainder of 7. So the mixed number is $1\frac{7}{8}$.

h) 14/9 = $1\frac{5}{9}$ We divide 14 by 9 to get 1 with a remainder of 5. So the mixed number is $1\frac{5}{9}$.

10. Convert the following mixed numbers to improper fractions.

a) $2\frac{8}{11}$ b) $1\frac{20}{21}$ c) $3\frac{1}{7}$ d) $6\frac{7}{12}$ e) $4\frac{6}{17}$

f) $5\frac{2}{5}$ g) $9\frac{3}{8}$ h) $10\frac{7}{10}$

Solution:

a) $2\frac{8}{11}$ = (2 × 11 + 8)/11 = 30/11 The improper fraction is 30/11.

b) $1\frac{20}{21}$ = (1 × 21 + 20)/21 = 41/21 The improper fraction is 41/21.

c) $3\frac{1}{7}$ = (3 × 7 + 1)/7 = 22/7 The improper fraction is 22/7.

d) $6\frac{7}{12}$ = (6 × 12 + 7)/12 = 79/12 The improper fraction is 79/12.

e) $4\frac{6}{17}$ = (4 × 17 + 6)/17 = 74/17 The improper fraction is 74/17.

f) $5\frac{2}{5}$ = (5 × 5 + 2)/5 = 27/5 The improper fraction is 27/5.

g) $9\frac{3}{8}$ = (9 × 8 + 3)/8 = 75/8 The improper fraction is 75/8.

h) 10 7/10 = (10 × 10 + 7)/10 = 107/10 The improper fraction is 107/10.

11. Compare the following numbers.

a) – 13 and +2 **b) +2 and +15** **c) – 24 and – 11**

d) +62 and +79 **e) 0.1 and 0.002** **f)** $\frac{2}{7}$ **and** $\frac{6}{11}$

g) $\frac{12}{7}$ **and** $\frac{15}{11}$ **h) – 62 and – 99** **i)** $-\frac{9}{14}$ **and** $+\frac{12}{19}$

Solution:

a) –13 and +2 To compare –13 and +2, we notice that –13 is negative and +2 is positive. Any negative number is less than any positive number, so –13 < +2.

b) +2 and +15 To compare +2 and +15, we notice that +15 is larger than +2, so +2 < +15.

c) –24 and –11 To compare –24 and –11, we notice that –11 is larger than –24, so –24 < –11.

d) +62 and +79 To compare +62 and +79, we notice that +79 is larger than +62, so +62 < +79.

e) 0.1 and 0.002 To compare 0.1 and 0.002, we notice that 0.1 is larger than 0.002, so 0.002 < 0.1.

f) $\frac{2}{7}$ and $\frac{6}{11}$ To compare $\frac{2}{7}$ and $\frac{6}{11}$, we need to find a common denominator. The least common multiple of 7 and 11 is 77, so we can write: $\frac{2}{7} = \frac{22}{77}\ \frac{6}{11} = \frac{42}{77}$ Now we see that $\frac{42}{77}$ is larger than $\frac{22}{77}$, so $\frac{2}{7} < \frac{6}{11}$.

g) 12/7 and 15/11 To compare 12/7 and 15/11, we need to find a common denominator. The least common multiple of 7 and 11 is 77, so we can write: 12/7 = 132/77 15/11 = 105/77 Now we see that 132/77 is larger than 105/77, so 12/7 > 15/11.

h) –62 and –99 To compare –62 and –99, we notice that –99 is smaller than –62, so –99 < –62.

i) -9/14 and +12/19 To compare -9/14 and +12/19, we need to find a common denominator. The least common multiple of 14 and 19 is 266, so we can write: -9/14 = -99/266 12/19 = 152/266 Now we see that -99/266 is smaller than 152/266, so -9/14 < 12/19

12. Compare and order the following numbers in ascending and descending order.

a) 87, 75, 92, 48, 67, 21

b) – 99, –102, –72, –35, –87, –68

c) +23, –42, –16, +31, +25, –55

d) 1.22, 3.1, 8.14, 1.5, 4.12, 2.69

e) $\frac{5}{4}, \frac{3}{7}, \frac{7}{2}, \frac{11}{14}, \frac{3}{8}, \frac{9}{5}$

f) $-\frac{6}{5}, -\frac{9}{5}, 3\frac{2}{5}, -1\frac{2}{5}, 4\frac{3}{5}, -8\frac{4}{5}$

Solution:

a) 87, 75, 92, 48, 67, 21 The smallest number is 21, then 48, 67, 75, 87, and finally 92. So the numbers arranged in ascending order are: 21, 48, 67, 75, 87, 92.

b) –99, –102, –72, –35, –87, –68 The smallest number is –102, then –99, –87, –72, –68, and finally –35. So the numbers arranged in ascending order are: –102, –99, –87, –72, –68, –35.

c) +23, –42, –16, +31, +25, –55 The smallest number is –55, then –42, –16, +23, +25, and finally +31. So the numbers arranged in ascending order are: –55, –42, –16, +23, +25, +31.

d) 1.22, 3.1, 8.14, 1.5, 4.12, 2.69 The smallest number is 1.22, then 1.5, 2.69, 3.1, 4.12, and finally 8.14. So the numbers arranged in ascending order are: 1.22, 1.5, 2.69, 3.1, 4.12, 8.14.

e) 5/4, 3/7, 7/2, 11/14, 3/8, 9/5 The smallest number is 3/7, then 3/8, 11/14, 5/4, 9/5, and finally 7/2. So the numbers arranged in ascending order are: 3/7, 3/8, 11/14, 5/4, 9/5, 7/2.

f) -6/5, -9/5, 3 2/5, -1 2/5, 4 3/5, -8 4/5 We can convert the mixed numbers to improper fractions: -6/5, -9/5, 17/5, -7/5, 23/5, -44/5 The smallest number is -44/5, then -9/5, -7/5, -6/5, 17/5, and finally 23/5. So the numbers arranged in ascending order are: -44/5, -9/5, -7/5, -6/5, 17/5, 23/5.

Learning Check 1.2

1. Round off the given numbers to the nearest 10, 100 and 1000.

a) 25,358 b) 310,219 c) 8,714,915 d) 3,616,749

e) 260,630 f) 74,293 g) 1,278,950 h) 6,926,017

Solution:

a) 25,358 rounded to the nearest 10:

- Look at the digit in the ones place, which is 8.
- Since 8 is greater than or equal to 5, we round up the tens digit to 6 and change all digits to the right to 0.
- Therefore, 25,358 rounded to the nearest 10 is 25,360.

25,358 rounded to the nearest 100:

- Look at the digit in the tens place, which is 5.
- Since 3 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 0, which gives us 25,400.

25,358 rounded to the nearest 1000:

- Look at the digit in the thousands place, which is 5.
- Since 3 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 5.
- Therefore, 25,358 rounded to the nearest 1000 is 25,000.

b) 310,219 rounded to the nearest 10:

- Look at the digit in the ones place, which is 9.
- Since 9 is greater than or equal to 5, we round up the tens digit to 2 and change all digits to the right to 0.
- Therefore, 310,219 rounded to the nearest 10 is 310,220.

310,219 rounded to the nearest 100:

- Look at the digit in the tens place, which is 1.
- Since 1 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 0.
- Therefore, 310,219 rounded to the nearest 100 is 310,200.

310,219 rounded to the nearest 1000:

- Look at the digit in the thousands place, which is 0.
- Since the digit in the hundreds place is 2, we leave the thousands digit as it is and change all digits to the right to 0.
- Therefore, 310,219 rounded to the nearest 1000 is 310,000.

c) 8,714,915 rounded to the nearest 10:

- Look at the digit in the ones place, which is 5.
- Since 5 is greater than or equal to 5, we round up the tens digit to 2 and change all digits to the right to 0.
- Therefore, 8,714,915 rounded to the nearest 10 is 8,714,920.

8,714,915 rounded to the nearest 100:

- Look at the digit in the tens place, which is 1.
- Since 1 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 0.
- Therefore, 8,714,915 rounded to the nearest 100 is 8,714,900.

8,714,915 rounded to the nearest 1000:

- Look at the digit in the thousands place, which is 4.
- Since the digit in the hundreds place is 9, we round up the thousands digit to 8,715,000.

d) 3,616,749 rounded to the nearest 10:

- Look at the digit in the ones place, which is 9.
- Since 9 is greater than or equal to 5, we round up the tens digit to 5 and change all digits to the right to 0.
- Therefore, 3,616,750 rounded to the nearest 10

3,616,749 rounded to the nearest 100:

- Look at the digit in the tens place, which is 4.
- Since 4 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 0.
- Therefore, 3,616,749 rounded to the nearest 100 is 3,616,700.

3,616,749 rounded to the nearest 1000:

- Look at the digit in the thousands place, which is 6.
- Since the digit in the hundreds place is 7, we round up the thousands digit to 3,617,000.

e) 260,630 rounded to the nearest 10:

- Look at the digit in the ones place, which is 0.
- Since 0 is less than 5, we leave the digit we want to round to (the tens digit) as it is and change all digits to the right to 0.
- Therefore, 260,630 rounded to the nearest 10 is 260,630.

260,630 rounded to the nearest 100:

- Look at the digit in the tens place, which is 3.
- Since 3 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 0.
- Therefore, 260,630 rounded to the nearest 100 is 260,600.

260,630 rounded to the nearest 1000:

- Look at the digit in the thousands place, which is 0.
- Since the digit in the hundreds place is 6, we round up the thousands digit to 261,000.

f) 74,293 rounded to the nearest 10:

- Look at the digit in the ones place, which is 3.
- Since 3 is less than 5, we leave the digit we want to round to (the tens digit) as it is and change all digits to the right to 0.
- Therefore, 74,293 rounded to the nearest 10 is 74,290.

74,293 rounded to the nearest 100:

- Look at the digit in the tens place, which is 9.
- Since 9 is greater than or equal to 5, we round up the hundreds digit to 3 and change all digits to the right to 0.
- Therefore, 74,293 rounded to the nearest 100 is 74,300.

74,293 rounded to the nearest 1000:

- Since the digit in the hundreds place is 2(less than 5), we round down the thousands digit to 74,000.

g) 1,278,950 rounded to the nearest 10:

- Look at the digit in the ones place, which is 0.

- Since 0 is less than 5, we leave the digit we want to round to (the tens digit) as it is and change all digits to the right to 0.
- Therefore, 1,278,950 rounded to the nearest 10 is 1,278,950.

1,278,950 rounded to the nearest 100:

- Look at the digit in the tens place, which is 5.
- Since 5 is greater than or equal to 5, we round up the thousands digit to 9 and change all digits to the right to 0.

  Therefore, 1,278,950 rounded to the nearest 100 is 1,279,000.

1,278,950 rounded to the nearest 1000:

- Look at the digit in the thousand places, which is 8.
- Since the digit in the hundreds place is 9(greater than 5), we round up the thousands digit to 9 and change all digits to the right to 0.
- Therefore, 1,278,950 rounded to the nearest 100 is 1,279,000.

h) 6,926,017 rounded to the nearest 10:

- Look at the digit in the ones place, which is 7.
- Since 7 is greater than or equal to 5, we round up the digit we want to round to (the tens digit) to 20 and change all digits to the right to 0.
- Therefore, 6,926,017 rounded to the nearest 10 is 6,926,020.

6,926,017 rounded to the nearest 100:

- Look at the digit in the tens place, which is 1.
- Since 1 is less than 5, we leave the digit we want to round to (the hundreds digit) as it is and change all digits to the right to 0.
- Therefore, 6,926,017 rounded to the nearest 100 is 6,926,000.

6,926,017 rounded to the nearest 1000:

- Look at the digit in the thousands place, which is 6.
- Since the digit in the hundreds place is 0, we round off the thousands digit to 6,927,000.

2. Round off the following decimals to the nearest tenth and hundredth.

a) 3.458 b) 1.502 c) 7.892 d) 8559.496

e) 1.892 f) 76.256 g) 6.445 h) 2.212

Solution:

a) 3.458 rounded to the nearest tenth:

- Look at the digit in the hundredths place, which is 4.
- Since 5 is greater than or equal to 5, we round up the digit we want to round to (the tenths digit) to 5.
- Therefore, 3.458 rounded to the nearest tenth is 3.5.

3.458 rounded to the nearest hundredth:

- Look at the digit in the hundredth place, which is 5.
- Since 8 is equal to 5, we round up the previous digit (the hundredth digit) to 6.
- Therefore, 3.458 rounded to the nearest hundredth is 3.46.

b) 1.502 rounded to the nearest tenth:

- Look at the digit in the tenth place, which is 5.
- Since 0 is less than 5, we leave the digit we want to round to (the tenths digit) as it is.
- Therefore, 1.502 rounded to the nearest tenth is 1.5.

1.502 rounded to the nearest hundredth:

- Look at the digit in the thousandths place, which is 0.
- Since 2 is less than 5, we leave the digit we want to round to (the hundredths digit) as it is.
- Therefore, 1.502 rounded to the nearest hundredth is 1.50.

c) 7.892 rounded to the nearest tenth:

- Look at the digit in the tenth place, which is 8.
- Since 9 is greater than or equal to 5, we round up the digit we want to round to (the tenths digit) to 9.
- Therefore, 7.892 rounded to the nearest tenth is 7.9.

7.892 rounded to the nearest hundredth:

- Look at the digit in the hundredth place, which is 9.

- Since 2 is less than 5, we leave the digit we want to round to (the hundredths digit) as it is.
- Therefore, 7.892 rounded to the nearest hundredth is 7.89.

d) 8559.496 rounded to the nearest tenth:

- Look at the digit in the tenth place, which is 4.
- Since 9 is greater than or equal to 5, we round up the digit we want to round to (the tenths digit) to 5 and carry over the additional 1 to the ones place.
- Therefore, 8559.496 rounded to the nearest tenth is 8559.5.

8559.496 rounded to the nearest hundredth:

- Look at the digit in the hundredth place, which is 9.
- Since 6 is greater than or equal to 5, we round up the digit we want to round to (the hundredth digit) to 5 and carry over the additional 1 to the hundredth place.
- Therefore, 8559.496 rounded to the nearest hundredth is 8559.50.

e) 1.892 rounded to the nearest tenth:

- Look at the digit in the hundredths place, which is 8.
- Since 9 is greater than or equal to 5, we round up the digit we want to round to (the tenths digit) to 5 and carry over the additional 1 to the tenth place Therefore, 1.892 rounded to the nearest tenth is 1.9.

1.892 rounded to the nearest hundredth:

- Look at the digit in the thousandths place, which is 9.
- Since 9 is greater than or equal to 5, we round up the digit we want to round to (the hundredths digit) to 0 and carry over the additional 1 to the tenths place
- Therefore, 1.892 rounded to the nearest hundredth is 1.89.

f) To round off 76.256 to the nearest tenth, we look at the digit in the second decimal place which is 5. Since 5 is greater than or equal to 5, we add 1 to the digit in the first decimal place, which gives us 76.3. Therefore, rounding off 76.256 to the nearest tenth gives us 76.3. To round off to the nearest hundredth, we simply keep the first two decimal places which gives us 76.26.

g) To round off 6.445 to the nearest tenth, we look at the digit in the second decimal place which is 4. Since 4 is less than 5, we leave the digit in the first decimal place as it is, which gives us 6.4. Therefore, rounding off 6.445 to the nearest tenth gives us 6.4. To round off to the nearest hundredth, we simply keep the first two decimal places which gives us 6.45.

h) To round off 2.212 to the nearest tenth, we look at the digit in the second decimal place which is 1. Since 1 is less than 5, we leave the digit in the first decimal place as it is, which gives us 2.2. Therefore, rounding off 2.212 to the nearest tenth gives us 2.2. To round off to the nearest hundredth, we simply keep the first two decimal places which gives us 2.21.

3. Round off the following numbers to 2 significant figures and to 3 significant figures

a) 67247 b) 99244 c) 623.78 d) 8.230 e) 53772 f) 238.796 g) 54298

Solution:

a) 67247 To round off to 2 significant figures, we consider the first two digits from the left, which are 67. The third digit, which is 2, is less than 5, so we round down. Therefore, 67247 rounded to 2 significant figures is 67000. To round off to 3 significant figures, we consider the first three digits from the left, which are 672. The fourth digit, which is 4, is less than 5, so we round down. Therefore, 67247 rounded to 3 significant figures is 67200.

b) 99244 To round off to 2 significant figures, we consider the first two digits from the left, which are 99. The third digit, which is 2, is less than 5, so we round down. Therefore, 99244 rounded to 2 significant figures is 99000. To round off to 3 significant figures, we consider the first three digits from the left, which are 992. The fourth digit, which is 4, is less than 5, so we round down. Therefore, 99244 rounded to 3 significant figures is 99200.

c) 623.78 To round off to 2 significant figures, we consider the first two digits from the left, which are 62. The third digit, which is 3, is less than 5, so we round down. Therefore, 623.78 rounded to 2 significant figures is 620. To round off to 3 significant figures, we consider the first three digits from the left, which are

624. The fourth digit, which is 7, is greater than 5, so we round up. Therefore, 623.78 rounded to 3 significant figures is 624.

d) 8.230 To round off to 2 significant figures, we consider the first two digits from the left, which are 8 and 2. The third digit, which is 3, is less than 5, so we round down. Therefore, 8.230 rounded to 2 significant figures is 8.2. To round off to 3 significant figures, we consider all three digits. The fourth digit, which is 0, is less than 5, so we round down. Therefore, 8.230 rounded to 3 significant figures is 8.23.

e) 53772 To round off to 2 significant figures, we consider the first two digits from the left, which are 54. The third digit, which is 7, is greater than 5, so we round up. Therefore, 53772 rounded to 2 significant figures is 54000. To round off to 3 significant figures, we consider the first three digits from the left, which are 538. The fourth digit, which is 7, is greater than 5, so we round up. Therefore, 53772 rounded to 3 significant figures is 53800.

f) 238.796 To round off to 2 significant figures, we consider the first two digits from the left, which are 24. The third digit, which is 7, is greater than 5, so we round up. Therefore, 238.796 rounded to 2 significant figures is 240. To round off to 3 significant figures, we consider the first three digits from the left, which are 239. The fourth digit, which is 7, is greater than 5, so we round up. Therefore, 238.796 rounded to 3 significant figures is 239.

g) 54298: To round off to 2 significant figures, we consider the first two digits from the left, which are 54. The third digit, which is 2, is less than 5, so we round down. Therefore, 54298 rounded to 2 significant figures is 54000. To round off to 3 significant figures, we consider the first three digits from the left, which are 543. The fourth digit, which is 9, is greater than 5, so we round up. Therefore, 54298 rounded to 3 significant figures is 54300.

Learning Check 1.3

1. Find the additive inverse of the following numbers.

a) $-\frac{5}{12}$ **b)** $\frac{18}{25}$ **c)** $-\frac{16}{33}$ **d)** $\frac{1}{2}$ **e)** $-\frac{79}{40}$ **f)** $3\frac{5}{11}$

g) $-\frac{19}{20}$ **h)** $\frac{3}{17}$

Solution:

The additive inverse of a number is the number that, when added to the original number, results in zero. In other words, the additive inverse of a number "a" is the number "-a". To find the additive inverse of a fraction, we simply change the sign of the numerator (the top number) and keep the denominator (the bottom number) the same.

a) Additive inverse of -5/12 is 5/12. (-5/12 + 5/12 = 0)

b) Additive inverse of 18/25 is -18/25. (18/25 + (-18/25) = 0)

c) Additive inverse of -16/33 is 16/33. (-16/33 + 16/33 = 0)

d) Additive inverse of 1/2 is -1/2. (1/2 + (-1/2) = 0)

e) Additive inverse of -79/40 is 79/40. (-79/40 + 79/40 = 0)

f) Additive inverse of 3 5/11 is -3 5/11. (3 5/11 + (-3 5/11) = 0)

g) Additive inverse of -19/20 is 19/20. (-19/20 + 19/20 = 0)

h) Additive inverse of 3/17 is -3/17. (3/17 + (-3/17) = 0)

2. Verify the commutative property for the following numbers.

a) 5, 7 **b) -3, 10** **c) 1.5, 0.2** **d)** $\frac{1}{4}, \frac{3}{8}$

e) $\frac{2}{7}, \frac{3}{8}$ **f)** $\frac{11}{16}, \frac{17}{64}$ **g)** $\frac{3}{13}, \frac{10}{13}$ **h)** $\frac{23}{25}, \frac{31}{70}$

Solution:

a) To verify the commutative property for addition with 5 and 7, we need to check if 5 + 7 = 7 + 5. 5 + 7 = 12 7 + 5 = 12 Since both sides are equal, the commutative property holds for addition with 5 and 7.

b) To verify the commutative property for addition with -3 and 10, we need to check if -3 + 10 = 10 + (-3). -3 + 10 = 7 10 + (-3) = 7 Since both sides are equal, the commutative property holds for addition with -3 and 10.

c) To verify the commutative property for addition with 1.5 and 0.2, we need to check if 1.5 + 0.2 = 0.2 + 1.5.

1.5 + 0.2 = 1.7

0.2 + 1.5 = 1.7

Since both sides are equal, the commutative property holds for addition with 1.5 and 0.2.

d) To verify the commutative property for addition with 1/4 and 3/8, we need to check if 1/4 + 3/8 = 3/8 + 1/4.

1/4 + 3/8 = 5/8

3/8 + 1/4 = 5/8 Since both sides are equal, the commutative property holds for addition with 1/4 and 3/8.

e) To verify the commutative property for addition with 2/7 and 3/8, we need to check if 2/7 + 3/8 = 3/8 + 2/7.

2/7 + 3/8 = 37/56

3/8 + 2/7 = 41/56.

Since both sides are not equal, the commutative property does not hold for addition with 2/7 and 3/8.

f) To verify the commutative property for addition with 11/16 and 17/64, we need to check if 11/16 + 17/64 = 17/64 + 11/16. 11/16 + 17/64 = 61/64 17/64 + 11/16 = 61/64 Since both sides are equal, the commutative property holds for addition with 11/16 and 17/64.

g) To verify the commutative property for addition with 3/13 and 10/13, we need to check if 3/13 + 10/13 = 10/13 + 3/13. 3/13 + 10/13 = 1 10/13 + 3/13 = 1 Since both sides are equal, the commutative property holds for addition with 3/13 and 10/13.

h) 23/25, 31/70:

23/25 + 31/70 = (23*14)/(25*14) + (31*5)/(70*5) = 322/350 + 155/350 = 477/350

31/70 + 23/25 = (31*5)/(70*5) + (23*14)/(25*14) = 155/350 + 322/350 = 477/350

Thus, we can see that 23/25 + 31/70 = 31/70 + 23/25 = 477/350. Therefore, the commutative property of addition is verified.

3. Verify the associative property under addition for the following rational numbers.

a) 20, 30, 40 b) 10.2, 15.1, 7.5 c) $\frac{4}{7}, \frac{11}{21}, \frac{3}{14}$

Solution:

a) 20, 30, 40:

To verify the associative property of addition, we need to show that:

(20 + 30) + 40 = 20 + (30 + 40)

We can start by simplifying each side of the equation:

(20 + 30) + 40 = 50 + 40 = 90

20 + (30 + 40) = 20 + 70 = 90

Therefore, we can see that (20 + 30) + 40 = 20 + (30 + 40) = 90. Hence, the associative property of addition is verified.

b) 10.2, 15.1, 7.5:

To verify the associative property of addition, we need to show that:

(10.2 + 15.1) + 7.5 = 10.2 + (15.1 + 7.5)

We can start by simplifying each side of the equation:

(10.2 + 15.1) + 7.5 = 25.3 + 7.5 = 32.8

10.2 + (15.1 + 7.5) = 10.2 + 22.6 = 32.8

Therefore, we can see that (10.2 + 15.1) + 7.5 = 10.2 + (15.1 + 7.5) = 32.8. Hence, the associative property of addition is verified.

c) 4/7, 11/21, 3/14:

To verify the associative property of addition, we need to show that:

(4/7 + 11/21) + 3/14 = 4/7 + (11/21 + 3/14)

We can start by simplifying each side of the equation:

(4/7 + 11/21) + 3/14 = (12/21 + 11/21) + 3/14 = 23/21 + 3/14 = 67/42

4/7 + (11/21 + 3/14) = 4/7 + (6/14 + 9/42) = 4/7 + 9/21 = 23/21

Therefore, we can see that (4/7 + 11/21) + 3/14 = 4/7 + (11/21 + 3/14) = 23/21 or 67/42. Hence, the associative property of addition is verified.

4. Find the multiplicative inverse of the following rational numbers.

a) $\frac{8}{15}$ **b)** $\frac{-12}{23}$ **c)** $\frac{3}{-8}$ **d)** $-\frac{12}{19}$ **e)** $\frac{10}{27}$ **f)** $-\frac{62}{79}$

Solution:

a) 8/15 The multiplicative inverse of 8/15 is 15/8. We can verify this by multiplying 8/15 by 15/8: 8/15 × 15/8 = (8 × 15)/(15 × 8) = 1/1 = 1

b) (–12)/23 The multiplicative inverse of (–12)/23 is –23/12. We can verify this by multiplying (–12)/23 by –23/12: (–12)/23 × (–23)/12 = (–12 × –23)/(23 × 12) = 1/1 = 1

c) 3/(–8) The multiplicative inverse of 3/(–8) is (–8)/3. We can verify this by multiplying 3/(–8) by (–8)/3: 3/(–8) × (–8)/3 = (3 × –8)/(–8 × 3) = 1/1 = 1

d) –12/19 The multiplicative inverse of –12/19 is –19/12. We can verify this by multiplying –12/19 by –19/12: –12/19 × –19/12 = (–12 × –19)/(19 × 12) = 1/1 = 1

e) 10/27 The multiplicative inverse of 10/27 is 27/10. We can verify this by multiplying 10/27 by 27/10: 10/27 × 27/10 = (10 × 27)/(27 × 10) = 1/1 = 1

f) –62/79 The multiplicative inverse of –62/79 is –79/62. We can verify this by multiplying –62/79 by –79/62: –62/79 × –79/62 = (–62 × –79)/(79 × 62) = 1/1 = 1

5. Verify the commutative property under multiplication for the following rational numbers.

a) 2, 8, b) 18 22 c) 0.2, 1.9 d) 15/56,28/50

Solution:

a) To verify the commutative property under multiplication for the rational numbers 2 and 8, we need to compute both 2 × 8 and 8 × 2 and check if they are equal.

2 × 8 = 16 8 × 2 = 16

Since 2 × 8 = 8 × 2 = 16, we can conclude that the commutative property holds for these two rational numbers.

b) To verify the commutative property under multiplication for the rational numbers 18 and 22, we need to compute both 18 × 22 and 22 × 18 and check if they are equal.

18 × 22 = 396 22 × 18 = 396

Since 18 × 22 = 22 × 18 = 396, we can conclude that the commutative property holds for these two rational numbers.

c) To verify the commutative property under multiplication for the rational numbers 0.2 and 1.9, we need to compute both 0.2 × 1.9 and 1.9 × 0.2 and check if they are equal.

0.2 × 1.9 = 0.38 1.9 × 0.2 = 0.38

Since 0.2 × 1.9 = 1.9 × 0.2 = 0.38, we can conclude that the commutative property holds for these two rational numbers.

d) To verify the commutative property under multiplication for the rational numbers 15/56 and 28/50, we need to compute both (15/56) × (28/50) and (28/50) × (15/56) and check if they are equal.

(15/56) × (28/50) = (15 × 28) / (56 × 50) = 3/20

(28/50) × (15/56) = (28 × 15) / (50 × 56) = 3/20

Since (15/56) × (28/50) = (28/50) × (15/56) = 3/20 we can conclude that the commutative property holds for these two rational numbers.

6. **Verify the associative property under multiplication for the following rational numbers.**

a) 10, 15, 20 b) 8, 3, 5 c) 2.5, 3.7, 7.2 d) 72/91, 13/18 , 35/44

Solution:

a) (10 x 15) x 20 = 150 x 20 = 3000 10 x (15 x 20) = 10 x 300 = 3000

Both products are equal to 3000, Hence proved.

b) (8 x 3) x 5 = 24 x 5 = 120 8 x (3 x 5) = 8 x 15 = 120

Both products are equal to 120, Hence proved.

c) (2.5 x 3.7) x 7.2 = 9.25 x 7.2 = 66.6

2.5 x (3.7 x 7.2) = 2.5 x 26.64 = 66.6

Both products are equal to 66.6, Hence proved.

d) (72/91 x 13/18) x 35/44 = 4/7 x 35/44 = 15/11

72/91 x (13/18 x 35/44) = 72/91 x 455/792 = 15/11

The two products are equal. Hence proved.

7. **Verify the distributive property of multiplication over addition and subtraction for the following rational numbers.**

   a) 13, 15, 17 **b) 39, 40, 44** **c) 10.1, 20.1, 30.1** **d)** $\frac{12}{15}, \frac{23}{30}, \frac{17}{25}$

   Solution:

   a) 13, 15, 17:

   13(15 + 17) = 13(32) = 416

   13(15) + 13(17) = 195 + 221 = 416

   Therefore, the distributive property is verified for these numbers.

   b) 39, 40, 44:

   40(39 + 44) = 40(83) = 3320

   40(39) + 40(44) = 1560 + 1760 = 3320

   Therefore, the distributive property is verified for these numbers.

   c) 10.1, 20.1, 30.1:

   20.1(10.1 + 30.1) = 20.1(40.2) = 808.02

   20.1(10.1) + 20.1(30.1) = 203.01 + 605.01 = 808.02

   Therefore, the distributive property is verified for these numbers.

   d) 12/15, 23/30, 17/25:

   (23/30)(12/15 + 17/25) = (23/30)(37/25) = 851/750

   (23/30)(12/15) + (23/30)(17/25) = (46/75) + (391/750) = 851/750

   Therefore, the distributive property is verified for these numbers.

Learning Check 1.4

1. **Bisma used 3 1/3 m of cloth on her shirt and 1 2/5 m on her scarf. How much cloth did she use in all?**

   Solution:

   To find the total cloth used by Bisma, we need to add the amount of cloth used in her shirt and scarf:

   - Cloth used in shirt = 3 1/3 meters
   - Cloth used in scarf = 1 2/5 meters
   - Total cloth used = 3 1/3 + 1 2/5 = 4 11/15 meters

2. **Ali has a rope that is 12 1/7 m long. He made a tent out of 9 1/2 m. How much of the rope is left?**

 Solution:

 To find the amount of rope left with Ali, we need to subtract the length of the tent from the total length of the rope:

 Total length of rope = 12 1/7 meters

 Length of tent = 9 1/2 meters

 Length of rope left = 12 1/7 - 9 1/2 = 2 9/14 meters

3. **Alisha drank 4 1/4 litres of water and Attia drank 3/4 litres more water than Alisha. How much water did Attia drink?**

 Solution:

 To find the amount of water drank by Attia, we need to add the amount of water drank by Alisha and 3/4 liters:

 Water drank by Alisha = 4 1/4 liters

 Water drank by Attia = 4 1/4 + 3/4 = 5 liters

4. **On Sunday, a shopkeeper sold 22 1/5 kg of apples and 13 2/3 kg of mangoes. How much fruit did he sell altogether?**

 Solution:

 To find the total amount of fruit sold by the shopkeeper, we need to add the weight of apples and mangoes sold:

 Weight of apples sold = 22 1/5 kg

 Weight of mangoes sold = 13 2/3 kg

 Total weight of fruit sold = 22 1/5 + 13 2/3 = 35 13/15 kg

5. **The length and width of a rectangular room are 8 3/4 m and 6 1/4 m, respectively. Find the perimeter of the room.**

 Solution:

 To find the perimeter of the rectangular room, we need to add the lengths of all sides:

 Length of the room = 8 3/4 meters

 Width of the room = 6 1/4 meters

Perimeter of the room = 2(8 3/4 + 6 1/4) = 2(15) = 30 meters

6. **Habib had a piece of wood 6 5/8 m long. If he divided the wood into pieces 7/8 m long, then how many pieces can the wood be divided into?**

   Solution:

   To find the number of pieces the wood can be divided into, we need to divide the total length of the wood by the length of each piece:

   Length of wood = 6 5/8 meters

   Length of each piece = 7/8 meters

   Number of pieces = (6 5/8) ÷ (7/8) = (53/8) ÷ (7/8) = 53/7 = 7 4/7 pieces

7. **Kinza bought 2 1/2 kg of potatoes and 1 3/4 kg of onions. How many kilograms of potatoes and onions did she buy altogether?**

   Solution:

   To find the total amount of potatoes and onions bought by Kinza, we need to add the weight of potatoes and onions:

   Weight of potatoes bought = 2 1/2 kg

   Weight of onions bought = 1 3/4 kg

   Total weight of potatoes and onions = 2 1/2 + 1 3/4 = 4 1/4 kg

8. **Kiran purchased 4 1/2 kg of sugar and consumed 2 1/4 kg. Find the mass of remaining sugar.**

   Solution:

   To find the amount of sugar remaining with Kiran, we need to subtract the amount of sugar consumed from the total amount of sugar purchased:

   Total amount of sugar purchased = 4 1/2 kg

   Amount of sugar consumed = 2 1/4 kg

   Amount of sugar remaining = 4 1/2 - 2 1/4 = 2 1/4 kg

Learning Check 1.5

1. Simplify the following.

a) 213 + 2[15 −3{13 + 25(35 – $\overline{18 - 15}$)}]

=213 + 2[15 −3{13 + 25(35 –3)}]

= 213 + 2[15 −3{13 + 25(32)}]

= 213 + 2[15 −3{13 + (800)}]

= 213 + 2[15− (2439)]

= 213 + [2(−2424)]

= 213 + (−4848)

= –4635

b) 357 − 35[12 + 25{(-27 + 18) ÷ 3(15 + 10)}]

=357 – 35[12 + 25{(-9) ÷ 3(15 + 10)}]

= 357 – 35[12 + 25{-9 ÷ 3 × 25}]

= 357 – 35[12 + 25{-3 x 25}]

= 357 – 35[12 + (25)(–75)]

= 357 – 35[12 + (–1875)]

= 357 – 35[-1863]

= 357 + 65205

= 65562

c) 150 − [12 − 3{5 – ($-\overline{10 + 7}$ + 8)}]

=150 – [12 – 3{5 – (-17 +8)}]

= 150 – [12 – 3{5 – (-9)}]

= 150 – [12 – (3)(14)]

= 150 – [12 – (42)]

= 150 – [–30]

= 180

d) 8(516 - 62 x 5) + ($\overline{32+5}$÷ 9) + 51

=8(516 - 62 × 5) + (37 ÷ 9) + 51

= 8(516 - (310)) + (4.11) + 51

= [8(206)] + (4.11) + 51

= (1648) + (4.11) + 51

= (1652.11) + (51)

= 1703.11

e) 672 – 85[26 – 11{120 – 5(1548 – 1530)}]

= 672 – 85[26 – 11{120 – (5)(18)}]

= 672 – 85[26 – (11)(30)]

= 672 – [85(-304)]

= 672 – (-25840)

= (672) + (25840)

= 26512

f) 42.5 – [(5.9 – $\overline{\mathbf{6.5 - 8.2}}$) + ($\overline{\mathbf{12.6 - 2.4}}$ + $\mathbf{4.5}$)] × $\mathbf{51 \div 17}$

=42.5 – [(5.9 + 1.7) + $(10.2 + 4.5)] \times 51 \div 17$

=42.5 – [(7.6) + (14.7)]× (51) ÷ (17)

= 42.5 – [(22.3)]× (51) ÷ (17)

= 42.5 – [(1137.3)] ÷ (17)

= 42.5 – (66.9)

= -24.4

g) 3.4 – [{2.1 – 1.5($\overline{\mathbf{4.6 - 3.9}}$ $-$ $\mathbf{5.6}$)}+ 2.4 × 1.6]

=3.4 – [{2.1 – 1.5(0.7 – 5.6)} + 2.4 × 1.6]

=3.4 – [{2.1 – 1.5(0.7 - 5.6)} + 2.4 × 1.6]

= 3.4 – [{2.1 – 1.5 × (–4.9)} + 2.4 × 1.6]

= 3.4 – [{2.1 + 7.35} + 3.84]

= 3.4 – [9.45 + 3.84]

= 3.4 – 13.29

= -9.89

h) 5.9 + 2.4[7.2 – 1.5{1.6 – (1.3 – 4.8 + 9.7)}]

= 5.9 + 2.4[7.2 – 1.5{1.6 – (6.2)}]

= 5.9 + 2.4[7.2 – 1.5 × (-4.6)]

= 5.9 + 2.4[7.2 + 6.9]

= 5.9 + 2.4 × [14.1]

= 5.9 + 33.84

= 39.74

i) 7/16 – [3/4 × 1/8{1/4 – (2/3 + 3/8)}]

= 7/16 – [3/4 × 1/8{1/4 – (25/24)}]

= 7/16 – [3/4 × 1/8 × (-19/24)]

= 7/16 – [–19/256]

= 7/16 + 19/256

= 131/256

j) 16 2/5 + 4[20 7/10 - 2{4 3/5 + (2 1/5 - 2 3/5)}]

$$= 16\frac{2}{5} + 4\ \left(20\frac{7}{10} - 2\ \left(4\frac{3}{5} + 2\frac{1}{5} - \frac{13}{5}\right)\right)$$
$$= 16\frac{2}{5} + 4\ \left(20\frac{7}{10} - 2\ \left(\frac{34}{5} - \frac{13}{5}\right)\right)$$
$$= 16\frac{2}{5} + 4\ \left(20\frac{7}{10} - 2\ \frac{21}{5}\right)$$
$$= 16\frac{2}{5} + 4\ \left(20\frac{7}{10} - \frac{42}{5}\right)$$
$$= 16\frac{2}{5} + 4\ \frac{123}{10}$$
$$= 16\frac{2}{5} + \frac{246}{5}$$
$$= \frac{328}{5}$$

= 65 3/5

k) 9/20 - (2/5 - 0.2) + 1/3 × 21/2

$$= \frac{9}{20} - \left(\frac{2}{5} - \frac{2}{10}\right) + \frac{21}{3} \cdot \frac{1}{2}$$
$$= \frac{9}{20} - \left(\frac{2}{5} - \frac{1}{5}\right) + \frac{21}{3} \cdot \frac{1}{2}$$
$$= \frac{9}{20} - \frac{1}{5} + \frac{21}{3} \cdot \frac{1}{2}$$
$$= \frac{9}{20} - \frac{1}{5} + \frac{1}{3} \cdot \frac{21}{2}$$
$$= \frac{9}{20} - \frac{1}{5} + \frac{7}{2}$$
$$= \frac{1}{4} + \frac{7}{2}$$
$$= \frac{15}{4}$$

= 3 3/4

l) $\mathbf{6 + \{\frac{4}{3} + (\frac{3}{4} - \frac{1}{3}) \div 3\frac{1}{4}\}}$

$= 6 + \{\frac{4}{3} + (\frac{5}{12}) \div \frac{13}{4}\}$

$=6 + \{\frac{4}{3} + \frac{5}{12} \times \frac{4}{13}\}$

$=6 + \{\frac{4}{3} + \frac{5}{39}\}$

$=6 + \{\frac{19}{13}\}$

$=\frac{97}{13}$

$=7\frac{6}{13}$

m) $\mathbf{1\frac{1}{5}\{(3\frac{1}{3} - 2\frac{1}{2}) \div (2\frac{5}{21} - 2)\}}$

= 6/5 {(10/3 - 5/2) ÷ (47/21 - 42/21)}

= 6/5 {(20/6 - 15/6) ÷ (5/21)}

= 6/5 {(5/6) ÷ (5/21)}

= 6/5 × (5/6) × (21/5)

= (6 × 5 × 21)/(5 × 6 × 5)

= 21/5

$= 4\frac{1}{5}$

n) $\mathbf{[\frac{3}{5} + \{(\frac{1}{7} \times \frac{2}{3}) \div \frac{10}{21}\} - \frac{4}{5}]}$

= [3/5 + {(2/21) ÷ 10/21} - 4/5]

= [3/5 + (2/21) × (21/10) - 4/5]

= [3/5 + (42/210) - 4/5]

= [126/210 + (42/210) - 168/210]

= (126 + 42 - 168)/210

= 0

o) $\mathbf{1 \div \frac{3}{7}(6 \div 8 \times 1) + (\frac{1}{5} \div \frac{7}{25} - \frac{8}{8})}$

$=1 \times \frac{7}{3}(0.75) + (\frac{1}{5} \times \frac{25}{7} - 1)$

$= \frac{7}{3}(\frac{3}{4}) + (\frac{5}{7} - 1)$

$= \frac{7}{4} + (\frac{-2}{7})$

$= \frac{7}{4} - \frac{2}{7}$

$= \frac{41}{28}$

$= 1\frac{13}{28}$

Unit Check

1. Encircle the correct option.

a) Mixed numbers can be converted into ________fractions.

i) proper ii) improper iii) unlike iv) like

b) Additive inverse of $-\frac{3}{5}$ is:

i) $-\frac{3}{5}$ ii) $-\frac{5}{3}$ iii) $\frac{3}{5}$ iv) $\frac{5}{3}$

c) Multiplicative inverse of $\frac{1}{3}$ is:

i) $-\frac{1}{3}$ ii) $\frac{1}{3}$ iii) -3 iv) 3

d) The additive identity of rational numbers is:

i) 0 ii) 1 iii) a iv) $\frac{1}{a}$

e) $2\frac{1}{5} - 1\frac{3}{10}$ = ______________.

i) $2\frac{2}{5}$ ii) $1\frac{9}{10}$ iii) $\frac{9}{10}$ iv) $2\frac{1}{10}$

2. Fill in the blanks.

a) $a + (-a)$ = ____0____.

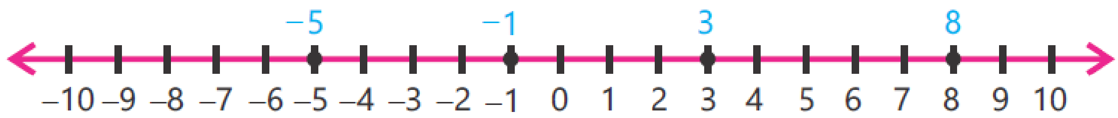
b) The multiplicative identity of rational numbers is ____1____.

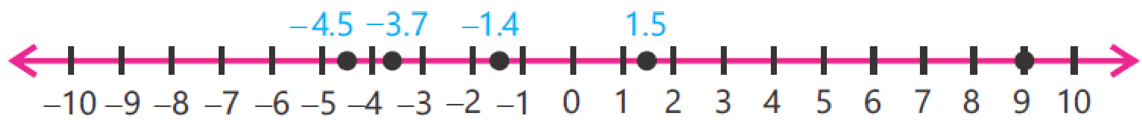
c) $\frac{3}{5} \times \frac{6}{7} = \frac{6}{7} \times \frac{3}{5}$ shows the __commutative__ property of rational numbers under multiplication.

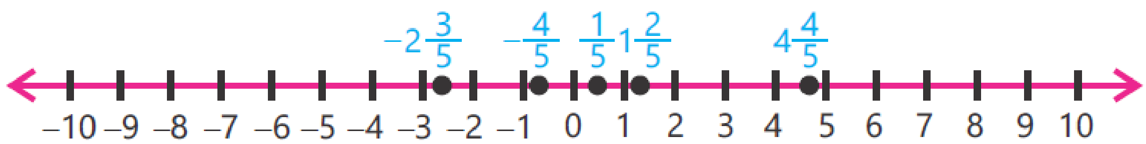
d) The absolute value of -5 is ____5____.

e) The total mass of $5\frac{3}{4}$ kg of sugar and $2\frac{3}{8}$ kg of rice is __8 1/8 kg.__.

3. Show the given rational numbers on a number line.

a)


b)


c)


4. Use the symbols <, > or = to compare the following rational numbers.

a) 67 and 76 **b) −18 and −9** **c) −5 and +2**

a) To compare 67 and 76, we can observe that both are positive integers. We can see that 76 is greater than 67, so we write:

67 < 76

b) To compare -18 and -9, we can observe that both are negative integers. The greater integer with negative sign is smaller than smaller integer with negative sign. So, we can write:

−18 > −9

c) To compare −5 and +2, we can observe that one is negative and one is positive. We know that any negative number is always less than any positive number, so we write:

−5 < 2.

d) To compare 5/10 and 0.5, we can simplify 5/10 to 1/2, which is the same as 0.5. Therefore, we have: 5/10 = 1/2 = 0.5

So, 5/10 is equal to 0.5, and we write: 5/10 = 0.5

e) To compare -4/5 and 1/2, we know that any negative number is always less than any positive number, so we write:

$-4/5 < 1/2$

f) To compare 3/7 and 11/14, we need to convert them into a common form. In this case, we can convert 3/7 into 6/14 by multiplying both the numerator and denominator by 2. Now we can compare these numbers directly.

So we have 6/14 and 11/14.

- Since the denominators are the same, we can compare the numerators directly.
- In this case, we have $6/14 < 11/14$.

  Therefore, we have:
- $3/7 < 11/14$

5. Compare and arrange the given rational numbers in ascending order.

a) −17, −2, +14, −4, +10 **b) 8.5, 3.4, 4.5, 7.2, 4.8, 6.7, 2.5**

c) 5/12, −2/3, −7/2, 2/3, 4/9, 8/9 **d) 14, 45, 34, 85, 73, 18, 48**

Solution:

a) To compare and arrange −17, −2, +14, −4, +10 in ascending order, we can simply write them in order from smallest to largest:

$-17 < -4 < -2 < 10 < 14$

So the ascending order is: −17, −4, −2, 10, 14

b) To compare and arrange 8.5, 3.4, 4.5, 7.2, 4.8, 6.7, 2.5 in ascending order, we can write them in order from smallest to largest:

$2.5 < 3.4 < 4.5 < 4.8 < 6.7 < 7.2 < 8.5$

So the ascending order is: 2.5, 3.4, 4.5, 4.8, 6.7, 7.2, 8.5

c) To compare and arrange 5/12, −2/3, −7/2, 2/3, 4/9, 8/9 in ascending order, we can first convert them to have the same denominator. The least common multiple of 2, 3, 9, and 12 is 36, so we can write:

5/12 = 15/36 -2/3 = -24/36 -7/2 = -126/36 2/3 = 24/36 4/9 = 16/36 8/9 = 32/36

Now we can write them in order from smallest to largest:

$-126/36 < -24/36 < 15/36 < 16/36 < 24/36 < 32/36$

We can simplify the fractions:

−7/2 < −2/3 < 5/12 < 4/9 < 2/3 < 8/9

So the ascending order is: −7/2, −2/3, 5/12, 4/9, 2/3, 8/9

d) To compare and arrange 14, 45, 34, 85, 73, 18, 48 in ascending order, we can write them in order from smallest to largest:

14 < 18 < 34 < 45 < 48 < 73 < 85

So the ascending order is: 14, 18, 34, 45, 48, 73, 85

6. Prove the commutative property for the given rational numbers under addition and multiplication.

a) 3.2 and 8.8 **b)** $\frac{3}{7}$ **and** $\frac{8}{21}$ **c)** $\frac{2}{11}$ **and** $\frac{5}{44}$ **d)** $\frac{1}{2}$ **and** $\frac{2}{5}$

Solution:

a) 3.2 and 8.8

- Addition: 3.2 + 8.8 = 12 and 8.8 + 3.2 = 12 Therefore, 3.2 + 8.8 = 8.8 + 3.2
- Multiplication: 3.2 × 8.8 = 28.16 and 8.8 × 3.2 = 28.16

  Therefore, 3.2 × 8.8 = 8.8 × 3.2

b) 3/7 and 8/21

- Addition: (3/7) + (8/21) = (9/21) + (8/21) = (17/21) and (8/21) + (3/7) = (8/21) + (9/21) = (17/21)

Therefore, (3/7) + (8/21) = (8/21) + (3/7)

- Multiplication: (3/7) × (8/21) = (24/147) and (8/21) × (3/7) = (24/147)

  Therefore, (3/7) × (8/21) = (8/21) × (3/7)

c) 2/11 and 5/44

- Addition: (2/11) + (5/44) = (8/44) + (5/44) = (13/44) and (5/44) + (2/11) = (5/44) + (8/44) = (13/44)

  Therefore, (2/11) + (5/44) = (5/44) + (2/11)
- Multiplication: (2/11) × (5/44) = (10 /484) and (5 /44)× (2 /11)=10 /484

  Therefore, (2 /11)× (5 /44)=(5 /44)× (2 /11)

d) 1 /2 and 2 /5

- Addition: (1 /2) + (2 /5) = (5+4)/10=9 /10 and (2 /5) + (1 /2) = ((4+5)/10) = 9 /10 Therefore, (1 /2) + (2 /5) = (2 /5)+(1 /2)

- Multiplication: ((1 /2)(2 /5))=1 /5 and (2 /5)(1 /2) = 1 /5.

  Therefore, (1 /2) × (2 /5) = (2 /5) × (1 /2)

7. Prove the associative property for the given rational numbers under addition and multiplication.

a) 7/8,3/16 and 5/8 b) 1/3, 1/6 and 1/9 c) 12/21 , 23/42 and 15/84

a) To prove the associative property for 7/8, 3/16, and 5/8 under addition, we have:

(7/8 + 3/16) + 5/8 = 14/16 + 5/8 = 21/16 7/8 + (3/16 + 5/8) = 7/8 + 11/16 = 21/16

Therefore, (7/8 + 3/16) + 5/8 = 7/8 + (3/16 + 5/8), and the associative property holds for addition.

To prove the associative property for 7/8, 3/16, and 5/8 under multiplication, we have:

(7/8 × 3/16) × 5/8 = 21/128 × 5/8 = 21/256 7/8 × (3/16 × 5/8) = 7/8 × 15/128 = 21/256

Therefore, (7/8 × 3/16) × 5/8 = 7/8 × (3/16 × 5/8), and the associative property holds for multiplication.

b) To prove the associative property for 1/3, 1/6, and 1/9 under addition, we have:

(1/3 + 1/6) + 1/9 = 2/6 + 1/9 = 5/9 1/3 + (1/6 + 1/9) = 1/3 + 3/18 = 5/9

Therefore, (1/3 + 1/6) + 1/9 = 1/3 + (1/6 + 1/9), and the associative property holds for addition.

To prove the associative property for 1/3, 1/6, and 1/9 under multiplication, we have:

(1/3 × 1/6) × 1/9 = 1/54 1/3 × (1/6 × 1/9) = 1/3 × 1/54 = 1/162

Therefore, (1/3 × 1/6) × 1/9 = 1/3 × (1/6 × 1/9), and the associative property holds for multiplication.

c) To prove the associative property for 12/21, 23/42, and 15/84 under addition, we have:

(12/21 + 23/42) + 15/84 = 24/42 + 15/84 = 29/42 12/21 + (23/42 + 15/84) = 12/21 + 31/84 = 29/42

Therefore, (12/21 + 23/42) + 15/84 = 12/21 + (23/42 + 15/84), and the associative property holds for addition.

To prove the associative property for 12/21, 23/42, and 15/84 under multiplication, we have:

(12/21 × 23/42) × 15/84 = 46/147 × 15/84 = 23/588 12/21 × (23/42 × 15/84)

8. Prove the distributive property of multiplication of rational numbers under addition and subtraction.

a) $\frac{8}{15}, \frac{5}{6}, \frac{7}{9}$ **b)** $\frac{11}{24}, \frac{19}{36}, \frac{5}{9}$ **c)** $\frac{2}{3}, \frac{7}{10}, \frac{17}{30}$

To prove the distributive property of multiplication of rational numbers under addition and subtraction, we need to show that:

a(b + c) = ab + ac and a(b - c) = ab - ac

where a, b, and c are rational numbers.

Let's prove the distributive property using the given rational numbers:

a) 8/15, 5/6, 7/9

8/15 × (5/6 + 7/9) = 8/15 × (15/18 + 14/18) = 8/15 × 29/18 = 116/135

(8/15 × 5/6) + (8/15 × 7/9) = 4/9 + 28/135 = (60 + 28) / 135 = 88/135

Therefore, a(b + c) = ab + ac, and the distributive property holds for these rational numbers.

For a(b - c), we have:

8/15 × (5/6 - 7/9) = 8/15 × (5/6 - 7/9 × 2/2) = 8/15 × (5/6 - 14/18) = 8/15 × (-1/9) = -8/135

And for ab - ac, we have:

(8/15 × 5/6) - (8/15 × 7/9) = 4/9 - 28/135 = (60 - 28) / 135 = 32/135

Therefore, a(b - c) = ab - ac, and the distributive property holds for these rational numbers as well.

b) 11/24, 19/36, 5/9

For a(b + c), we have:

11/24 × (19/36 + 5/9) = 11/24 × (19/36 + 20/36) = 11/24 × 39/36 = 429/864

And for ab + ac, we have:

(11/24 × 19/36) + (11/24 × 5/9) = 209/864 + 55/864 = 264/864 = 11/36

Therefore, a(b + c) = ab + ac, and the distributive property holds for these rational numbers.

For a(b - c), we have:

11/24 × (19/36 - 5/9) = 11/24 × (19/36 - 20/36 × 2/2) = 11/24 × (-1/9) = -11/216

And for ab - ac, we have:

(11/24 × 19/36) - (11/24 × 5/9) = 209/864 - 55/864 = 154/864 = 77/432

Therefore, a(b - c) = ab - ac, and the distributive property holds for these rational numbers as well.

c) 2/3, 7/10, 17/30

For a(b + c), we have:

2/3 × (7/10 + 17/30) = 2/3 × (21/30 + 17/30) = 2/3 × 38/30 = 76/90 = 38/45

And for ab + ac, we have:

(2/3 × 7/10) + (2/3 × 17/30) = 14/30 + 34/90 = 7/15 + 17/45 = (21 + 17)/45 = 38/45

Therefore, a(b + c) = ab + ac, and the distributive property holds for these rational numbers.

For a(b - c), we have:

2/3 × (7/10 - 17/30) = 2/3 × (7/10 - 17/30) = 2/3 × (2/15) = 4/45

And for ab - ac, we have:

(2/3 × 7/10) - (2/3 × 17/30) = 14/30 - 34/90 = 7/15 - 17/45 = (21 - 17)/45 = 4/45

Therefore, a(b - c) = ab - ac, and the distributive property holds for these rational numbers as well.

9. Simplify the following.

a) $\frac{9}{20} - [\frac{1}{5} + \{\frac{1}{4} + (\frac{5}{6} - \overline{\frac{1}{3} + \frac{1}{2}})\}]$

$= \frac{9}{20} - [\frac{1}{5} + \{\frac{1}{4} + (\frac{5}{6} - \frac{5}{6})\}]$

$= \frac{9}{20} - [\frac{1}{5} + \{\frac{1}{4} + (0)\}]$

$= \frac{9}{20} - [\frac{1}{5} + \frac{1}{4}]$

$=\frac{9}{20} - \frac{9}{20}$

$= 0$

b) $5\frac{1}{4} \div \frac{3}{7} (\frac{45}{24} - \frac{2}{3} + \frac{5}{6}) \times \frac{2}{5}$

$=\frac{21}{4} \times \frac{7}{3} (\frac{49}{24}) \times \frac{2}{5}$

$=\frac{7}{2} \times \frac{7}{1} (\frac{49}{24}) \times \frac{1}{5}$

= 2401/240

= 10 1/240

10. Ali has two pipes that are 7 1/6 m and 4 3/4 m long, respectively. What is the total length of the pipes altogether? What is the difference between the lengths of the two pipes?

To find the total length of the two pipes, we can simply add their lengths:

Total length = 7 1/6 m + 4 3/4 m = 43/6 m + 19/4 m = 143/12 m = 11 11/12 m

Difference between lengths = 7 1/6 m − 4 3/4 m = 43/6 – 19/4 = 29/12 = 2 5/12 m

11. Ammar purchased 5 1/2 litres of oil. He divides the oil equally into 4 cans. How much oil will there be in each can?

Total oil = 5 ½ litre

No. of cans = 4

Oil in each can = 5 1/2 l ÷ 4 = 11/2 l × 1/4 = 11/8 l = 1 3/8 l

Learning Check 2.1

1. **Find the LCM of the following.**
   Solution:

   a) 6, 8, 10

2	6	8	10
2	3	4	5
2	3	2	5
3	3	1	5
5	1	1	5
	1	1	1

LCM of $6, 8, 10 = 2 \times 2 \times 2 \times 3 \times 5 = 120$

   b) 32, 48, 72

2	32	48	72
2	16	24	36
2	8	12	18
2	4	6	9
2	2	3	9
3	1	3	9
3	1	1	3
	1	1	1

LCM of $32, 48, 72 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$

   c) 15, 30, 90

2	15	30	90
3	15	15	45
3	5	5	15
5	5	5	5
	1	1	1

LCM of $15, 30, 90 = 2 \times 3 \times 3 \times 5 = 90$

d) 15, 60, 144

2	15	60	144
2	15	30	72
2	15	15	36
2	15	15	18
3	15	15	9
3	5	5	3
5	5	5	1
	1	1	1

LCM of $15, 60, 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 720$

e) 8, 24, 40

2	8	24	40
2	4	12	20
2	2	6	10
3	1	3	5
5	1	1	5
	1	1	1

LCM of $8, 24, 40 = 2 \times 2 \times 2 \times 3 \times 5 = 120$

f) 30, 42, 54

2	30	42	54
3	15	21	27
3	5	7	9
3	5	7	3
5	5	7	1
7	1	7	1
	1	1	1

LCM of $30, 42, 54 = 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 1890$

g) 12, 14, 86

2	12	14	86
2	6	7	43
3	3	7	43
7	1	7	43
43	1	1	43
	1	1	1

LCM of $12, 14, 86 = 2 \times 2 \times 3 \times 7 \times 43 = 3612$

h) 100, 150, 200

2	100	150	200
2	50	75	100
2	25	75	50
3	25	75	25
5	25	25	25
5	5	5	5
	1	1	1

LCM of $100, 150, 200 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 600$

i) 48, 108, 140

2	48	108	140
2	24	54	70
2	12	27	35
2	6	27	35
3	3	27	35
3	1	9	35
3	1	3	35
5	1	1	35
7	1	1	7
	1	1	1

LCM of $48, 108, 140 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 7 = 15120$

j) 105, 220, 345

2	105	220	345
2	105	110	345
3	105	55	345
5	35	55	115
7	7	11	23
11	1	11	23
23	1	1	23
	1	1	1

LCM of $105, 220, 345 = 2 \times 2 \times 3 \times 5 \times 7 \times 11 \times 23 = 106260$

k) 159, 265, 371

3	159	265	371
5	53	265	371
7	53	53	371
53	53	53	53
	1	1	1

LCM of $159, 265, 371 = 3 \times 5 \times 7 \times 53 = 5565$

l) 520, 780, 1300

2	520	780	1300
2	260	390	650
2	130	195	325
3	65	195	325
5	65	65	325
5	13	13	65
13	13	13	13
	1	1	1

LCM of $520, 780, 1300 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 13 = 7800$

2. Find the HCF of the following.

a) 8, 12, 24

Factor of 8,12,24 are as follows...

2	8
2	4
2	2
	1

2	12
2	6
3	3
	1

2	24
2	12
2	6
3	3
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.
Find HCF

$8 = 2 \times 2 \times 2$

$12 = 2 \times 2 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

$\text{HCF} = 2 \times 2 = 4$

HCF of 8, 12, 24 is 4

b) 14, 20, 28

Factor of 14,20,28 are as follows...

2	14
7	7
	1

2	20
2	10
5	5
	1

2	28
2	14
7	7
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.
Find HCF

$14 = 2 \times 7$

$20 = 2 \times 2 \times 5$

$28 = 2 \times 2 \times 7$

$\text{HCF} = 2 = 2$

HCF of 14, 20, 28 is 2

c) 23, 45, 60

Factor of 23,45,60 are as follows...

23	23
	1

3	45
3	15
5	5
	1

2	60
2	30
3	15
5	5
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$23 = 23$

$45 = 3 \times 3 \times 5$

$60 = 2 \times 2 \times 3 \times 5$

HCF = = 1

HCF of 23, 45, 60 is 1

d) 32, 96, 112

Factor of 32,96,112 are as follows...

2	32
2	16
2	8
2	4
2	2
	1

2	96
2	48
2	24
2	12
2	6
3	3
	1

2	112
2	56
2	28
2	14
7	7
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$112 = 2 \times 2 \times 2 \times 2 \times 7$

HCF $= 2 \times 2 \times 2 \times 2 = 16$

HCF of 32, 96, 112 is 16

e) 27, 72, 90

Factor of 27,72,90 are as follows...

3	27
3	9
3	3
	1

2	72
2	36
2	18
3	9
3	3
	1

2	90
3	45
3	15
5	5
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.
Find HCF

27 =				3	× 3	× 3			
72 =	2	× 2	× 2	× 3	× 3				
90 =	2			× 3	× 3		× 5		
HCF =				3	× 3			= 9	

HCF of 27, 72, 90 is 9

f) 150, 210, 320

Factor of 150,210,320 are as follows...

2	150
3	75
5	25
5	5
	1

2	210
3	105
5	35
7	7
	1

2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.
Find HCF

150 =	2					× 3	× 5	× 5		
210 =	2					× 3	× 5		× 7	
320 =	2	× 2	× 2	× 2	× 2	× 2	× 5			
HCF =	2						× 5			= 10

HCF of 150, 210, 320 is 10

g) 18, 54, 82

Factor of 18,54,82 are as follows...

2	18
3	9
3	3
	1

2	54
3	27
3	9
3	3
	1

2	82
41	41
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$18 = 2 \times 3 \times 3$

$54 = 2 \times 3 \times 3 \times 3$

$82 = 2 \times 41$

$\text{HCF} = 2 = 2$

HCF of 18, 54, 82 is 2

h) 6, 42, 78

Factor of 6,42,78 are as follows...

2	6
3	3
	1

2	42
3	21
7	7
	1

2	78
3	39
13	13
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$6 = 2 \times 3$

$42 = 2 \times 3 \times 7$

$78 = 2 \times 3 \times 13$

$\text{HCF} = 2 \times 3 = 6$

HCF of 6, 42, 78 is 6

i) 150, 200, 500

Factor of 150,200,500 are as follows...

2	150
3	75
5	25
5	5
	1

2	200
2	100
2	50
5	25
5	5
	1

2	500
2	250
5	125
5	25
5	5
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$150 = 2 \times 3 \times 5 \times 5$

$200 = 2 \times 2 \times 2 \times 5 \times 5$

$500 = 2 \times 2 \times 5 \times 5 \times 5$

$\text{HCF} = 2 \times 5 \times 5 = 50$

HCF of 150, 200, 500 is 50

j) 140, 245, 385

Factor of 140,245,385 are as follows...

2	140
2	70
5	35
7	7
	1

5	245
7	49
7	7
	1

5	385
7	77
11	11
	1

$140 = 2 \times 2 \times 5 \times 7$

$245 = 5 \times 7 \times 7$

$385 = 5 \times 7 \times 11$

$\text{HCF} = 5 \times 7 = 35$

HCF of 140, 245, 385 is 35

k) 246, 656, 984

Factor of 246,656,984 are as follows...

2	246
3	123
41	41
	1

2	656
2	328
2	164
2	82
41	41
	1

2	984
2	492
2	246
3	123
41	41
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$246 = 2 \times 3 \times 41$

$656 = 2 \times 2 \times 2 \times 2 \times 41$

$984 = 2 \times 2 \times 2 \times 3 \times 41$

$\text{HCF} = 2 \times 41 = 82$

HCF of 246, 656, 984 is 82

l) 104, 208, 416

Solution:

Factor of 104,208,416 are as follows...

2	104
2	52
2	26
13	13
	1

2	208
2	104
2	52
2	26
13	13
	1

2	416
2	208
2	104
2	52
2	26
13	13
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$104 = 2 \times 2 \times 2 \times 13$

$208 = 2 \times 2 \times 2 \times 2 \times 13$

$416 = 2 \times 2 \times 2 \times 2 \times 2 \times 13$

$\text{HCF} = 2 \times 2 \times 2 \times 13 = 104$

HCF of 104, 208, 416 is 104

Learning Check 2.2

1. Find the square of the following numbers.

a) 57	b) 72	c) 102	d) 125	e) 205
f) 248	g) 9a	h) 50b	i) 330	j) 700
k) 0.2	l) 1.5	m) 4/5	n) 3/7	o) 3.7

Solution:

a) $57^2 = 57 \times 57 = 3249$

b) $72^2 = 72 \times 72 = 5184$

c) $102^2 = 102 \times 102 = 10404$

d) $125^2 = 125 \times 125 = 15625$

e) $205^2 = 205 \times 205 = 42025$

f) $248^2 = 248 \times 248 = 61504$

g) $(9a)^2 = 9a \times 9a = 81a^2$

h) $(50b)^2 = 50b \times 50b = 2500b^2$

i) $330^2 = 330 \times 330 = 108900$

j) $700^2 = 700 \times 700 = 490000$

k) $0.2^2 = 0.2 \times 0.2 = 0.04$

l) $1.5^2 = 1.5 \times 1.5 = 2.25$

m) $(4/5)^2 = 4/5 \times 4/5 = 16/25$

n) $(3/7)^2 = 3/7 \times 3/7 = 9/49$

o) $3.7^2 = 3.7 \times 3.7 = 13.69$

2. The length of a square-shaped table is 60 cm. Find the area of the surface of the table.

Length of square shaped table= 60 cm

Area of square = length x length

= 60 cm x 60 cm

= 3600 cm^2

Therefore, the area of the surface of the table is 3600 square cm.

3. A square-shaped chessboard has a length of 22 cm. What is the area of the chessboard?

Length of square shaped chessboard= 22 cm

Area of square = length x length

= 22 cm x 22 cm

= 484 cm^2

Therefore, the area of the surface of the chessboard is 484 square cm.

4. The length of a square-shaped plot is 141 m. Find the cost of planting grass in the plot at the rate of Rs 140 per m^2.

Length of square shaped plot= 141 m

Area of square = length x length

= 141 m x 141 m

= 19881 m^2

Rate of planting grass= Rs 140 per m^2

Cost of planting grass = Rs 140 x 19881

= Rs 2783340

Therefore, the cost of planting grass in the square-shaped plot at the rate of Rs 140 per square meter is Rs 2,783,340.

5. A cube-shaped room has a length of 42 m. Find the cost of cementing the walls of the room at the rate of Rs 600 per m_2.
(Hint: The walls of the room are also square-shaped.)

Length of square shaped room= 42 m

Area of square (wall)= length x length

= 42 m x 42 m

= 1764 m^2

So, area of one wall = 1764 m^2

Area of 4 walls = 1764 x 4 = 7056 m^2

Rate of cementing= Rs 600 per m^2

Cost of cementing = Rs 600 x 7056

= Rs 4233600

Therefore, the cost of cementing the walls of the cube-shaped room at the rate of Rs 600 per square meter is Rs 4,233,600.

Learning Check 2.3

1. Find the square root of the following using the factorization method.

a) 25 b) 81 c) 121 d) 169 e) 256
f) 324 g) 441 h) 0.36 i) 0.09 j) 0.49
k) 1.96 l) 100/225 m)16/529

Solution:

a) 25

First find factors of 25

5	25
5	5
	1

Write the prime factors in the form of pairs.

25 = $\overline{5 \text{ x } 5}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{25}$ = 5

b) 81

First find factors of 81

3	81
3	27
3	9
3	3
	1

Write the prime factors in the form of pairs.

81 = $\overline{3 \text{ x } 3}$ x $\overline{3 \text{ x } 3}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{81}$ = 3 x 3

= 9

c) 21

First find factors of 121

11	121
11	11
	1

Write the prime factors in the form of pairs.

121 = $\overline{11 \text{ x } 11}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{121}$ = 11

d) 169

First find factors of 169

13	169
13	13
	1

Write the prime factors in the form of pairs.

169 = $\overline{13 \text{ x } 13}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{169}$ = 13

e) 256

First find factors of 256

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Write the prime factors in the form of pairs.

256 = $\overline{2 \text{ x } 2}$ x $\overline{2 \text{ x } 2}$ x $\overline{2 \text{ x } 2}$ x $\overline{2 \text{ x } 2}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{256}$ = 2 x2 x 2 x 2

= 16

f) 324

First find factors of 324

2	324
2	162
3	81
3	27
3	9
3	3
	1

Write the prime factors in the form of pairs.

324 = $\overline{2 \text{ x } 2}$ x $\overline{3 \text{ x } 3}$ x $\overline{3 \text{ x } 3}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{324}$ = 2 x 3 x 3 = 18

g) 441

First find factors of 441

3	441
3	147
7	49
7	7
	1

Write the prime factors in the form of pairs.

441 = $\overline{3 \text{ x } 3}$ x $\overline{7 \text{ x } 7}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{441}$ = 3 x 7

= 21

h) 0.36

First, change the decimal number into a fraction and then find the prime factors of the numerator and the denominator separately and write them in pairs.
0.36= 36/100

2	36
2	18
3	9
3	3
	1

2	100
2	50
5	25
5	5
	1

Write the prime factors in the form of pairs.

36 = $\overline{2 \text{ x } 2}$ x $\overline{3 \text{ x } 3}$

100 = $\overline{2 \text{ x } 2}$ x $\overline{5 \text{ x } 5}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{36}$ = 2 x 3 = 6

$\sqrt{100}$ = 2 x 5 = 10

So, $\sqrt{0.36}$ = 6/10 = 0.6

i) 0.09

First, change the decimal number into a fraction and then find the prime factors of the numerator and the denominator separately and write them in pairs.
0.09= 9/100

First find factors of 100

2	100
2	50
5	25
5	5
	1

First find factors of 9

3	9
3	3
	1

Write the prime factors in the form of pairs.

9 = $\overline{3 \times 3}$

100 = $\overline{2 \times 2} \times \overline{5 \times 5}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{9}$ = 3

$\sqrt{100}$ = 2 x 5 = 10

So, $\sqrt{0.09}$ = 3/10 = 0.3

j) 0.49

First, change the decimal number into a fraction and then find the prime factors of the numerator and the denominator separately and write them in pairs.
0.49= 49/100

First find factors of 100

2	100
2	50
5	25
5	5
	1

First find factors of 49

7	49
7	7
	1

Write the prime factors in the form of pairs.

49 = $\overline{7 \times 7}$

100 = $\overline{2 \times 2} \times \overline{5 \times 5}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{49}$ = 7

$\sqrt{100}$ = 2 x 5 = 10

So, $\sqrt{0.49}$ = 7/10 = 0.7

k) 1.96

First, change the decimal number into a fraction and then find the prime factors of the numerator and the denominator separately and write them in pairs.
1.96= 196/100

First find factors of 196

2	196
2	98
7	49
7	7
	1

First find factors of 100

2	100
2	50
5	25
5	5
	1

Write the prime factors in the form of pairs.

196 = $\overline{2 \times 2}$ x $\overline{7 \times 7}$

100 = $\overline{2 \times 2}$ x $\overline{5 \times 5}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{196}$ = 2 x 7 = 14

$\sqrt{100}$ = 2 x 5 = 10

So, $\sqrt{1.96}$ = 14/10 = 1.4

l) 100/225

Find the prime factors of the numerator and the denominator separately and write them in pairs.

3	225
3	75
5	25
5	5
	1

2	100
2	50
5	25
5	5
	1

Write the prime factors in the form of pairs.

100 = $\overline{2 \times 2}$ x $\overline{5 \times 5}$

225 = $\overline{3 \times 3}$ x $\overline{5 \times 5}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{100}$ = 2 x 5 = 10 $\sqrt{225}$ = 3 x 5 = 15

So, $\sqrt{100/225}$ = 10/15

m) 16/529

Find the prime factors of the numerator and the denominator separately and write them in pairs.

2	16
2	8
2	4
2	2
	1

23	529
23	23
	1

Write the prime factors in the form of pairs.

16 = $\overline{2 \times 2}$ x $\overline{2 \times 2}$ 529 = $\overline{23 \times 23}$

Take only one number from each pair and find their product. This will be the square root of the number.

$\sqrt{16}$ = 2 x 2 = 4 $\sqrt{529}$ = 23

So, $\sqrt{16/529}$ = 4/23

2. Find the square root of the following using the division method.

a) 625 b) 676 c) 841 d) 144 e) 196
f) 0.04 g) 0.25 h) 0.01 i) 4/36 j) 784/900
k) 576/729 l) 49/484

Solution:

a) 625

$$
\begin{array}{r|l}
 & 2\ \ 5 \\
\hline
2 & \overline{6}\ \overline{25} \\
 & 4 \\
\hline
45 & 2\ 25 \\
 & 2\ 25 \\
\hline
50 & 0
\end{array}
$$

Number = 625
Square Root = 25

b) 676

$$
\begin{array}{r|l}
 & 2\ \ 6 \\
\hline
2 & \overline{6}\ \overline{76} \\
 & 4 \\
\hline
46 & 2\ 76 \\
 & 2\ 76 \\
\hline
52 & 0
\end{array}
$$

Number = 676
Square Root = 26

c) 841

$$
\begin{array}{r|l}
 & 2\ \ 9 \\
\hline
2 & \overline{8}\ \overline{41} \\
 & 4 \\
\hline
49 & 4\ 41 \\
 & 4\ 41 \\
\hline
58 & 0
\end{array}
$$

Number = 841
Square Root = 29

d) 144

$$\begin{array}{r|rr} & 1 & 2 \\ \hline 1 & \overline{1} & \overline{44} \\ & 1 & \\ \hline 22 & & 44 \\ & & 44 \\ \hline 24 & & 0 \end{array}$$

Number = 144
Square Root = 12

e) 196

$$\begin{array}{r|rr} & 1 & 4 \\ \hline 1 & \overline{1} & \overline{96} \\ & 1 & \\ \hline 24 & & 96 \\ & & 96 \\ \hline 28 & & 0 \end{array}$$

Number = 196
Square Root = 14

f) 0.04

First, change the decimal number into a fraction and then find square root of the numerator and the denominator separately.
0.04= 4/100

$$\begin{array}{r|r} & 2 \\ \hline 2 & \overline{4} \\ & 4 \\ \hline 4 & 0 \end{array}$$

Number = 4
Square Root = 2

$$\begin{array}{r|rr} & 1 & 0 \\ \hline 1 & \overline{1} & \overline{00} \\ & 1 & \\ \hline 20 & & 0 \\ & & 0 \\ \hline 20 & & 0 \end{array}$$

Number = 100
Square Root = 10

$\sqrt{0.04} = \sqrt{4/100}$ =2/10 = 0.2

g) 0.25

First, change the decimal number into a fraction and then find square root of the numerator and the denominator separately.
0.25= 25/100

$$\begin{array}{r|l} & 5 \\ 5 & \overline{25} \\ & 25 \\ \hline 10 & 0 \end{array} \qquad \begin{array}{r|ll} & 1 & 0 \\ 1 & \overline{1} & \overline{00} \\ & 1 & \\ \hline 20 & & 0 \\ & & 0 \\ \hline 20 & & 0 \end{array}$$

Number = 25
Square Root = 5

Number = 100
Square Root = 10

$\sqrt{0.25} = \sqrt{25/100}$ =5/10 = 0.5

h) 0.01

First, change the decimal number into a fraction and then find square root of the numerator and the denominator separately.
0.01= 1/100

$$\begin{array}{r|l} & 1 \\ 1 & \overline{1} \\ & 1 \\ \hline 2 & 0 \end{array} \qquad \begin{array}{r|ll} & 1 & 0 \\ 1 & \overline{1} & \overline{00} \\ & 1 & \\ \hline 20 & & 0 \\ & & 0 \\ \hline 20 & & 0 \end{array}$$

Number = 1
Square Root = 1

Number = 100
Square Root = 10

$\sqrt{0.01} = \sqrt{1/100}$ =1/10 = 0.1

i) 4/36

Find square root of the numerator and the denominator separately.

$$\begin{array}{r|l} & 2 \\ 2 & \overline{4} \\ & 4 \\ \hline 4 & 0 \end{array} \qquad \begin{array}{r|r} & 6 \\ 6 & \overline{36} \\ & 36 \\ \hline 12 & 0 \end{array}$$

Number = 4
Square Root = 2

Number = 36
Square Root = 6

$\sqrt{4/36}$ =2/6

j) 784/900

Find square root of the numerator and the denominator separately.

$$\begin{array}{r|rr} & 3 & 0 \\ 3 & \overline{9} & \overline{00} \\ & 9 & \\ \hline 60 & & 0 \\ & & 0 \\ \hline 60 & & 0 \end{array} \qquad \begin{array}{r|rr} & 2 & 8 \\ 2 & \overline{7} & \overline{84} \\ & 4 & \\ \hline 48 & 3 & 84 \\ & 3 & 84 \\ \hline 56 & & 0 \end{array}$$

Number = 900
Square Root = 30

Number = 784
Square Root = 28

$\sqrt{784/900}$ =28/30

k) 576/729

Find square root of the numerator and the denominator separately.

$$\begin{array}{r|rr} & 2 & 4 \\ 2 & \overline{5} & \overline{76} \\ & 4 & \\ \hline 44 & 1 & 76 \\ & 1 & 76 \\ \hline 48 & & 0 \end{array} \qquad \begin{array}{r|rr} & 2 & 7 \\ 2 & \overline{7} & \overline{29} \\ & 4 & \\ \hline 47 & 3 & 29 \\ & 3 & 29 \\ \hline 54 & & 0 \end{array}$$

Number = 576
Square Root = 24

Number = 729
Square Root = 27

$\sqrt{576/729}$ =24/27

l) 49/484

Find square root of the numerator and the denominator separately.

	2	2
2	$\overline{4}$	$\overline{84}$
	4	
42		84
		84
44		0

	7
7	$\overline{49}$
	49
14	0

Number = 484
Square Root = 22

Number = 49
Square Root = 7

$\sqrt{49/484}$ =7/22

1. **Find the length of a side of a square if its area is equal to the area of a rectangle whose width is 6 cm and length is 24 cm.**

Solution:

The area of a rectangle is given by its length multiplied by its width, so the area of the given rectangle is:

Area of rectangle = length × width = 24 cm × 6 cm = 144 cm²

Since the area of the square is equal to the area of the rectangle, we can set up an equation:

Area of square = side × side = 144 cm²

Taking the square root of both sides of the equation, we get:

Side = √144 cm

	1	2
1	$\overline{1}$	$\overline{44}$
	1	
22		44
		44
24		0

Evaluating the square root of 144, we get:

side = 12 cm

Therefore, the length of a side of the square is 12 cm

2. The area of a square-shaped hall of a masjid is 961 m2. Find the perimeter of the masjid.

If the area of a square-shaped hall of a masjid is 961 m^2, then the length of each side of the square can be found by taking the square root of the area:

Side of square = $\sqrt{\text{(Area of square)}} = \sqrt{961 \text{ m}^2} = 31$ m

	3	1
3	9	61
	9	
61		61
		61
62		0

The perimeter of a square is the sum of the lengths of all its sides, so the perimeter of the masjid can be found by multiplying the length of one side by 4:

Perimeter of masjid = 4 × Side of square = 4 × 31 m = 124 m

Therefore, the perimeter of the masjid is 124 meters.

3. The area of a square-shaped pool is 400 m^2. Find the cost of tiling around the pool's boundary at the rate of Rs 525 per metre.

If the area of a square-shaped pool is 400 m^2, then the length of each side of the square can be found by taking the square root of the area:

Side of square = $\sqrt{\text{(Area of square)}} = \sqrt{400 \text{ m}^2} = 20$ m

	2	0
2	4	00
	4	
40		0
		0
40		0

To find the perimeter of the square-shaped pool, we multiply the length of one side by 4:

Perimeter of pool = 4 × Side of square = 4 × 20 m = 80 m

The cost of tiling around the pool's boundary at the rate of Rs 525 per metre is:

Cost of tiling = Perimeter of pool × Cost per metre = 80 m × Rs 525/m = Rs 42,000

Therefore, the cost of tiling around the pool's boundary at the rate of Rs 525 per metre is Rs 42,000.

4. Find the length of a square-shaped wooden piece if its area is 1156 cm^2.

If the area of a square-shaped wooden piece is 1156 cm², then the length of each side of the square can be found by taking the square root of the area:

	3 4
3	$\overline{11}\ \overline{56}$
	9
64	2 56
	2 56
68	0

Side of square = √(Area of square) = √1156 cm² = 34 cm

Therefore, the length of a side of the square-shaped wooden piece is 34 cm.

5. **The area of a square-shaped park is 900 m². Find the cost of fencing around the park if the rate is Rs 215 per metre.**

If the area of a square-shaped park is 900 m², then the length of each side of the square can be found by taking the square root of the area:

Side of square = √(Area of square) = √900 m² = 30 m

To find the perimeter of the square-shaped park, we multiply the length of one side by 4:

Perimeter of park = 4 × Side of square = 4 × 30 m = 120 m

The cost of fencing around the park at the rate of Rs 215 per metre is:

Cost of fencing = Perimeter of park × Cost per metre = 120 m × Rs 215/m = Rs 25,800

Therefore, the cost of fencing around the park at the rate of Rs 215 per metre is Rs 25,800.

	3 0
3	$\overline{9}\ \overline{00}$
	9
60	0
	0
60	0

6. **Find the perimeter of a square-shaped frame whose area is 784 cm².**

If the area of a square-shaped frame is 784 cm², then the length of each side of the square can be found by taking the square root of the area:

Side of square = √(Area of square) = √784 cm² = 28 cm

To find the perimeter of the square-shaped frame, we multiply the length of one side by 4:

Perimeter of frame = 4 × Side of square = 4 × 28 cm = 112 cm

Therefore, the perimeter of the square-shaped frame is 112 cm.

	2 8
2	$\overline{7}\ \overline{84}$
	4
48	3 84
	3 84
56	0

7. What is the length of a square-shaped tile if its area is 0.49 m^2?

If the area of a square-shaped tile is 0.49 m^2, then the length of each side of the square can be found by taking the square root of the area:

Side of square = √(Area of square) = √0.49 m^2 = 0.7 m

Therefore, the length of a side of the square-shaped tile is 0.7 meters

	7
7	49
	49
14	0

Unit Check

1. Encircle the correct option.

a) The HCF of 24, 42 and 84 is:
i) 4 ii) 6 iii) 12 iv) 24

b) The LCM of 36 and 64 is:
i) 4 ii) 144 iii) 362 iv) 576

c) The square of 40 is:
i) 400 ii) 160 iii) 1600 iv) 4000

d) The square root of 2.25 is:
i) 1.5 ii) 1.25 iii) 2.5 iv) 5.5

e) The square of 19 is:
i) 256 ii) 316 iii) 306 iv) 361

f) 225 is the square of:
i) 25 ii) 15 iii) 35 iv) 22

g) The square root of $\frac{81}{900}$ is:
i) $\frac{9}{30}$ ii) $\frac{300}{100}$ iii) $\frac{10}{300}$ iv) $\frac{30}{10}$

h) The square root of 8.41 is:
i) 0.29 ii) 29.0 iii) 0.029 iv) 2.9

2. Fill in the blanks.

a) The square of 4.7 is 22.09.

b) 1156 is the square of 34.

c) $\sqrt{\frac{169}{289}}$ = 13/17.

d) The square root of 5.29 is 2.3.

e) HCF of 9, 41, 72 is 1.

3. Find the LCM of the following.

a) 8, 38, 52

2	8	38	52
2	4	19	26
2	2	19	13
13	1	19	13
19	1	19	1
	1	1	1

LCM of $8, 38, 52 = 2 \times 2 \times 2 \times 13 \times 19 = 1976$

b) 15, 60, 80

2	15	60	80
2	15	30	40
2	15	15	20
2	15	15	10
3	15	15	5
5	5	5	5
	1	1	1

LCM of $15, 60, 80 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = 240$

c) 32, 96, 128

2	32	96	128
2	16	48	64
2	8	24	32
2	4	12	16
2	2	6	8
2	1	3	4
2	1	3	2
3	1	3	1
	1	1	1

LCM of $32, 96, 128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 384$

d) 14, 84, 112

2	14	84	112
2	7	42	56
2	7	21	28
2	7	21	14
3	7	21	7
7	7	7	7
	1	1	1

LCM of $14, 84, 112 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 336$

4. Find the HCF of the following.

a) 4, 8, 16

Factor of 4,8,16 are as follows...

2	4
2	2
	1

2	8
2	4
2	2
	1

2	16
2	8
2	4
2	2
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.
Find HCF

$4 = 2 \times 2$
$8 = 2 \times 2 \times 2$
$16 = 2 \times 2 \times 2 \times 2$

HCF = 2×2 = 4

HCF of 4, 8, 16 is 4

b) 11, 77, 121

Factor of 11,77,121 are as follows...

11	11
	1

7	77
11	11
	1

11	121
11	11
	1

The HCF of a, b is the largest positive number that exactly divides both a and b.
Find HCF

$11 = 11$
$77 = 7 \times 11$
$121 = 11 \times 11$

HCF = 11 = 11

HCF of 11, 77, 121 is 11

c) 108, 324, 648

Factor of 108,324,648 are as follows...

2	108
2	54
3	27
3	9
3	3
	1

2	324
2	162
3	81
3	27
3	9
3	3
	1

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

d) 36, 144, 288

Factor of 36,144,288 are as follows...

$$\begin{array}{r|r} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 288 \\ \hline 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

The HCF of a, b is the largest positive number that exactly divides both a and b.

Find HCF

$$\begin{array}{l} 36 = 2 \times 2 \times 3 \times 3 \\ 144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ 288 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \\ \hline \text{HCF} = 2 \times 2 \times 3 \times 3 = 36 \end{array}$$

HCF of 36, 144, 288 is 36

5. Which of these are perfect squares of odd numbers? Explain your answer.

a) 81 b) 121 c) 256 d) 324 e) 49 f) 625 g) 25

Solution:

Here are the perfect squares of odd numbers from the given list:

a) 81

We know that the square of an odd number is always odd. So, 81 is a perfect square of an odd number because 81 is odd.

b) 121

We know that the square of an odd number is always odd. So, 121 is a perfect square of an odd number because 121 is odd.

e) 49

We know that the square of an odd number is always odd. So, 49 is a perfect square of an odd number because 49 is odd.

f) 625

We know that the square of an odd number is always odd. So, 625 is a perfect square of an odd number because 625 is odd.

g) 25

We know that the square of an odd number is always odd. So, 25 is a perfect square of an odd number because 25 is odd.

6. Verify that the square of each of the following proper fractions is less than the fraction.

a) 4/11 b) 6/13 c) 11/21 d) 45/46 e) 99/100 f) 34/45

Solution:

a) 4/11 We have $(4/11)^2 = 16/121 < 4/11$.

b) 6/13 We have $(6/13)^2 = 36/169 < 6/13$.

c) 11/21 We have $(11/21)^2 = 121/441 < 11/21$.

d) 45/46 We have $(45/46)^2 = 2025/2116 < 45/46$.

e) 99/100 We have $(99/100)^2 = 9801/10000 < 99/100$.

f) 34/45 We have $(34/45)^2 = 1156/2025 < 34/45$.

Therefore, the square of each of the given proper fractions is less than the fraction itself.

7. Check if the square of each of the following decimals is smaller than the decimal.

a) 0.2 b) 5.1 c) 2.1 d) 6.8 e) 1.02 f) 0.05

Solution:

a) 0.2
$(0.2)^2 = 0.04$.
As 0.04 < 0.2, so the square of 0.2 is smaller than 0.2.

b) 5.1
$(5.1)^2 = 26.01$.
As 26.01 > 5.1, so the square of 5.1 is not smaller than 5.1.

c) 2.1
$(2.1)^2 = 4.41$.
As 4.41 > 2.1, so the square of 2.1 is not smaller than 2.1.

d) 6.8
$(6.8)^2 = 46.24$.
As $46.24 > 6.8$, so the square of 6.8 is not smaller than 6.8.

e) 1.02
$(1.02)^2 = 1.0404$.
As $1.0404 > 1.02$, so the square of 1.02 is not smaller than 1.02.

f) 0.05
$(0.05)^2 = 0.0025$. As $0.0025 < 0.05$, so the square of 0.05 is smaller than 0.05.

Hence it can be seen that the square of a decimal smaller than 1 is smaller than the decimal.

a) Find the square of the following numbers.

a) 11 b) 27 c) 108 d) 214 e) 715 f) 923

Solution:

a) $(11)^2 = 11 \times 11 = 121$

b) $(27)^2 = 27 \times 27 = 729$

c) $(108)^2 = 108 \times 108 = 11664$

d) $(214)^2 = 214 \times 214 = 45796$

e) $(715)^2 = 715 \times 715 = 511225$

f) $(923)^2 = 923 \times 923 = 851929$

b) Find the square root of the following numbers.

a) 1.44 b) 5.29 c) 324 d) 256 e) 289/36 f) 441/961

Solution:

a) 1.44

First, change the decimal number into a fraction and then find square root of the numerator and the denominator separately.
1.44= 144/100

	1 2
1	$\overline{1}\ \overline{44}$
	1
22	44
	44
24	0

Number = 144
Square Root = 12

	1 0
1	$\overline{1}\ \overline{00}$
	1
20	0
	0
20	0

Number = 100
Square Root = 10

$\sqrt{1.44} = \sqrt{144/100}$ =12/10 = 1.2

b) 5.29

First, change the decimal number into a fraction and then find square root of the numerator and the denominator separately.
5.29= 529/100

	2 3
2	$\overline{5}\ \overline{29}$
	4
43	1 29
	1 29
46	0

Number = 529
Square Root = 23

	1 0
1	$\overline{1}\ \overline{00}$
	1
20	0
	0
20	0

Number = 100
Square Root = 10

$\sqrt{5.29} = \sqrt{529/100}$ =23/10 = 2.3

c) 324

	1 8
1	$\overline{3}\ \overline{24}$
	1
28	2 24
	2 24
36	0

Number = 324
Square Root = 18

d) 256

	1 6
1	$\overline{2}\ \overline{56}$
	1
26	1 56
	1 56
32	0

Number = 256
Square Root = 16

e) 289/36

Find square root of the numerator and the denominator separately.

	1 7
1	$\overline{2}\ \overline{89}$
	1
27	1 89
	1 89
34	0

Number = 289
Square Root = 17

	6
6	$\overline{36}$
	36
12	0

Number = 36
Square Root = 6

$\sqrt{289/36}$ =17/6

f) 441/961

Find square root of the numerator and the denominator separately.

	2 1
2	$\overline{4}\ \overline{41}$
	4
41	41
	41
42	0

Number = 441
Square Root = 21

	3 1
3	$\overline{9}\ \overline{61}$
	9
61	61
	61
62	0

Number = 961
Square Root = 31

$\sqrt{441/961}$ =21/31

c) The base and altitude of a parallelogram are 16 cm and 9 cm, respectively. If the area of a square is equal to the area of this parallelogram, find the length of the side of the square.

The area of the parallelogram is given by:

Area of parallelogram = base × altitude = 16 cm × 9 cm = 144 cm^2

Since the area of the square is equal to the area of the parallelogram, the area of the square is also 144 cm². Therefore, we can find the length of a side of the square by taking the square root of the area of the square:

Side of square = √(Area of square) = √144 cm² = 12 cm

Therefore, the length of a side of the square is 12 cm.

$$\begin{array}{r|l} & 1\quad 2 \\ \hline 1 & \overline{1}\ \overline{44} \\ & 1 \\ \hline 22 & 44 \\ & 44 \\ \hline 24 & 0 \end{array}$$

Number = 144
Square Root = 12

d) A square-shaped garage has a length of 6.5 m. What is the area of the surface of the garage?

Since the garage is square-shaped, all of its sides are equal in length. Therefore, the width of the garage is also 6.5 m.

The formula for the area of a surface of a square is:

Area of surface = length x length

=6.5 m × 6.5 m

= 42.25 m²

e) The length of a square-shaped sheet of paper is 20 cm. What is the area of the surface of the sheet of paper?

Since the sheet of paper is square-shaped, all of its sides are equal in length.

The formula for the area of a surface of a square is:

Area of surface = length x length

=20 cm × 20 cm

= 400 cm²

f) The length of a square-shaped garden is 35 m. Find the cost of planting grass in the garden at the rate of Rs 430 per m^2 .

Since the garden is square-shaped, all of its sides are equal in length.

The formula for the area of a square is:

Area of square = length x length

= $(35\text{ m})^2$ = 1225 m²

To find the cost of planting grass in the garden, we need to multiply the area of the garden by the cost per square meter:

Cost of planting grass = Area of garden × Cost per square meter

Cost of planting grass = 1225 m² × Rs 430/m² = Rs 526750

Therefore, the cost of planting grass in the garden at the rate of Rs 430 per m² is Rs 526750.

g) The area of a square-shaped carrom board is 7.29 cm². Find the perimeter of the carrom board.

Side of carrom board = √(Area of carrom board)

= √7.29 cm²

=√729/100 cm²

=27/10 cm

= 2.7 cm

	2 7
2	$\overline{7}\ \overline{29}$
	4
47	3 29
	3 29
54	0

	1 0
1	$\overline{1}\ \overline{00}$
	1
20	0
	0
20	0

Since the carrom board is a square, all four sides are equal in length. Therefore, the perimeter of the carrom board is:

Perimeter of carrom board = 4 × Side of carrom board Perimeter of carrom board

= 4 × 2.7 cm

Perimeter of carrom board = 10.8 cm

Therefore, the perimeter of the carrom board is 10.8 cm.

h) The area of a square-shaped mirror is 625 cm². Find the cost of the mirror's frame at the rate of Rs 65 per cm.

Side of mirror = √(Area of mirror) = √625 cm² = 25 cm

The perimeter of the mirror is:

Perimeter of mirror = 4 × Side of mirror = 4 × 25 cm = 100 cm

To find the cost of the frame, we need to multiply the perimeter of the mirror by the cost per centimeter:

	2 5
2	$\overline{6}\ \overline{25}$
	4
45	2 25
	2 25
50	0

Cost of frame = Perimeter of mirror × Cost per centimeter

Cost of frame = 100 cm × Rs 65/cm

Cost of frame = Rs 6500

Therefore, the cost of the mirror's frame at the rate of Rs 65 per cm is Rs 6500.

i) Find the length of the floor of a square-shaped bookstore whose area is 625 m^2.

Area of floor= 625 m^2

Length of a side of square = $\sqrt{625\ m^2} = 25$ m

	2 5
2	$\overline{6}\ \overline{25}$
	4
45	2 25
	2 25
50	0

So, the length of the floor is 25 m.

Learning Check 3.1

1. **Find the average rate for the following.**

   a) 54 pages read in 180 minutes
   b) Rs 1500 wage for 3 days.
   c) 51 math problems solved in 3 days
   d) Earning of Rs 2000 in 3 hours.
   e) 100 km covered in 5 litres of petrol.

   Solution:

   a) To find the average rate of reading, we need to divide the number of pages by the time taken in minutes:

   Average rate = 54 pages / 180 minutes = 0.3 pages per minute

   b) To find the average wage per day, we need to divide the total wage by the number of days worked:

   Average wage = Rs 1500 / 3 days = Rs 500 per day

   c) To find the average number of math problems solved per day, we need to divide the total number of problems by the number of days:

   Average problems solved per day = 51 problems / 3 days = 17 problems per day

   d) To find the average earning per hour, we need to divide the total earnings by the number of hours worked:

   Average earning = Rs 2000 / 3 hours = Rs 666.67 per hour (rounded to two decimal places)

   e) To find the average distance covered per litre, we need to divide distance covered by the fuel consumed:

   Average fuel consumption = 100 km/5 litre = 20 km/litre.

2. **A bookbinder can bind 105 books in 35 minutes. Find the average rate of binding books per minute.**

   To find the average rate of binding books per minute, we need to divide the number of books bound by the time taken in minutes:

   Average rate = 105 books / 35 minutes = 3 books per minute

3. **In 8 days, Ali recited 72 ayats of the Holy Quran. Find his average rate of reciting the ayats.**

   To find the average rate of reciting ayats per day, we need to divide the number of ayats by the number of days:

   Average rate = 72 ayats / 8 days = 9 ayats per day

4. **For Rs 540, Asim purchased 45 prayer caps for a masjid. Find the unit rate of the caps.**

   To find the unit rate of the prayer caps, we need to divide the total cost by the number of caps:

   Unit rate = Rs 540 / 45 caps = Rs 12 per cap

5. **Madiha counted her pulse and found that it was 360 beats in 5 minutes. What is her average pulse rate?**

   To find Madiha's average pulse rate, we need to divide the number of beats by the time taken in minutes:

   Average pulse rate = 360 beats / 5 minutes = 72 beats per minute

6. **Zaeem skipped the rope 175 times in 7 minutes. Find the average rate of skipping rope per minute.**

   To find the average rate of skipping rope per minute, we need to divide the number of times skipped by the time taken in minutes:

   Average rate = 175 skips / 7 minutes = 25 skips per minute

7. **Rabia designs 18 pages of a book in 144 minutes. What is her average rate for designing a page?**

   To find Rabia's average rate of designing a page, we need to divide the number of pages designed by the time taken in minutes:

   Average rate = 18 pages / 144 minutes = 0.125 pages per minute

8. **Usman climbed 150 steps in 3 minutes. How many steps did he climb per minute?**

   To find the number of steps Usman climbed per minute, we need to divide the total number of steps by the time taken in minutes:

Steps per minute = 150 steps / 3 minutes = 50 steps per minute

Learning Check 3.2

1. **Calculate the increase or decrease in ratio for the given quantities.**

Old Quantity	New Quantity
156	148
62	24
75	85
236	202
300	320
462	560

a) New Quantity : Old Quantity

148 : 156

148/4 : 156/4

37 : 39 (Simplified form)

As the ratio of new quantity to the old quantity in fraction form is 37/39 (Proper fraction) so, the new quantity has been decreased in ratio 37:39.

b) New Quantity : Old Quantity

24 : 62

24/2 : 62/2

12 : 31 (Simplified form)

As the ratio of new quantity to the old quantity in fraction form is 12/31 (Proper fraction) so, the new quantity has been decreased in ratio 12:31.

c) New Quantity : Old Quantity

85 : 75

85/5 : 75/5

17 : 15 (Simplified form)

As the ratio of new quantity to the old quantity in fraction form is 17/15 (Improper fraction) so, the new quantity has been increased in ratio 17 : 15.

d) New Quantity : Old Quantity

202 : 236

202/2 : 236/2

101 : 118 (Simplified form)

As the ratio of new quantity to the old quantity in fraction form is 101/118 (Proper fraction) so, the new quantity has been decreased in ratio 101:118.

e) New Quantity : Old Quantity

320 : 300

320/20 : 300/20

16 : 15 (Simplified form)

As the ratio of new quantity to the old quantity in fraction form is 16/15 (Improper fraction) so, the new quantity has been increased in ratio 16 : 15.

f) New Quantity : Old Quantity

560 : 462

560/14 : 46/14

40 : 33 (Simplified form)

As the ratio of new quantity to the old quantity in fraction form is 40/33 (Improper fraction) so, the new quantity has been increased in ratio 40:33.

2. The number of students in a school decreased from 230 to 210. In what ratio did the number decrease?

Current No. of students : Previous No. of students

210 : 230

21 : 23 (Simplified form)

So, number of students decreased in the ratio 21:23. We can say that current number of students are 21/23 times the previous no. of students.

3. Alina got 40 fewer marks in the final term as compared to the midterm. If she got 715 marks in the midterm, in what ratio did her marks decrease?

Marks in final Term : Marks in mid Term

675 : 715

135 : 143 (Simplified form)

So, her marks decreased in the ratio 135:143. We can say that marks in final Term are 135/143times the Marks in mid Term.

4. **The number of employees increased in a factory from 42 to 77. In what ratio did the number of employees increase?**

New employees	:	Old employees	
77	:	42	
11	:	6	(Simplified form)

So, number of employees increase in the ratio 11:6. We can say that no. of new employees are 11/6 times the no. of old employees.

5. **The price of a shirt increased from Rs 1250 to Rs 1500. In what ratio has the price of the shirt increased?**

New price	:	Old price	
1500	:	1250	
6	:	5	(Simplified form)

So, price of shirt increases in the ratio 6:5. We can say that new price is 6/5 times the old price.

6. **Ali recited 52 ayats of the Holy Quran on Saturday and 74 ayats on Sunday. In what ratio does he recite more ayats on Sunday?**

No. of Ayats recited on Sunday	:	No. of Ayats recited on Saturday	
74	:	52	
37	:	26	(Simplified form)

So, No. of Ayats recited increase in the ratio 37:26. We can say that No. of Ayats recited on Sunday is 37/26 times the No. of Ayats recited on Saturday.

7. **Faria's weight increased from 48 kg to 50 kg. In what ratio does her weight increase?**

Current weight	:	weight previously	
50	:	48	
25	:	24	(Simplified form)

So, Faria's weight increased in the ratio of 25 : 24. We can say that her current weight is 25/24 times her weight previously.

8. **A baker baked 360 cupcakes in a day. If he increased the number of cupcakes he baked in the ratio of 7 : 6, what is the new quantity of cupcakes he would be baking daily?**

   Express the given ratio in the form of a fraction

   $$7:6 = 7/6$$

   Find the product of the previous no. of cupcakes and the fraction formed.

   $$360 \times 7/6 = 420$$

   So he could be baking 420 cupcakes now.

9. **There are 456 workers in a factory. The factory owner wants to decrease the number of workers. He wants to decrease the number of workers by a ratio of 7: 8. How many workers will be in the factory now?**

   Express the given ratio in the form of a fraction

   $$7:8=7/8$$

   Find the product of the old number of workers and the fraction formed.

   $$456 \times 7/8=399$$

   So, now there will be 399 workers in the factory.

Learning Check 3.3

1. **Find the missing term in the following proportions.**

   Solution:

   a) 88 : 11 : : x : 9

   As in a proportion, the product of means= product of extremes, so:

   $88 \times 9 = 11 \times x$

   $x = (88 \times 9)/11$

   $x = 72$

   Therefore, the missing term is 72.

   b) 13 : 52 : : 15 : x

   As in a proportion, the product of means= product of extremes, so:

   $13 \times x = 52 \times 15$

x = (52 × 15)/13

x = 60

Therefore, the missing term is 60.

c) x : 200 : : 65 : 100

As in a proportion, the product of means= product of extremes, so:

x × 100 = 200 × 65

x = (200 × 65)/100

x = 130

Therefore, the missing term is 130.

d) 512 : 32 : : x : 43

As in a proportion, the product of means= product of extremes, so:

512 × 43 = 32 × x

x = (512 × 43)/32

x = 688

Therefore, the missing term is 688.

2. 12 students complete a task in 7 hours. How many students will complete the same task in 3 hours?

Here, the number of students is inversely proportional to time. So,

Hours	:	No. of Students
7	:	12
3	:	x

$$\frac{7}{3} = \frac{x}{12}$$

3 × x = 7 × 12

x = (7 x 12)/3

= 84/3

=28

Hence, 28 students will complete the same task in 3 hours**.**

3. **Asad washed 56 bikes in 4 hours. How many bikes can be washed in 7 hours?**

   Here, the number of bikes is directly proportional to time. So,

Hours	:	No. of Bikes
4 ↑	:	56 ↑
7	:	x

$$\frac{4}{7} = \frac{56}{x}$$

$4 \times x = 7 \times 56$

x = (7 x 56)/4

= 392/4

=98

Hence, he will wash 98 bikes in 7 hours.

4. **Ahad paid Rs 630 for 7 packs of biscuits. How many biscuit packs can he buy for Rs 1260?**

   Here, the number of packs is directly proportional to amount. So,

Amount (in Rs)	:	No. of Packs
630 ↑	:	7 ↑
1260	:	x

$$\frac{630}{1260} = \frac{7}{x}$$

$630 \times x = 7 \times 1260$

x = (7 x 1260)/630

=14

Hence, he can buy 14 packs in Rs 1260.

5. **14 tube wells can fill a water tank in 45 minutes. If 4 tube wells are out of order, how long will the remaining tube wells take to fill the tank?**

   Here, the number of tube wells is inversely proportional to the time. As there were 14 tube wells, out of which 4 are out of order, so there are 14 – 4 = 10 tube wells now.

   So,

No. of tube wells	:	Time (in min.)
14	:	45
10	:	x

$$\frac{14}{10} = \frac{x}{45}$$

$10 \times x = 45 \times 14$

$x = (45 \times 14)/10$

$= 63$

Hence, 10 tube wells will fill the tank in 63 min.

6. **12 men build a wall in 9 days. How many men can build the same wall in 4 days?**

   Here, the number of men is inversely proportional to the no. of days.

   So,

No. of men	:	No. of days
12	:	9
x	:	4

$$\frac{12}{x} = \frac{4}{9}$$

$4 \times x = 12 \times 9$

$x = (12 \times 9)/4$

= 27

Hence, 27 men can build the same wall in 4 days.

7. If 5 boxes occupy a space of 0.625 m^3, how much space will be required for 180 such boxes?

Here, the number of boxes is directly proportional to the space they occupy. So,

No. of boxes	:	Space occupied(in m^3)
5 ↑	:	0.625 ↑
180	:	x

$$\frac{5}{180} = \frac{0.625}{x}$$

$5 \times x = 0.625 \times 180$

x = (0.625 × 180)/5

=22.5

Hence, 22.5 m^3 will be required for 180 such boxes.

8. There was enough food for 40 days in a medical camp for 1833 people. How many people may leave the camp so that the same food is sufficient for 60 days?

Here, the number of days is inversely proportional to the no. of people.

So,

No. of days	:	No. of people
40 ↑	:	1833 ↓
60	:	x

$$\frac{40}{60} = \frac{x}{1833}$$

$60 \times x = 40 \times 1833$

x = (40 x 1833)/60

= 1222

So, there is enough food for 60 days for only 1222 people. To know how many people may leave, subtract 1222 from 1833.

1833 – 1222 = 611

Hence, 611 people may leave the camp so that the same food is sufficient for 60 days.

9. 12 machines are used to prepare 120 shirts in a day. How many machines can prepare 460 shirts in a day?

Here, the number of machines is directly proportional to the number of shirts they prepare. So,

No. of machines	:	No. of shirts
12	:	120
x	:	460

$$\frac{12}{x} = \frac{\quad}{460}$$

120 x x = 12 x 460

x = (12 x 460)/120

=46

Hence, 46 machines can prepare 460 shirts in a day.

Learning Check 3.4

1. Encircle the correct option.

a) Comparing two quantities with different units of measure is known as:

i) rate ii) ratio iii) proportion iv) none of these

b) Two quantities will be called directly proportional to each other if the decrease in one quantity causes the other quantity to:

i) increase ii) decrease iii) remain the same iv) add up

c) The missing term in the proportion of 3 : 27 :: 8 : $\boxed{}$ is:

i) 24 ii) $\frac{8}{9}$ iii) 72 iv) 216

d) The ratio of 500 m to 1 km is:

i) 500 : 1 ii) 1 : 500 iii) 1 : 2 iv) 2 : 1

e) The price of an item increased from Rs 15 to Rs 30. In what ratio has the price increased?

i) 4 : 5 ii) 2 : 1 iii) 5 : 6 iv) 1 : 2

2. **Fill in the blanks.**
   a) The ratio of 30 cm to 5 m is <u>3:50</u>.
   b) Nimra's average eyeblink rate per minute is <u>6 times/min</u> if she blinks 90 times in 15 minutes.
   c) The ratio between 620 and 800 is <u>31:40</u>.
   d) If 42 : 7 : : x : 13, then the value of x is <u>78</u>.
   e) Product of means = <u>Product of extremes.</u>

3. **A soldier does his duty for 150 hours in 25 days. What are his average duty hours per day?**

   To find the average duty hours per day, we can divide the total duty hours by the number of days worked.

   Average duty hours per day = Total duty hours / Number of days worked

   Here, the soldier does his duty for 150 hours in 25 days, so:

   Average duty hours per day = 150 hours / 25 days

   Simplifying this expression, we get:

   Average duty hours per day = 6 hours/day

   Therefore, the soldier's average duty hours per day is 6 hours.

4. **A shopkeeper sold 5 items for Rs 540. What is the average price per item?**

   To find the average price per item, we can divide the total cost by the number of items sold.

   Average price per item = Total cost / Number of items sold

   Here, the shopkeeper sold 5 items for Rs 540, so:

   Average price per item = 540 / 5

   Simplifying this expression, we get:

   Average price per item = 108 Rs/item

   Therefore, the average price per item is Rs 108.

5. **The number of pages in an updated book increases from 250 to 300. In what ratio does the number of pages in the updated book increase?**

New Pages		: Old pages
300	:	250
6	:	5

So, number of pages in the updated book increase in the ratio 6:5. We can say that current pages are 6/5 times the old pages.

6. **Asim learned 150 new words on Monday. He learned 120 new words on Tuesday. In what ratio did the number of new words decrease?**

Words learnt on Tuesday : Words learnt on Monday

120 : 150

4 : 5

So, number of words learnt decreased in the ratio 4:5. We can say that number of words learnt on Tuesday are 4/5 times the number of words learnt on Monday.

7. **Hira earns Rs 12,000 in 10 days. How much will she earn in 42 days?**

Here, the earning is directly proportional to the number of days. So,

Earning (in Rs)	:	No. of Days
12000 ↑	:	10 ↑
x	:	42

$$\frac{12000}{x} = \frac{10}{42}$$

10 x x = 12000 x 42

x = (12000 x 42)/10

=50400

Hence, she will earn Rs 50400 in 42 days.

8. Rashid sold 2 dozen eggs for Rs 480. How many eggs did he sell in Rs 720?

Here, the number of eggs is directly proportional to amount. So,

No. of eggs	:	Amount (in Rs)
24 ↑	:	480 ↑
x	:	720

$$\frac{24}{x} = \frac{480}{720}$$

$480 \times x = 720 \times 24$

$x = (720 \times 24)/480$

$=36$

Hence, he sold 36(3 dozen) eggs in Rs 720.

9. If 28 people consume food in 15 days, how long can 60 people consume the same food?

Here, the number of people is inversely proportional to the no. of days. So,

No. of people	:	No. of Days
28 ↑	:	15 ↓
60	:	x

$$\frac{28}{60} = \frac{x}{15}$$

$60 \times x = 28 \times 15$

$x = (28 \times 15)/60$

$= 7$

So 60 people will consume the same amount of food in 7 days.

10. Ali reads 16 pages of a storybook in 2 days. How many pages can be read in 7 days at the same rate?

Here, the number of pages is directly proportional to the number of days. So,

No. of pages	:	No. of Days
16	:	2
x	:	7

$\frac{16}{x}$:

2 x x = 16 x 7

x = (16 x 7)/2

=56

Hence, he can read 56 pages in 7 days.

11. A camp of 250 people has enough food for 30 days. How long will the food last if the number of people in the camp is reduced to 150?

Here, the number of people is inversely proportional to the no. of days. So,

No. of people	:	No. of Days
250	:	30
150	:	x

$$\frac{250}{150} = \frac{x}{30}$$

150 × x = 30 x 250

x = (30 x 250)/150

= 50

So, the food will last for 50 days if the number of people in the camp is reduced to 150

Learning check 4.1

1. **Find the profit or loss percentage if:**

a) Cost price = Rs 2400, Selling price = Rs 2160

b) Cost price = Rs 3500, Selling price = Rs 4200

c) Cost price = Rs 45200, Selling price = Rs 46104

d) Cost price = Rs 9800, Selling price = Rs 9408

Solution:

a) In this case, the selling price is less than the cost price, which means there is a loss.

Loss = cost price - selling price

=Rs 2400 - Rs 2160

= Rs 240.

Loss % = Loss/cost Price x 100%

= (240/2400) x 100%

= 10%.

Therefore, the loss percentage is 10%.

b) In this case, the selling price is greater than the cost price, which means there is a profit. Profit = selling price - cost price

= Rs 4200 - Rs 3500

= Rs 700.

Profit % = Profit/cost Price x 100%

= (700/3500) x 100

= 20%.

Therefore, the profit percentage is 20%.

c) In this case, the selling price is greater than the cost price, which means there is a profit. Profit = selling price - cost price

= Rs 46104 - Rs 45200

= Rs 904.

Profit % = Profit/cost Price x 100%

=(904/45200) x 100 = 2%.

Therefore, the profit percentage is 2%.

d) In this case, the selling price is less than the cost price, which means there is a loss.

Loss = cost price - selling price

=Rs 9800 - Rs 9408

= Rs 392.

Loss % = Loss/cost Price x 100%

=(392/9800) x 100

= 4%.

Therefore, the loss percentage is 4%.

2. The cost price of a shirt is Rs 1,800 and its selling price is Rs 1,836. Find the profit percentage of this shirt.

Profit = Selling Price - Cost Price

= Rs 1836 - Rs 1800

= Rs 36

To find the profit percentage, we need to calculate what percentage of the cost price the profit represents:

Profit Percentage = (Profit / Cost Price) x 100%

= (36 / 1800) x 100%

= 2%

Therefore, the profit percentage on the shirt is 2%.

3. Ali paid Rs 32,500 for an old bike and spent Rs 2,000 to repair it. Then he sold it for Rs 35,880. Find out his profit and profit percentage.

Profit = Selling price - Cost price

= Rs 35,880 - (Rs 32,500 + Rs 2,000)

= Rs 1,380

Profit percentage = (Profit / Cost price) x 100%

= (1,380 / 34,500) x 100%

= 4%

Therefore, Ali's profit on the old bike is Rs 1,380 and the profit percentage is 4%.

4. **Madiha purchases a toy cycle for Rs 1,260. She sold it for Rs 1,200. Find out her loss and loss percentage.**

Loss = Cost price - Selling price

= Rs 1,260 - Rs 1,200

= Rs 60 Loss percentage

= (Loss / Cost price) x 100%

= (60 / 1,260) x 100%

= 4.76%

Therefore, Madiha's loss on the toy cycle is Rs 60 and the loss percentage is 4.76%.

5. **Saad bought a house for Rs 2,756,000. After a few years of staying there, he sold it for a profit of Rs 152,000. Find out the selling price of the house.**

Selling price = Cost price + Profit

= Rs 2,756,000 + Rs 152,000

= Rs 2,908,000

Therefore, the selling price of the house is Rs 2,908,000.

6. **The selling price of a pair of shoes is Rs 4,560 and the shopkeeper suffered a loss of Rs 912. Find out how much the shoes cost.**

Cost price = Selling price + Loss

= Rs 4,560 + Rs 912

= Rs 5,472

Therefore, the shoes cost Rs 5,472.

7. **Find the discount offered by a wholesaler if the marked price of an air conditioner is Rs 16,000 and its selling price is Rs 14,500.**

Discount = Marked price - Selling price

= Rs 16,000 - Rs 14,500

= Rs 1,500

Therefore, the discount offered by the wholesaler is Rs 1,500.

8. **The marked price of a scientific calculator is Rs 4,500. If it is sold for Rs 4,200, find the offered discount and the percentage of the discount.**

Discount = Marked price - Selling price

= Rs 4,500 - Rs 4,200

= Rs 300 Discount percentage

= (Discount / Marked price) x 100%

= (300 / 4,500) x 100%

= 6.67%

Therefore, the offered discount is Rs 300 and the discount percentage is 6.67%.

Learning check 4.2

1. **Maida earns Rs 552,000 annually. Find her income tax at the rate of 5%, if the amount of rebate is Rs 60,000.**

Maida's taxable income after the rebate is Rs 552,000 - Rs 60,000 = Rs 492,000.

Income tax = Tax rate x Taxable income

= 5% x Rs 492,000

= Rs 24,600

Therefore, Maida's income tax is Rs 24,600.

2. **A house is worth Rs 3,500,000. Find the property tax if the tax rate is 7%.**

Property tax = Tax rate x Property value

= 7% x Rs 3,500,000

= Rs 245,000

Therefore, the property tax is Rs 245,000.

3. **Wajid owns a plot worth Rs 7,000,000. What is the amount of property tax to be paid if the tax rate is 3%?**

Property tax = Tax rate x Property value

= 3% x Rs 7,000,000

= Rs 210,000

Therefore, the amount of property tax to be paid is Rs 210,000.

4. **A laptop is sold for Rs 96,500, including 10% GST. Find the amount of tax paid.**

Let the cost of laptop = Rs x

Selling price=10 % of x + x

96500 = 10/100 of x + x

=0.1 x + x

96500= 1.1 x

or x = 96500/1.1

= 87727.27

Therefore, the cost of the laptop is Rs 87,727.27 and the amount of tax paid is:

Amount of tax paid = 10% of Cost of laptop

= 10/100 x 87727.27

= Rs 8,772.72

Therefore, the amount of tax paid is Rs 8,772.7.

5. **Asma bought a mobile and paid GST of Rs 47,800 at the rate of 18%. Find out the actual price of the mobile.**

Let the actual price of the mobile be x. Then, we have:

GST = 18% of x

= 0.18x

Given that Asma paid Rs 47,800 as GST, we have:

0.18x = Rs 47,800

x = Rs 265,555.56

Therefore, the actual price of the mobile is Rs 265,555.56.

6. **Hira bought a car for Rs 3,200,000. She paid 17% tax on it. Hira sold the car for Rs 3,550,000 and take 18% tax on it. Find the amount of value-added tax she paid.**

Price of car = Rs 3,200,000

Tax paid = 17 %

= 17/100 × 3200000

= Rs 544000

She sold that car in Rs 3,550,000.

The tax on selling the car = 18% x 3550000 = 639000

Value added tax = tax recovered on sale – tax paid on purchase

= Rs 639000 – 544000

= Rs 95000

7. **Munazza works for a firm. If she sold a printer for Rs 14,500, she got a 4% commission on it. How much commission did she get?**

Commission = 4% of Rs 14,500

= 0.04 x Rs 14,500

= Rs 580

Therefore, Munazza got a commission of Rs 580.

8. **Usama works in a washing machine shop. He received a commission of 1.5% on each washing machine sold. How much commission did he receive if he sold four washing machines for a total of Rs 340,000?**

The total cost of four washing machines sold by Usama is Rs 340,000. Therefore, the cost of one washing machine is:

Cost of one washing machine = Rs 340,000 / 4

= Rs 85,000

Usama received a commission of 1.5% on the cost of each washing machine.

Therefore, the commission he received on the sale of four washing machines is:

Commission = 1.5% of (4 x Rs 85,000)

= 1.5% x Rs 340,000

= Rs 5,100

Therefore, Usama received a commission of Rs 5,100.

Learning check 4.3

1. **Usman had an annual savings of Rs 1,200,000. If he paid Zakat at the rate of 2.5%, find the amount paid as Zakat.**

The amount of Zakat paid by Usman is:

Zakat = 2.5% of Rs 1,200,000

= 0.025 x Rs 1,200,000

= Rs 30,000

Therefore, Usman paid Rs 30,000 as Zakat.

2. **Ayesha paid Rs 10,000 as Zakat. What were her yearly savings?**

Let Ayesha's yearly savings be x. Then, the amount of Zakat paid by her is:

2.5% of x = Rs 10,000

0.025x = Rs 10,000

x = Rs 10,000 / 0.025

x = Rs 400,000

Therefore, Ayesha's yearly savings are Rs 400,000.

3. **Find the Zakat on gold of the amount of Rs 1,550,000.**

The Zakat on gold is 2.5% of the value of the gold. Therefore, the Zakat on gold worth Rs 1,550,000 is:

Zakat = 2.5% of Rs 1,550,000

= 0.025 x Rs 1,550,000

= Rs 38,750

Therefore, the Zakat on gold worth Rs 1,550,000 is Rs 38,750.

4. **Find the amount of Zakat on 12 tolas of gold and 80 tolas of silver if the rate of gold is Rs 96,000 per tola and the rate of silver is Rs 2000 per tola.**

The Zakat on gold is 2.5% of the value of the gold. Therefore, the Zakat on 12 tolas of gold worth Rs 96,000 per tola is:

Value of 12 tolas of gold = 12 x Rs 96,000

= Rs 1,152,000

Zakat on gold = 2.5% of Rs 1,152,000

= 0.025 x Rs 1,152,000

= Rs 28,800

The Zakat on 80 tolas of silver worth Rs 2000 per tola is:

Value of 80 tolas of silver = 80 x Rs 2000

= Rs 160,000

Zakat on silver = 2.5% of Rs 160,000

= 0.025 x Rs 160,000

= Rs 4,000

Therefore, the amount of Zakat on 12 tolas of gold and 80 tolas of silver is Rs 28,800 + Rs 4,000 = Rs 32,800.

5. **If Huma earned Rs 48,500 by selling a wheat crop and paid Ushr at the rate of 10%, what amount did she pay as Ushr?**

The amount of Ushr paid by Huma is:

Ushr = 10% of Rs 48,500

= 0.10 x Rs 48,500

= Rs 4,850

Therefore, Huma paid Rs 4,850 as Ushr.

6. **Find the Ushr of a rice crop grown worth Rs 85,000 in natural resources.**

The Ushr on a rice crop grown in natural resources is 10% of the value of the crop. Therefore, the Ushr on a rice crop worth Rs 85,000 is:

Ushr = 10% of Rs 85,000

= 0.1 x Rs 85,000

= Rs 8500

Therefore, the Ushr on a rice crop worth Rs 85,000 is Rs 8500.

7. **Find the Ushr of a wheat crop grown with artificial resources worth Rs 455,000.**

The Ushr on a wheat crop grown with artificial resources is 10% of the value of the crop. Therefore, the Ushr on a wheat crop worth Rs 455,000 is:

Ushr = 10% of Rs 455,000

= 0.10 x Rs 455,000

= Rs 45,500

Therefore, the Ushr on a wheat crop worth Rs 455,000 is Rs 45,500.

8. **The amount of Ushr paid by Ahmad was Rs 11,200 at a rate of 5% on his annual crop production. Calculate the worth of his crops.**

Let the worth of Ahmad's crops be x. Then, the amount of Ushr paid by him is:

5% of x = Rs 11,200

0.05x = Rs 11,200

x = Rs 11,200 / 0.05

x = Rs 224,000

Therefore, the worth of Ahmad's crops is Rs 224,000.

Unit check

1. Encircle the correct option.

a) Ali purchased a television for Rs 12,000 and sold it at Rs 10,500. The loss percentage is:

i) 11% ii) 12.5% iii) 15.3% iv) 8%

b) Find the GST on purchasing an item for Rs 6,000 at the rate of 17%.

i) Rs 600 ii) Rs 1020 iii) Rs 2000 iv) Rs 2400

c) Zakat is deducted at the rate of:

i) 0.5% ii) 2.5% iii) 5% iv) 10%

d) The amount of Ushr on the production of wheat at the rate of 10% of the total amount of Rs 50,000 is:

i) Rs 500 ii) Rs 5,000 iii) Rs 10,000 iv) Rs 15,000

e) GST stands for:

i) Generic Sales Tax ii) General Sold Tax

iii) General Sales Tax iv) General Standard Tax

2. Fill in the blanks.

a) Profit % = $\dfrac{\text{profit}}{\boxed{\text{Cost Price}}} \times 100.$

b) Property tax at the rate of 7% on land worth Rs 1,520,000 is Rs 106400.

c) If the annual income of an employee is Rs 525,000, then income tax at the rate of 3% is Rs Rs 15750.

d) Some companies pay their employees an additional amount in addition to salary as commission.

e) According to Islam, the fixed rate of Zakat is 2.5%.

3. Find the percentage of loss or profit if:

a) Cost price = Rs 7250, Selling price = Rs 8265

b) Cost price = Rs 1965, Selling price = Rs 1572

c) Cost price = Rs 3582, Selling price = Rs 5373

d) Cost price = Rs 9840, Selling price = Rs 7995

Solution:

a) Cost price = Rs 7250, Selling price = Rs 8265

Profit = Selling price - Cost price

= Rs 8265 - Rs 7250

= Rs 1015

Profit percentage = (Profit / Cost price) x 100%

= (Rs 1015 / Rs 7250) x 100%

= 14%

Therefore, the percentage of profit is 14%.

b) Cost price = Rs 1965, Selling price = Rs 1572

Loss = Cost price - Selling price

= Rs 1965 - Rs 1572

= Rs 393

Loss percentage = (Loss / Cost price) x 100%

= (Rs 393 / Rs 1965) x 100%

= 20%

Therefore, the percentage of loss is 20%.

c) Cost price = Rs 3582, Selling price = Rs 5373

Profit = Selling price - Cost price

= Rs 5373 - Rs 3582

= Rs 1791

Profit percentage = (Profit / Cost price) x 100%

= (Rs 1791 / Rs 3582) x 100%

= 50%

Therefore, the percentage of profit is 50%.

d) Cost price = Rs 9840, Selling price = Rs 7995

Loss = Cost price - Selling price

= Rs 9840 - Rs 7995

= Rs 1845

Loss percentage = (Loss / Cost price) x 100%

= (Rs 1845 / Rs 9840) x 100%

= 18.75%

Therefore, the percentage of loss is 18.75%.

4. Find the discount and discount percentage on a ceiling fan if the marked price is Rs 8215 and the selling price is Rs 5550.

Discount = Marked price - Selling price

= Rs 8215 - Rs 5550

= Rs 2665

The discount percentage can be calculated as:

Discount percentage = (Discount / Marked price) x 100%

= (Rs 2665 / Rs 8215) x 100%

= 32.44%

Therefore, the discount on the ceiling fan is Rs 2665 and the discount percentage is 32.44%.

5. A shopkeeper earned a 15% profit by selling a camera for Rs 20,500. Find out the actual price of the camera.

Let the actual price of the camera be x.

Selling price = Actual price + Profit

Rs 20500 = x + 15% of x

Rs 20500 = $x + 15/100 \times x$

Rs 20500 = $x + 0.15x$

$20500 = 1.15x$

Or x = Rs 20500/1.15

= Rs 17826.08

Hence, the actual price of the camera is Rs 17826 approximately.

6. **Ahad paid property tax of Rs 65,800 at a rate of 17%. What is the worth of his property?**

   Let the worth of Ahad's property be x.

   Given, he paid property tax of Rs 65,800 at a rate of 17%.

   Therefore, (17/100) × x = Rs 65,800

   Or, x = Rs (65,800 × 100) / 17

   Or, x = Rs 387058.82

   Hence, the worth of Ahad's property is Rs 387058.82.

7. **Ahmad bought two products for Rs 25,300 and Rs 32,700. Find the GST paid on each product at the rate of 10%.**

   To find the GST paid on each product at the rate of 10%, we need to calculate 10% of the cost of each product as:

   GST on product 1 = (10/100) × Rs 25,300

   = Rs 2,530

   GST on product 2 = (10/100) × Rs 32,700

   = Rs 3,270

   Therefore, the GST paid on the first product is Rs 2,530 and the GST paid on the second product is Rs 3,270.

   Total GST paid is Rs 2,530 + Rs 3,270 = Rs 5800

8. **Madiha purchased an oven for Rs 15,200. She paid a tax of 5% and sold that oven for Rs 18,300 and charged a tax of 7%. How much value-added tax did she pay?**

   Tax paid on purchase = 5% of Rs 15200

   = Rs (5/100) x Rs 15,200

   = Rs 760.

   She then sold the oven for Rs 18,300 and charged a tax of 7%.

   Tax charged= 7% of 18300

=Rs (7/100) x Rs 18,300

= Rs 1,281.

Therefore, the value-added tax she paid is:

Rs 1,281 - Rs 760 = Rs 521.

9. A house was sold for Rs 5,250,000 through an agent. If the percentage of commission is 1.5%, find the amount which the agent received as commission.

The amount of commission received by the agent is calculated as:

Commission = 1.5% of Rs 5,250,000

=(1.5/100) x Rs 5,250,000

= Rs 78,750

Therefore, the amount which the agent received as commission is Rs 78,750.

10. Saadia's yearly savings are Rs 650,000. What is the amount of Zakat to be paid by her?

The amount of Zakat to be paid is 2.5% of the total wealth owned by the person.

Saadia's yearly savings = Rs 650,000.

Zakat to be paid = 2.5% of Rs 650,000

=(2.5/100) x Rs 650,000

= Rs 16,250

Hence, the amount of Zakat to be paid by Saadia is Rs 16,250.

11. Ali's land is irrigated by a natural resource. How much did his corn sell for if he paid Rs 5800 as Ushr on it?

Let the selling price = Rs x

Amount paid as Usher = 10% of Rs x

Rs 5800 = 10/100 × x

Or x = (Rs 5800 x 100)/10

x = 58000

Therefore, Ali's corn sold for Rs 58,000

Learning Check 5.1

1. Write the following sets in tabular form.

a) The set of months of the year beginning with J.

b) The set of the provincial capitals of Pakistan.

c) The set of the first five square natural numbers.

d) The set of all even positive integers less than 16.

e) The set of all odd numbers less than 15.

f) The set of even numbers between 2 and 4.

g) The set of prime numbers less than 20.

h) The set of the letters in the word MATHEMATICS.

Solution:

a) {January, June, July}

b) {Quetta, Peshawar, Lahore, Karachi, Islamabad}

c) {1, 4, 9, 16, 25}

d) {2, 4, 6, 8, 10, 12, 14}

e) {1, 3, 5, 7, 9, 11, 13}

f) { }

g) {2, 3, 5, 7, 11, 13, 17, 19}

h) {M, A, T, H, E, I, C, S}

2. Write the following sets in descriptive form and set builder form.

a) A = {0, 2, 4, 6, 8, 10}

b) B = {2, 3, 5, 7, 11}

c) J = {2, 4, 8, 16}

d) D = {6, 12, 18, 24, 30}

e) F = {-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}

f) G = {4, 8, 12, 16, 20, 24}

Solution:

a) Descriptive form: The set A is the set of even numbers from 0 to 10.

Set builder form: A = {*x* | *x* Î E and 0 £ *x* £ 10}

b) Descriptive form: The set B is the set of prime numbers from 2 to 11.

Set builder form: B = {*x* | *x* is a prime number from 2 to 11}

c) Descriptive form: The set J is the set of powers of 2 from 21 to 24.

Set builder form: J = {2*x* | *x* = 1, 2, 3, 4}

d) Descriptive form: The set D is the set of multiples of 6 from 6 to 30.

Set builder form: D = {6*x* | *x* = 1, 2, 3, 4, 5}

e) Descriptive form: The set F is the set of integers from -5 to 5.

Set builder form: F = {*x* | *x* is an integer, -5 £ *x* £ 5}

f) Descriptive form: The set G is the set of multiples of 4 from 4 to 24.

Set builder form: G = {4*x* | *x* = 1, 2, 3, 4, 5, 6}

3. Separate the equal sets and the equivalent sets from the following:

a) A = {2, 3, 5, 6, 8}, B = {8, 6, 5, 3, 2}

b) C = {3, 6, 9, 12, 15}, D = the set of the first five multiples of the number 5.

c) E = {a, e, i, o, u}, F = {1, 2, 3, 4, 5}

d) G = the set of four cities of Pakistan, H = {Lahore, Multan, Rawalpindi, Karachi}

e) I = {2, 6, 10, 16}, J = {2, 4, 6, 8}

f) K = {1, 2, 3, 4}, L = {2, 3, 1, 4}

Solution:

a) A and B are equivalent sets because they have the same number of elements.

A and B are equal as well because they have same elements in them, even though they are in a different order.

b) C and D are equivalent sets as they have same number of elements.

C and D are not equal as the elements are different in both.

c) E and F are equivalent sets as they have same number of elements.

E and F are not equal as the elements are different in both.

d) G and H are equivalent sets as they have same number of elements.

G and H can be equal if G has same four cities as H has, otherwise they will not be equal.

e) I and J are equivalent sets as they have same number of elements.

I and J are not equal as the elements are different in both.

f) K and L are equivalent sets because they have the same number of elements.

K and L are equal as well because they have same elements in them, even though they are in a different order.

4. **Fill in the blanks by using the terms proper subset, superset, or improper subset.**

   a) A = {2, 3, 7}, B = {1, 2, 3, 5, 7}

   A is proper subset of B.

   Reason: All elements of A are in B, but B contains elements that are not in A.

   b) R = the set of integers from -3 to +3, S = {-4, -3, -2, -1, 0, 1, 2, 3, 4}

   S is superset of R.

   Reason: S has all the elements of R.

   c) M =Φ, N = {a, b, c, d}

   M is proper subset of N.

   Reason: M is an empty set and empty set is a proper subset of every set(and improper subset of itself).

   d) E = the set of English alphabets between W and Z, F = {X, Y}

   E is improper subset of F.

   Reason: All elements of E are in F and all the elements of F are in E.

   e) P = the set of four quadrilaterals.

   Q = {square, rhombus, rectangle, kite, parallelogram, trapezium}

   Q is superset of P.

   Reason: Q has all the elements of P.

5. **Separate the disjoint sets and the overlapping sets.**

   a) X = {13, 15, 17, 19, 21}, Y = {12, 14, 16, 18, 20}

   b) P = {2, 3, 6, 9, 12, 15}, Q = {1, 2, 4, 8, 12, 16}

   c) A = the set of even numbers, B = the set of odd numbers.

   d) M = {3, 6, 9, 12, ⋯}, N = {5, 10, 15, 20, ⋯}

   e) E = the set of prime numbers less than 30, F = {10, 20, 25, 35, 45, 60}

Solution:

a) X and Y are disjoint sets because they do not share any elements. The sets only contain odd or even numbers respectively, and there are no numbers that are both odd and even.

b) b) P and Q are overlapping sets because they share the element 12. Both sets contain the number 12, so they have at least one element in common.

c) c) A and B are disjoint sets because even and odd numbers have no elements in common. An even number is divisible by 2, while an odd number is not divisible by 2. Therefore, there are no numbers that are both even and odd.

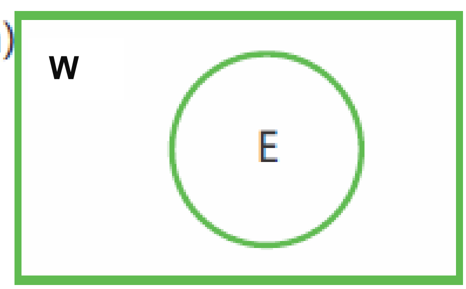
d) d) M and N are overlapping sets because they share the element 15, which is a multiple of both 3 and 5. M contains all multiples of 3, while N contains all multiples of 5. Since 15 is a multiple of both 3 and 5, it is an element of both sets.

e) e) E and F are disjoint sets because they do not share any elements. The set of prime numbers less than 30 contains only prime numbers, while F contains composite numbers. Therefore, there are no numbers that are in both sets.

Learning Check 5.2

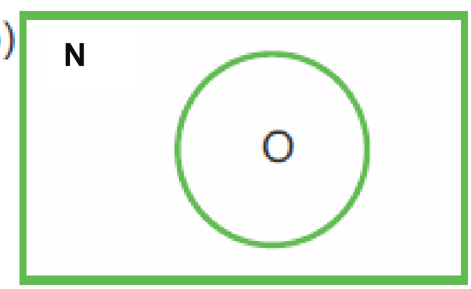
1. Use a Venn diagram to present the relationships between the given sets.

a)

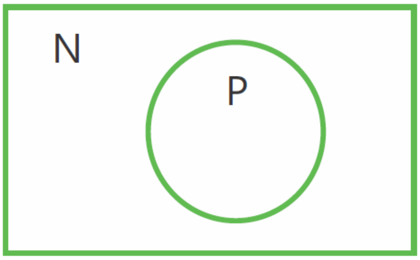


E is a subset of W

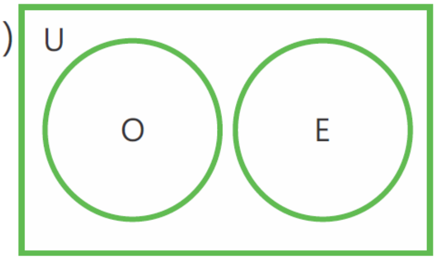
b)



O is a subset of N

c)


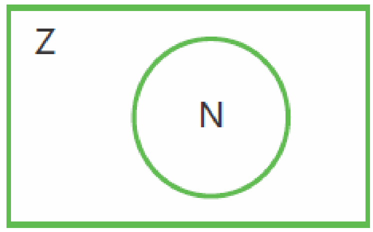
P is a subset of N

d)


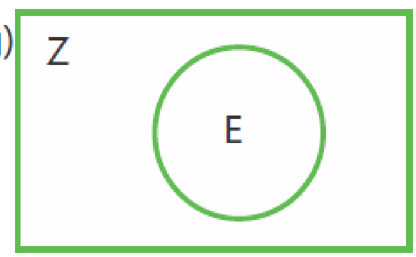
O and E are Disjoint sets

e)

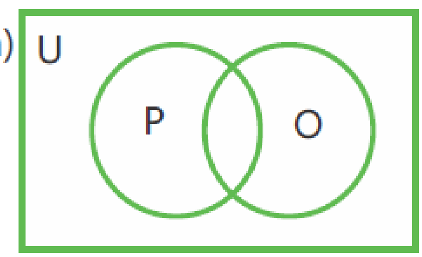

W is a subset of Z

f)


N is a subset of Z

g)


E is a subset of Z

h)


P and O are overlapping sets

Learning Check 5.3

1. Find the union of the given sets.

a) A = {1, 2, 4, 7, 10}, B = {1, 3, 5, 6, 8, 9}

b) X = {4, 8, 12, 16}, Y = {6, 12, 18, 24, 30}

c) P = {2, 7, 12, 17, 23}, Q = {1, 7, 13, 19, 25}

d) M = {10, 20, 30, 40, 50}, N = {20, 50, 80, 110}

e) E = {1, 3, 5, 7, 9, 11, 15}, F = {2, 3, 5, 7, 11, 13}

Solution:

To find the union of two sets, we simply combine all the elements of both sets, making sure to remove any duplicates.

a) A = {1, 2, 4, 7, 10}, B = {1, 3, 5, 6, 8, 9}.

The union of A and B is:

A ∪ B = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

b) X = {4, 8, 12, 16}, Y = {6, 12, 18, 24, 30}.

The union of X and Y is:

X ∪ Y = {4, 6, 8, 12, 16, 18, 24, 30}.

c) P = {2, 7, 12, 17, 23}, Q = {1, 7, 13, 19, 25}.

The union of P and Q is:

P ∪ Q = {1, 2, 7, 12, 13, 17, 19, 23, 25}.

d) M = {10, 20, 30, 40, 50}, N = {20, 50, 80, 110}

The union of M and N is:

M ∪ N = {10, 20, 30, 40, 50, 80, 110}.

e) E = {1, 3, 5, 7, 9, 11, 15}, F = {2, 3, 5, 7, 11, 13}.

The union of E and F is:

E ∪ F = {1, 2, 3, 5, 7, 9, 11, 13, 15}.

2. Find the intersection of the given sets.

a) A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8, 10}

b) C = {2, 8, 14, 20, 26}, D = {4, 8, 12, 16, 20}

c) E = {40, 45, 50, 55, 60}, F = {30, 40, 50, 60, 70}

d) G = {2, 3, 5, 7, 11, 13, 17}, H = {2, 6, 10, 14, 18, 22, 26}

e) K = {a, e, i, o, u}, L = {m, n, o, p, q}

Solution:

a) A = {1, 3, 5, 7, 9}, B = {2, 4, 6, 8, 10}

The intersection of A and B is:

A ∩ B = {}.

Since A and B have no elements in common, their intersection is an empty set.

b) C = {2, 8, 14, 20, 26}, D = {4, 8, 12, 16, 20}

The intersection of C and D is:

$C \cap D = \{8, 20\}$.

The elements 8 and 20 are the only ones that are in both sets.

c) E = {40, 45, 50, 55, 60}, F = {30, 40, 50, 60, 70}.

The intersection of E and F is:

$E \cap F = \{40, 50, 60\}$.

These are the only elements that are in both sets.

d) G = {2, 3, 5, 7, 11, 13, 17}, H = {2, 6, 10, 14, 18, 22, 26}.

The intersection of G and H is:

$G \cap H = \{2\}$.

This is the only element that is in both sets.

e) K = {a, e, i, o, u}, L = {m, n, o, p, q}.

The intersection of K and L is:

$K \cap L = \{o\}$.

This is the only element that is in both sets.

3. **If O = The set of odd numbers less than 10 and P = The set of prime numbers less than 15, then find the union and intersection of sets O and P.**

Solution:

O = {1, 3, 5, 7, 9} (The set of odd numbers less than 10)

P = {2, 3, 5, 7, 11, 13} (The set of prime numbers less than 15)

To find the union of O and P, we list all the elements of both sets, making sure to remove any duplicates. The union of O and P is:

$O \cup P = \{1, 2, 3, 5, 7, 9, 11, 13\}$.

To find the intersection of O and P, we list only the elements that are in both sets. The intersection of O and P is:

$O \cap P = \{3, 5, 7\}$.

These are the only elements that are in both sets.

4. **If X = {a, c, e, g, i}, Y = {b, d, f, h, o} and Z = {a, e, i, o, u} then find the following:**

   a) X – Y   b) X – Z   c) Y – Z   d) Y – X   e) Z – Y

   Solution:

   a) X - Y means we need to find all the elements that are in X but not in Y.

   X - Y = {a, c, e, g, i} - {b, d, f, h, o}

   = {a, c, e, g, i}.

   b) X - Z means we need to find all the elements that are in X but not in Z.

   X - Z = {a, c, e, g, i} - {a, e, i, o, u}

   = {c, g}.

   c) Y - Z means we need to find all the elements that are in Y but not in Z.

   Y - Z = {b, d, f, h, o} - {a, e, i, o, u}

   = {b, d, f, h}.

   d) Y - X means we need to find all the elements that are in Y but not in X.

   Y - X = {b, d, f, h, o} - {a, c, e, g, i}

   = {b, d, f, h, o}.

   e) Z - Y means we need to find all the elements that are in Z but not in Y.

   Z - Y = {a, e, i, o, u} - {b, d, f, h, o}

   = {a, e, i, u}.

5. **If A = {1, 3, 5, 7, 11}, B = {1, 5, 12} and C = {1, 2, 3, 4}, then find $A \cup B \cup C$ and $A \cap B \cap C$.**

   Solution:

   To find AUB, first we list all the elements of A and B together making sure to remove any duplicates. Therefore:

   AUB = {1, 3, 5, 7, 11, 12}.

   Next, to find AUBUC, we list all the elements of AUB and C together making sure to remove any duplicates. Therefore:

   AUBUC = {1, 3, 5, 7, 11, 12} U {1, 2, 3, 4}

   = {1, 2, 3, 4, 5, 7, 11, 12}.

To find $A \cap B$, first we need to find the elements that are common to both sets A and B. Therefore:

$A \cap B = \{1, 5\}$

Next, to find $A \cap B \cap C$, we need to find the elements that are common to all three sets. Therefore:

$A \cap B \cap C = \{1, 5\} \cap \{1, 2, 3, 4\}$

$= \{1\}$

This is the only element that is in all three sets.

6. If $U = \{1, 2, 3, \ldots, 20\}$, $X = \{2, 4, 6, \ldots, 20\}$, $Y =$ $\{3, 6, 9, 12, 15, 18\}$ **and $Z = \{2, 3, 5, 7, 11, 13, 17, 19\}$, then find:**

a) $X - Y$ b) $Y - Z$ c) X^c d) Y^c e) Z^c f) U^c

Solution:

a) $X - Y$ means we need to find all the elements that are in X but not in Y. Therefore,

$X - Y = \{2, 4, 6, \ldots, 20\} - \{3, 6, 9, 12, 15, 18\}$

$= \{2, 4, 8, 10, 14, 16, 20\}$.

b) $Y - Z$ means we need to find all the elements that are in Y but not in Z. Therefore,

$Y - Z = \{3, 6, 9, 12, 15, 18\} - \{2, 3, 5, 7, 11, 13, 17, 19\}$

$= \{6, 9, 15, 18\}$.

c) X^c means we need to find all the elements of U that are not in X. Therefore:

$X^c = U - X$

$= \{1, 2, 3, \ldots, 20\} - \{2, 4, 6, \ldots, 20\}$

$= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$.

d) Y^c means we need to find all the elements of U that are not in Y. Therefore:

$Y^c = U - Y$

$= \{1, 2, 3, \ldots, 20\} - \{3, 6, 9, 12, 15, 18\}$

$= \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$.

e) Z^c means we need to find all the elements of U that are not in Z. Therefore:

$Z^c = U - Z$

$=\{1, 2, 3, \ldots, 20\} - \{2, 3, 5, 7, 11, 13, 17, 19\}$

$= \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$.

f) U^c means we need to find all the elements of U that are not in U. Therefore:

$U^c = U - U$

$=\{1, 2, 3, \ldots, 20\} - \{1, 2, 3, \ldots, 20\}$

$= \{\ \}$.

So, there are no elements that are not in U since U contains all the integers from 1 to 20.

7. If P = {1, 4, 7, 10}, Q = {2, 6, 10}, and U = {1, 2, 3, ..., 12}, then find:

a) P^c b) Q^c c) $P^c - Q^c$ d) $P^c \cup Q$ e) $Q^c - P^c$

Solution:

a) P^c means we need to find the complement of P. Therefore, we need to find all the elements that are in U but not in P. We get:

$P^c = U - P$

$= \{1, 2, 3, 5, 6, 8, 9, 11, 12\} - \{1, 4, 7, 10\}$

$= \{2, 3, 5, 6, 8, 9, 11, 12\}$.

b) Q^c means we need to find the complement of Q. Therefore, we need to find all the elements that are in U but not in Q. We get:

$Q^c = U - Q =$

$= \{1, 3, 4, 5, 7, 8, 9, 11, 12\} - \{2, 6, 10\}$

$= \{1, 3, 4, 5, 7, 8, 9, 11, 12\}$.

c) $P^c - Q^c$ means we need to find all the elements that are in P^c but not in Q^c. Therefore, we need to subtract Q^c from P^c. We get:

$P^c - Q^c = \{2, 3, 5, 6, 8, 9, 11, 12\} - \{1, 3, 4, 5, 7, 8, 9, 11, 12\}$

$= \{2, 6\}$.

d) $P^c U Q$ means we need to find all the elements that are in either P or Q. Therefore, we need to combine elements of P and Q. We get:

$P^c \cup Q = \{2, 3, 5, 6, 8, 9, 11, 12\} \cup \{2, 6, 10\}$

$= \{2, 3, 5, 6, 8, 9, 10, 11, 12\}$

e) Q^c - P^c means we need to find all the elements that are in Q^c but not in P^c.

Therefore, we need to subtract P^c from Q^c. We get:

$Q^c - P^c$ = {1, 3, 4, 5, 7, 8, 9, 11, 12} - {2, 3, 5, 6, 8, 9, 11, 12}

= {1, 4, 7}.

8. If A = {2, 3, 5, 7}, B = {1, 3, 5, 7, 9} and U = {1, 2, 3, ..., 10}, then verify the following properties:

a) $(A \cap B)^c = A^c \cup B^c$ b) $(A \cup B)^c = A^c \cap B^c$

Solution:

a) To find the complement of $(A \cap B)$, we first need to find the intersection of A and B.

A = {2, 3, 5, 7}

B = {1, 3, 5, 7, 9}

$A \cap B$ = {3, 5, 7}

To find the complement of $(A \cap B)$, we need to find all the elements in U that are not in $(A \cap B)$. Therefore, we subtract the elements of $(A \cap B)$ from U to get:

$(A \cap B)^c$ = {1, 2, 4, 6, 8, 9, 10} - {3, 5, 7}

= {1, 2, 4, 6, 8, 9, 10}

A^c = {1, 4, 6, 8, 9, 10}

B^c = {2, 4, 6, 8, 10}

$A^c U B^c$ = {1, 2, 4, 6, 8, 9, 10}

We can see that $(A \cap B)^c = A^c U B^c$, so the property is verified.

b) To find the complement of (AUB), we need to find the union of A and B.

A = {2, 3, 5, 7}

B = {1, 3, 5, 7, 9}

AUB = {1, 2, 3, 5, 7, 9}

To find the complement of (AUB), we need to find all the elements in U that are not in (AUB). Therefore, we subtract the elements of (AUB) from U to get:

$(AUB)^c$ = {1, 2, 4, 6, 8, 10}

A^c = {1, 4, 6, 8, 9, 10}

B^c = {2, 4, 6, 8, 10}

$A^c \cap B^c = \{4, 6, 8, 10\}$

We can see that $(A \cup B)^c = A^c \cap B^c$, so the property is verified.

Unit Check

1. Encircle the correct option.

a) The set of even numbers between 12 and 14 is:

i) {13} ii) 13 iii) {14} iv) $\varnothing$

b) If two sets A and B have no common elements, they are called:

i) overlapping sets ii) disjoint sets iii) universal sets iv) null sets

c) If two sets contain at least one element in common, they are called:

i) overlapping sets ii) disjoint sets iii) universal sets iv) null sets

d) If $A \cap B = \varnothing$, then $A - B =$

i) A ii) B iii) $\varnothing$ iv) $B \cup A$

e) $A \cap A^c =$ __________.

i) A ii) U iii) $\varnothing$ iv) $A - U$

2. Fill in the blanks.

a) In descriptive form, a set is represented in the form of statements using well-defined words.

b) $A \cap A^c =$ Φ.

c) The intersection of two given sets is a set which consists of all the common elements of both sets.

d) A universal set is the set of all elements of the sets which are under consideration in a particular context.

e) If A = {2, 5, 10, 15} and B = {3, 5, 7, 16}, then B – A = {3, 7, 16}.

3. Write the following sets in descriptive form.

a) A = {13, 17, 19, 23}

b) B = {7, 14, 21, 28, 35, ⋯}

c) C = {red, blue, green, black, white}

d) D = {pen, pencil, eraser, notebook}

e) E = {⋯, -4, -3, -2, -1, 0, 1, 2, 3, 4, ⋯}

f) F = {0, 1, 2, 3, 4, ⋯, 10}

Solution:

a) The set of prime numbers from 13 to 23.

b) The set of all multiples of the number 7

c) The set of five colours

d) The set of different stationary items

e) The set of all integers

f) The set of whole numbers up to 10

4. Write the following sets in tabular form.

a) The set of all natural numbers. b) The set of integers between 16 and 30.

c) The set of prime numbers from 30 to 50. d) The set of days of the week.

e) The set of months of the year.

f) The set of the letters in the word ENGLISH.

g) The set of all square numbers less than 150.

Solution:

a) {1, 2, 3, 4, ⋯}

b) {17, 18, 19, ⋯, 29}

c) {31, 37, 41, 43, 47}

d) {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

e) {January, February, March, April, May, June, July, August, September, October, November, December}

f) {E, N, G, L, I, S, H}

g) {1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144}

5. If A = {4, 5, 6, 7}, B = {1, 3, 5, 7, 9}, C = {2, 4, 6, 8, 10}, and D = {1, 2, 3, …, 10} then prove the following statements.

a) C is a subset of D　　b) D is a superset of A

c) B and C are disjoint sets　　d) B is equivalent to C

Solution:

a) To prove that C is a subset of D, we need to show that every element of C is also an element of D.
The elements in C are {2, 4, 6, 8, 10} and the elements in D are {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.
We can see that all the elements in C are also in D. Therefore, C is a subset of D.

b) b) To prove that D is a superset of A, we need to show that every element of A is also an element of D.

The elements in A are {4, 5, 6, 7} and the elements in D are {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

We can see that all the elements in A are also in D. Therefore, D is a superset of A.

c) c) To prove that B and C are disjoint sets, we need to show that they have no elements in common.

The elements in B are {1, 3, 5, 7, 9} and the elements in C are {2, 4, 6, 8, 10}.

$B \cap C = \{1, 3, 5, 7, 9\} \cap \{2, 4, 6, 8, 10\}$

$= \{ \}$

We can see that there are no common elements between B and C. Therefore, B and C are disjoint sets.

d) d) To prove that B is equivalent to C, we need to show that they have the same number of elements.

The elements in B are {1, 3, 5, 7, 9} and the elements in C are {2, 4, 6, 8, 10}.

We can see that both sets have five elements. Therefore, B is equivalent to C.

6. If U = {1, 2, 3, …, 15} and X^c = {2, 4, 6, …, 14}, then find X.

Xc is the complement of X, which means it contains all the elements that are not in X but are in U.

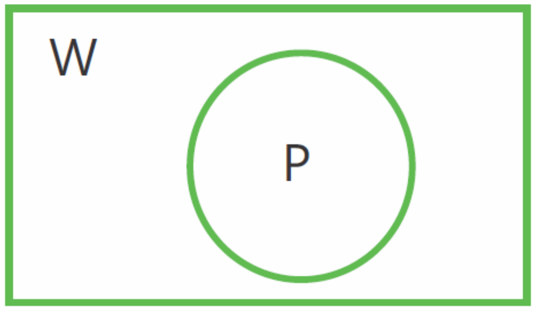
To find X, we need to determine the elements that are in Xc and remove them from U.

We can see that Xc contains all the even numbers between 2 and 14 (inclusive).

Therefore, X contains all the odd numbers between 1 and 15 (inclusive).

X = {1, 3, 5, 7, 9, 11, 13, 15}

7. Use a Venn diagram to represent the relationship between a set of prime numbers and a set of whole numbers.



Set of prime numbers is a subset of the set of whole numbers.

8. If U = {1, 2, 3, …, 20}, A = {11, 13, 17, 19}, B = {1, 3, 5, 10, 15, 20}, C = {2, 6, 10, 14, 18, 20} and D = {3, 6, 9, 12, 13, 18}, then find the following:

a) $A \cup C$ **b)** $B \cap A$ **c)** $A \cap B^c$ **d)** $A^c \cup B^c$

e) $B^c \cap D$ **f)** $C \cup D^c$ **g)** $C^c \cup D^c$ **h)** $C^c \cap D$

Solution:

a) AUC represents the union of sets A and C. To find AUC, we need to combine all the elements that are in either A or C.

A = {11, 13, 17, 19} and C = {2, 6, 10, 14, 18, 20}.

AUC = {2, 6, 10, 11, 13, 14, 17, 18, 19, 20}

b) $B \cap A$ represents the intersection of sets B and A. To find $B \cap A$, we need to determine the elements that are in both B and A.

A = {11, 13, 17, 19} and B = {1, 3, 5, 10, 15, 20}.

$B \cap A$ = { }

c) $A \cap B^c$ represents the intersection of sets A and B^c. To find $A \cap B^c$, we need to determine the elements that are in A but not in B.

A = {11, 13, 17, 19} and B^c = {2, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19}.

$A \cap B^c$ = {17, 13, 17, 19}

d) A^cUB^c represents the union of the complements of sets A and B. To find AcUBc, we need to determine the elements that are not in A or B.

A^c = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 18, 20} and B^c = {2, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19}.

A^cUB^c = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

e) $B^c \cap D$ represents the intersection of sets B^c and D. To find $B^c \cap D$, we need to determine the elements that are in both B^c and D.

B^c = {2, 4, 6, 7, 8, 9, 11, 12, 13, 14, 16, 17, 18, 19} and D = {3, 6, 9, 12, 13, 18}.

$B^c \cap D$ = {6, 9, 12, 13, 18}

f) CUD^c represents the union of sets C and the complement of D. To find CUD^c, we need to determine the elements that are in either C or not in D.

C = {2, 6, 10, 14, 18, 20} and D^c = {1, 2, 4, 5, 7, 8, 10, 11, 14, 15, 16, 17, 19, 20}.

CUD^c = {1, 2, 4, 5, 6, 7, 8, 10, 11, 14, 15, 16, 17, 18, 19, 20}

g) C^cUD^c represents the union of the complements of sets C and D. To find C^cUD^c, we need to determine the elements that are not in C and not in D.

C^c = {1, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19} and D^c = {1, 2, 4, 5, 7, 8, 10, 11, 14, 15, 16, 17, 19, 20}.

C^cUD^c = {1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 19, 20}

h) $C^c \cap D$ represents the intersection of sets C^c and D. To find $C^c \cap D$, we need to determine the elements that are in both C^c and D.

C^c = {1, 3, 4, 5, 7, 8, 9, 11, 12, 13, 15, 16, 17, 19} and D = {3, 6, 9, 12, 13, 18}.

$C^c \cap D$ = {3, 9, 12, 13}

9. **If $A = \{1, 2, 3, \ldots, 8\}$, $B = \{1, 4, 7\}$ and $C = \{2, 4, 6, 8\}$, then find $A - (B - C)$.**

First, we need to find the set B - C, which contains all the elements in B that are not in C. We can write:

$B - C = \{1, 7\}$

Next, we can find A - (B - C) by removing the elements in B - C from A. Since B - C contains only 1 and 7, we need to remove these two elements from A to get the desired set. We can write:

$A - (B - C) = \{2, 3, 4, 5, 6, 8\}$

Therefore, $A - (B - C) = \{2, 3, 4, 5, 6, 8\}$.

10. **If $U = \{a, b, c, \ldots, m\}$ and $A = \{a, b, d, f, h, k, l\}$, then prove that $A^c \cap A = \Phi$.**

We know that A^c contains all the elements that are in U but not in A.

We are given that $U = \{a, b, c, \ldots, m\}$ and $A = \{a, b, d, f, h, k, l\}$.

To prove that $A^c \cap A = \Phi$, we need to show that there are no elements that are in both A^c and A.

A^c contains all the elements in U except for those in A. Therefore:

$A^c = \{c, e, g, i, j, m\}$

We can see that there are no elements in A^c that are also in A. Therefore:

$A^c \cap A = \Phi$.

Hence, we have proved that $A^c \cap A$ is an empty set.

Learning check 6.1

1. **Find the term-to-term rule in the following patterns. Also find the next 3 terms of the given patterns.**

   a) **5, 17, 29, 41, 53, …**

   Solution:

   The given number sequence is.

   5, 17, 29, 41, 53, …

   Firstly, work out the difference in the terms. This pattern is going up by 12 each time, so we will add 12 to the last term to find the next term in the pattern.

   So the term-to-term rule is: **adding 12**

   So, the next three terms of the given sequence are:

   5, 17, 29, 41, 53, <u>65</u>, <u>77</u>, <u>89</u>.

   So, 65, 77 and 89 are the next three terms of the given sequence.

   b) **12, 23, 34, 45, …**

   Solution:

   The given number sequence is.

   12, 23, 34, 45, …

   Firstly, work out the difference in the terms. This pattern is going up by 11 each time, so we will add 11 to the last term to find the next term in the pattern.

   So the term-to-term rule is: **adding 11**

   So, the next three terms of the given sequence are:

   12, 23, 34, 45, <u>56</u>, <u>67</u>, <u>78</u>.

   So, 56, 67 and 78 are the next three terms of the given sequence.

   c) **112, 170, 228, 344, …**

   Solution:

   The given number sequence is.

   112, 170, 228, 344, …

Firstly, work out the difference in the terms. This pattern is going up by 58 each time, so we will add 58 to the last term to find the next term in the pattern.

So the term-to-term rule is: **adding 58**

So, the next three terms of the given sequence are:

112, 170, 228, 344, <u>392</u>, <u>450</u>, <u>508</u>.

So, 392, 450 and 508 are the next three terms of the given sequence.

d) 3, 8, 13, 18, 23, …

Solution:

The given number sequence is.

3, 8, 13, 18, 23, …

Firstly, work out the difference in the terms. This pattern is going up by 5 each time, so we will add 5 to the last term to find the next term in the pattern.

So the term-to-term rule is: **adding 5**

So, the next three terms of the given sequence are:

3, 8, 13, 18, 23, <u>28</u>, <u>33</u>, <u>38</u>.

So, 28, 33 and 38 are the next three terms of the given sequence.

e) 347, 340, 333, …

Solution:

The given number sequence is.

347, 340, 333, …

Firstly, work out the difference in the terms. This pattern is going down by 7 each time, so we will subtract 7 from the last term to find the next term in the pattern.

So the term-to-term rule is: **Subtract 7**

So, the next three terms of the given sequence are:

347, 340, 333, <u>326</u>, <u>319</u>, <u>312</u>.

So, 326, 319 and 312 are the next three terms of the given sequence.

f) $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, \frac{5}{4} \ldots$

Solution:

The given number sequence is.

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, \frac{5}{4}, \ldots$

Firstly, work out the difference in the terms. This pattern is going up by $\frac{1}{4}$ each time, so we will add $\frac{1}{4}$ to the last term to find the next term in the pattern.

So the term-to-term rule is: **adding $\mathbf{\frac{1}{4}}$**

So, the next three terms of the given sequence are:

$\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}, \frac{7}{4}, 2.$

So, $\frac{6}{4}, \frac{7}{4}$ and 2 are the next three terms of the given sequence.

2. Identify the position-to-term rules for the following patterns and find the 12th and 25th terms.

a) 1, 2, 3, 4, 5, …

Solution:

First, write the terms and their positions in the pattern in form of a table.

Position	1st	2nd	3rd	4th	5th
Term	1	2	3	4	5

Now find the rule for the terms.

Position	1st	2nd	3rd	4th	5th
Term	1	2	3	4	5
Rule	1 + 0	2 + 0	3 + 0	4 + 0	5 + 0

Now, to find the 12th term of this pattern, we will add 0 to 12.

So, the 12th term is: 12 + 0 = 12.

Now, to find the 25th term of this pattern, we will add 0 to 25.

So, the 25th term is: 25 + 0 = 25.

Similarly, if we denote any general term as n, then the position to term rule here is n + 0.

b) 2, 4, 6, 8, …

Solution:

First, write the terms and their positions in the pattern in form of a table.

Position	1st	2nd	3rd	4th
Term	2	4	6	8

Now find the rule for the terms.

Position	1st	2nd	3rd	4th
Term	2	4	6	8
Rule	1 × 2	2 × 2	3 × 2	4 × 2

Now, to find the 12th term of this pattern, we will multiply 12 by 2.

So, the 12th term is: 12 × 2 = 24.

Now, to find the 25th term of this pattern, we will multiply 25 by 2.

So, the 25th term is: 25 × 2 = 50.

Similarly, if we denote any general term as n, then the position to term rule here is 2n.

c) $\frac{1}{2}$, 1, $\frac{3}{2}$, 2, $\frac{5}{2}$…

Solution:

First, write the terms and their positions in the pattern in form of a table.

Position	1st	2nd	3rd	4th	5th
Term	$\frac{1}{2}$	2	$\frac{3}{2}$	2	$\frac{5}{2}$

Now find the rule for the terms.

Position	1st	2nd	3rd	4th	5th
Term	$\frac{1}{2}$	2	$\frac{3}{2}$	2	$\frac{5}{2}$
Rule	$1 \times \frac{1}{2}$	$2 \times \frac{1}{2}$	$3 \times \frac{1}{2}$	$4 \times \frac{1}{2}$	$5 \times \frac{1}{2}$

Now, to find the 12th term of this pattern, we will multiply $\frac{1}{2}$ to 12.

So, the 12th term is: $12 \times \frac{1}{2} = 6$.

Now, to find the 25th term of this pattern, we will multiply $\frac{1}{2}$ to 25.

So, the 25th term is: $25 \times \frac{1}{2} = \frac{25}{2}$.

Similarly, if we denote general term as n, then the position to term rule here is $\frac{n}{2}$.

d) 11, 12, 13, 14, 15…

Solution:

First, write the terms and their positions in the pattern in form of a table.

Position	1st	2nd	3rd	4th	5th
Term	11	12	13	14	15

Now find the rule for the terms.

Position	1st	2nd	3rd	4th	5th
Term	11	12	13	14	15
Rule	1 + 10	2 + 10	3 + 10	4 + 10	5 + 10

Now, to find the 12th term of this pattern, we will add 10 to 12.

So, the 12th term is: 12 + 10 = 22.

Now, to find the 25th term of this pattern, we will add 10 to 25.

So, the 25th term is: 25 + 10 = 35.

Similarly, if we denote general term as n, then the position to term rule is n + 10.

e) 9, 18, 27, …

Solution:

First, write the terms and their positions in the pattern in form of a table.

Position	1st	2nd	3rd
Term	9	18	27

Now find the rule for the terms.

Position	1st	2nd	3rd
Term	9	18	27
Rule	1 × 9	2 × 9	3 × 9

Now, to find the 12th term of this pattern, we will multiply 9 by 12.

So, the 12th term is: 12 × 9 = 108.

Now, to find the 25th term of this pattern, we will multiply 9 by 25.

So, the 25th term is: 25 × 9 = 225.

Similarly, if we denote general term as n, then the position to term rule here is 9n.

3. **Marwa read 12 pages of the book on the first day, 18 pages on the second day and 24 pages on the third day. If she follows this pattern, how many pages will she read on the 4th and 5th days, respectively?**

   Solution:

   Let's arrange the number of pages read by Merwa in sequence.

   12, 18, 24, _______, _______

   Firstly, work out the difference in the terms. This pattern is going up by 6 each time, so

   we will add 6 to the last term to find the next term in the pattern.

   12, 18, 24, 30, 36.

   So, she will read 30 and 36 pages on the 4th and 5th days respectively.

4. **Mohid has begun working out. He gradually increased his exercise time. On Monday, he worked out for 10 minutes, 13 minutes on Tuesday, and 16 minutes on Wednesday. How many minutes will he exercise on Thursday if he keeps increasing the time in the same manner?**

   Solution:

   Let's arrange the exercise time of Mohid in sequence.

   10, 13, 16, _______

Firstly, work out the difference in the terms. This pattern is going up by 3 each time, so we will add 3 to the last term to find the next term in the pattern.

10, 13, 16, <u>19</u>.

So, Mohid worked out for 19 minutes on Thursday.

5. If the nth term of a sequence is 2n + 6, find the 5th, 10th and 15th terms of this sequence.

Solution:

The nth term of a sequence = 2n + 6

To find the 5th term of the sequence Put n = 5 in 2n + 6.

2n + 6 = 2(5) + 6 = 10 + 6 = 16

So, the 10th term of this sequence is 16.

To find the 10th term of the sequence Put n = 10 in 2n + 6.

2n + 6 = 2(10) + 6 = 20 + 6 = 26

So, the 10th term of this sequence is 26.

To find the 15th term of the sequence Put n = 15 in 2n + 6.

2n + 6 = 2(15) + 6 = 30 + 6 = 36

So, the 15th term of this sequence is 36.

6. If the nth term of a sequence is 5n – 1, then find the 3rd and 8th terms of this sequence.

Solution:

The nth term of a sequence = 5n – 1

To find the 3rd term of the sequence Put n = 3 in 5n – 1.

5n – 1 = 5(3) – 1 = 15 – 1 = 14

So, the 3rd term of this sequence is 14.

To find the 8th term of the sequence Put n = 8 in 5n – 1.

5n – 1 = 5(8) – 1 = 40 – 1 = 39

So, the 8th term of this sequence is 39.

7. If the nth term of a sequence is 4 + 7n, then find the 9th and 24th terms of this sequence.

Solution:

The nth term of a sequence = 4 + 7n

To find the 9th term of the sequence Put n = 9 in 4 + 7n.

4 + 7n = 4 + 7(9) = 4 + 63 = 67

So, the 9th term of this sequence is 67.

To find the 24th term of the sequence Put n = 24 in 4 + 7n.

4 + 7n = 4 + 7(24) = 4 + 168 = 172

So, the 24th term of this sequence is 172.

8. If the nth term of a sequence $\frac{\mathbf{n}}{\mathbf{2n+1}}$ then find the 30th term of this sequence.

Solution:

The nth term of a sequence = $\frac{n}{2n+1}$

To find the 30th term of the sequence put n = 30 in $\frac{n}{2n+1}$.

$\frac{n}{2n+1} = \frac{30}{2(30)+1} = \frac{30}{60+1} = \frac{30}{61}$

So, the 30th term of this sequence is $\frac{30}{61}$.

Learning check 6.2

1. Write down whether the following sentences are open or closed.

a) 3 + 5 = 8

Solution:

As the given equation contain no variable and the given sentence is true so the given sentence is a closed sentence.

b) *x* – y = 1

Solution:

Here we cannot say that the statement is true or false, because we do not know the value of the variables x and y. so the given sentence is an open sentence.

c) a + 2 = 6

Solution:

Here we cannot say that the statement is true or false, because we do not know the value of the variable “a”. so the given sentence is an open sentence.

d) 7 – y = 12

Solution:

Here we cannot say that the statement is true or false, because we do not know the value of the variable "y". so the given sentence is an open sentence.

e) $3t + 5 = 9$

Solution:

Here we cannot say that the statement is true or false, because we do not know the value of the variable "t". so the given sentence is an open sentence.

f) $x + 7 = 2x$

Solution:

Here we cannot say that the statement is true or false, because we do not know the value of the variable "x". so the given sentence is an open sentence.

g) $2 - t = 1$

Solution:

Here we cannot say that the statement is true or false, because we do not know the value of the variable "t". so the given sentence is an open sentence.

h) $7 - 12 = -5$

Solution:

As the given equation contain no variable and the given sentence is true so the given sentence is a closed sentence.

2. Separate the terms of the expressions from the following.

a) $3x + 7y - 2$

Solution:

3x, 7y and -2 are the three terms of the given expression.

b) $a + 3b + c$

Solution:

a, 3b and c are the three terms of the given expression.

c) $x + 2y - 7$

Solution:

x, 2y and -7 are the three terms of the given expression.

d) $x^2 - 2x + 1$

Solution:

x^2, –2x and 1 are the three terms of the given expression.

e) $2 + x - 8y$

Solution:

2, x and $-8y$ are the three terms of the given expression.

f) $a^3 + 8a + 2$

Solution:

a^3, 8a and 2 are the three terms of the given expression.

g) $u^2 + v^2 - 2$

Solution:

u^2, v^2 and –2 are the three terms of the given expression.

h) $6x^3 + 2x^2 - 2x + 1$

Solution:

$6x^3$, $2x^2$, $-2x$ and 1 are the four terms of the given expression.

3. Separate the like and unlike terms from the expressions.

a) $32x^2 + 10x + x^2$

Solution:

Here $32x^2$ and x^2 are like terms. In the term $10x$, the variable is the same but its exponent is 1 which is different from the exponents of the other terms. So, $10x$ and $32x^2$ as well as $10x$ and x^2 are unlike terms.

b) 5w + 12 w

Solution:

Here 5w and 12w are like terms. As the variable and exponent of both terms are same so two terms of the given expression are like terms.

c) $3a^3 + 6b^3 + c^3$

Solution:

As the exponent of three terms are same but the variables of three terms are different so, $3a^3$, $6b^3$, c^3 are unlike terms.

d) 3y + 7y – 10y

Solution:

As the three terms of the given expression has variable y and the exponent of three terms of the expression is 1 so, here 3y and 7y and – 10y are like terms.

4. Write the constant, variable, coefficient and exponent of the variables in the expression: $3p^2 - 2q^2 + t + 7$.

Solution:

The constant in the given expression is 7 because it is a fixed number that does not change like variable.

The variable in the given expression are: p, q and t. As the value of p, q and t are not known.

As the number which is multiplied by a variable is called a coefficient so,

3 is the coefficient of p^2 and 2 is the coefficient of q^2 and 1 is the coefficient of the variable t.

As we know that the exponent is the number which represents repeated multiplication of the same variable so 2 is the exponent of variable p, 2 is the exponent of variable q and 1 is the exponent of variable t.

Learning Check 6.3

1. Identify the polynomial expressions from the following.

a) $2x^2 + \sqrt{x}$

Solution:

Look at the expression $2x^2 + \sqrt{x}$. Here, x is the variable that has a fractional exponent $\frac{1}{2}$. So, the expression is not a polynomial.

b) $2x^3 - 4x^2 + x + 1$

Solution:

Observe the expression $2x^3 - 4x^2 + x + 1$. Here, x is a variable and have positive exponents, i.e., 3, 2 and 1, respectively. So, the expression is a polynomial.

c) $3x^2 + \frac{1}{x}$

Solution:

Look at the expression $3x^2 + \frac{1}{x}$. Here, x is the variable that is in denominator so its exponent is –1 So, the expression is not a polynomial.

d) $\sqrt{x}$ **+ 2y + 7**

Solution:

Look at the expression $\sqrt{x} + 2y + 7$. Here, x is the variable that has a fractional exponent $\frac{1}{2}$. So, the expression is not a polynomial.

e) **2a – b + 10**

Solution:

Observe the expression 2a – b + 10. Here, a and b are the variables and have positive exponents, i.e., 1 and 1, respectively. So, the expression is a polynomial.

f) $\mathbf{p^2 + q + 1}$

Solution:

Observe the expression $p^2 + q + 1$. Here, p and q are the variables and have positive exponents, i.e., 2 and 1, respectively. So, the expression is a polynomial.

g) $\mathbf{2y^2 + 2y - 4}$

Solution:

Observe the expression $2y^2 + 2y - 4$. Here, y is the variable and have positive exponents, i.e., 2 and 1, respectively. So, the expression is a polynomial.

h) $\mathbf{9 - \frac{5}{x^2 + x + 1}}$

Solution:

Look at the expression $\mathbf{9} - \frac{5}{x^2 + x + 1}$. Here, x is the variable that is in denominator. So, the expression is not a polynomial.

i) $\mathbf{xy + 7x^2 + 2y^2 - 1}$

Solution:

Observe the expression $\mathbf{xy + 7x^2 + 2y^2 - 1}$. Here, x and y are the variables and have positive exponents. So, the expression is a polynomial.

2. Identify the monomials, binomials and trinomials.

a) $-4x^3 + x^2 + 2$

Solution:

As the given expression contain three terms $4x^3$, x^2 and 2 so, the given expression is a trinomial.

b) $x^4 + xy^2$

Solution:

As the given expression contain two terms x^4 and xy^2 so, the given expression is a binomial.

c) 2b

Solution:

As the given expression contain only one term so, the given expression is monomial.

d) x^3y^2

Solution:

As the given expression contain only one term so, the given expression is monomial.

e) $a^2b + 2b^3 + c$

Solution:

As the given expression contain three terms a^2b, $2b^3$ and c so, the given expression is a trinomial.

f) 3x + y

Solution:

As the given expression contain two terms 3x and y so, the given expression is a binomial.

g) $2x^3 + x^2 + xy$

Solution:

As the given expression contain three terms $2x^3$, x^2 and xy so, the given expression is a trinomial.

h) $w^2 + 2w$

Solution:

As the given expression contain two terms w^2 and w so, the given expression is a binomial.

i) $2z^2$

Solution:

As the given expression contain only one term so, the given expression is monomial.

Learning Check 6.4

1. Solve the following.

a) $(x^2 + x + 5) + (3x^2 – 2x – 7)$

Solution:

We can add the given expression horizontally and vertically. Let us add first horizontally and then vertically.

Horizontal method:

$(x^2 + x + 5) + (3x^2 – 2x – 7)$

Remove the bracket and group like terms, then add and subtract them.

$= x^2 + x + 5 + 3x^2 – 2x – 7$

$= x^2 + 3x^2 + x – 2x + 5 – 7$

$= 4x^2 – x – 2$

Vertical method:

$$\begin{array}{r} x^2 + x + 5 \\ 3x^2 - 2x - 7 \\ \hline 4x^2 - x - 2 \end{array}$$

b) $(x + 3) + (4 – 5x)$

Solution:

Let us add first horizontally and then vertically.

Horizontal method:

$(x + 3) + (4 – 5x)$

Remove the bracket and group like terms, then add and subtract them.

$= x + 3 + 4 – 5x$

$= x – 5x + 3 + 4$

$= – 4x + 7$

Vertical method:

$$\begin{array}{r} x + 3 \\ -5x + 4 \\ \hline -4x + 7 \end{array}$$

c) $(a^2 + 1) + (a + 7)$

Let us add first horizontally and then vertically.

Horizontal method:

$(a^2 + 1) + (a + 7)$

Remove the bracket and group like terms, then add and subtract them.

$= a^2 + 1 + a + 7$

$= a^2 + a + 1 + 7$

$= a^2 + a + 8$

Vertical method:

$$\begin{array}{r} a^2 + \quad 1 \\ a + 7 \\ \hline a^2 + a + 8 \end{array}$$

d) $(4w^2 – 4w + 1) + (w^2 – 7)$

Let us add first horizontally and then vertically.

Horizontal method:

$(4w^2 – 4w + 1) + (w^2 – 7)$

Remove the bracket and group like terms, then add and subtract them.

$= 4w^2 – 4w + 1 + w^2 – 7$

$= 4w^2 + w^2 – 4w + 1 – 7$

$= 5w^2 – 4w – 6$

Vertical method:

$$\begin{array}{|lcr|}\hline 4w^2 & – 4w & + 1 \\ w^2 & & – 7 \\ \hline 5w^2 & – 4w & – 6 \\ \hline \end{array}$$

e) **$(3x^3 – 2x^2 + 7x – 1) + (x^3 + x – 5)$**

Let us add first horizontally and then vertically.

Horizontal method:

$(3x^3 – 2x^2 + 7x – 1) + (x^3 + x – 5)$

Remove the bracket and group like terms, then add and subtract them.

$= 3x^3 – 2x^2 + 7x – 1 + x^3 + x – 5$

$= 3x^3 + x^3 – 2x^2 + 7x + x – 5 – 1$

$= 4x^3 – 2x^2 + 8x – 6$

Vertical method:

$$\begin{array}{|lccr|}\hline 3x^3 & – 2x^2 & + 7x & – 1 \\ x^3 & & + x & – 5 \\ \hline 4x^3 & – 2x^2 & + 8x & – 6 \\ \hline \end{array}$$

f) **$(–3x^2 + 3 + 4x) + (2x – 5 + 4x^2)$**

Let us add first horizontally and then vertically.

Horizontal method:

$= (-3x^2 + 3 + 4x) + (2x - 5 + 4x^2)$

Remove the bracket and group like terms, then add and subtract them.

$= -3x^2 + 3 + 4x + 2x - 5 + 4x^2$

$= -3x^2 + 4x^2 + 4x + 2x - 5 + 3$

$= x^2 + 6x - 2$

Vertical method:

$$\begin{array}{r} -3x^2 + 4x + 3 \\ +\ 4x^2 + 2x - 5 \\ \hline x^2 + 6x - 2 \end{array}$$

g) $(9m^3 - 5m^2 - 4) + (-9m^3 + 6m^2 + 7)$

Let us add first horizontally and then vertically.

Horizontal method:

$= (9m^3 - 5m^2 - 4) + (-9m^3 + 6m^2 + 7)$

Remove the bracket and group like terms, then add and subtract them.

$= 9m^3 - 5m^2 - 4 - 9m^3 + 6m^2 + 7$

$= 9m^3 - 9m^3 - 5m^2 + 6m^2 - 4 + 7$

$= m^2 + 3$

Vertical method:

$$\begin{array}{r} 9m^3 - 5m^2 - 4 \\ -9m^3 + 6m^2 + 7 \\ \hline m^2 + 3 \end{array}$$

h) $(-y^3 - 19) + (y^3 + 4 + y^2)$

Let us add first horizontally and then vertically.

Horizontal method:

$= (-y^3 - 19) + (y^3 + 4 + y^2)$

Remove the bracket and group like terms, then add and subtract them.

$= -y^3 - 19 + y^3 + 4 + y^2$

$= -y^3 + y^3 + y^2 - 19 + 4$

$= y^2 - 15$

Vertical method:

$$\begin{array}{r} -y^3 \qquad - 19 \\ y^3 + y^2 + 4 \\ \hline y^2 - 15 \end{array}$$

2. **Solve the following.**

   a) **$(3x^2 + 2x - 8) - (2x^2 - 3x + 2)$**

   Solution:

   We can subtract the given expression horizontally and vertically. Let us subtract first horizontally and then vertically.

   Horizontal method:

   $(3x^2 + 2x - 8) - (2x^2 - 3x + 2)$

   Remove the brackets and change the sign of the terms which are to be subtracted.

   $= 3x^2 + 2x - 8 - 2x^2 + 3x - 2$

   Now group like terms, then add and subtract them.

   $= 3x^2 - 2x^2 + 2x + 3x - 8 - 2$

   $= x^2 + 5x - 10$

   Vertical method:

$$\begin{array}{r} 3x^2 + 2x - 8 \\ \pm 2x^2 \mp 3x \pm 2 \\ \hline x^2 + 5x - 10 \end{array}$$

b) $(w^2 + 4) - (w^2 - w^3 + 1)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(w^2 + 4) - (w^2 - w^3 + 1)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= w^2 + 4 - w^2 + w^3 - 1$

Now group like terms, then add and subtract them.

$= w^3 + w^2 - w^2 - 1 + 4$

$= w^3 + 3$

Vertical method:

$$\begin{array}{r}
w^2 + 4 \\
\mp w^3 \pm w^2 \pm 1 \\
\hline
w^3 \qquad + 3
\end{array}$$

c) $(3 - 4y^2) - (-6y^2 + 7)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(3 - 4y^2) - (-6y^2 + 7)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 3 - 4y^2 + 6y^2 - 7$

Now group like terms, then add and subtract them.

$= -4y^2 + 6y^2 - 7 + 3$

$= 2y^2 - 4$

Vertical method:

$$\begin{array}{r} -4y^2 + 3 \\ \mp 6y^2 \pm 7 \\ \hline 2y^2 - 4 \end{array}$$

d) $(x^3 - 3x^2 + 10) - (x^3 + 2x^2 + x - 1)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(x^3 - 3x^2 + 10) - (x^3 + 2x^2 + x - 1)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= x^3 - 3x^2 + 10 - x^3 - 2x^2 - x + 1$

Now group like terms, then add and subtract them.

$= x^3 - x^3 - 3x^2 - 2x^2 - x + 10 + 1$

$= - 5x^2 - x + 11$

Vertical method:

$$\begin{array}{rrrr} x^3 - 3x^2 & & + 10 \\ \pm x^3 \pm 2x^2 & \pm x & \mp 1 \\ \hline -5x^2 & - x & + 11 \end{array}$$

e) $(2x - 2x^2y + y) - (x^3 - 4x^2y + y)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(2x - 2x^2y + y) - (x^3 - 4x^2y + y)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 2x - 2x^2y + y - x^3 + 4x^2y - y$

Now group like terms, then add and subtract them.

$= 2x - 2x^2y + 4x^2y - x^3 - y + y$

$= -x^3 + 2x + 2x^2y$

Vertical method:

$$\begin{array}{rr} & 2x - 2x^2y + y \\ \pm x^3 & \mp 4x^2y \pm y \\ \hline \multicolumn{2}{l}{-x^3 + 2x + 2x^2y} \end{array}$$

f) **$(4t^3 - 4st + s^3) - (-2t^3 - s^3)$**

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(4t^3 - 4st + s^3) - (-2t^3 - s^3)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 4t^3 - 4st + s^3 + 2t^3 + s^3$

Now group like terms, then add and subtract them.

$= 4t^3 + 2t^3 - 4st + s^3 + s^3$

$= 6t^3 - 4st + 2s^3$

Vertical method:

$$\begin{array}{rcr} 4t^3 & - 4st & + s^3 \\ \mp 2t^3 & & \mp s^3 \\ \hline 6t^3 & - 4st & + 2s^3 \end{array}$$

g) $(3 - x - x^2 - 2x^3) - (x^3 - 3x^2 + x - 2)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(3 - x - x^2 - 2x^3) - (x^3 - 3x^2 + x - 2)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 3 - x - x^2 - 2x^3 - x^3 + 3x^2 - x + 2$

Now group like terms, then add and subtract them.

$= - 2x^3 - x^3 - x^2 + 3x^2 - x - x + 3 + 2$

$= - 3x^3 + 2x^2 - 2x + 5$

Vertical method:

$$\begin{array}{r} - 2x^3 - x^2 - x + 3 \\ \pm x^3 \mp 3x^2 \pm x \mp 2 \\ \hline - 3x^3 + 2x^2 - 2x + 5 \end{array}$$

h) $(w^2 - 2w + 1) - (2w^2 - 5w + 2)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(w^2 - 2w + 1) - (2w^2 - 5w + 2)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= w^2 - 2w + 1 - 2w^2 + 5w - 2$

Now group like terms, then add and subtract them.

$= w^2 - 2w^2 - 2w + 5w + 1 - 2$

$= - w^2 + 3w - 1$

Vertical method:

$$\begin{array}{r} w^2 - 2w + 1 \\ \pm 2w^2 \mp 5w \pm 2 \\ \hline -w^2 + 3w - 1 \end{array}$$

3. **Subtract the sum of the polynomials $(3x^2 - 4y + 1)$ and $(x^2 + 4)$ from $(x^2 - 3y + 10)$.**

   Solution:

   First add the polynomials $(3x^2 - 4y + 1)$ and $(x^2 + 4)$.

   $= 3x^2 - 4y + 1 + x^2 + 4$

   Now group like terms, then add and subtract them.

   $= 3x^2 + x^2 - 4y + 1 + 4$

   $= 4x^2 - 4y + 5$

   Now subtract $(4x^2 - 4y + 5)$ from $(x^2 - 3y + 10)$.

   $= (x^2 - 3y + 10) - (4x^2 - 4y + 5)$

   Remove the brackets and change the sign of the terms which are to be subtracted.

   $= x^2 - 3y + 10 - 4x^2 + 4y - 5$

   Now group like terms, then add and subtract them.

   $= x^2 - 4x^2 - 3y + 4y + 10 - 5$

   $= -3x^2 + y + 5$

4. **What should be added to $(-9t^3 - t^2 - 3t^5 - t^4 - t)$ to get $(-6t^4 - t - t^5 - t^3 - 3t^2)$?**

   Solution:

   Arrange the polynomial $-6t^4 - t - t^5 - t^3 - 3t^2$ as $-t^5 - 6t^4 - t^3 - 3t^2 - t$, then subtract $-9t^3 - t^2 - 3t^5 - t^4 - t$ from $-t^5 - 6t^4 - t^3 - 3t^2 - t$.

   $= (-t^5 - 6t^4 - t^3 - 3t^2 - t) - (-9t^3 - t^2 - 3t^5 - t^4 - t)$

   Remove the brackets and change the sign of the terms which are to be subtracted.

$= -t^5 - 6t^4 - t^3 - 3t^2 - t + 9t^3 + t^2 + 3t^5 + t^4 + t$

Group like terms, then add and subtract them.

$= -t^5 + 3t^5 - 6t^4 + t^4 - t^3 + 9t^3 - 3t^2 + t^2 - t + t$

$= 2t^5 - 5t^4 + 8t^3 - 2t^2$

Thus, $2t^5 - 5t^4 + 8t^3 - 2t^2$ is the required polynomial.

5. **What should be subtracted from ($11m^5 - 25m - 4m^4 + 3m^2 + m^6$) to get ($-7m^3 - 2m^4 + 15m$)?**

Solution:

First arrange the polynomial:

$11m^5 - 25m - 4m^4 + 3m^2 + m^6$ as $m^6 + 11m^5 - 4m^4 + 3m^2 - 25m$,

then subtract ($-7m^3 - 2m^4 + 15m$) from ($m^6 + 11m^5 - 4m^4 + 3m^2 - 25m$).

$= (m^6 + 11m^5 - 4m^4 + 3m^2 - 25m) - (-7m^3 - 2m^4 + 15m)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= m^6 + 11m^5 - 4m^4 + 3m^2 - 25m + 7m^3 + 2m^4 - 15m$

Group like terms, then add and subtract them.

$= m^6 + 11m^5 - 4m^4 + 2m^4 + 7m^3 + 3m^2 - 25m - 15m$

$= m^6 + 11m^5 - 2m^4 + 7m^3 + 3m^2 - 40m$

Thus, $m^6 + 11m^5 - 2m^4 + 7m^3 + 3m^2 - 40m$ is the required polynomial.

Learning Check 6.5

1. **Find product of the following polynomials.**

   a) **$4x^2$, $3x$**

   Solution:

   To multiply a monomial by another monomial, find the product of the coefficients of the involved variable and add the exponents of the same variables.

   $= 4x^2 \times 3x = (4 \times 3)(x^2 \times x)$

   $= 12x$

   b) **x, 4y, 7**

   Solution:

To multiply a monomial by another monomial, find the product of the coefficients of the involved variable and add the exponents of the same variables.

$= x \times 4y \times 7 = (1 \times 4 \times 7)\ (x \times y)$

$= 28xy$

c) 4a, (2a – 1)

Solution:

The term $3a^2$ is multiplied by each term of the binomial.

$4a \times (2a - 1) = (4a \times 2a) - (4a \times 1)$

Multiply the coefficients of the variables and subtract the exponents of the same base.

$= (4 \times 2)\ a^{1+1} - (4 \times 1)\ a$

$= 8a^2 - 4a$

d) m^2, (2m + 3mn)

Solution:

The term m^2 is multiplied by each term of the binomial.

$m^2 \times (2m + 3mn) = (m^2 \times 2m) + (m^2 \times 3mn)$

Multiply the coefficients of the variables and add the exponents of the same base.

$= (1 \times 2)\ m^{2+1} + (1 \times 3)\ (m^{2+1} \times n)$

$= 2m^3 + 3m^3n$

e) $(4x^2 - 5)$, $(x^2 - 2x + 1)$

Solution:

Multiply the 1st term of the binomial by each term of the trinomial and multiple the 2nd term of the binomial by each term of the trinomial. Then simplify by adding or subtracting the like terms.

$= (4x^2 - 5)\ (x^2 - 2x + 1)$

$= 4x^2 \times (x^2 - 2x + 1) - 5(x^2 - 2x + 1)$

$= (4x^{2+2} + 8x^{2+1} + 4x^2\ ) - (\ 5x^2 - 10x + 5)$

$= (4x^4 + 8x^3 + 4x^2) - (5x^2 - 10x + 5)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 4x^4 + 8x^3 + 4x^2 – 5x^2 + 10x – 5$

$= 4x^4 + 8x^3 – x^2 + 10x – 5$

f) **$(6t – 1), (t – 3t^2)$**

Solution:

Multiply the 1st term of the binomial by each term of the binomial and multiple the 2nd term of the binomial by each term of the binomial. Then simplify by adding or subtracting the like terms.

$= (6t – 1)\ (t – 3t^2)$

$= 6t\ (t – 3t^2) – 1(t – 3t^2)$

$= (6t^{1+1} – 18t^{1+2}) – (t – 3t^2)$

$= (6t^2 – 18t^3) – (t – 3t^2)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 6t^2 – 18t^3 – t + 3t^2$

$= – 18t^3 + 6t^2 + 3t^2 – t$

$= – 18t^3 + 9t^2 – t$

g) **$(t + 2), (2t^2 – tu + 4t – 9)$**

Solution:

Multiply the 1st term of the binomial by each term of the trinomial and multiple the 2nd term of the binomial by each term of the trinomial. Then simplify by adding or subtracting the like terms

$= t \times (2t^2 – tu + 4t – 9) + 2 \times (2t^2 – tu + 4t – 9)$

$= 2t^{1+2} – t^{1+1}u + 4t^{1+1} – 9t + 4t^2 – 2tu + 8t – 18$

$= 2t^3 – t^2u + 4t^2 – 9t + 4t^2 – 2tu + 8t – 18$

$= 2t^3 – t^2u + 8t^2 – 9t + 8t – 2tu – 18$

$= 2t^3 – t^2u + 8t^2 – t – 2tu – 18$

h) **$2x^2, (x^2 – 2xy + 3y)$**

The term $2x^2$ is multiplied by each term of the trinomial.

$= 2x^2 \times (x^2 – 2xy + 3y)$

Multiply the coefficients of the variables and subtract the exponents of the same base.

$= 2x^2 \times x^2 - 2x^2 \times 2xy + 2x^2 \times 3y$

$= 2x^{2+2} - (2 \times 2)\ x^{2+1} \times y + (2 \times 3)\ x^2y$

$= 2x^4 - 4x^3y + 6x^2y$

i) $(3xy^3 - y), (x^3y + x)$

Solution:

Multiply the 1st term of the binomial "$3xy^3$" by each term of the binomial and multiple the 2nd term of the binomial "y" by each term of the binomial. Then simplify by adding or subtracting the like terms.

$= (3xy^3 - y)\ (x^3y + x)$

$= 3xy^3\ (x^3y + x) - y(x^3y + x)$

$= (3x^{3+1}y^{3+1} + 3x^{1+1}y^3) - (x^3y^{1+1} + xy)$

$= (3x^4y^4 + 3x^2y^3) - (x^3y^2 + xy)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 3x^4y^4 + 3x^2y^3 - x^3y^2 - xy$

$= 3x^4y^4 - x^3y^2 + 3x^2y^3 - xy$

j) $(a^2 - b^2), (ab^2 - a^2b)$

Solution:

Multiply the 1st term of the binomial "a^2" by each term of the binomial and multiple the 2nd term "b^2" of the binomial by each term of the binomial. Then simplify by adding or subtracting the like terms.

$= (a^2 - b^2)\ (ab^2 - a^2b)$

$= a^2\ (ab^2 - a^2b) - b^2\ (ab^2 - a^2b)$

$= (a^{2+1}\ b^2 - a^{2+2}b) - (ab^{2+2} - a^2b^{1+2})$

$= (a^3b^2 - a^4b) - (ab^4 - a^2b^3)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= a^3b^2 – a^4b – ab^4 + a^2b^3$

$= – a^4b + a^3b^2 + a^2b^3 – ab^4$

k) $(–3p^2 + 4pq + 1), (–4q + 3pq)$

Solution:

Multiply the 1st term of the binomial by each term of the trinomial and multiple the 2nd term of the binomial by each term of the trinomial. Then simplify by adding or subtracting the like terms.

$= (–4q + 3pq) (–3p^2 + 4pq + 1)$

$= –4q \times (–3p^2 + 4pq + 1) + 3pq (–3p^2 + 4pq + 1)$

$= – (–12qp^2 + 16pq^{1 + 1} + 4q) + (–9qp^{2+1} + 12p^{1 + 1}q^{1 + 1} + 3pq)$

$= +12qp^2 – 16pq^2 – 4q –9qp^3 + 12p^2q^2 + 3pq$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= +12qp^2 – 16pq^2 – 4q –9qp^3 + 12p^2q^2 + 3pq$

$= – 9p^3q + 12p^2q^2 + 12p^2q – 16pq^2 + 3pq – 4q$

l) $(x^2 – 4xy + 5), (–2x^2 – 3xy + 1)$

Solution:

Multiply the 1st term of the trinomial "x^2" by each term of the trinomial, multiple the 2nd term "4xy" of the trinomial by each term of the trinomial and multiple the 3rd term "5" of the trinomial by each term of the trinomial. Then simplify by adding or subtracting the like terms.

$= (x^2 – 4xy + 5) (–2x^2 – 3xy + 1)$

$= x^2 (–2x^2 – 3xy + 1) – 4xy (–2x^2 – 3xy + 1) + 5(–2x^2 – 3xy + 1)$

$= (–2x^{2+2} – 3x^{2+1}y + x^2) – (–8x^{1+2}y – 12x^{1+1} y^{1+1} + 4xy) + (–10x^2 –15xy + 5)$

$= (–2x^4 – 3x^3y + x^2) – (–8x^3y – 12x^2 y^2 + 4xy) + (–10x^2 –15xy + 5)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= –2x^4 – 3x^3y + x^2 + 8x^3y + 12x^2 y^2 – 4xy –10x^2 –15xy + 5$

$= –2x^4 – 3x^3y + 8x^3y + 12x^2 y^2–10x^2 + x^2 –15xy – 4xy + 5$

$= -2x^4 + 5x^3y + 12x^2y^2 - 9x^2 - 19xy + 5$

m) $(y^2 - 2y + 1), (3y^2 + 2y - 6)$

Solution:

Multiply the 1st term of the trinomial by each term of the trinomial, multiple the 2nd term of the trinomial by each term of the trinomial and multiple the 3rd term of the trinomial by each term of the trinomial. Then simplify by adding or subtracting the like terms.

$= (y^2 - 2y + 1)(3y^2 + 2y - 6)$

$= y^2(3y^2 + 2y - 6) - 2y(3y^2 + 2y - 6) + 1(3y^2 + 2y - 6)$

$= (3y^{2+2} + 2y^{2+1} - 6y^2) - (6y^{1+2} + 4y^{1+1} - 12y) + (3y^2 + 2y - 6)$

$= (3y^4 + 2y^3 - 6y^2) - (6y^3 + 4y^2 - 12y) + (3y^2 + 2y - 6)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 3y^4 + 2y^3 - 6y^2 - 6y^3 - 4y^2 + 12y + 3y^2 + 2y - 6$

$= 3y^4 + 2y^3 - 6y^3 - 6y^2 - 4y^2 + 3y^2 + 12y + 2y - 6$

$= 3y^4 - 4y^3 - 7y^2 + 14y - 6$

n) $6x^2y^2, (x^2 - 7xy + y^2)$

Solution:

The term $6x^2y^2$ is multiplied by each term of the trinomial.

$= 6x^2y^2 \times (x^2 - 7xy + y^2)$

Multiply the coefficients of the variables and subtract the exponents of the same base.

$= (6x^2y^2 \times x^2) - (6x^2y^2 \times 7xy) + (6x^2y^2 \times y^2)$

$= 6x^{2+2}y^2 - (6 \times 7)\, x^{2+1} \times y^{2+1} + 6x^2y^{2+2}$

$= 6x^4y^2 - 42x^3y^3 + 6x^2y^4$

o) $(2r^2 - 3r + 1), (2r - 6)$

Solution:

Multiply the 1st term of the binomial by each term of the trinomial and multiple the 2nd term of the binomial by each term of the trinomial. Then simplify by adding or subtracting the like terms.

$= (2r – 6) \times (2r^2 – 3r + 1)$

$= 2r \times (2r^2 – 3r + 1) – 6(2r^2 – 3r + 1)$

$= (4r^{1+2} – 6r^{1+1} + 2r) – (12r^2 –18r + 6)$

$= (4r^3 – 6r^2 + 2r) – (12r^2 –18r + 6)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 4r^3 – 6r^2 + 2r – 12r^2 +18r – 6$

$= 4r^3 – 6r^2 – 12r^2 + 2r +18r – 6$

$= 4r^3 – 18r^2 + 20r – 6$

Learning Check 6.6

1. **Simplify the following algebraic expressions.**

a) (3x – 2y) – (x – y)

Solution:

$(3x – 2y) – (x – y) = 3x – 2y – x + y$

$= 3x – x – 2y + y$

$= 2x – y$

b) (4a – 3b) + (6a + 1)

Solution:

$(4a – 3b) – (6a + 1) = 4a – 3b – 6a – 1$

$= 4a – 6a – 3b – 1$

$= – 2a – 3b – 1$

2t – 5 (2t – 5)

Solution:

$2t – 5 (2t – 5) = 2t – 10t + 25$

$= – 8t +25$

c) $-3(2x - 5y) + 2y(x - 4)$

Solution:

$$\begin{aligned} -3(2x - 5y) + 2y(x - 4) &= -3(2x - 5y) + 2y(x - 4) \\ &= -6x + 15y + 2xy - 8y \\ &= -6x + 15y - 8y + 2xy \\ &= -6x + 7y + 2xy \end{aligned}$$

d) $6p\{(-2p + 3q) - (-3p + q)\}$

Solution:

$6p\{(-2p + 3q) - (-3p + q)\} = 6p\{-2p + 3q + 3p - q\}$ (first solve the operation within the parentheses)

$= 6p\{-2p + 3p + 3q - q\}$ (next solve the curly brackets)

$= 6p\{p + 2q\}$

$= 6p^2 + 12pq\}$

e) $10p - 2(11 + 6p - 3)$

Solution:

$$\begin{aligned} 10p - 2(11 + 6p - 3) &= 10p - 22 - 12p + 6 \\ &= 10p - 2p - 22 + 6 \\ &= 8p - 16 \end{aligned}$$

f) $5(3a - 7) + (9x + 12) + 4$

Solution:

$$\begin{aligned} 5(3a - 7) + (9x + 12) + 4 &= 15a - 35 + 9x + 12 + 4 \\ &= 15a + 9x + 12 + 4 - 35 \\ &= 15a + 9x + 16 - 35 \\ &= 15a + 9x - 19 \end{aligned}$$

g) $4pq[2p - \{3q - (p - 3q)\}]$

Solution:

$4pq\ [2p - \{3q - (p - 3q)\}] = 4pq\ [2p - \{3q - p + 3q\}]$ (first solve the operation within the parentheses)

$= 4pq\ [2p - \{2q + 3q\}]$ (next solve the curly brackets)

$= 4pq\ [2p - 2q - 3q]$ (next solve the square brackets)

$= 4pq\ [2p - 5q]$

$= 8p^2q - 20pq^2$

h) $6a\{(4a + b) - (a - 4b)\}$

Solution:

$6a\ \{(4a + b) - (a - 4b)\} = 6a\ \{(4a + b) - (a - 4b)\}$

$= 6a\ \{4a + b - a - 4b\}$

$= 6a\ \{4a - a + b - 4b\}$

$= 6a\ \{3a - 3b\}$

$= 18a^2 - 18ab\}$

i) $[8x\ \{4x - (3x - 2) - 8\}]$

Solution:

$[8x\ \{4x - (3x - 2) - 8\}] = [8x\ \{4x - 3x + 2 - 8\}]$

$= [8x\ \{x - 6\}]$

$= 8x^2 - 48x$

j) $12y\ (2y^2 - 9y) \div 6y^2$

Solution:

$12y\ (2y^2 - 9y) \div 6y^2 = 12y\ (2y^2 - 9y) \div 6y^2$

$= (24y^3 - 108y^2) \div 6y^2$

$= 4y - 18$

k) $-8ab\ (2ab + 3b + 2a)$

Solution:

$-8ab\ (2ab + 3b + 2a) = (-8ab \times 2ab) + (-8ab \times 3b) + (-8ab \times 2a)$

$= -16a^2b^2 - 24ab^2 - 16a^2b$

l) $2x(x - 3) - 4a\ (a + 3)$

Solution:

$2x(x - 3) - 4a\ (a + 3) = 2x^2 - 6x - 4a^2 + 12a$

$= -4a^2 + 2x^2 + 12a - 6x$

m) $3t\ [t + \{12t^2 - (4t^2 - 3t + 1)\}$

Solution:

$3t\ [t + \{12t^2 - (4t^2 - 3t + 1)\} = 3t\ [t + \{12t^2 - 4t^2 + 3t - 1\}$

$= 3t\ [t + \{8t^2 + 3t - 1\}]$

$= 3t\ [t + 8t^2 + 3t - 1]$

$= 3t\ [t + 3t + 8t^2 - 1]$

$= 3t\ [4t + 8t^2 - 1]$

$= 12t^2 + 24t^3 - 3t$

Learning Check 6.7

1. Find the square of the following binomials by using identity.

a) $(x - 2y)$

Solution:

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$(x - 2y)^2 = x^2 - 2(x)\ (2y) + (2y)^2$

$= x^2 + 4xy + 25y^2$

b) $(2m + 3)$

Solution:

By using the identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$(2m + 3)^2 = (2m)^2 - 2(2m)\ (3) + (3)^2$

$= 4m^2 + 12m + 9$

c) $(p - 4q)$

Solution:

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$(p - 4q)^2 = (p)^2 - 2(p)(4q) + (4q)^2$

$= p^2 - 8pq + 16q^2$

d) (a + b)

Solution:

$(a + b)^2 = (a + b)(a + b)$

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$= a^2 + 2ab + b^2$

e) (5u – 2v)

Solution:

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$(5u - 2v)^2 = (5u)^2 - 2(5u)(2v) + (2v)^2$

$= 25u^2 - 20uv + 4v^2$

f) (3x + 5y)

Solution:

By using the identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$(3x + 5y)^2 = (3x)^2 + 2(3x)(5y) + (5y)^2$

$= 9x^2 + 30xy + 25y^2$

g) (a – 5)

Solution:

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$(a - 5)^2 = (a)^2 - 2(a)(5) + (5)^2$

$= a^2 - 10a + 25$

h) (2r + s)

Solution:

By using the identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$(2r + s)^2 = (2r)^2 + 2(2r)(s) + (s)^2$

$= 4r^2 + 4rs + s^2$

i) (5y – 3z)

Solution:

By using the identity,

$(a – b)^2 = a^2 – 2ab + b^2$

$(5y – 3z)^2 = (5y)^2 – 2(5y)(3z) + (3z)^2$

$= 25y^2 – 30xy + 9z^2$

j) (8f + g)

Solution:

By using the identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$(8f + g)^2 = (8f)^2 + 2(8f)(g) + (g)^2$

$= 64f^2 + 16fg + g^2$

k) (4a – b)

Solution:

By using the identity,

$(a – b)^2 = a^2 – 2ab + b^2$

$(4a – b)^2 = (4a)^2 – 2(4a)(b) + (b)^2$

$= 16a^2 – 8ab + b^2$

l) (9x – 7y)

Solution:

By using the identity,

$(a – b)^2 = a^2 – 2ab + b^2$

$(9x – 7y)^2 = (9x)^2 – 2(9x)(7y) + (7y)^2$

$= 81x^2 – 126xy + 49y^2$

2. Using identity, find the product of the following binomials.

a) (u + v) (u – v)

Solution:

(u + v) (u – v)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(u + v)(u - v) = (u)^2 - (v)^2$

$= u^2 - v^2$

b) (2s – t) (2s + t)

Solution:

(2s – t) (2s + t)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(2s - t)(2s + t) = (2s)^2 - (t)^2$

$= 4s^2 - t^2$

c) (4x + y) (4x – y)

Solution:

(4x + y) (4x – y)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(4x + y)(4x - y) = (4x)^2 - (y)^2$

$= 16x^2 - y^2$

d) $\left(x + \frac{2}{x}\right)\left(x - \frac{2}{x}\right)$

Solution:

$\left(x + \frac{2}{x}\right)\left(x - \frac{2}{x}\right)$

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$\left(x + \frac{2}{x}\right)\left(x - \frac{2}{x}\right) = (x)^2 - \left(\frac{2}{x}\right)^2$

$= x^2 - \frac{4}{x^2}$

e) (a + 4b) (a – 4b)

Solution:

(a + 4b) (a – 4b)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(a + 4b)(a - 4b) = (a)^2 - (4b)^2$

$= a^2 - 16b^2$

f) (2p + 3q) (2p – 3q)

Solution:

(2p + 3q) (2p – 3q)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(2p + 3q)(2p - 3q) = (2p)^2 - (3q)^2$

$= 4p^2 - 9q^2$

g) (2a + 7) (2a – 7)

Solution:

(2a + 7) (2a – 7)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(2a + 7)(2a - 7) = (2a)^2 - (7)^2$

$= 4a^2 - 49$

h) (2x + 5y) (2x – 5y)

Solution:

(2x + 5y) (2x – 5y)

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(2x + 5y)(2x - 5y) = (2x)^2 - (5y)^2$

$= 4x^2 - 25y^2$

i) $(2a + \frac{1}{b})(2a - \frac{1}{b})$

Solution:

$(2a + \frac{1}{b})(2a - \frac{1}{b})$

By using the identity,

$(a + b)(a - b) = a^2 - b^2$

$(2a + \frac{1}{b})(2a - \frac{1}{b}) = (2a)^2 - (\frac{1}{b})^2$

$= 4a^2 - \frac{1}{b^2}$

3. Find the value of $(a + b)^2$ if $a^2 + b^2 = 7$ and $ab = 4$.

Solution:

As given, $a^2 + b^2 = 7$, $ab = 4$

By using the identity,

$(a + b)^2 = a^2 + 2ab + b^2$

$= a^2 + b^2 + 2ab$

$(a + b)^2 = 7 + 2(4) = 7 + 8 = 15$

4. Find the value of $(a - b)^2$ if $a^2 + b^2 = 12$ and $ab = 6$.

Solution:

As given, $a^2 + b^2 = 12$, $ab = 6$

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 - 2ab$

$(a + b)^2 = 12 + 2(6) = 12 + 12 = 24$

5. Find the value of ab if $(a - b)^2 = 5$ and $a^2 + b^2 = 8$.

Solution:

As given, $a^2 + b^2 = 8$, $(a - b)^2 = 5$

By using the identity,

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 + b^2 - 2ab$

$5 = 8 - 2ab$

$2ab = 8 - 5$

$2ab = 3$

$ab = \frac{3}{2}$

6. **Find the value of $a^2 + b^2$ if $(a - b) = 9$ and $ab = 12$.**

   Solution:

   As given, $ab = 8$, $(a - b) = 9$

   By using the identity,

   $(a - b)^2 = a^2 - 2ab + b^2$

   $= a^2 + b^2 - 2ab$

   $(9)^2 = a^2 + b^2 - 2(12)$

   $81 = a^2 + b^2 - 24$

   $a^2 + b^2 = 81 + 24$

   $a^2 + b^2 = 105$

Learning Check 6.8

1. **Factorize the following by common factor method.**

   a) **$42x^3 - 14x^2$**

   Solution:

   $= 42x^3 - 14x^2$

   $= 14x^2(3x - 1)$ by taking $14x^2$ as common factor.

   So, $14x^2$, $(3x - 1)$ are the factors of the expression $42x^3 - 14x^2$.

   b) **$5uv - 15u^2v^2$**

   Solution:

   $= 5uv - 15u^2v^2$

   $= 5uv(1 - 3uv)$ (by taking $5uv$ as common factor.)

   So, $5uv$, $(1 - 3uv)$ are the factors of the expression $5uv - 15u^2v^2$.

c) $51 + 17a^2$

Solution:

$= 51 – 17a^2$

$= 17(3 – a^2)$ by taking 17 as common factor.

So, 17, $(3 – a^2)$ are the factors of the expression $51 – 17a^2$.

d) $10y^3 – 9y^2 + y$

Solution:

$= 10y^3 – 9y^2 + y$

$= y\ (10y^2 – 9y^1 + 1)$ by taking y as common factor.

So, y, $(10y^2 – 9y + 1)$ are the factors of the expression $10y^3 – 9y^2 + y$.

e) $21u^3 + 28u^2 – 14u$

Solution:

$= 21u^3 + 28u^2 – 14u$

$= 7u\ (3u^2 + 4u – 2)$ by taking 7u as common factor.

So, 7u, $(3u^2 + 4u – 2)$ are the factors of the expression $21u^3 + 28u^2 – 14u$.

f) $–90x^4 + 60$

Solution:

$= –90x^4 + 60$

$= 30(3x^4 + 2)$ by taking 30 as common factor.

So, 30, $(3x^4 + 2)$ are the factors of the expression $–90x^4 + 60$.

g) $15a^5 – 3a^2$

Solution:

$= 15a^5 – 3a^2$

$= 3a^2(5a^3 – 1)$ by taking $3a^2$ as common factor.

So, $3a^2$, $(5a^3 – 1)$ are the factors of the expression $15a^5 – 3a^2$.

h) $\mathbf{6p^5q + 2pq^3 - 6q}$

Solution:

$= 6p^5q + 2pq^3 - 6q$

$= 2q(3p^5 + pq^2 - 3)$ by taking 2q as common factor.

So, 2q, $(3p^5 + pq^2 - 3)$ are the factors of the expression $6p^5q + 2pq^3 - 6q$.

i) $\mathbf{32yz^2 - 48yz + 24}$

Solution:

$= 32yz^2 - 48yz + 24$

$= 8(4yz^2 - 6yz + 3)$ by taking 8 as common factor.

So, 8, $(4yz^2 - 6yz + 3)$ are the factors of the expression $32yz^2 - 48yz + 24$.

2. Factorize the following by regrouping method.

j) **ax – by – bx + ay**

Solution:

= ax – by – bx + ay

Rearrange the terms and make groups for taking the common factor.

= ax – bx + ay – by

= x(a – b) + y(a – b)

Here, (a – b) is the common factor in both groups

= (a – b) (x + y)

k) **2x + 3y + 3x + 2y**

Solution:

= 2x + 3y + 3x + 2y

Rearrange the terms and make groups for taking the common factor.

= 2x + 2y + 3y + 3x

= 2(x + y) + 3(x + y)

Here, (x + y) is the common factor in both groups.

= (x + y)(2 + 3)

= 5 (x + y)

l) $ax - 4y + 4x - ay$

Solution:

$= ax - 4y + 4x - ay$

Rearrange the terms and make groups for taking the common factor.

$= ax + 4x - 4y - ay$

$= x(a + 4) + y(a + 4)$

Here, $(a + 4)$ is the common factor in both groups.

$= (a + 4)(x + y)$

m) $mx + nx - my - ny$

Solution:

$= mx + nx - my - ny$

Rearrange the terms and make groups for taking the common factor.

$= mx + nx - my - ny$

$= x(m + n) - y(m + n)$

Here, $(m + n)$ is the common factor in both groups.

$= (m + n)(x - y)$

n) $5pq - 2p - 5q^2 + 2q$

Solution:

$= 5pq - 2p - 5q^2 + 2q$

Rearrange the terms and make groups for taking the common factor.

$= 5pq - 5q^2 - 2p + 2q$

$= 5q(p - q) - 2(p - q)$

Here, $(p - q)$ is the common factor in both groups.

$= (p - q)(5q - 2)$

o) $4mn - 7n + 12m - 21$

Solution:

$= 4mn - 7n + 12m - 21$

Rearrange the terms and make groups for taking the common factor.

$= 4mn + 12m - 7n - 21$

$= 4m(n + 3) – 7(n + 3)$

Here, $(n + 3)$ is the common factor in both groups.

$= (n + 3) (4m – 7)$

p) $\mathbf{y^3 + y^2 + y + 1}$

Solution:

$= y^3 + y^2 + y + 1$

Rearrange the terms and make groups for taking the common factor.

$= y^3 + y^2 + y + 1$

$= y^2 (y +1) + 1(y + 1)$

Here, $(y + 1)$ is the common factor in both groups.

$= (y + 1) (y^2 + 1)$

q) $\mathbf{y^2 – 13y – y + 13}$

Solution:

$= y^2 – 13y – y + 13$

Rearrange the terms and make groups for taking the common factor.

$= y^2 – y – 13y + 13$

$= y(y – 1) – 13(y – 1)$

Here, $(y – 1)$ is the common factor in both groups.

$= (y – 1) (y – 13)$

r) $\mathbf{t^2 + 3t + 4t + 12}$

Solution:

$= t^2 + 3t + 4t + 12$

Rearrange the terms and make groups for taking the common factor.

$= t^2 + 3t + 4t + 12$

$= t(t + 3) + 4(t + 3)$

Here, $(t + 3)$ is the common factor in both groups.

$= (t + 3) (t + 4)$

Learning Check 6.9

1. Factorize the following by breaking the middle term.

a) $\mathbf{t^2 – 2t + 1}$

Solution:

In order to factorize $t^2 - 2t + 1$, we have to find two numbers such that their sum is -2 and their product is 1, as $(+1) \times (+1) = +1$

Clearly, $(-1) + (-1) = -2$ and $(-1) \times (-1) = +1$

We now split the middle term 2t of $t^2 - 2t + 1$ as $-t -t$.

$$t^2 - 2t + 1 = t^2 - t - t + 1$$
$$= t(t - 1) - 1(t - 1)$$
$$= (t - 1)(t - 1)$$

So, the factors of $t^2 - 2t + 1$ are $(t - 1)$ and $(t - 1)$.

b) $m^2 + 7m + 12$

Solution:

In order to factorize $m^2 + 7m + 12$, we have to find two numbers such that their sum is +7 and their product is 12, as $(+1) \times (+12) = +12$

Clearly, $(+4) + (+3) = +12$ and $(+4) \times (+3) = +12$

We now split the middle term 7m of $m^2 + 7m + 12$ as $4m + 3m$.

$$m^2 + 7m + 12 = m^2 + 4m + 3m + 12$$
$$= m(m + 4) + 3(m + 4)$$
$$= (m + 4)(m + 3)$$

So, the factors of $m^2 + 7m + 12$ are $(m + 4)$ and $(m + 3)$.

c) $x^2 + 2x - 35$

Solution:

In order to factorize $x^2 + 2x - 35$, we have to find two numbers such that their sum is 2 and their product is 1, as $(+1) \times (-35) = -35$

Clearly, $(7) + (-5) = -35$ and $(-7) \times (5) = -35$

We now split the middle term "2" of $x^2 + 2x - 35$ as $+7x -5x$.

$$x^2 + 2x - 35 = x^2 + 7x - 5x - 35$$
$$= x(x + 7) - 5(x + 7)$$
$$= (x + 7)(x - 5)$$

So, the factors of $x^2 + 2x - 35$ are $(x + 7)$ and $(x - 5)$.

d) $5u^2 - 13u + 8$

Solution:

In order to factorize $5u^2 - 13u + 8$, we have to find two numbers such that their sum is -13 and their product is 40, as $(+5) \times (+8) = +40$

Clearly, $(-8) + (-5) = -13$ and $(-8) \times (-5) = +40$

We now split the middle term $-13u$ of $5u^2 - 13u + 8$ as $-8u - 5u$.

$$5u^2 - 13u + 8 = 5u^2 - 13u + 8$$
$$= 5u^2 - 8u - 5u + 8$$
$$= 5u^2 - 8u - 5u + 8$$
$$= u(5u - 8) - 1(5u - 8)$$
$$= (5u - 8)(u - 1)$$

So, the factors of $5u^2 - 13u + 8$ are $(5u - 8)$ and $(u - 1)$.

e) $3y^2 + 11yz + 8z^2$

Solution:

In order to factorize $3y^2 + 11yz + 8z^2$, we have to find two numbers such that their sum is $+11$ and their product is 24, as $(+3) \times (+8) = +24$

Clearly, $(+8) + (+3) = +11$ and $(+8) \times (+3) = +24$

We now split the middle term $11yz$ of $3y^2 + 11yz + 8z^2$ as $+8yz + 3yz$.

$$3y^2 + 11yz + 8z^2 = 3y^2 + 3yz + 8yz + 8z^2$$
$$= 3y(y + z) + 8z(y + z)$$
$$= (y + z)(3y + 8z)$$

So, the factors of $3y^2 + 11yz + 8z^2$ are $(y + z)$ and $(3y + 8z)$).

f) $y^2 + 14y - 51$

Solution:

In order to factorize $y^2 + 14y - 51$, we have to find two numbers such that their sum is$+14$ and their product is 51, as $(+1) \times (-51) = -51$

Clearly, $(+17) + (-3) = +14$ and $(+17) \times (-3) = -51$

We now split the middle term $14y$ of $y^2 + 14y - 51$as $+17y - 3y$.

$y^2 + 14y - 51 = y^2 + 17y - 3y - 51$

$= y(y + 17) - 3(y + 17)$

$= (y + 17)(y - 3)$

So, the factors of $y^2 + 14y - 51$ are $(y + 17)$ and $(y - 3)$.

g) $11a^2 + ab - 12b^2$

Solution:

In order to factorize $11a^2 + ab - 12b^2$, we have to find two numbers such that their sum is 1 and their product is -132, as $(+11) \times (-12) = -132$

Clearly, $(+12) + (-11) = +1$ and $(11) \times (-12) = -132$

We now split the middle term ab of $11a^2 + ab - 12b^2$ as $+12ab - 11ab$.

$11a^2 + ab - 12b^2 = 11a^2 + 12ab - 11ab - 12b^2$

$= a(11a + 12b) - b(11a + 12b)$

$= (11a + 12b)(a - b)$

So, the factors of $11a^2 + ab - 12b^2$ are $(11a + 12b)$ and $(a - b)$

h) $6x^2 + 29xy + 23y^2$

Solution:

In order to factorize $6x^2 - 29xy + 23y^2$, we have to find two numbers such that their sum is $+29$ and their product is 138, as $(+6) \times (+23) = +138$

Clearly, $(23) + (6) = +29$ and $(+23) \times (+6) = +138$

We now split the middle term 29xy of $6x^2 - 29xy + 23y^2$ as $+23xy + 6xy$.

$6x^2 - 29xy + 23y^2 = 6x^2 + 6xy + 23xy + 23y^2$

$= 6x(x + y) + 23y(x + y)$

$= (x + y)(6x + 23y)$

So, the factors of $6x^2 - 29xy + 23y^2$ are $(x + y)$ and $(6x + 23y)$

i) $6a^2 + 11ab + 3b^2$

Solution:

In order to factorize $6a^2 - 11ab + 3b^2$, we have to find two numbers such that their sum is 11 and their product is 18, as $(+6) \times (+3) = +18$

Clearly, $(-9) + (-2) = +18$ and $(-9) \times (-2) = +18$

We now split the middle term 11ab of $6a^2 - 11ab + 3b^2$ as $-9ab - 2ab$.

$$6a^2 - 11ab + 3b^2 = 6a^2 - 9ab - 2ab + 3b^2$$
$$= 3a(2a - 3b) - b(2a - 3b)$$
$$= (2a - 3b)\,(3a - b)$$

So, the factors of $6a^2 - 11ab + 3b^2$ are $(2a - 3b)$ and $(3a - b)$.

j) $6x^2 + x - 1$

Solution:

In order to factorize $6x^2 + x - 1$, we have to find two numbers such that their sum is $+1$ and their product is -6, as $(+6) \times (-1) = -6$

Clearly, $(+3) + (-2) = +1$ and $(-3) \times (+2) = -6$

We now split the middle term x of $6x^2 + x - 1$ as $+3x - 2x$.

$$6x^2 + x - 1 = 6x^2 + 3x - 2x - 1$$
$$= 3x\,(2x + 1) - 1(2x + 1)$$
$$= (2x + 1)\,(3x - 1)$$

So, the factors of $6x^2 + x - 1$are $(2x + 1)$ and $(3x - 1)$.

k) $3x^2 - 11x - 4$

Solution:

In order to factorize $3x^2 - 11x - 4$, we have to find two numbers such that their sum is -11 and their product is -12, as $(+3) \times (-4) = -12$

Clearly, $(-12) + (+1) = -11$ and $(-12) \times (+1) = -12$

We now split the middle term 11x of $3x^2 - 11x - 4$ as $-12x + x$.

$$3x^2 - 11x - 4 = 3x^2 - 12x + x - 4$$
$$= 3x(x - 4) + 1(x - 4)$$
$$= (x - 4)\,(3x + 1)$$

So, the factors of $3x^2 - 11x - 4$ are $(x - 4)$ and $(3x + 1)$.

l) $u^2 - u - 30$

Solution:

In order to factorize $u^2 - u - 30$, we have to find two numbers such that their sum is -1 and their product is -30, as $(+1) \times (-30) = -30$

Clearly, $(-6) + (+5) = -1$ and $(-6) \times (+5) = -30$

We now split the middle term -1 of $u^2 - u - 30$ as $-6u + 5u$.

$$u^2 - u - 30 = u^2 - 6u + 5u - 30$$
$$= u(u - 6) + 5(u - 6)$$
$$= (u - 6)(u + 5)$$

So, the factors of $u^2 - u - 30$ are $(u - 6)$ and $(u + 5)$.

Unit Check 6

1. Encircle the correct option.

a) The 7th term of the sequence 13, 35, 57, 79, 101, … is:

i) 112 ii) 123 iii) 145 iv) 167

b) If the nth term of a sequence is $2(n^2 - 4)$, then the 4th term of the sequence is:

i) 0 ii) 10 iii) 24 iv) 42

c) The degree of the polynomial $x^2y^3 + y^4 + x^2 + 1$ is:

i) 2 ii) 3 iii) 4 iv) 5

d) Factors of the expression $x^2 - 8x + 15$ are:

i) $(x - 8)$ and $(x + 1)$ ii) $(x - 3)$ and $(x + 5)$

iii) $(x - 3)$ and $(x - 5)$ iv) $(x + 8)$ and $(x - 1)$

e) Polynomials which contain only two terms are known as:

i) monomials ii) binomials iii) trinomials iv) quadrinomials

2. Fill in the blanks.

a) The 20th term of the sequence 3, 7, 11, 15, 19, … is _79__.

b) $(a + b)(a - b)$ = ___$a^2 - b^2$_____.

c) The expression $2a^2 + 3bc \times d$ has ____two_______ number of terms.

d) Factors of $3x^2 + 4x$ are x and ___3x + 4__________.

e) In the expression $2x^2 + 3x + 1$, ___1_____ is the constant.

3. Ibrahim learnt three vocabulary words in the first week, five in the second week, seven in the third week, nine words in the fourth week and eleven words in the fifth week. Find the number of words he would have learned by the sixth week if he continued to learn new words in the same pattern.

Solution:

Let's arrange the number of vocabulary words read by Ibrahim in sequence.

3, 5, 7, 9, 11 __

Firstly, work out the difference in the terms. This pattern is going up by 2 each time, so

we will add 2 to the last term to find the next term in the pattern.

3, 5, 7, 9, 11 13.

So, he will learn 13 vocabulary words in sixth week.

4. **If the nth term of a sequence is $8n^2 – 5$, find the 4th, 11th and 18th terms of this sequence.**

   Solution:

   The nth term of a sequence = $8n^2 – 5$

   To find the 4th term of the sequence Put n = 4 in $8n^2 – 5$.

   $8n^2 – 5 = 8(4)^2 – 5 = 128 – 5 = 123$

   So, the 4th term of this sequence is 123.

   To find the 11th term of the sequence Put n = 11 in $8n^2 – 5$.

   $8n^2 – 5 = 8(11)^2 – 5 = 968 – 5 = 963$

   So, the 11th term of this sequence is 963.

   To find the 18th term of the sequence Put n = 18 in $8n^2 – 5$.

   $8n^2 – 5 = 8(18)^2 – 5 = 2592 – 5 = 2587$

   So, the 18th term of this sequence is 2587.

5. **Solve the following.**

   a) **$(2 – 3x^2) + (6x^2 + 7)$**

      Solution:

      Let us add first horizontally and then vertically.

      Horizontal method:

      $= (2 – 3x^2) + (6x^2 + 7)$

      Remove the bracket and group like terms, then add and subtract them.

      $= 2 – 3x^2 + 6x^2 + 7$

      $= – 3x^2 + 6x^2 + 7 + 2$

$= 3x^2 + 9$

Vertical method:

$-3x^2 + 2$
$+6x^2 + 7$
$3x^2 + 9$

b) $(2x^2 + 5x - 1) + (-x^2 - 6x + 40)$

Solution:

Let us add first horizontally and then vertically.

Horizontal method:

$= (2x^2 + 5x - 1) + (-x^2 - 6x + 40)$

Remove the bracket and group like terms, then add and subtract them.

$= 2x^2 + 5x - 1 - x^2 - 6x + 40$

$= 2x^2 - x^2 + 5x - 6x - 1 + 40$

$= x^2 - x + 39$

Vertical method:

$2x^2 + 5x - 1$
$-x^2 - 6x + 40$
$x^2 - x + 39$

c) $(3x^2 - 4xy - x) + (5x^2 - 8xy + y)$

Solution:

Let us add first horizontally and then vertically.

Horizontal method:

$= (3x^2 - 4xy - x) + (5x^2 - 8xy + y)$

Remove the bracket and group like terms, then add and subtract them.

$= 3x^2 - 4xy - x + 5x^2 - 8xy + y$

$= 3x^2 + 5x^2 - 4xy - 8xy - x + y$

$= 8x^2 - 12xy - x + y$

Vertical method:

$3x^2 - 4xy - x$
$5x^2 - 8xy \quad + y$
$8x^2 - 12xy - x + y$

d) $(t^3 - 2t^2 + t - 7) + (t^2 - 2t + 7)$

Solution:

Let us add first horizontally and then vertically.

Horizontal method:

$= (t^3 - 2t^2 + t - 7) + (t^2 - 2t + 7)$

Remove the bracket and group like terms, then add and subtract them.

$= t^3 - 2t^2 + t - 7 + t^2 - 2t + 7$

$= t^3 - 2t^2 + t^2 + t - 2t - 7 + 7$

$= t^3 - t^2 - t$

Vertical method:

$t^3 - 2t^2 + t - 7$
$t^2 - 2t + 7$
$t^3 - t^2 \quad - t + 0$

6. Solve the following.

a) $(x^2 - 4) - (-2x^2 + 3x - 1)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(x^4 - 4) - (-2x^2 + 3x - 1)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= x^4 - 4 + 2x^2 - 3x + 1$

Now group like terms, then add and subtract them.

$= x^4 + 2x^2 - 3x - 4 + 1$

$= x^4 + 2x^2 - 3x - 3$

Vertical method:

$$\begin{array}{r} x^4 \qquad\quad -4 \\ \mp 2x^2 \pm 3x \mp 1 \\ \hline x^4 + 2x^2 - 3x - 3 \end{array}$$

b) $(7 - 4x - 3x^2) - (3x - 3 - 5x^2)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(7 - 4x - 3x^2) - (3x - 3 - 5x^2)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 7 - 4x - 3x^2 - 3x + 3 + 5x^2$

Now group like terms, then add and subtract them.

$= 5x^2 - 3x^2 - 4x - 3x + 3 + 7$

$= 2x^2 - 7x + 10$

Vertical method:

$$\begin{array}{r} -3x^2 - 4x + 7 \\ \mp 5x^2 \pm 3x \mp 3 \\ \hline 2x^2 - 7x + 10 \end{array}$$

c) $(4x^3 - 3x^2 + 1) - (3x^3 - 3x^2 + 2x - 10)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(4x^3 - 3x^2 + 1) - (3x^3 - 3x^2 + 2x - 10)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 4x^3 - 3x^2 + 1 - 3x^3 + 3x^2 - 2x + 10$

Now group like terms, then add and subtract them.

$= 4x^3 - 3x^3 - 3x^2 + 3x^2 - 2x + 1 + 10$

$= x^3 - 2x + 11$

Vertical method:

$$\begin{array}{rrrr} 4x^3 & -\ 3x^2 & & +\ 1 \\ \pm\ 3x^3 & \mp\ 3x^2 & \pm\ 2x & \mp\ 10 \\ \hline x^3 & & -\ 2x & +\ 11 \end{array}$$

d) $(x^4 - 2x + 1) - (-2x^4 - 6x^3 + 4x - 1)$

Solution:

Let us subtract first horizontally and then vertically.

Horizontal method:

$(x^4 - 2x + 1) - (-2x^4 - 6x^3 + 4x - 1)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= x^4 - 2x + 1 + 2x^4 + 6x^3 - 4x + 1$

Now group like terms, then add and subtract them.

$= x^4 + 2x^4 + 6x^3 - 2x - 4x + 1 + 1$

$= 3x^4 + 6x^3 - 6x + 2$

Vertical method:

$$\begin{array}{rrrr} x^4 & & -2x & +\ 1 \\ \mp 2x^4 & \mp\ 6x^3 & \pm\ 4x & \mp\ 1 \\ \hline 3x^4 & +\ 6x^3 & -6x & +\ 2 \end{array}$$

7. The sum of two expressions is $4x^3 + 6x^2 + x - 7$. If one of the expressions is $-2x^3 - 3x + 5$, find the other.

Solution:

To find the other expression subtract $(-2x^3 - 3x + 5)$ from $(4x^3 + 6x^2 + x - 7)$.

$= (4x^3 + 6x^2 + x - 7) - (-2x^3 - 3x + 5)$

Remove the brackets and change the sign of the terms which are to be subtracted.

$= 4x^3 + 6x^2 + x - 7 + 2x^3 + 3x - 5$

Now group like terms, then add and subtract them.

$= 4x^3 + 2x^3 + 6x^2 + x + 3x - 7 - 5$

$= 6x^3 + 6x^2 + 4x - 12$

8. Multiply the following polynomials.

a) $2x^2, (x^2 - 9)$

The term $2x^2$ is multiplied by each term of the binomial.

$2x^2 \times (x^2 - 9) = (2x^2 \times x^2) - (2x^2 \times 9)$

Multiply the coefficients of the variables and add the exponents of the same base.

$= (2 \times 1)x^2 - (2 \times 9)x^2$

$= 2x^2 - 18x^2$

b) $4xy, 3x^2y^4$

Solution:

$(4xy)(3x^2y^4) = (4 \times 3)\ x^{2+1}y^{1+4}$

$= 12x^3y^5$

c) $(3x^2 - y), (x^2 + 3y)$

Solution:

Multiply the 1st term of the binomial by each term of the binomial and multiple the 2nd term of the binomial by each term of the binomial. Then simplify by adding or subtracting the like terms.

$= (3x^2 - y)\ (x^2 + 3y)$

$= (3x^2\ (x^2 + 3y) - y(x^2 + 3y)$

$= (3x^{2+2} - 9x^2y) - (yx^2 + 3y^2)$

$= (3x^4 - 9x^2y) - (yx^2 + 3y^2)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 3x^4 - 9x^2y - yx^2 - 3y^2$

d) $(9xy - 2y^2), (2x^2y + 1)$

Solution:

Multiply the 1st term of the binomial by each term of the binomial and multiple the 2nd term of the binomial by each term of the binomial. Then simplify by adding or subtracting the like terms.

$= (9xy - 2y^2)\ (2x^2y + 1)$

$= 9xy\ (2x^2y + 1) - 2y^2\ (2x^2y + 1)$

$= (18x^{1+2}\ y^{1+1} + 9xy) - (4x^2y^{2+1} + 2y^2)$

$= (18x^3\ y^2 + 9xy) - (4x^2y^3 + 2y^2)$

Change the sign of each term in the second polynomial because – sign exists before the parenthesis.

$= 18x^3y^2 + 9xy - 4x^2y^3 - 2y^2$

$= 18x^3y^2 - 4x^2y^3 + 9xy - 2y^2$

9. Simplify the following.

a) $2x\ (7xy - 8)$

Solution:

$2x\ (7xy - 8) = 14xy - 16x$

b) $7t\ \{3t + (4t - 1)\}$

Solution:

$7t\{3t + (4t - 1)\} = 7t\{3t + 4t - 1\}$

$= 7t\{7t - 1\}$

$= 49t^2 - 7t$

c) $3uv\ \{(2u - 3uv) + (6v + 2uv)\}$

Solution:

$3uv\ \{(2u - 3uv) + (6v + 2uv)\} = 3uv\ \{2u - 3uv + 6v + 2uv\}$

$= 3uv\ \{2u - uv + 6v\}$

$= 6u^2v - 3u^2v^2 + 18uv^2$

10. Factorize the following.

a) $a^2 - 8a + 16$

Solution:

In order to factorize $a^2 - 8a + 16$, we have to find two numbers such that their sum is -8 and their product is $+16$, as $(+1) \times (+16) = +16$

Clearly, $(-4) + (-4) = -8$ and $(-4) \times (-4) = +16$

We now split the middle term -8 of $a^2 - 8a + 16$ as $-4a - 4a$.

$$\begin{aligned} a^2 - 8a + 16 &= a^2 - 8a + 16 \\ &= a^2 - 4a - 4a + 16 \\ &= a(a - 4) - 4(a - 4) \\ &= (a - 4)(a - 4) \end{aligned}$$

So, the factors of $a^2 - 8a + 16$ are $(a - 4)$ and $(a - 4)$.

b) $4x^2 + 12x + 9$

Solution:

By using identity: $(a + b)^2 = a^2 + b^2 + 2ab$

$$\begin{aligned} 4x^2 + 12x + 9 &= (2x)^2 + 2(2x)(3) + 3^2 \\ &= (2x + 3)^2 \end{aligned}$$

c) $5x^2 - 125y^2$

By using identity: $(a + b)(a - b) = a^2 - b^2$

$$5x^2 - 125y^2 = 5(x^2 - 25y^2)$$

$$5((x)^2 - (5y)^2) = 5(x + 5)(x - 5)$$

d) d) $3x^2 + 2x - 1$

Solution:

In order to factorize $3x^2 + 2x - 1$, we have to find two numbers such that their sum is $+2$ and their product is -3, as $(+3) \times (-1) = -3$

Clearly, $(+3) + (-1) = +2$ and $(+3) \times (-1) = -3$

We now split the middle term $+2$ of $3x^2 + 2x - 1$ as $+3x - x$.

$$\begin{aligned} 3x^2 + 2x - 1 &= 3x^2 + 3x - x - 1 \\ &= 3x(x + 1) - 1(x + 1) \\ &= (x + 1)(3x - 1) \end{aligned}$$

So, the factors of $3x^2 + 2x - 1$ are $(x + 1)$ and $(3x - 1)$

Learning check 7.1

1. **Solve the following linear equations.**

   a) **x + 3.4 = 7.5**

   Solution:

   x + 3.4 = 7.5

   To isolate the variable x, subtract 3.4 from both sides.

   x + 3.4 – 3.4 = 7.5 – 3.4

   x = 4.1

   b) **2x – 12 = 2**

   Solution:

   2x – 12 = 2

   To isolate the variable x, add 12 on both sides.

   2x – 12 + 12 = 2 + 12

   x = 14

   c) **7x = 28**

   Solution:

   7x = 28

   To isolate the variable x, divide both side by 7 on both sides.

   $\frac{7x}{7} = \frac{28}{7}$

   x = 4

   d) **–4x + 3 = –7**

   Solution:

   –4x + 3 = –7

   To isolate the variable x, subtract 3 from both sides.

   –4x + 3 – 3 = –7 – 3

   –4x = –10

   To isolate the variable x, divide both side by –4 on both sides.

   $\frac{-4x}{-4} = \frac{-10}{-4}$

   x = $\frac{5}{2}$

e) $\frac{x}{2} - 1 = 3$

Solution:

$\frac{x}{2} - 1 = 3$

To isolate the variable x, add 1 on both sides.

$\frac{x}{2} - 1 + 1 = 3 + 1$

$\frac{x}{2} = 4$

To isolate the variable x, multiply both side by 2.

$\frac{x}{2} \times 2 = 4 \times 2$

x = 8

f) $\frac{1}{(2x-9)} + 5 = 3$

Solution:

$\frac{1}{(2x-9)} + 5 = 3$

To isolate the variable x, subtract 5 from both sides.

$\frac{1}{(2x-9)} + 5 - 5 = 3 - 5$

$\frac{1}{(2x-9)} = -2$

1 = –2(2x – 9)

1 = –4x + 9

To isolate the variable x, subtract 9 from both sides.

1 – 9 = –4x + 9 – 9

– 8 = –4x

To isolate the variable x, divide both side by –4.

$\frac{-4x}{-4} = \frac{-8}{-4}$

x = 2

g) $\frac{(2x+1)}{2} - \frac{1}{4} = \frac{3x}{8}$

Solution:

$\frac{(2x+1)}{2} - \frac{1}{4} = \frac{3x}{8}$

Take LCM of the given equation.

$\frac{2(2x+1)-1}{4} = \frac{3x}{8}$

$\frac{4x+2-1}{4} = \frac{3x}{8}$

$\frac{4x+1}{4} = \frac{3x}{8}$

By cross multiplying we have:

8(4x + 1) = 12x

32x + 8 = 12 x

By subtracting 12x from both side we have:

32x –12x + 8 = 12x –12x

20x + 8 = 0

To isolate the variable x, subtract 8 from both sides.

20x + 8 – 8 = 0 – 8

20x = –8

To isolate the variable x, divide both side by 20.

$\frac{20x}{20} = \frac{-8}{20}$

$x = \frac{-2}{5}$

h) **2x + 8 = 6x**

Solution:

2x + 8 = 6x

By subtracting 2x from both side we have:

2x + 8 –2x = 6x –2x

4x = 8

To isolate the variable x, divide both side by 4.

$\frac{4x}{4} = \frac{8}{4}$

x = 2

i) $\mathbf{12x - \frac{3}{2} = 6}$

Solution:

$12x - \frac{3}{2} = 6$

By adding $\frac{3}{2}$ from both side we have:

$12x - \frac{3}{2} + \frac{3}{2} = \frac{3}{2} + 6$

$12x = \frac{3+12}{2}$

$12x = \frac{15}{2}$

To isolate the variable x, divide both side by 12.

$\frac{12x}{12} = \frac{15}{24}$

$x = \frac{15}{24}$

2. The sum of two numbers is 102. One number is three times the other number. What are the numbers?

Solution:

Suppose the number is = x

Other number = 3x

According to the condition given:

x + 3x = 102

4x = 102

Divide both side by 4

$\frac{4x}{4} = \frac{102}{4}$

x = 25.5

3x = 3 (25.5) = 76.5

3. The price of 5 chairs and 3 tables is Rs 33000. If the price of a chair is half of the price of a table, find the price of a chair.

Solution:

Suppose the price of table (in Rs) = *x*

The cost of 3 chairs = 3*x*

The chair costs half of the price of table so:

The cost of a geometry box = (*x* - 180)

The cost of 5 chairs = 5x/2

Total cost of 5 chair and 3 tables = Rs 33000

According to the condition given:

$3x + \frac{5x}{2} = 33000$

$\frac{6x+5x}{2} = 33000$

$11x = 66000$

$x = 6000$

So the price of one table is Rs 6000

As the price of the chair is half of the price of the table so,

$\frac{x}{2} = \frac{6000}{2} =$ Rs 3000

4. The sum of two consecutive numbers is 32. Find the numbers.

Solution:

Suppose the two consecutive numbers are x and x +1

According to the condition:

x + x + 1 = 32

2x + 1 = 32

2x = 31

x = 15.5

x + 1 = 15.5 + 1

x + 1 = 16.5

so the two consecutive numbers are 15.5 and 16.5.

5. **The age of Rimsha is three times the age of Saadia. If the sum of their ages is 64, find their ages.**

   Solution:

   Suppose the age of Saadia = x

   Then Rimsha age = $3x$

   According to the condition:

   $x + 3x = 64$

   $4x = 64$

   $x = 16$

   the age of Saadia is 16 years.

   Rimsha' s age = 3(16) = 48 years

6. **If 8 is less than 5 times a number is 7, find the number.**

   Solution:

   Suppose the number = x

   According to the given condition:

   $5x - 8 = 7$

   $5x = 15$

   $x = 3$

7. **Alina and Sidra participated in a quiz. They obtained 30 marks in all. If Alina obtained 6 marks more than Sidra, how many marks did Sidra obtain?**

   Solution:

   Suppose Sidra's marks = x

   Alina Marks = $x + 6$

   According to the given condition:

   $x + x + 6 = 30$

   $2x + 6 = 30$

   $2x = 24$

   $x = 12$

   Sidra's marks = 12

   Alina's marks = 12 + 6 = 18

Learning check 7.2

1. **Identify the linear equations in two variables. Justify your answer.**
   a) **$2x + y = 22$**

   Solution:

   it is the linear equation in two variables as x and y are two variables.

   b) **$5x – 6 = 14$**

   Solution:

   It is not the linear equation in two variables as the only variable is x.

   c) **$x – 7y = 2$**

   Solution:

   it is the linear equation in two variables as x and y are two variables.

   d) **$3 – 8x = 2$**

   Solution:

   It is not the linear equation in two variables as the only variable is x.

   e) **$7x – 2 = 11y$**

   Solution:

   It is the linear equation in two variables as x and y are two variables.

   f) **$3x + 11 = 2y$**

   Solution:

   it is the linear equation in two variables as x and y are two variables.

2. **Construct a linear equation in two variables for the following.**
   a) **4 added 2 twice a number equals another number.**

   Solution:

   Suppose the one number = x

   Twice a number = 2x

   Another number = y

   According to the given condition:

   $4 + 2x = y$

b) 3 less than 5 times the first number is equal to 2 less than 5 times another number.

Solution:

Suppose the first number = x

Twice a number = 5x

Another number = y

According to the given condition:

5x – 3 = 5y – 2

c) One fourth of a number, when added to 16, equals three times another number.

Solution:

Suppose the first number = x

One fourth of the number is = 1/4x

Another number = y

3 times of another number = 3y

According to the given condition:

$\frac{1}{4}x + 16$ = 3y

d) A number increased by 15 is five times another number.

Solution:

Suppose the first number = x

Number increased by 15 = x + 15

Another number = y

five times of another number = 5y

According to the given condition:

$x + 15 = 5y$

e) Twice a number subtracted from half of another number equals 3.

Solution:

Suppose the first number = x

Twice of the number = 2x

Another number = y

Half of the number = $\frac{1}{2}$ y

According to the given condition:

$$\frac{1}{2}\,y - 2x = 3$$

3. **The total price of 5 prayer caps and 3 prayer mats is Rs 3550. Construct a linear equation in two variables for this statement.**

Solution:

Suppose the cost of a prayer cap =x

Suppose the cost of a prayer mat = y

According to the given condition the linear equation in two variables is:

5x + 3y = Rs3550

4. **The number of seats in two buses and five cars is 90. Construct a linear equation two variables for this statement.**

Solution:

Suppose the number of seat in a bus =x

Suppose the number of seat in a car = y

According to the given condition the linear equation in two variables is:

2x + 5y = 90

5. **The sum of 2 less than a number and 8 less than another number is 20. Construct linear equation in two variables for this statement.**

Solution:

Suppose one number = x

Other number = y

According to the given condition the linear equation in two variables is:

(x – 2) + (y – 8) = 20

6. **The price of 3 kg of apples and 4 kg of grapes is Rs 410. Construct a linear equation in two variables for this statement.**

Solution:

Suppose price of one kg apple = x

Suppose price of one kg grapes = y

According to the given condition the linear equation in two variables is:

$3x + 4y = 410$

7. **The weight of 5 boys and 10 girls in grade 7 is 600 kg. Construct a linear equation two variables for this statement.**

Solution:

Suppose weight of a boy = x

Suppose weight of a girl = y

According to the given condition the linear equation in two variables is:

$5x + 10y = 600$kg

8. **The difference between two numbers is 77. Construct a linear equation in two variables for this statement.**

Solution:

Suppose one number = x

Other number = y

According to the given condition the linear equation in two variables is:

$x - y = 77$

Learning check 7.3

1. **List 5 ordered pairs for each of the four quadrants.**

1st quadrant: (2, 3), (1, 4), (1, 3), (2, 8), (1, 2)

2nd quadrant: (–2, 3), (–1, 4), (–1, 3), (–2, 8), (–1, 2)

3rd quadrant: (–2, –3), (–1, –4), (–1, –3), (–2, –8), (–1, –2)

4th quadrant: (2, –3), (1, –4), (1, –3), (2, –8), (1, –2)

2. Identify the quadrant in which each of these ordered pairs lies.

a) (–2, 3) Quadrant: __________

Solution: As in the given ordered pair the x is negative and y is positive so lies in the quadrant II.

b) (2, –3) Quadrant: __________

Solution: As in the given ordered pair the x is positive and y is negative so lies in the quadrant IV.

c) (2, 7) Quadrant: __________

Solution: As in the given ordered pair the x and y is positive so lies in the quadrant I.

d) (–2, –5) Quadrant: _________

Solution: As in the given ordered pair the x is negative and y is also negative so lies in the quadrant III.

e) (–8, 3) Quadrant: _________

Solution: As in the given ordered pair the x is negative and y is positive so lies in the quadrant II.

f) (–4, 4) Quadrant: _________

Solution: As in the given ordered pair the x is negative and y is positive so lies in the quadrant II.

g) (5, –6) Quadrant: _________

Solution: As in the given ordered pair the x is positive and y is negative so lies in the quadrant IV.

h) (7, –6) Quadrant: _________

Solution: As in the given ordered pair the x is positive and y is negative so lies in the quadrant IV.

3. Plot the given points on a coordinate plane.

a) A (3, –6) b) B (0, –4) c) C (–3, –7) d) D (–4, 2)

e) E (–6, 0) f) F (3, 9) g) G (0, 0) h) H (1, 4)

i) I (0, 5) j) J (9, 0)

Solution:

a) In A (3, –6), x coordinate is positive and y coordinate is negative. So, this ordered pair lies in Quadrant IV. Start at the origin and count 3 units along the x-axis in the right (positive) direction and then count –6 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

b) In B (0, –4)), x coordinate is at origin and y coordinate is negative. Start at the origin as x = 0 then count –4 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

c) In C (–3, –7), x and y coordinate both are negative. So, this ordered pair lies in Quadrant III. Start at the origin and count –3 units along the x-axis in the left (negative) direction and then count –7 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

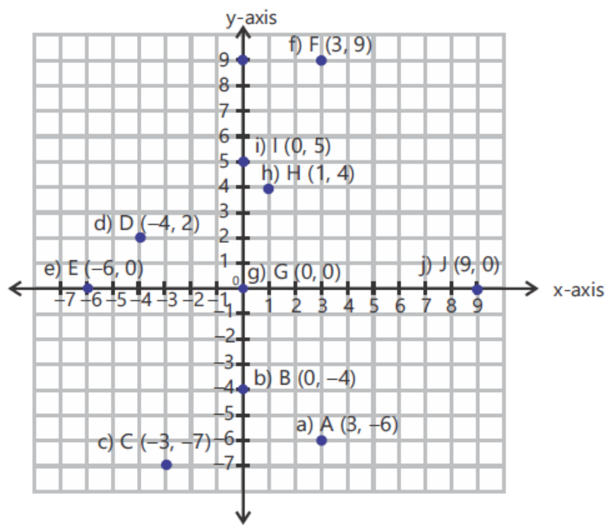
d) In D (–4, 2), x coordinate is negative and y coordinate is positive. So, this ordered pair lies in Quadrant II. Start at the origin and count –4 units along the x-axis in the left (negative) direction and then count 2 units along the y-axis in the upward (positive) direction. Mark the point and mention the ordered pair.

e) e) In E (–6, 0), x coordinate is negative and y coordinate is at origin. So, Start at the origin and count –6 units along the x-axis in the left (negative) direction. Mark the point and mention the ordered pair.

f) In F (3, 9), x and y coordinate both are positive. So, this ordered pair lies in Quadrant I. Start at the origin and count 3 unit along the x-axis in the right (positive) direction and then count 9 units along the y-axis in the upward (positive) direction. Mark the point and mention the ordered pair.

g) In G (0, 0), x and y coordinate both are 0. So, mark the point at origin and mention the ordered pair.

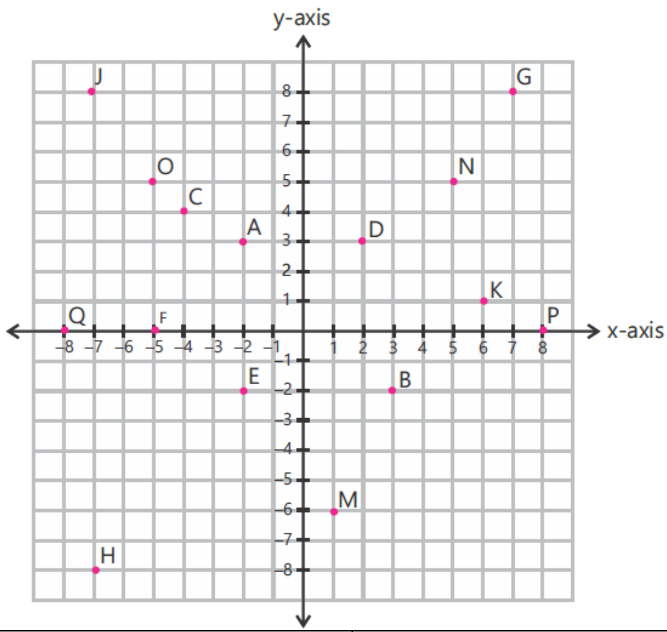
h) In H (1, 4), x and y coordinate both are positive. So, this ordered pair lies in Quadrant I. Start at the origin and count 1 unit along the x-axis in the right (positive) direction and then count 4 units along the y-axis in the upward (positive) direction. Mark the point and mention the ordered pair.



i) In I (0, 5), x coordinate is at origin and y coordinate is positive. Start at the origin as x= 0 and then count 5 units along the y-axis in the upward (positive) direction. Mark the point and mention the ordered pair.

j) In J (9, 0) x coordinate is positive and y coordinate is at origin. Start at the origin and count 9 units along the x-axis in the right (positive) direction. Mark the point and mention the ordered pair.

4. Identify the coordinates of the points on the coordinate plane and then draw a table to show the ordered pairs.



A (-2, 3)	J(-7, 8)
B(3, -2)	K(6, 1)
C(-4, 4)	M(1, -6)
D(2, 3)	N(5, 5)
E(-2, -2)	O(-5, 5)
F(-5, 0)	P(8, 0)
G(7, 8)	Q(-8, 0)
H(-7, -8)	

Learning check 7.4

1. Plot the graph of the following equations in one variable.

a) x + 6 = 12

Solution:

First find the value of x:

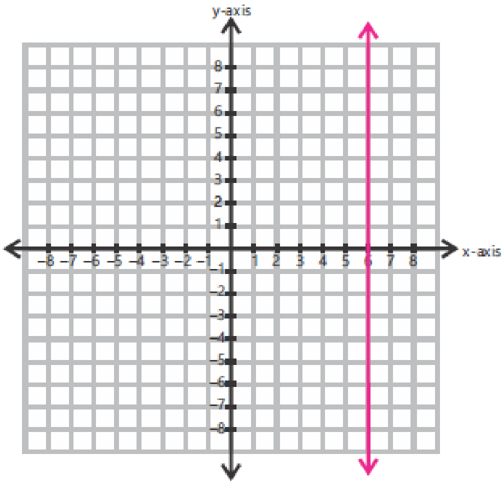
x = 12 - 6 = 6

Now for the required graph, the value of *x* will always be 6. As *x* is always 6, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	6	6	6	6	6
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (6, 1), (6, 2), (6, 3), (6, 4) and (6, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through 6 on the x-axis.

This is the required graph of the equation x + 6 = 12. This graph is showing **an equation of a vertical line**.



b) 4x + 9 = 17

Solution:

First find the value of x:

4x = 17 - 9 = 8

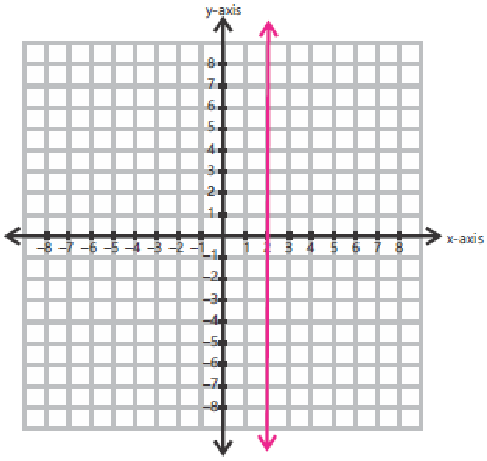
x = 2

Now for the required graph, the value of *x* will always be 2. As *x* is always 2, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	2	2	2	2	2
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (2, 1), (2, 2), (2, 3), (2, 4) and (2, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through 2 on the x-axis. This is the required graph of the equation 4x + 9 = 17.

This graph is showing **an equation of a vertical line**.



c) $5x - 2 = 8$

Solution:

First find the value of x:

$5x = 8 + 2 = 10$

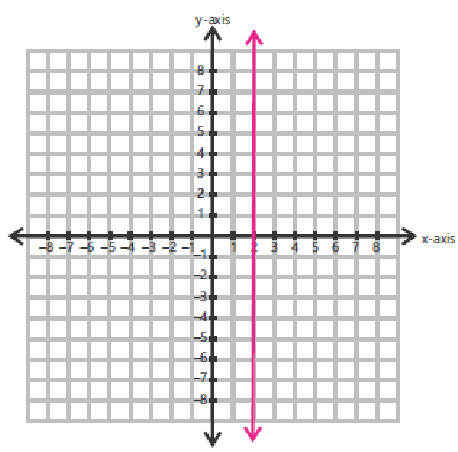
$x = 2$

Now for the required graph, the value of *x* will always be 2. As *x* is always 2, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	2	2	2	2	2
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (2, 1), (2, 2), (2, 3), (2, 4) and (2, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through 2 on the x-axis. This is the required graph of the equation $4x + 9 = 17$.

This graph is showing **an equation of a vertical line**.



d) 5 – 7x = –23

Solution:

First find the value of x:

5 + 23 = 7x

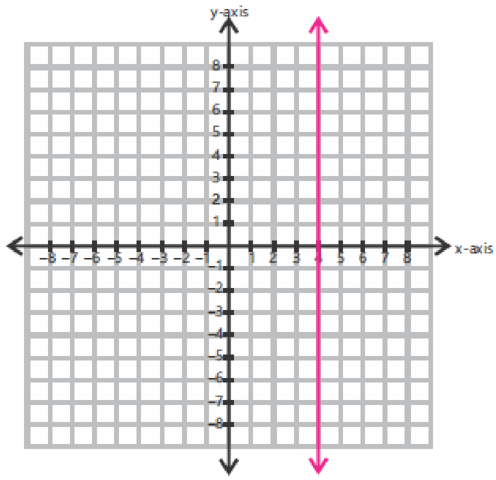
28 = 7x

x = 4

Now for the required graph, the value of *x* will always be 4. As *x* is always 4, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	4	4	4	4	4
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (4, 1), (4, 2), (4, 3), (4, 4) and (4, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through 4 on the x-axis. This is the required graph of the equation 5 – 7x = –23. This graph is showing **an equation of a vertical line**.



e) –4x + 2 = 14

Solution:

First find the value of x:

–4x = 14 – 2 = 12

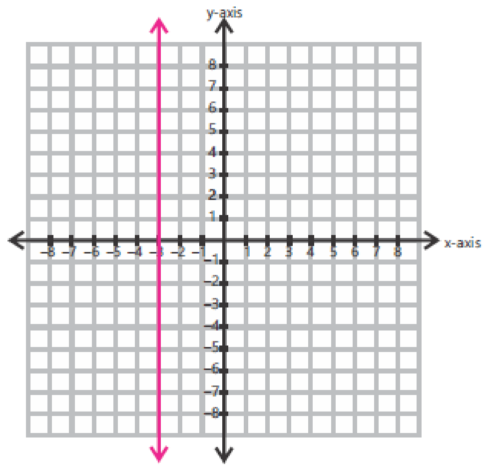
–4x = 12

x = –3

Now for the required graph, the value of *x* will always be –3. As *x* is always 4, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	–3	–3	–3	–3	–3
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (–3, 1), (–3, 2), (–3, 3), (–3, 4) and (–3, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through –3 on the x-axis. This is the required graph of the equation –4x +2 = 14. This graph is showing **an equation of a vertical line**.



f) 14 – 2x = 3x + 9

Solution:

First find the value of x:

14 – 2*x* = 3*x* + 9

14 – 9 = 3*x* + 2*x*

5 = 5*x*

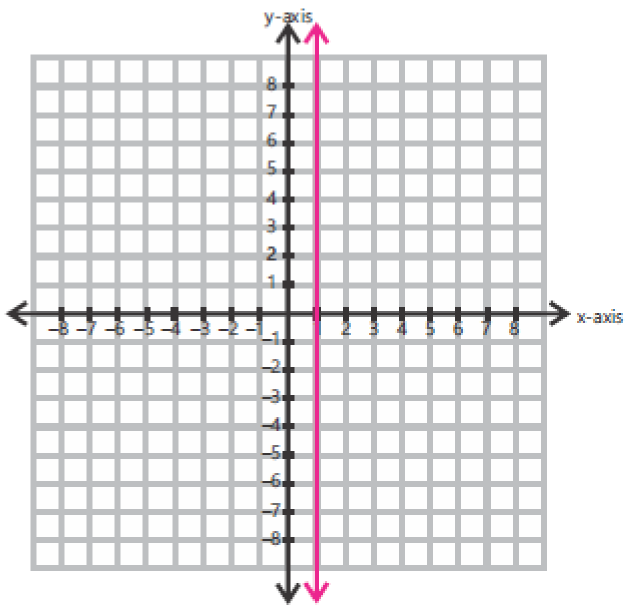
x = 1

Now for the required graph, the value of *x* will always be 1. As *x* is always 1, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	1	1	1	1	1
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (1, 1), (1, 2), (1, 3), (1, 4) and (1, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through 1 on the x-axis.

This is the required graph of the equation 14 – 2*x* = 3*x* + 9. This graph is showing **an equation of a vertical line**.



2. **Plot the graph of the following equations in two variables.**

   a) **y = 2x**

   Solution:

   We will find a few pairs of numbers for x and y which satisfy the equation y = 2x. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

   Put x = 1

   y = 2(1) = 2

   Put x = 2

y = 2(2) = 4

Put x = –1

y = 2(–1) = –2

Put x = –2

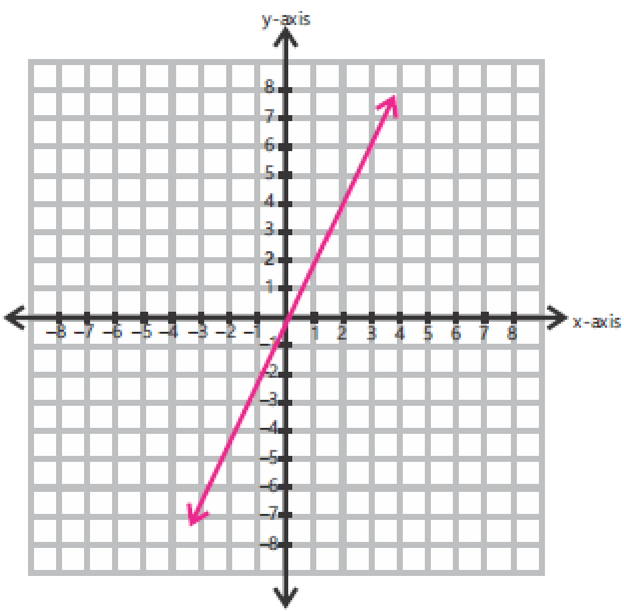
y = 2(–2) = –4

Put x = –3

y = 2(–3) = –6

x	1	2	–1	–2	–3
y	2	4	–2	–4	–6

So, some of the ordered pairs for the above equation are (1, 2), (2, 4), (–1, –2), (–2, –4) and (–3, –6). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation y = 2x.



b) y = –3x

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation y = 2x. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put x = 1

y = –3(1) = –3

Put x = 2

y = –3(2) = –6

Put x = –1

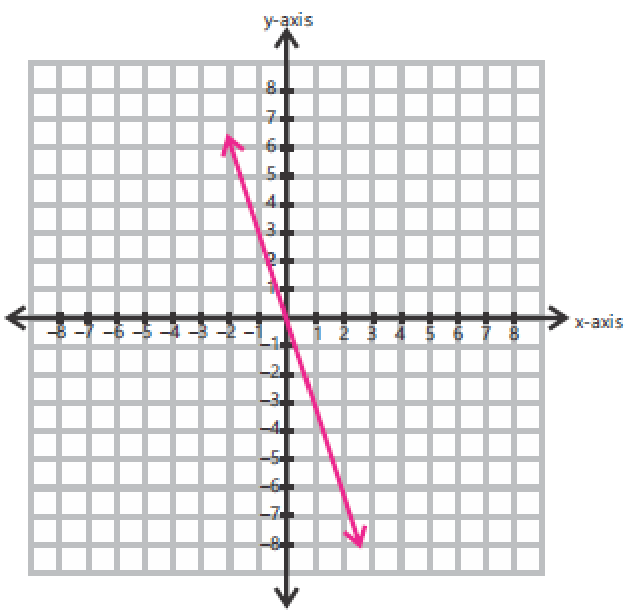
y = –3(–1) = 3

Put x = –2

y = –3(–2) = 6

x	1	2	–1	–2
y	–3	–6	3	6

So, some of the ordered pairs for the above equation are (1, –3), (2, –6), (–1, 3) and (–2, 6). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation y = 2x.



c) $y = x - 5$

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation $y = 2x$. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put $x = 1$

$y = 1 - 5$

$= -4$

Put $x = 2$

$y = 2 - 5$

$= -3$

Put $x = -1$

$y = -1 - 5$

$= -6$

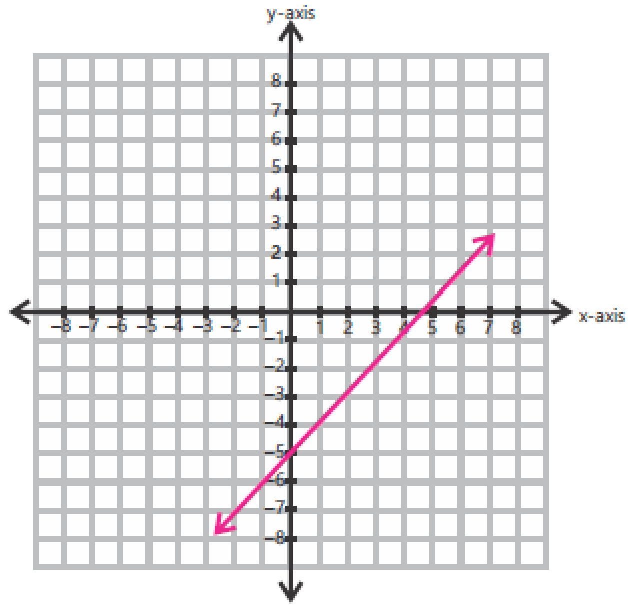
Put $x = -2$

$y = -2 - 5$

$= -7$

X	1	2	–1	–2
y	–4	–3	–6	–7

So, some of the ordered pairs for the above equation are (1, –4), (2, –3), (–1, –6) and (–2, –7). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation y = x – 5 .



d) x – 2 = 3y

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation y = 2x. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put x = 1

1 – 2 = 3y

–1 = 3y

$y = -\frac{1}{3}$

Put x = 2

2 – 2 = 3y

0 = 3y

y = 0

Put x = –1

–1 – 2 = 3y

–3 = 3y

y = -1

Put x = –2

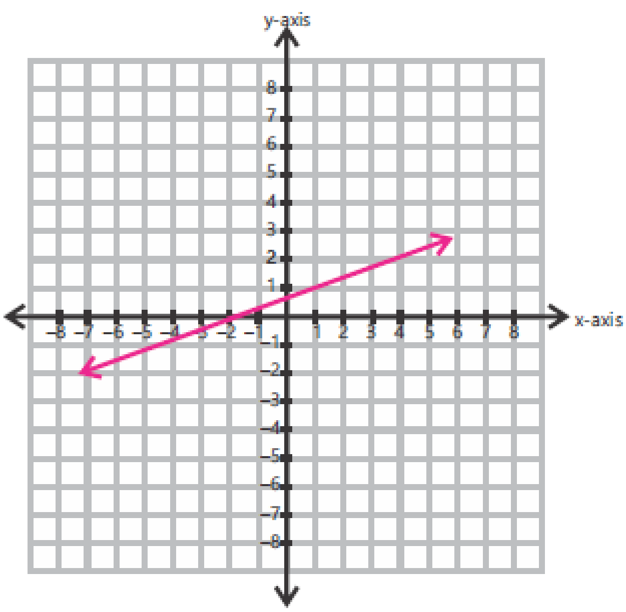
–2 – 2 = 3y

–4 = 3y

y = $-\frac{4}{3}$

x	1	2	–1	–2
y	$-\frac{1}{3}$	0	–1	$\frac{4}{3}$

So, some of the ordered pairs for the above equation are (1, $-\frac{1}{3}$), (2, 0), (–1, –1) and (–2, $\frac{4}{3}$). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation x – 2 = 3y.



e) $\mathbf{2x + y = \frac{1}{2}}$

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation y = 2x. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put x = 1

$2(1) + y = \frac{1}{2}$

$2 + y = \frac{1}{2}$

$y = \frac{1}{2} - 2 = -\frac{3}{2} = -1.5$

Put x = 2

$2(2) + y = \frac{1}{2}$

$4 + y = \frac{1}{2}$

$y = \frac{1}{2} - 4 = -\frac{7}{2} = -3.5$

Put x = –1

$2(-1) + y = \frac{1}{2}$

$-2 + y = \frac{1}{2}$

$y = \frac{1}{2} + 2 = \frac{5}{2} = 2.5$

Put x = –2

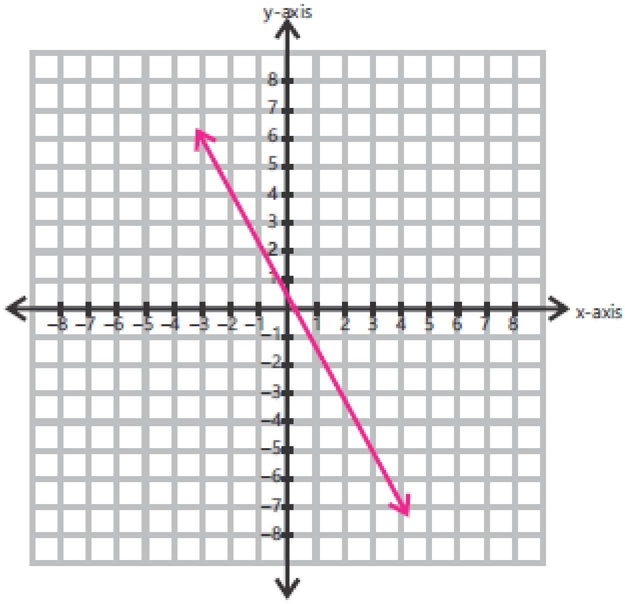
$2(-2) + y = \frac{1}{2}$

$-4 + y = \frac{1}{2}$

$y = \frac{1}{2} + 4 = \frac{9}{2} = 4.5$

x	1	2	–1	–2
y	–1.5	–3.5	2.5	4.5

So, some of the ordered pairs for the above equation are (1, –1.5), (2, –3.5), (–1, 2.5) and (–2, 4.5). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation $2x + y = \frac{1}{2}$.



f) **x + 2y = 5**

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation x +2y = 5. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put x = 1

$1 + 2y = 5$

$2y = 5 – 1$

$2y = 4$

$y = 2$

Put x = 2

$2 + 2y = 5$

$2y = 5 – 2$

2y = 3

y = $\frac{3}{2}$

Put x = –1

–1 + 2y = 5

2y = 5 + 1

2y = 6

y = 3

Put x = –2

–2 + 2y = 5

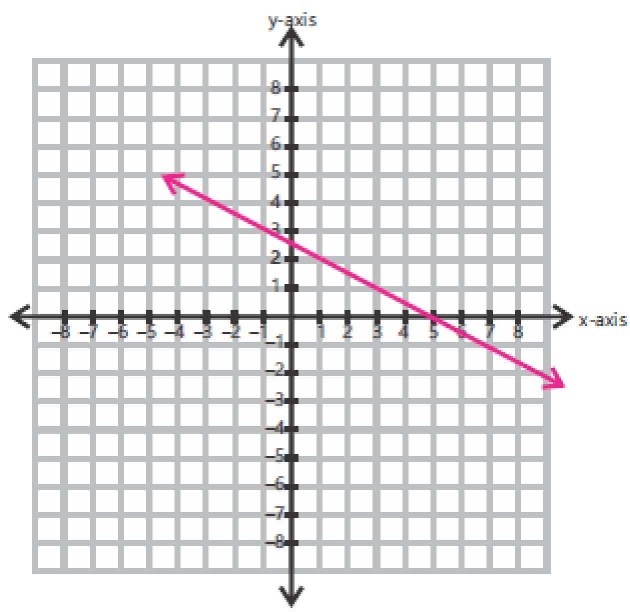
2y = 5 + 2

2y = 7

y = $\frac{7}{2}$

x	1	2	–1	–2
y	2	$\frac{3}{2}$	3	$\frac{7}{2}$

So, some of the ordered pairs for the above equation are (1, 2), (2, $\frac{3}{2}$), (–1, 3) and (–2, $\frac{7}{2}$). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation x + 2y = 5.

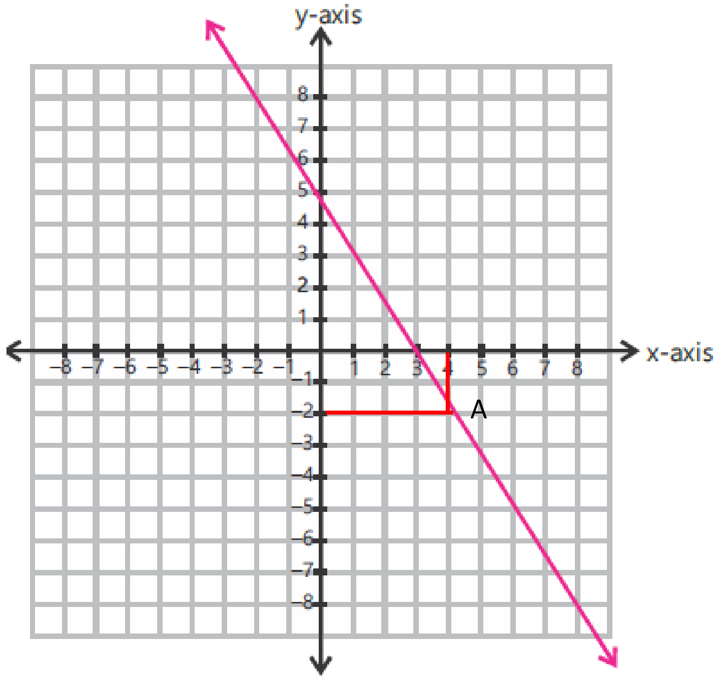


3. Observe the graph and find:

a) The value of y when x = 4.

Solution:

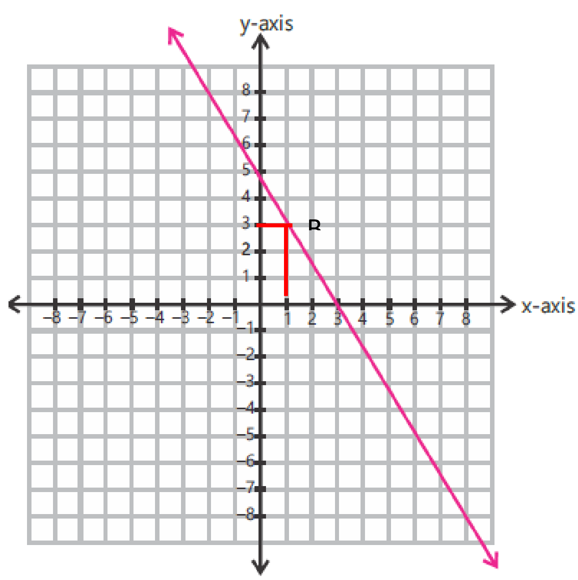
To find y when x = 4, locate 4 on the y-axis in a positive direction on the graph. Mark the point and name it A. Join this point to the y-axis using a straight line. We can see that the line meets at –2 on the y-axis. So, when x = 4 then y = –2 on this graph.



b) The value of x when y = 3.

Solution:

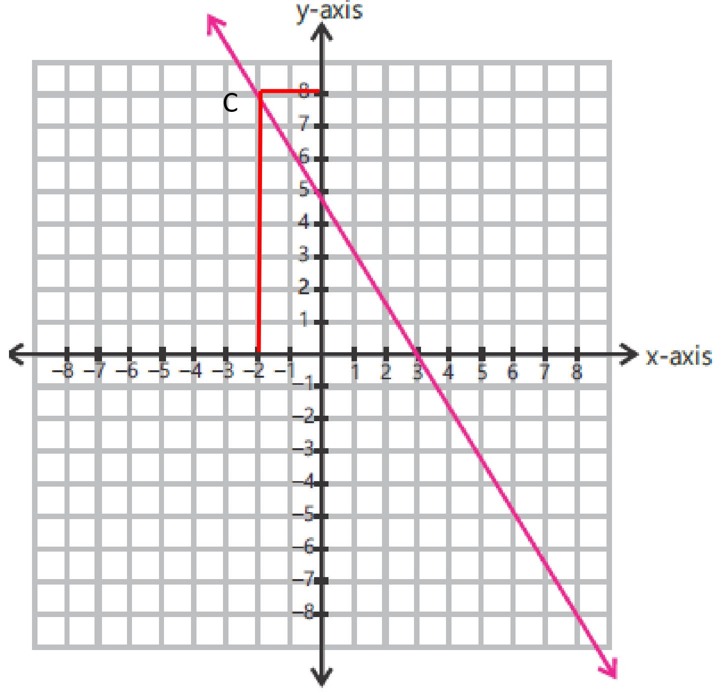
To find x when y = 3, locate 3 on the y-axis in a positive direction on the graph. Mark the point and name it B. Join this point to the y-axis using a straight line. We can see that the line meets at +1 on the x-axis. So, when y = 3 then x = +1 on this graph.



c) The value of *x* when y = 8.

Solution:

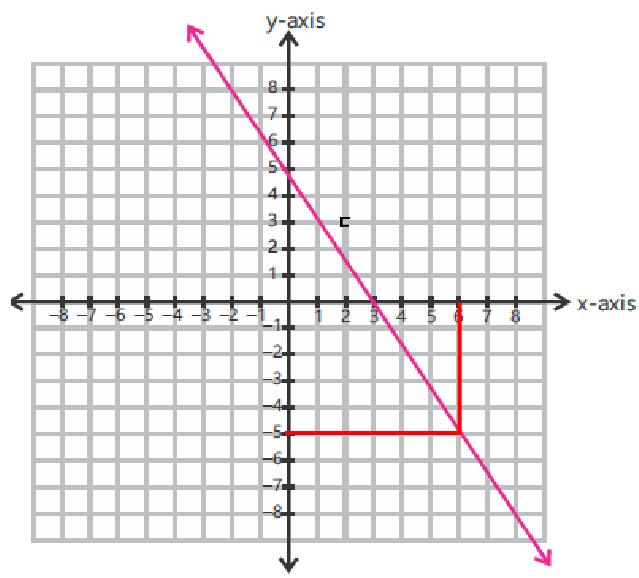
To find x when y = 8, locate 8 on the y-axis in a positive direction on the graph. Mark the point and name it C. Join this point to the y-axis using a straight line. We can see that the line meets at –2 on the x-axis. So, when y = 8 then *x* = –2 on this graph.



d) The value of *x* when *y* = –5.

Solution:

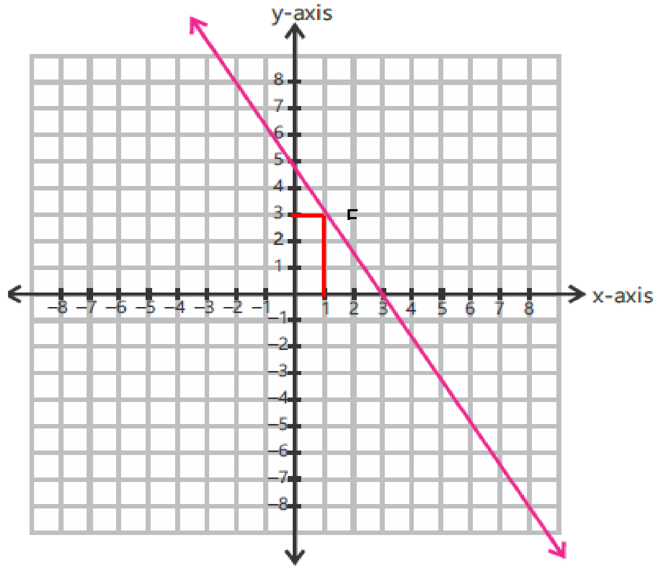
To find x when y = –5, locate –5 on the y-axis in a negative direction on the graph. Mark the point and name it D. Join this point to the y-axis using a straight line. We can see that the line meets at –2 on the x-axis. So, when y = –5 then *x* = 6 on this graph.



e) The value of y when $x = 1$.

Solution:

To find y when $x = 1$, locate 1 on the x-axis in a positive direction on the graph. Mark the point and name it E. Join this point to the x-axis using a straight line. We can see that the line meets at 3 on the y-axis. So, when x = 1 then $y = 3$ on this graph.



Unit check 7

1. Encircle the correct option.

a) How many solutions exist for the linear equation 2x + 3 = 4?

i) 1 ii) 2 iii) 3 iv) 4

b) The point where the two axes intersect is called the:

i) x-axis ii) y-axis iii) origin iv) equation

c) Which of the points lies in quadrant IV?

i) (4, 2) ii) (2, –3) iii) (–3, –2) iv) (–3, 1)

d) Which one is an equation of a horizontal line?

i) x = 3 ii) x + 3 = –2 iii) y = 5 iv) x – y =2

e) Half of a number is added to two times another number, equals 5 is shown by:

i) $\frac{1}{2}x = 2y + 5$ ii) $\frac{1}{2}x + 2y = 5$ iii) $\frac{1}{2}x + 5 = 2y$ iv) $\frac{1}{2}x - 5 = 2y$

2. Fill in the blanks.

a) The value of x in the equation $8x - 1 = \frac{1}{2}$ is ______$\frac{3}{16}$________.

b) The point (–3, 7) lies in quadrant ___II__________.

c) The general form of the linear equation in one variable is __ax + b = c__.

d) The ordered pair for the __origin__ is always (0,0).

e) The solution of 0.4x + 0.1 = 0.2x + 0.3 is __1___.

3. Solve the following linear equations.

a) x + 11 = 3

Solution:

$x + 11 = 3$

To isolate the variable x, subtract 11 from both sides.

$x + 11 - 11 = 3 - 11$

$x = -8$

b) 2x – 6 = 4

Solution:

$2x - 6 = 4$

To isolate the variable x, add 6 on both sides.

$2x - 6 + 6 = 4 + 6$

$2x = 10$

To isolate the variable x, divide both sides by 2.

$\frac{2}{2}x = \frac{10}{2}$

$x = 5$

c) 3 – 5x = 6

Solution:

$3 - 5x = 6$

To isolate the variable x, subtract 3 from both sides.

$3 - 5x - 3 = 6 - 3$

$-5x = 3$

To isolate the variable x, divide both sides by –5.

$\frac{-5}{-5}x = -\frac{3}{5}$

$x = -\frac{3}{5}$

d) 8x – 5 = 60

Solution:

$8x - 5 = 60$

To isolate the variable x, add 5 to both sides.

$8x - 5 + 5 = 60 + 5$

$8x = 65$

To isolate the variable x, divide both sides by 8.

$\frac{8}{8}x = \frac{65}{8}$

$x = \frac{65}{8}$

e) $\boldsymbol{\frac{1}{2}x + \frac{2}{3} = \frac{5}{6}}$

Solution:

$\frac{1}{2}x + \frac{2}{3} = \frac{5}{6}$

To isolate the variable x, subtract $\frac{2}{3}$ from both sides.

$\frac{1}{2}x + \frac{2}{3} - \frac{2}{3} = \frac{5}{6} - \frac{2}{3}$

$\frac{1}{2}x = \frac{5-4}{6}$

$\frac{1}{2}x = \frac{1}{6}$

To isolate the variable x, multiply both sides by 2.

$2 \times \frac{1}{2}x = 2 \times \frac{1}{6}$

$x = \frac{1}{3}$

f) 4.5 + 3.4x = –6.5

Solution:

$4.5 + 3.4x = -6.5$

To isolate the variable x, subtract 4.5 from both sides.

$4.5 + 3.4x - 4.5 = -6.5 - 4.5$

$3.4x = -11$

To isolate the variable x, divide both sides by 3.4.

$\frac{3.4}{3.4}x = \frac{-11}{3.4}$

$x = 3.24$

4. **Nimra's age is one-third the age of her sister. If the difference between their ages is 24 years, find their ages.**

Solution:

Suppose the age of Sister = x

Then Nimra's age = $\frac{1}{3}x$

According to the condition:

$x - \frac{1}{3}x = 24$

$\frac{2}{3}x = 24$

$x = 36$

the age of sister is 36 years.

Nimra' s age = $\frac{1}{3}(36) = 12$ years

5. **Two geometry boxes and five sticky notes cost Rs 660 in total. If the price of a geometry box is 3 times the price of the sticky notes, find their price.**

Solution:

Suppose the cost of a sticky notes (in Rs) = x

The cost of 5 sticky notes = $5x$

The geometry box costs is three times the cost of the sticky notes so,

The cost of a geometry box = $3x$

The cost of 2 geometry boxes = $6x$

Total cost of 5 sticky notes and 2 geometry boxes = Rs 660

According to the condition given:

$5x + 6x = 660$

$11x = 660$

$x = 60$

the price of a sticky not is Rs 60.

The price of the geometry box = $3x = 3(60) = 180$

6. **Construct a linear equation in two variables for the following.**

a) Rs 20 added to the price of half kg apples equals to price of one kg of peaches.

Solution:

Suppose price of 1 kg apple = x

Suppose price of 1 kg peach = y

According to the given condition the linear equation in two variables is:

$20 + \frac{1}{2}x = y$

b) One-fourth of a number, when added to another number, equals 10.

Solution:

Suppose one number = x

Suppose another number = y

According to the given condition the linear equation in two variables is:

$\frac{1}{4}x + y = 10$

c) The width is one-half the length of a rectangle.

Solution:

Suppose length of rectangle = x

Suppose width of rectangle = y

According to the given condition the linear equation in two variables is:

$\frac{1}{2}x = y$

d) A number decreased by 6 equals 3 times another number.

Solution:

Suppose one number = x

Suppose another number = y

According to the given condition the linear equation in two variables is:

$$x - 6 = 3y$$

e) 7 decreased by a number is equal to another number.

Solution:

Suppose one number = x

Suppose another number = y

According to the given condition the linear equation in two variables is:

$$7 - x = y$$

7. Identify the quadrant in which each of the given ordered pairs lies.

a) (2, –9) Quadrant: ______________
b) (2, –5) Quadrant: ______________
c) (–1, –4) Quadrant: ______________
d) (-5, 1) Quadrant: ______________

a) (2, –9) Quadrant: __________

Solution: As in the given ordered pair the x is positive and y is negative so lies in the quadrant IV.

b) (2, –5) Quadrant: __________

Solution: As in the given ordered pair the x is positive and y is negative so lies in the quadrant IV.

c) (–1, –4) Quadrant: _________

Solution: As in the given ordered pair the x is negative and y is also negative so lies in the quadrant III.

d) (–5, 1) Quadrant: __________

Solution: As in the given ordered pair the x is negative and y is positive so lies in the quadrant II.

8. Plot the given points on a coordinate plane.

a) (4, 7) b) (2, –6) c) (0, –4) d) (–8, –5)
e) (–5, 2) f) (2, –3) g) (4, 1) h) (–3, –5)

Solution:

a) In (4, 7), x and y coordinate both are positive. So, this ordered pair lies in Quadrant I. Start at the origin and count 4 units along the x-axis in the right (positive) direction and then count 7 units along the y-axis in the upward (positive) direction. Mark the point A and mention the ordered pair.

b) In (2, –6), x coordinate is positive and y coordinate is negative. So, this ordered pair lies in Quadrant IV. Start at the origin and count 2 units along the x-axis in the right (positive) direction and then count –6 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

c) In (0, –4)), x coordinate is at origin and y coordinate is negative. Start at the origin as x = 0 then count –4 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

d) In C (–8, –5), x and y coordinate both are negative. So, this ordered pair lies in Quadrant III. Start at the origin and count –8 units along the x-axis in the left (negative) direction and then count –5 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

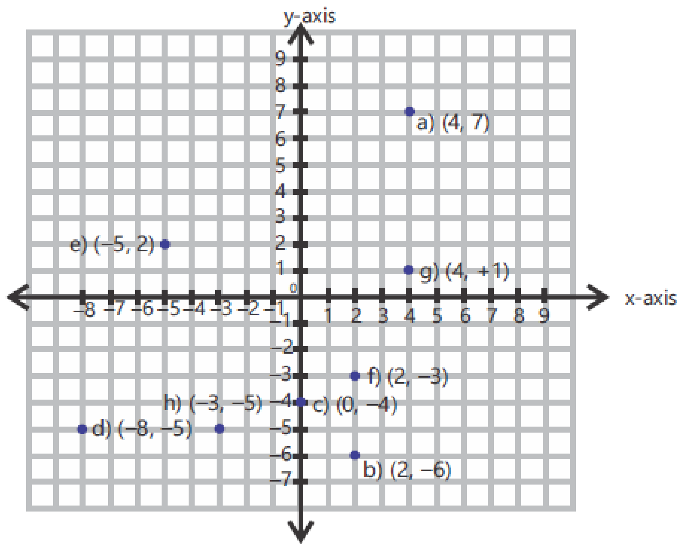
e) In D (–5, 2), x coordinate is negative and y coordinate is positive. So, this ordered pair lies in Quadrant II. Start at the origin and count –5 units along the x-axis in the left (negative) direction and then count 2 units along the y-axis in the upward (positive) direction. Mark the point and mention the ordered pair.

f) In (2, –3), x coordinate is positive and y coordinate is negative. So, this ordered pair lies in Quadrant IV. Start at the origin and count 2 units along the x-axis in the right (positive) direction and then count –3 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

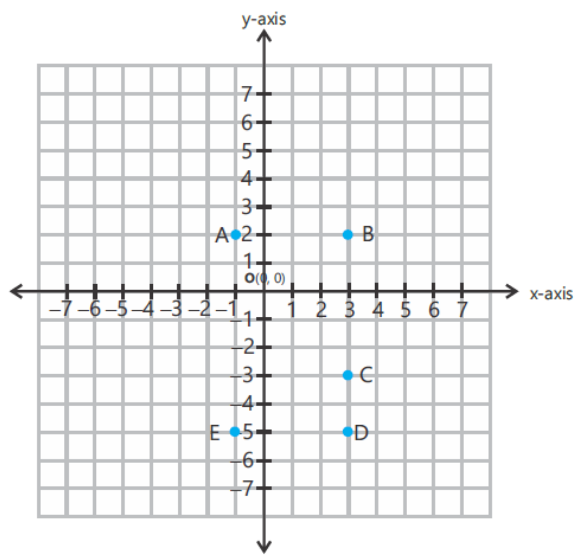
g) In (4, 1), x and y coordinate both are positive. So, this ordered pair lies in Quadrant I. Start at the origin and count 4 units along the x-axis in the right (positive) direction and then count 1 unit along the y-axis in the upward (positive) direction. Mark the point and mention the ordered pair.

h) In C (–3, –5), x and y coordinate both are negative. So, this ordered pair lies in Quadrant III. Start at the origin and count –3 units along the x-axis in the left (negative) direction and then count –5 units along the y-axis in the downward (negative) direction. Mark the point and mention the ordered pair.

i)



9. Identify the coordinates of the points on the coordinate plane.



A (-1, 2)	D(3, -5)
B(3, 2)	E(-1, -5)
C(3, -3)	

10. Plot the graph of the following equations.

a) x + 3 = y

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation x +3 =y. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put x = 1

1 + 3 = y

y = 4

Put x = 2

2 + 3 = y

y = 5

Put x = –1

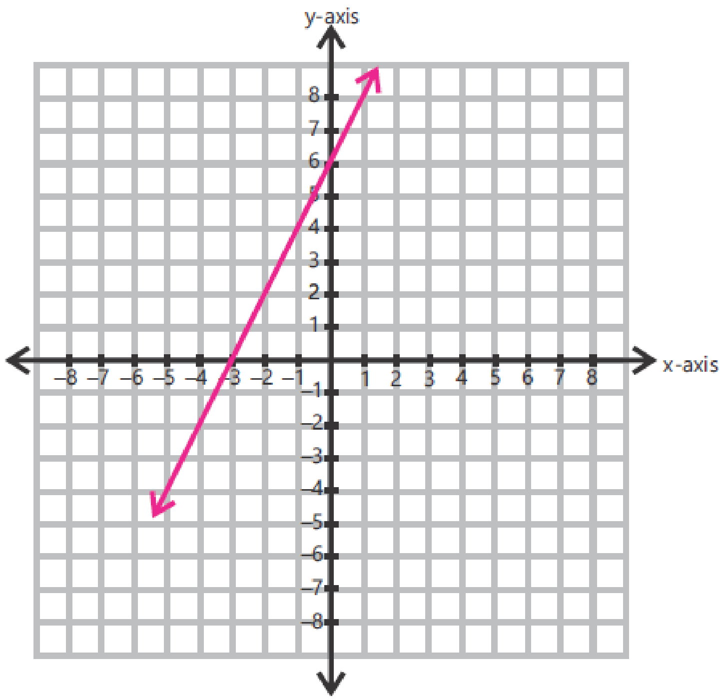
–1 + 3 = y

y = 2

Put x = –2

–2 + 3 = y

y = 1



x	1	2	–1	–2
y	4	5	2	1

So, some of the ordered pairs for the above equation are (1, 4), (2, 5), (–1, 2) and (–2,

1). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation x + 3 = y.

b) 3x + 8 = 5

Solution:

First find the value of x:

3x + 8 = 5

3x = 5 – 8

3x = – 3

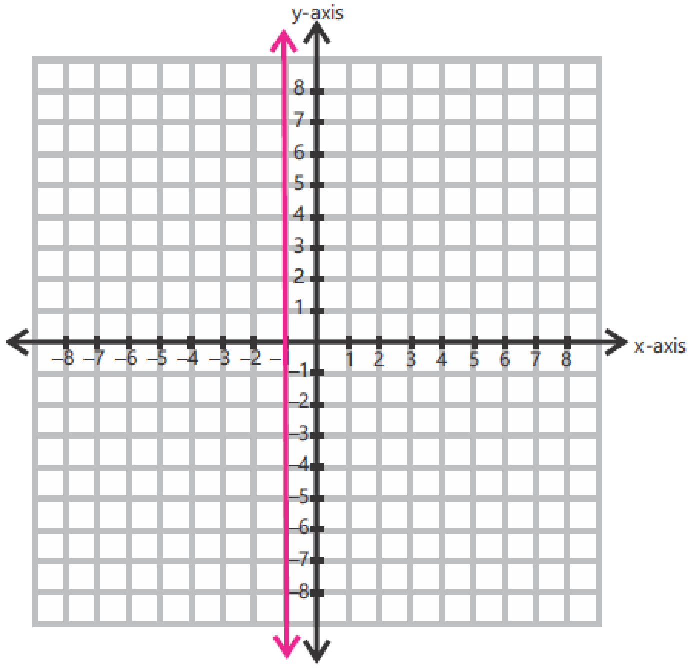
x = –1

Now for the required graph, the value of *x* will always be –1. As *x* is always –1, *y* can take any value and it will never affect *x*. Let's put some values for y.

x	–1	–1	–1	–1	–1
y	1	2	3	4	5

So, some of the ordered pairs for the above equation are (–1, 1), (–1, 2), (–1, 3), (–1, 4) and (–1, 5). Plot all of these ordered pairs to get the required graph. The vertical line goes through 2 on the x-axis. This is the required graph of the equation 3x + 8 = 5.

This graph is showing **an equation of a vertical line**.



c) y = 4x

Solution:

We will find a few pairs of numbers for x and y which satisfy the equation y = 4x. We can find the points by choosing a few values for x and putting those in the equation to find the respective y values.

Put x = 1

y = 4(1)

y = 4

Put x = 2

y = 4(2)

y = 8

Put x = –1

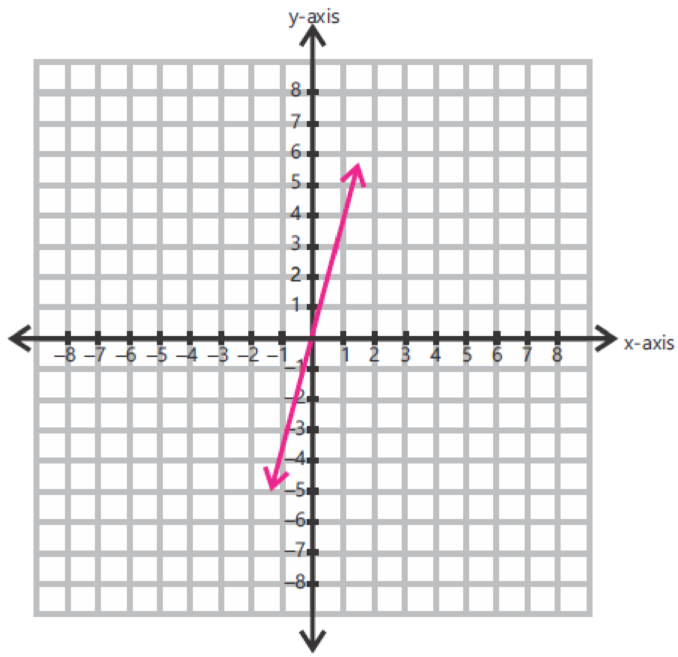
y = 4(–1)

y = –4

Put x = –2

y = 4(–2)

y = –8



x	1	2	–1	–2
y	4	8	–4	–8

So, some of the ordered pairs for the above equation are (1, 4), (2, 8), (–1, –4) and (–2,

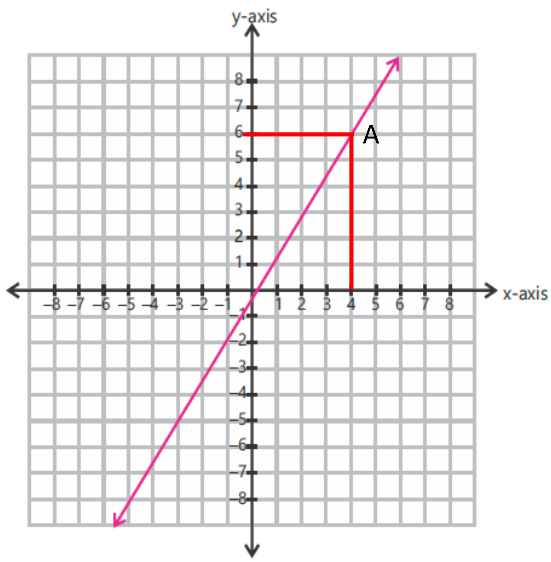
–8). Plot all of these ordered pairs to get the required graph. This is the required graph for the equation y = 4x.

11. Observe the graph and find:

a) The value of *x* when y = 6.

Solution:

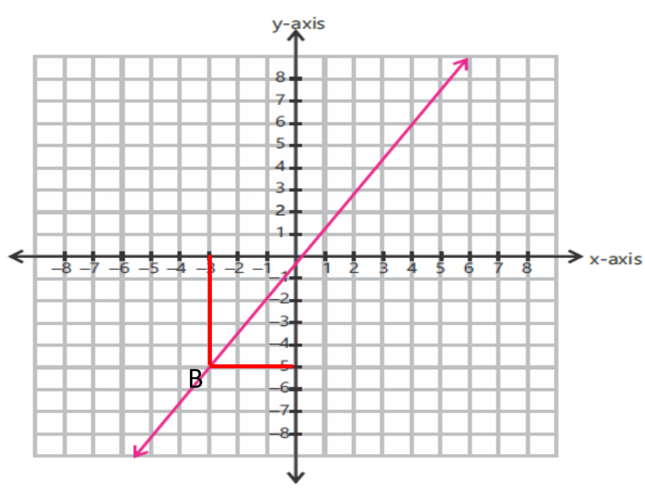
To find x when y = 6, locate 6 on the y-axis in a positive direction on the graph. Mark the point and name it A. Join this point to the x-axis using a straight line. We can see that the line meets at 4 on the x-axis. So, when y = 3 then *x* = +4 on this graph.



b) The value of y when x = –3.

Solution:

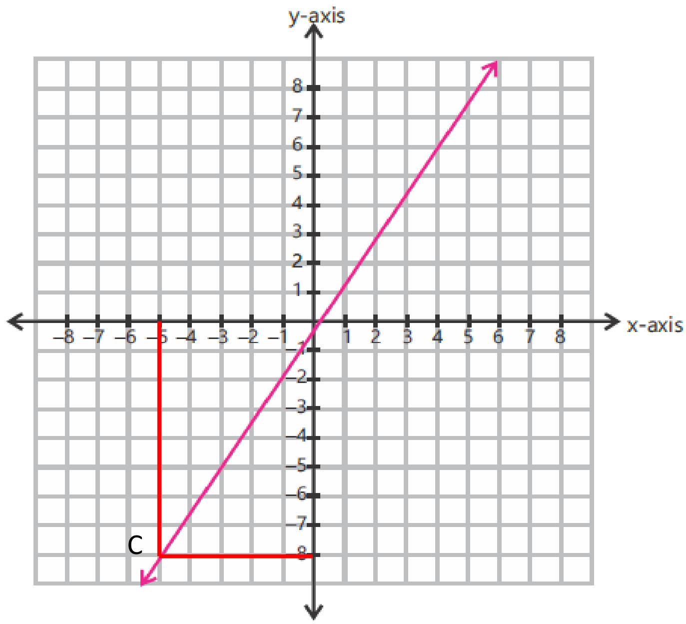
To find y when x = 6, locate –3 on the x-axis in a negative direction on the graph. Mark the point and name it B. Join this point to the x-axis using a straight line. We can see that the line meets at –5 on the y-axis. So, when x = 3 then *y* = –5 on this graph.



c) The value of *x* when y = –8.

Solution:

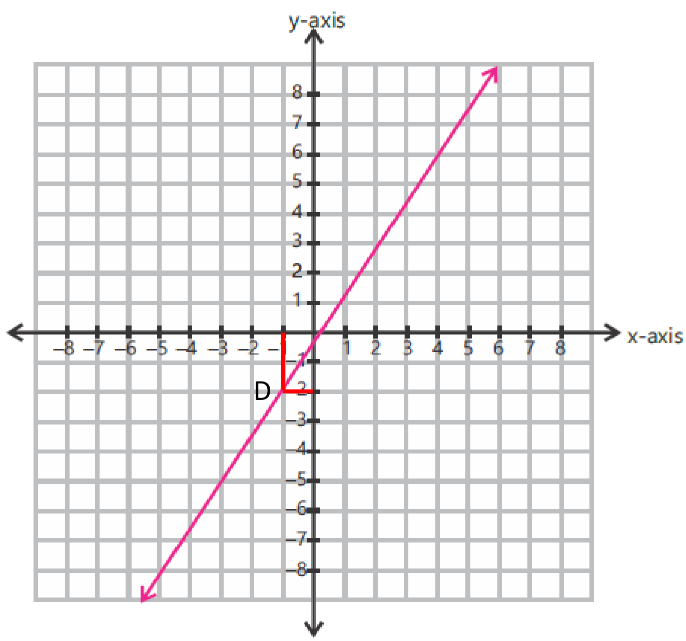
To find x when y = –8, locate –8 on the y-axis in a negative direction on the graph. Mark the point and name it C. Join this point to the x-axis using a straight line. We can see that the line meets at –5 on the x-axis. So, when y = –8 then *x* = –5 on this graph.



d) The value of y when x = –1.

Solution:

To find y when x = –1, locate –1 on the x-axis in a negative direction on the graph. Mark the point and name it D. Join this point to the x-axis using a straight line. We can see that the line meets at –2 on the y-axis. So, when x = –1 then *y* = –2 on this graph.



Learning Check 8.1

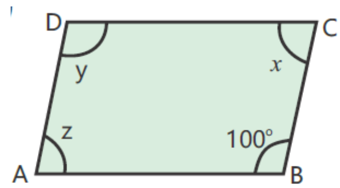
2. Find the unknown angles marked in the following parallelograms.

a) Here ∠B=100°

Opposite angles in a parallelogram are equal. So,

∠B=∠y=100°

The sum of adjacent angles between two parallel sides is 180°, so:



∠B+∠x=180°

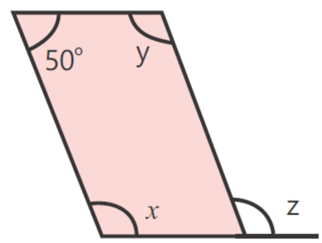
100° + ∠x = 180°

∠x= 180° – 100°

∠x=80°

Again, ∠x=∠z as they are opposite angles of the parallelogram. So:

∠z= 80°.



b) The sum of adjacent angles between two parallel sides is 180°, so:

∠50°+∠y=180°

∠y= 180° – 50°

∠y=130°

Again, ∠x=∠y as they are opposite angles of the parallelogram. So:

∠x= 130°.

To find <z, we need to find the fourth unknown angle.

Let the fourth angle be "a".

Then: <a = 50° as they are opposite angles of the parallelogram.

The sum of all angles on a straight line equal to 180°.

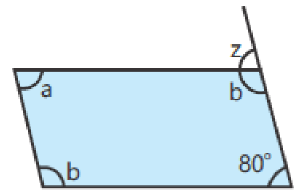
So, <z + <a=180°

<z + 50° =180°

<z= 180° - 50°

<z=130°

c) The sum of adjacent angles between two parallel sides is 180°, so:



$\angle 80^\circ + \angle b = 180^\circ$

$\angle b = 180^\circ - 80^\circ$

$\angle b = 100^\circ$

Again, $\angle a = 80^\circ$ as they are opposite angles of the parallelogram. So:

The sum of all angles on a straight line equal to 180°.

So, $<z + <b = 180^\circ$

$<z + 100^\circ = 180^\circ$

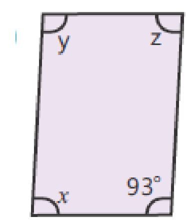
$<z = 180^\circ - 100^\circ$

$<z = 80^\circ$

d) Opposite angles in a parallelogram are equal. So,

$\angle y = 93^\circ$

The sum of adjacent angles between two parallel sides is 180°, so:



$\angle y + \angle z = 180^\circ$

$93^\circ + \angle z = 180^\circ$

$\angle z = 180^\circ - 93^\circ$

$\angle z = 87^\circ$

Again, $\angle x = \angle z$ as they are opposite angles of the parallelogram.

So:

$\angle x = 87^\circ$.

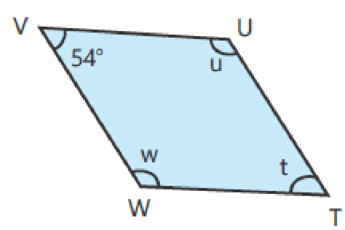
3. Find the unknown marked angles in the following rhombuses.

a) Opposite angles in a rhombus are equal. So,

$\angle t = 54^\circ$

The sum of adjacent angles between two parallel sides is 180°, so:

$\angle t + \angle w = 180^\circ$



$54^\circ + \angle w = 180^\circ$

$\angle w = 180^\circ - 54^\circ$

$\angle w = 126^\circ$

Again, $\angle u = \angle w$ as they are opposite angles of the rhombus. So:

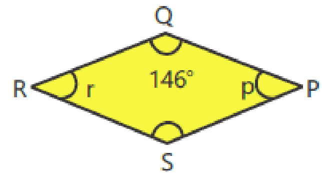
$\angle u = 126^o$.

b) The sum of adjacent angles between two parallel sides is 180^o, so:

$\angle 146^o + \angle p = 180^o$

$\angle p = 180^o - 146^o$

$\angle p = 34^o$

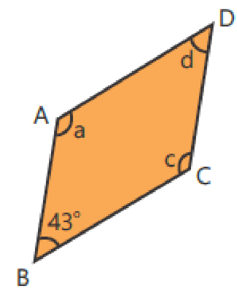


Again, $\angle r = \angle p$ as they are opposite angles of the rhombus. So:

$\angle r = 34^o$.

c) Opposite angles in a rhombus are equal. So,

$\angle d = 43^o$



The sum of adjacent angles between two parallel sides is 180^o, so:

$\angle d + \angle c = 180^o$

$43^o + \angle c = 180^o$

$\angle c = 180^o - 43^o$

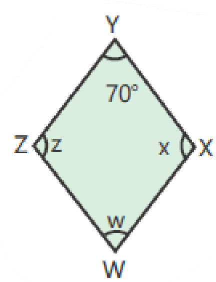
$\angle c = 137^o$

Again, $\angle a = \angle c$ as they are opposite angles of the rhombus. So:

$\angle a = 137^o$.

d) Opposite angles in a rhombus are equal. So,

$\angle w = 70^o$



The sum of adjacent angles between two parallel sides is 180^o, so:

$\angle w + \angle x = 180^o$

$70^o + \angle x = 180^o$

$\angle x = 180^o - 70^o$

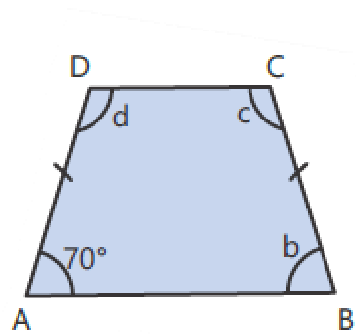
$\angle x = 110^o$

Again, $\angle z = \angle x$ as they are opposite angles of the rhombus. So:

$\angle z = 110^o$.

4. Find the unknown marked angles in the following trapeziums.

a) Here the trapezium ABCD is an isosceles trapezium as AD and BC are equal in measure. In an isosceles trapezium, the angles adjacent to each parallel side are congruent



So, <b=70^{o} and <d=<c.

In a trapezium, the sum of angles between the parallel sides is equal to 180^{o}. So:

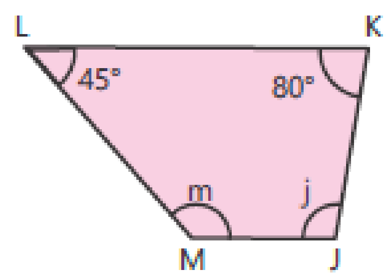
<d + 70^{o}=180^{o}

<d =180^{o} - 70^{o}

<d = 110^{o}

So, <c=<d= 110^{o}

b) In a trapezium, the sum of angles between the parallel sides is equal to 180^{o}. So:



<m + 45^{o}=180^{o}

<m =180^{o} - 45^{o}

<m=135^{o}

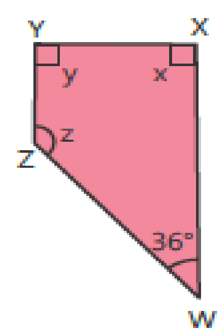
Similarly:

<j + 80^{o}=180^{o}

<j =180^{o} - 80^{o}

<j = 100^{o}

c) In a trapezium, the sum of angles between the parallel sides is equal to 180^{o}. So:



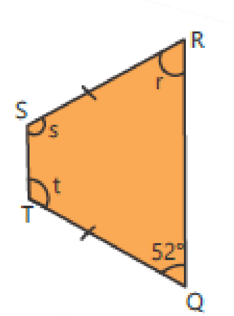
<z + 36^{o}=180^{o}

<z =180^{o} - 36^{o}

<z =144^{o}

< x and <y both are right angles i.e. they measure 90^{o} as mentioned in the figure.

d) Here the trapezium QRST is an isosceles trapezium as QT and RS are equal in measure. In an isosceles trapezium, the angles adjacent to each parallel side are congruent. So,



<r=52° and <s=<t

In a trapezium, the sum of angles between the parallel sides is equal to 180°. So:

<t + 52°=180°

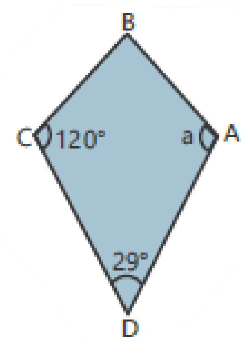
<t =180° - 52°

<t = 128°

So, <s=<t= 128°

5. Find the unknown angles in the following kites.

a) One pair of opposite angles is equal in kites.



So, <a=120°

The sum of all angles of a quadrilateral is equal to 360°, so we can find <B by subtracting the sum of the other three angles from 360°.

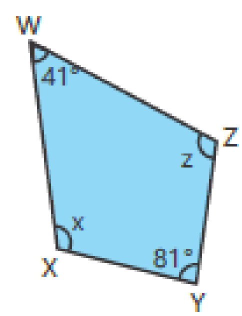
<B = 360° – (120° + 120° + 29°)

<B =360° – 269°

= 91°

b) The sum of all angles of a quadrilateral is equal to 360°. So,

<x + <y + 41° + 81° = 360°.



Also, <x=<y. So, by putting x=y, we get:

<x + <x + 41° + 81° = 360°

2(<x) + 122° = 360°

2(<x) = 360° - 122°

2(<x) = 238°

<x = 238°/2

<x= 119°

So, <x=<y=119°

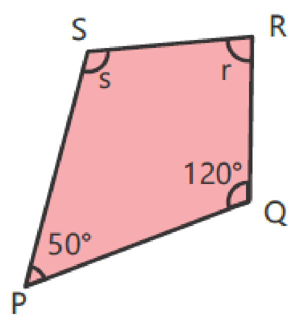
c) Here <s=<120º (pair of opposite angles is equal)

The sum of all angles of a quadrilateral is equal to 360º, so we can find <r by subtracting the sum of the other three angles from 360º.

<r = 360º – (120º + 120º + 50º)

<r =360º – 290º

= 70º



d) One pair of opposite angles is equal in kites.

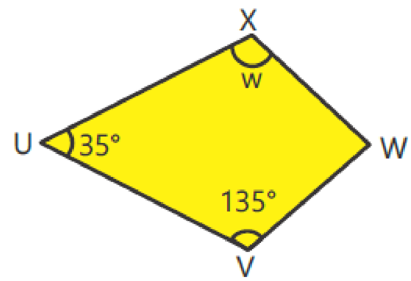
So, <w=135º

The sum of all angles of a quadrilateral is equal to 360º, so we can find <B by subtracting the sum of the other three angles from 360º.

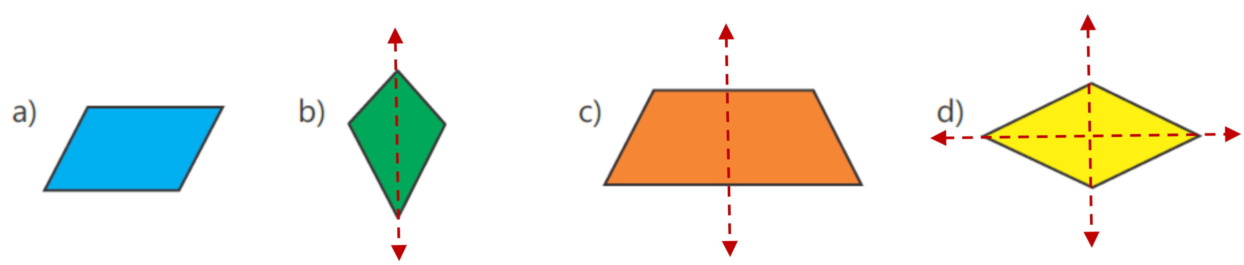
<W = 360º – (135º + 135º + 35º)

<W =360º – 305º

= 55º



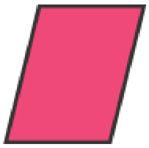
6. Draw the lines of symmetry for each of the following (if any).



No symmetry	1 line of symmetry	1 line of symmetry	2 lines of symmetry

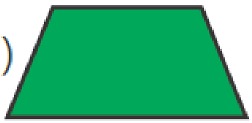
Learning Check 8.2

Identify the regular and irregular polygons among the following. Also, tell if they are concave or convex. Justify your answer.

a) 

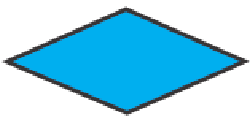
It is an **irregular** polygon as its interior angles and sides are not equal.

It is a **convex** polygon as all angles are less than 180º.

b) 

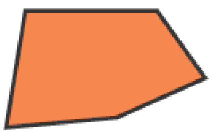
It is an **irregular** polygon as its interior angles and sides are not equal.

It is a **convex** polygon as all angles are less than 180º.

c) 

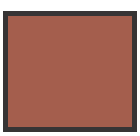
It is an **irregular** polygon as its interior angles are not equal.

It is a **convex** polygon as all angles are less than 180º.

d) 

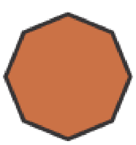
It is an **irregular** polygon as its interior angles and sides are not equal.

It is a **convex** polygon as all angles are less than 180º.

e) 

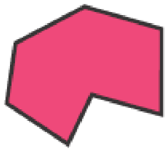
It is a **regular** polygon as its interior angles and sides are equal.

It is a **convex** polygon as all angles are less than 180º.

f) 

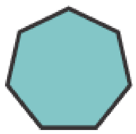
It is a **regular** polygon as its interior angles and sides are equal.

It is a **convex** polygon as all angles are less than 180º.

g) 

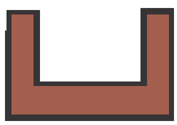
It is an **irregular** polygon as its interior angles and sides are not equal.

It is a **concave** polygon as one of its interior angles is greater than 180º.

h) 

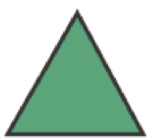
It is a **regular** polygon as its interior angles and sides are equal.

It is a **convex** polygon as all angles are less than 180º.

i) 

It is an **irregular** polygon as its interior angles and sides are not equal.

It is a **concave** polygon as one of its interior angles is greater than 180º.

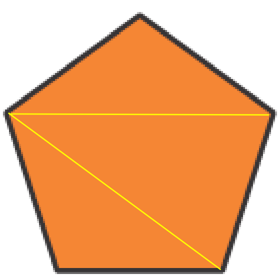
j) 

It is a **regular** polygon as its interior angles and sides are equal.

It is a **convex** polygon as all angles are less than 180º.

Learning Check 8.3

1. **Find the sum of the interior angles of the polygons by counting the triangles formed in the following figures.**

a) 

The shape is a regular pentagon. A pentagon is a polygon with four sides. When we join one of its vertices to all other non-adjacent vertices, we get 3 triangles as shown in the image.

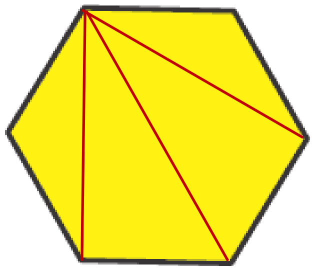
Sum of interior angles of a pentagon $=180^{\circ} \times$number of triangles formed

= 180º x 3

= 540º

So, the sum of interior angles of a pentagon equals 540º.

b)



The shape is a regular hexagon. A hexagon is a polygon with six sides. When we join one of its vertices to all other non-adjacent vertices, we get 4 triangles.

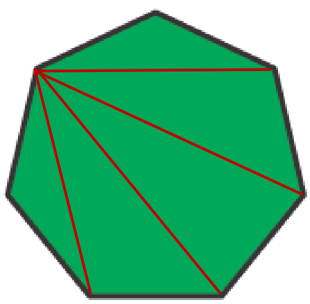
Sum of interior angles of a hexagon=180º ×number of triangles formed

= 180º x 4

= 720º

So, the sum of interior angles of a hexagon equals 720º.

c)



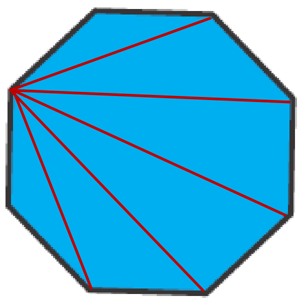
The shape is a regular heptagon. A heptagon is a polygon with seven sides. When we join one of its vertices to all other non-adjacent vertices, we get 5 triangles.

Sum of interior angles of a heptagon=180º ×number of triangles formed

= 180º x 5

= 900º

So, the sum of interior angles of a heptagon equals 900º.

d) 

The shape is a regular octagon. An octagon is a polygon with seven sides. When we join one of its vertices to all other non-adjacent vertices, we get 6 triangles.

Sum of interior angles of a octagon =180º × number of triangles formed

= 180º x 6

= 1080º

So, the sum of interior angles of an octagon equals 1080º.

2. The number of sides of the polygons is given. Find the sum of the interior angles of the polygons using the formula.

a) 10 **b) 13** **c) 16** **d) 21**

Solution:

a) 10

We know that:

Sum of interior angles of a polygon = 180º × (n – 2)

Putting n=10 in the formula, we get:

Sum of interior angles of a 10-sided polygon = 180º × (10 – 2)

= 180º × 8

= 1440º

So, the sum of the interior angles of a 10-sided polygon is 1440º.

b) 13

We know that:

Sum of interior angles of a polygon = 180º × (n – 2)

Putting n=13 in the formula, we get:

Sum of interior angles of a 13-sided polygon = $180^o \times (13 - 2)$

$= 180^o \times 11$

$= 1980^o$

So, the sum of the interior angles of a 13-sided polygon is 1980^o.

b) 16

We know that:

Sum of interior angles of a polygon = $180^o \times (n - 2)$

Putting n=16 in the formula, we get:

Sum of interior angles of a 16-sided polygon = $180^o \times (16 - 2)$

$= 180^o \times 14$

$= 2520^o$

So, the sum of the interior angles of a 16-sided polygon is 2520^o.

b) 21

We know that:

Sum of interior angles of a polygon = $180^o \times (n - 2)$

Putting n=21 in the formula, we get:

Sum of interior angles of a 21-sided polygon = $180^o \times (21 - 2)$

$= 180^o \times 19$

$= 3420^o$

So, the sum of the interior angles of a 21-sided polygon is 3420^o.

3. Find the measure of each interior angle of a regular polygon having:

a) 5 sides **b) 10 sides** **c) 15 sides**

Solution:

a) 5 sides

Each interior angle of a regular polygon= $\frac{\text{Sum of interior angles of the pentagon}}{\text{Number of interior angles of the polygon}}$

Each interior angle of a regular polygon = $\frac{180\text{o} \times (\text{n} - 2)}{\text{Number of interior angles of the polygon}}$

Putting n=5 and the number of angles, we get:

Each interior angle of a regular 5-sided polygon = $\frac{180o \times (5-2)}{5}$

$= \frac{180o \times 3}{5}$

$= \frac{540^0}{5}$

$= 108^o$

b) 10 sides

Each interior angle of a regular polygon= $\frac{\text{Sum of interior angles of the pentagon}}{\text{Number of interior angles of the polygon}}$

Each interior angle of a regular polygon = $\frac{180o \times (n-2)}{\text{Number of interior angles of the polygon}}$

Putting n = 10 and the number of angles, we get:

Each interior angle of a regular 10-sided polygon = $\frac{180o \times (10-2)}{10}$

$= \frac{180o \times 8}{10}$

$= \frac{1440^0}{10}$

$= 144^o$

c) 15 sides

Each interior angle of a regular polygon= $\frac{\text{Sum of interior angles of the pentagon}}{\text{Number of interior angles of the polygon}}$

Each interior angle of a regular polygon = $\frac{180o \times (n-2)}{\text{Number of interior angles of the polygon}}$

Putting n = 15 and the number of angles, we get:

Each interior angle of a regular 15-sided polygon = $\frac{180o \times (15-2)}{15}$

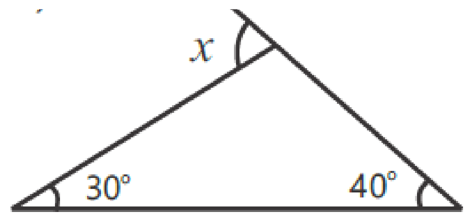
$= \frac{180o \times 13}{15}$

$= \frac{2340^0}{15}$

$= 156^o$

4. Using the exterior angle property of triangles to find the unknown angles.

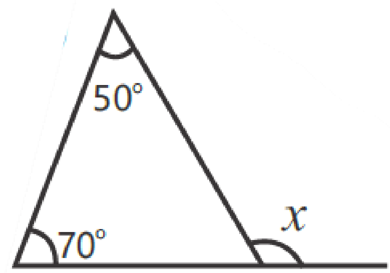
a) According to exterior angle property of triangles, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles.



Applying this property on the given triangles, we can write that:

$x = 30^o + 40^o = 70^o.$

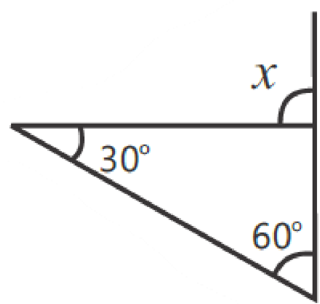
So, $x = 70^o$.



b) By applying exterior angle property of triangles, we get:

$x = 70^o + 50^o = 120^o.$

So, $x = 120^o$.



c) By applying exterior angle property of triangles, we get:

$x = 30^o + 60^o = 90^o.$

So, $x = 90^o$.

5. Find the measure of the exterior angles of a regular 10-sided polygon.

The sum of the exterior angles of any polygon is always 360°. In a regular polygon, each exterior angle has the same measure.

To find the measure of the exterior angles of a regular 10-sided polygon, we can divide 360° by the number of sides:

Measure of an exterior angle of any polygon = 360°/ Number of sides of the polygon.

$= 360^o/10 = 36^o$

Therefore, each exterior angle of a regular 10-sided polygon has a measure of 36^o.

6. **The measures of 4 interior angles of an irregular pentagon are 100°, 108°, 80° and 135°. Find the measure of the 5th interior angle.**

Since it's a pentagon (5 sides), we can find the sum of its interior angles by using formula:

Sum of interior angles of pentagon = $(n-2) \times 180^o$

$= (5-2) \times 180°$

$= 540°$

The sum of the given angles is $100° + 108° + 80° + 135° = 423°$.

Therefore, the measure of the fifth interior angle is:

$540° - 423° = 117°$

So, the measure of the 5th interior angle is 117°.

7. **The measures of six interior angles of an irregular heptagon are 105°, 140°, 90°, 130°, 157° and 115°. Find the measure of the 7th interior angle.**

Since it's a heptagon (7 sides), we can find the sum of its interior angles by using formula:

Sum of interior angles of heptagon = $(n-2) \times 180^o$

$= (7-2) \times 180°$

$= 900°$

The sum of the given angles is $105° + 140° + 90° + 130° + 157° + 115° = 737°$.

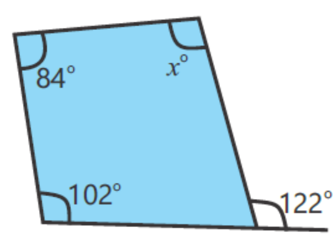
Therefore, the measure of the seventh interior angle is:

$900° - 737° = 163°$.

So, the measure of the 7th interior angle is 163°.

8. **Find the unknown angles.**

a) The three interior angles given are 84^o, 102^o and x^o. To find the value of $x^{o,}$ fist we need to find the measure of fourth angle using the adjacent exterior angle i.e 122^o.



The sum of angles on a straight line equals 180^o.

So the fourth angle = $180^o – 122^o = 58^o$

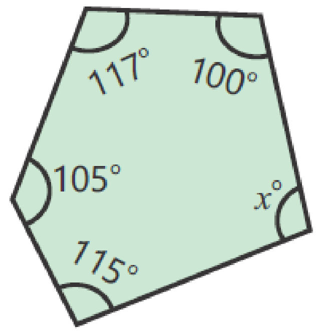
The sum of all angles of a quadrilateral is equal to 360^o, so we can find $<x^o$ by subtracting the sum of the other three angles from 360^o.

$< x^o = 360^o – (102^o + 84^o + 58^o)$

$< x^o =360^o – 244^o$

$= 116^o$

b) Since it's a pentagon (5 sides), we can find the sum of its interior angles by using formula:



Sum of interior angles of pentagon = $(n-2) \times 180^o$

$= (5-2) \times 180°$

$= 540°$

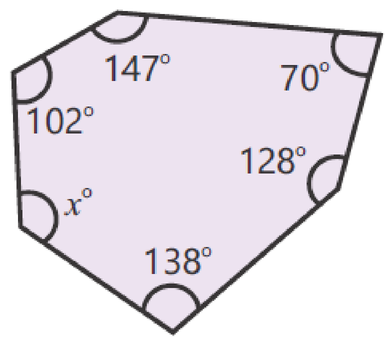
The sum of the given angles is $115° + 105° + 117° + 100° = 437°$.

Therefore, the measure of the fifth interior angle i.e $< x^o$ is:

$< x^o = 540° - 437° = 103°$

So, the measure of the 5th interior angle is 103°.

c) Since it's a hexagon (6 sides), we can find the sum of its interior angles by using formula:



Sum of interior angles of hexagon = $(n-2) \times 180^o$

$= (6-2) \times 180°$

$= 720°$

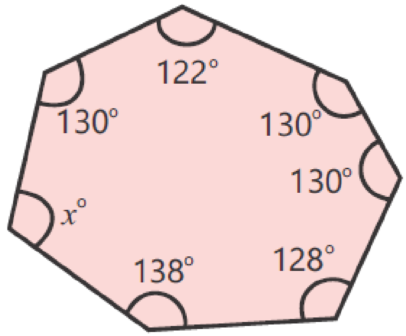
The sum of the given angles is $102° + 147° + 70° + 128° + 138^o = 585°$.

Therefore, the measure of the 6th interior angle i.e $< x^o$ is:

$< x^o = 720° - 585° = 135°$

So, the measure of the 6th interior angle is 135°.

d) Since it's a heptagon (7 sides), we can find the sum of its interior angles by using formula:



Sum of interior angles of heptagon = (n-2) x 180°

= (7-2) x 180°

= 900°

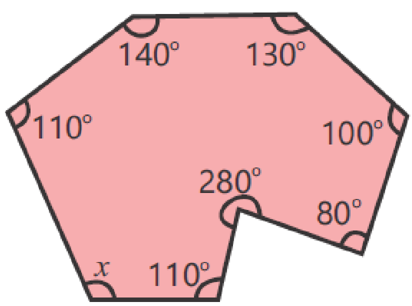
The sum of the given angles is: 130° + 122° + 130° + 130° + 128° + 138°= 778°.

Therefore, the measure of the 7th interior angle i.e < x° is:

< x° = 900° - 778° = 122°

So, the measure of the 7th interior angle is 122°.

e) Since it's a octagon (8 sides), we can find the sum of its interior angles by using formula:



Sum of interior angles of heptagon = (n-2) x 180°

= (8-2) x 180°

= 1080°

The sum of the given angles is: 280° + 80° + 100° + 130° + 140° + 110° + 110° = 950°.

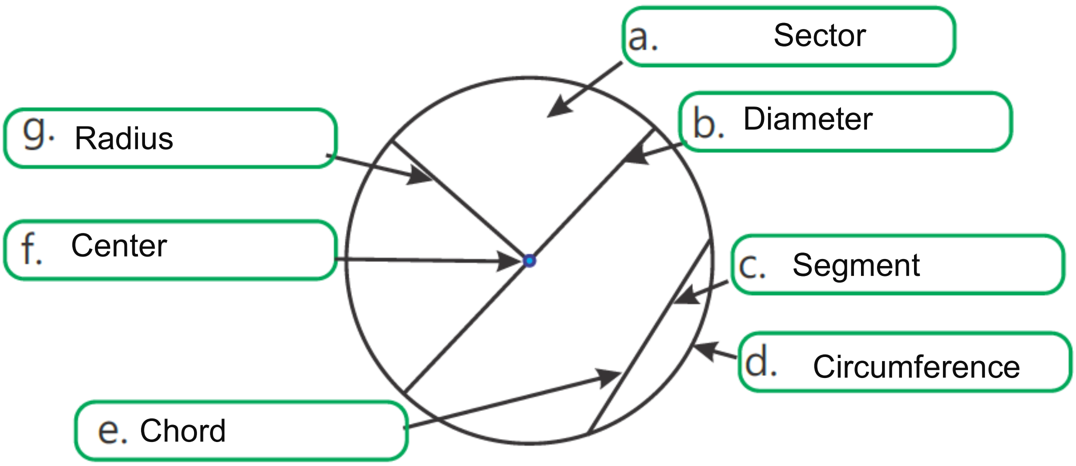
Therefore, the measure of the 8th interior angle i.e < x° is:

< x° = 1080° - 950° = 130°

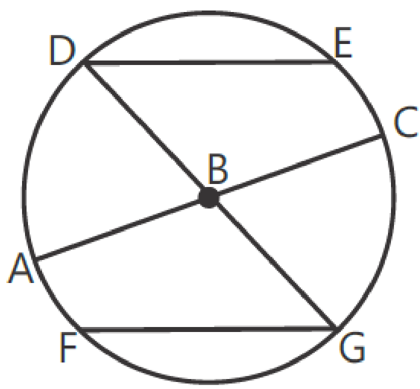
So, the measure of the 8th interior angle is 130°.

Learning Check 8.4

1. **Label the parts of the following circle.**



2. **Consider the following figure:**



a) Name the two chords on this circle that are not diameters.

DE and FG are two chords as they are the line segments joining two points on the boundary of the circle.

b) Name all the radii of the given circle.

AB, BC, BD, BG are the radii as they are showing the distance from the center of the circle to any point of the boundary of the circle.

c) What are AC and DG?

AC and DG are the diameters of the circle as they are passing through the centre of the circle and touching two points on its boundary.

d) If DG is 5 cm long, then how long is DB?

DG is the diameter and DB is the radius which is half the radius in length.

So DB = ½ x DG = ½ x 5 cm = 2.5 cm.

Unit Check

1. Encircle the correct option.

a) A chord passing through the centre of the circle is known as:
i) segment ii) arc iii) radius iv) diameter

b) A polygon that has all of its vertices pointing outward is called:
i) concave polygon ii) convex polygon
iii) symmetric polygon iv) adjacent polygon

c) All sides of a regular polygon are:
i) parallel ii) equal in length iii) not parallel iv) not equal

d) The sum of the interior angles of an octagonal convex polygon is:
i) 720° ii) 900° iii) 1080° iv) 1260°

e) Which of these has no line of symmetry?
i) square ii) parallelogram iii) rectangle iv) rhombus

2. Fill in the blanks:

a) The distance from the _center_ to any point on the circumference of the circle is known as the "radius".

b) A polygon with the least number of sides is called _triangle_.

c) The opposite sides of a _kite_ are not equal in length and only one pair of opposite angles is equal.

d) The opposite angles of a parallelogram are _equal_.

e) The measure of an exterior angle of a regular _hexagon_ is 60°.

3. Find the measure of each interior and exterior angle of a regular 14 sided-polygon

The sum of the interior angles of a polygon with 14 sides=(n-2) x 180°

=(14-2) x 180°

$= 2160^o$

The measure of each interior angle of a regular 14-sided polygon =

$= (n-2) \times 180^o/14$

$=(14-2) \times 180^o/14$

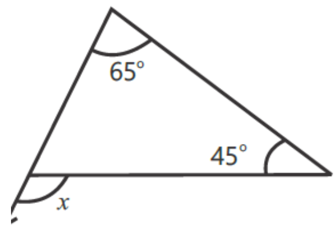
$= 154.28^o$

The exterior angle of a polygon with n sides = $360^o/14$

$= 25.71^o$.

4. **Use the exterior angle property of triangles to find the unknown angles.**

a) According to exterior angle property of triangles, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles.

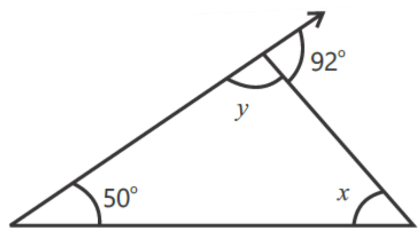


Applying this property on the given triangle, we can write that:

$x = 65^o + 45^o = 110^o$.

So, $x = 110^o$.

b) According to exterior angle property of triangles, the measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles



Applying this property on the given triangles, we can write that:

$50^o + x = 92^o$.

$x = 92^o - 50^o$

$= 42^o$

To find y, we can either subtract 92^o from 180^o or use the angle sum property of triangles:

Sum of all interior angles in a triangle = 180^o

$50^o + x + y = 180^o$

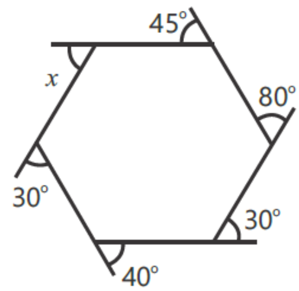
$50^o + 42^o + y = 180^o$

$y = 180^o – (50^o + 42^o)$

$= 180^o – 92^o$

$= 88^o$

5. Find the measure of angle *x* in the given irregular polygon.



Sum of all exterior angles of a polygon=360^o

$45^o + 80^o + 30^o + 40^o + 30^o + x = 360^o$

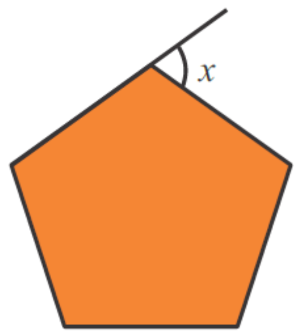
$x = 360^o – (45^o + 80^o + 30^o + 40^o + 30^o)$

$x = 360^o – 225^o$

$x = 135^o$

6. Find the marked angles in each of the following regular polygons.

a) The sum of the exterior angles of any polygon is always 360^o. In a regular polygon, each exterior angle has the same measure.



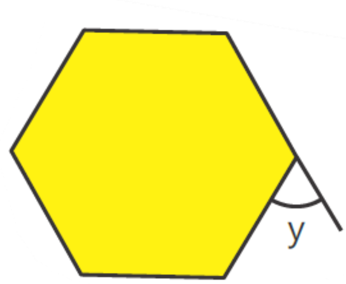
The given polygon is a regular pentagon. To find the measure of the exterior angles of a regular pentagon (5-sided polygon), we can divide 360^o by the number of sides i.e 5:

Exterior angle of any polygon = x = 360°/ Number of sides of the polygon.

$= 360^o/5$

$= 72^o$

b) The sum of the exterior angles of any polygon is always 360^o. In a regular polygon, each exterior angle has the same measure.



The given polygon is a regular hexagon. To find the measure of the exterior angles of a regular hexagon (6-sided polygon), we can divide 360^o by the number of sides i.e 6:

Exterior angle of any polygon = x = 360°/ Number of sides of the polygon.

= 360°/6

= 60°

7. The measures of four interior angles of a pentagon are 115°, 92°, 107° and 83°. Find the measure of the fifth interior angle.

Since it's a pentagon (5 sides), we can find the sum of its interior angles by using formula:

Sum of interior angles of pentagon = (n-2) x 180°

= (5-2) x 180°

= 540°

The sum of the given angles is 115°+ 92° + 107° + 83°= 397°. Therefore, the measure of the 5th interior angle is:

540° - 397° = 143°.

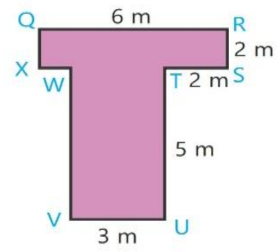
So, the measure of the 5th interior angle is 143°.

Learning check 9.1

1. **Find the perimeter and area of these composite shapes.**

Solution:

a)



To find the area of the given composite figure, divide it into 2 rectangles and name them as A and B.

Find the Perimeter and area of each rectangle separately.

Perimeter of Rectangle A = L + L +L +L + L = 6 + 2 +2 + 6 =16m

Perimeter of Rectangle A = L + L +L +L + L = 5+5+3+3 =16m

Perimeter of Composite figure = Perimeter of Rectangle A+ Perimeter of Rectangle B

= 16 + 16 = 32m

Area of Rectangle = Length x Width

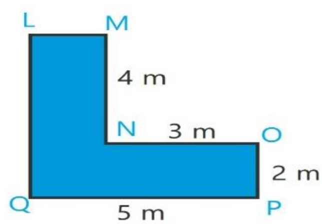
Area of Rectangle A = 6 x 2 = 12m^2

Area of Rectangle B = 5 x 3 = 15m^2

Area of Composite figure = Area of Rectangle A+ Area of Rectangle B

= 12 + 15 = 27m^2

b)



To find the area of the given composite figure, divide it into 2 rectangles and name them as A and B.

Find the Perimeter and area of each rectangle separately.

Perimeter of Rectangle A = L + L +L +L + L = 4 +4 +2 +2 =12m

Perimeter of Rectangle A = L + L +L +L + L = 5 +5 +2 +2 =14m

Perimeter of Composite figure = Perimeter of Rectangle A+ Perimeter of Rectangle B

= 12 + 14 = 26m

Area of Rectangle = Length x Width

Area of Rectangle A = 4 x 2 = 8m^2

Area of Rectangle B = 5 x 2 = 10m^2

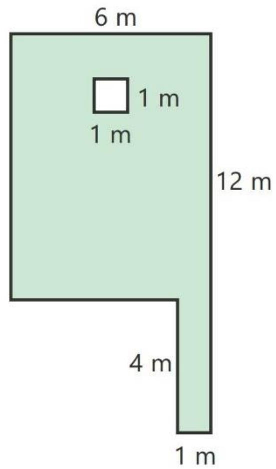
Area of Composite figure = Area of Rectangle A+ Area of Rectangle B

= 8 + 10 = 18m^2

2. Find the area of the shaded region

Solution:

a)



To find the area of the given composite figure, divide it into 2 rectangles and name them as A and B.

Width of Rectangle A = 12 – 4 = 8m

Area of Rectangle = Length x Width

Area of Rectangle A = 8 x 6 = 42m^2

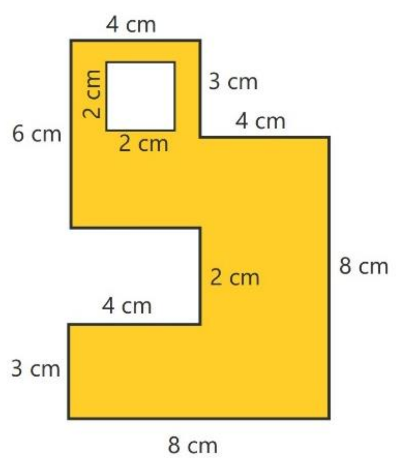
Area of Rectangle B = 1 x 4 = 4m^2

Area of Rectangle which is not shaded = 1 x 1 = 1m^2

Area of Composite figure = Area of Rectangle A+ Area of Rectangle B - Area of Rectangle which is not shaded

= 42 + 4 - 1 = 43m^2

b)



To find the area of the given composite figure, divide it into 2 rectangles and name them as A and B.

Area of Rectangle = Length x Width

Area of Rectangle A = 6 x 4 = 32cm^2

Area of Rectangle B = 4 x 3 = 12cm^2

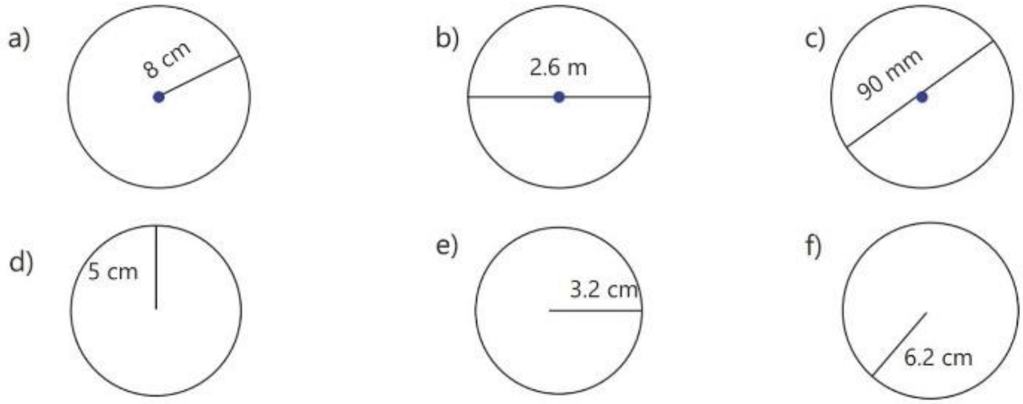
Area of square which is not shaded = 2 x 2 = 4cm^2

Area of Composite figure = Area of Rectangle A+ Area of Rectangle B - Area of square which is not shaded

= 32 + 12 - 4 = 68 – 4 = 64cm^2

Learning Check 9.2

1. Find the circumference and area of each of the following circles if p = 3.14.



a) Solution:

Area of the circle=?

Circumference of a circle =?

We know that:

Area of a circle = πr^2

$= 3.14 \times (8)^2 = 3.14 \times 64 = 200.96cm^2$

Circumference of a circle= $2\Phi r$

$2 \times 3.14 \times 8 = 50.24cm$

b) Solution:

Area of the circle=?

Circumference of a circle =?

We know that:

Area of a circle =πr^2

$= 3.14 \times (2.6)^2 = 3.14 \times 6.76 = 21.23m^2$

Circumference of a circle= $2\Phi r$

$2 \times 3.14 \times 2.6 = 16.3m$

c) Solution:

Area of the circle=?

Circumference of a circle =?

We know that:

Area of a circle = πr^2

$= 3.14 \times (90)^2 = 3.14 \times 8100 = 25434mm^2$

Circumference of a circle= $2\Phi r$

$2 \times 3.14 \times 90 = 565.2mm$

d) Solution

Area of the circle=?

Circumference of a circle =?

We know that:

Area of a circle = πr^2

$= 3.14 \times (5)^2 = 3.14 \times 25 = 78.5cm^2$

Circumference of a circle= $2\Phi r$

$= 2 \times 3.14 \times 5 = 31.4cm$

e) Solution

Area of the circle=?

Circumference of a circle =?

We know that:

Area of a circle = πr^2

$= 3.14 \times (3.2)^2 = 3.14 \times 10.24 = 32.15cm^2$

Circumference of a circle= $2\Phi r$

$= 2 \times 3.14 \times 3.2 = 20.096cm$

f) Solution:

Area of the circle=?

Circumference of a circle =?

We know that:

Area of a circle = πr^2

$= 3.14 \times (6.2)^2 = 3.14 \times 38.44 = 120.70mm^2$

Circumference of a circle= $2\Phi r$

$=2 \times 3.14 \times 6.2 = 38.94mm$

2. Find the circumference and area of the circles having these radii if p = 3.14

a) Solution:

Circumference of a circle= $2\Phi r$

=2 x 3.14 x 2.2 = 13.82cm

Area of a circle = πr^2

= 3.14 x $(2.2)^{2}$ = 3.14 x 4.84 = 15.19cm^2

b) Solution:

Circumference of a circle= $2\Phi r$

=2 x 3.14 x 75 = 471mm

Area of a circle = πr^2

= 3.14 x $(75)^{2}$ = 3.14 x 5625 = 17662.5mm^2

c) Solution:

Circumference of a circle= $2\Phi r$

=2 x 3.14 x 0.12 = 0.7536m

Area of a circle = πr^2

= 3.14 x $(0.12)^{2}$ = 3.14 x 4.84 = 0.0452m^2

d) Solution:

Circumference of a circle= $2\Phi r$

=2 x 3.14 x 15 = 94.2mm

Area of a circle = πr^2

= 3.14 x $(15)^{2}$ = 3.14 x 225 = 706.5mm^2

e) Solution:

Circumference of a circle= $2\Phi r$

=2 x 3.14 x 12.5 = 78.5cm

Area of a circle = πr^2

= 3.14 x $(12.5)^{2}$ = 3.14 x156.25 = 490.65cm^2

3. The diameter of a circular disc is 3.5 cm. Find the circumference of the disc.

Solution:

Diameter = 3.5cm

Circumference =?

As we know

D = C/π

= 3.5/3.14 = 10.99cm

4. **The circumference of a large pizza is 113.04 cm. Find the radius of the pizza.**

Solution:

C = 113.04cm

r =?

As we know

r = c/2π

r = 113.04/2 x 3.14 = 18cm

5. **The radius of a vehicle wheel is 42 cm. Find the area of the wheel in m^2 .**

Solution:

r = 42cm = 0.42m

Area =?

Area of a circle = πr^2

= 3.14 x $(0.42)^2$ = 3.14 x 0.1764 = 0.554m^2

6. **The radius of a circular flower bed is 15 m. Find the cost of iron fencing around the flower bed at the rate of Rs 1050 per meter.**

Solution:

r = 15m

Rate of fencing = 1050/m

As we know,

C = 2Φr

So after putting values

C = 2 x 3.14 x 15

C = 94.25m

Length of fencing =94.25m

Cost of fencing = Length of fencing x Rate/meter

= 94.25 x 1050

= 98912.5Rs

7. **The diameter of a clock is 64 cm. Find the circumference and area of the circular clock.**

Solution:

Diameter of clock = 64cm

Circumference of the circular clock =?

Area of the circular clock =?

Circumference = π d

= 3.14 x 64 = 201.06cm

We know,

Radius = diameter/2

= 64/2 = 32cm

Area of the circular clock = π r^2

= 3.14 x $(32)^2$ = 3216.99cm^2

8. **The circumference of a circular field is 109.9 m. Find the cost of fencing around the field at the rate of Rs 240 per m.**

Solution:

The circumference of a circular field =109.9 m

cost of fencing around the field =?

rate = Rs 24 /m.

Circumference =2πr

Radius = C/2π

= 109.9/2 x 3.14

= 17.47m

cost of fencing around the field = Total length of fencing x Rate/m

= 109.9 x 240 = 26,376Rs

9. **The radius of a circular area is 110 m. Calculate the cost of tiling it at 2 the rate of Rs 980 per m.**

Solution:

The radius of a circular area =110 m

cost of tiling it =?

Rate =?

Area = πr^2

$= 3.14 \times (110)^2 = 37994m^2$

cost of tiling the circular area = total length of circular area x rate of tiling

= 37994m x 980/m

= 37,234,120Rs

10. The radius of a circular well is 4.3 m. Find:

a) The area of the circular base.

b) The cost of polishing the base of the circular well at the rate of Rs 400/$m^{2.}$

Solution:

Radius of circular well = 4.3m

a) Area =?

$A = \pi r^2$

$= 3.14 \times (4.3)^2$

$= 3.14 \times 18.49$

$= 58.06m^2$

b) cost of polishing the base of the circular well =?

Length of circular well = 4.3m

Rate = 400/m^2

cost of polishing the base of the circular well = Length of circular well x Rate

= 58.06m x 400/m^2

= 23235.2Rs

Learning Check 9.3

1. Find the surface area and volume of the rectangular prisms having the given dimensions.

Solution:

a) l = 7 cm, w = 4 cm, h = 3.5 cm

Solution:

Total surface area of the box = 2 (lw + wh + hl)

= 2(7 x 4 + 4 x 3.5 + 3.5 x 7)

= $133cm^2$

Volume of a rectangular prism = length × width × height

= 7 x 4 x 3.5 = $98cm^3$

b) l = 9.4 cm, w = 5.8 cm, h = 6 cm

Solution:

Total surface area of the box = 2 (lw + wh + hl)

= 2x 9.4 x 5.8 + 2 x 5.8 x 6 + 2 x 6x 9.4

= 109.04 + 69.6 + 112.8 = $291.44cm^2$

Volume of a rectangular prism = length × width × height

= 9.4 x 5.8 x 6

= $327.12cm^3$

c) l = 5.1 m, w = 2.7 cm, h = 3.2 m

Solution:

Convert width into m to have equal units

We get w =0.027m

Total surface area of the box = 2 (lw + wh + hl)

= 2 x 5.1 x 0.027 + 2 x 0.027 x 3.2 + 2 x 3.2 x5.1

= 0.275 + 0.17 + 32.64

= $33.085cm^2$

Volume of a rectangular prism = length × width × height

= 5.1 x 0.027 x 3.2 = $0.441cm^3$

d) l = 9 cm, w = 6.5 cm, h = 5 cm

Solution:

Total surface area of the box = 2 (lw + wh + hl)

= 2 x 9 x 6.5 + 2 x 6.5 x 5 + 2 x 5 x 9

= 117+ 65 + 90 = $272cm^2$

Volume of a rectangular prism = length × width × height

= $9 \times 6.5 \times 5 = 292.5cm^3$

2. Find the surface area and volume of the square prisms having the given dimensions.

a) l = w = h = 4.5 m

Solution:

Surface area of a square prism = $6l^2 = 6(4.5)^2 = 121.5m^2$

The volume of a square prism (with square vertical faces) = length × width × height

The volume of a square prism (with square vertical faces) $= l^3$

$= (4.5)^3 = 91.13m^3$

b) l = w = 5 cm, h = 8 cm

Solution:

Surface area of a square prism = $2l^2 + 4lh = 2 \times (5)^2 + 4 \times 5 \times 8 = 210cm^2$

The volume of a square prism (with rectangular vertical faces) $= l^2h$

$= (5)^2 \times 8 = 200cm^3$

c) l = w = 5.6 cm, h = 9.2 cm

Solution:

Surface area of a square prism = = $2l^2 + 4lh = 2 \times (5.6)^2 + 4 \times 5.6 \times 9.2 = 268.8cm^2$

The volume of a square prism (with rectangular vertical faces) = l^2 h

$= (5.6)^2 \times 9.2$

$= 288.512cm^3$

d) l = w = h = 5.9 m

Solution:

Surface area of a square prism = $6l^2 = 6(5.9)^2 = 208.86m^2$

The volume of a square prism (with square vertical faces) = length × width × height

The volume of a square prism (with square vertical faces) $= l^3$

$= (5.9)^3 = 205.379m^3$

3. Find the surface area and volume of the triangular prisms having the given dimensions.

a) S1 = b = 8 cm, S2 = 6.5 cm S3 = 6.5 cm, L = 9.5 cm, h = 6 cm

Solution:

Total surface area = b x h + (s1 + s2 + s3) x L

= 8 x 6 +(8+6.5+6.5) x 9.5

= $247.5cm^2$

Volume = Base Area × Length of Prism

= b × h × L

= ½ x8 x 6 x9.5 = $228cm^3$

b) S1 = b = 7.2 cm, S2 = 5.8 cm, S3 = 6 cm, L = 10 cm, h = 5.8 cm

Solution:

Total surface area = b x h + (s1 + s2 + s3) x L

Total surface area = b x h + (s1 + s2 + s3) x L

= 7.2 x 5.8 +(7.2+5.8+6) x 10

= $231.76cm^2$

Volume = Base Area × Length of Prism

= ½ × b × h × L

= ½ x 7.2 x 5.8 x 10 = $208cm^3$

c) S1 = b = 3 m, S2 = 5 m, S3 = 4 m, L = 7 m, h = 9 cm

Solution:

Total surface area = b x h + (s1 + s2 + s3) x L

= 3 x 9 +(3+5+4) x 7

= $111m^2$

Volume = Base Area × Length of Prism

=1/2 × b × h × L

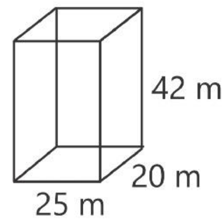
= ½ x 3 x 9 x 7 = $94.5m^3$

d) S1 = b = 5 cm, S2 = 8 cm, S3 = 9 cm, L = 12 cm, h = 8 cm

Solution:

4. Find the surface area and volume of the following prisms.

a)



Solution:

Total surface area of the box = 2 (lw + wh + hl)

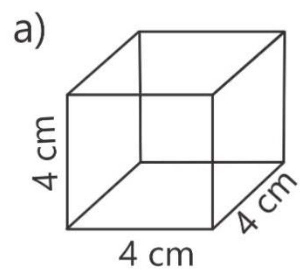
= 2 x 20 x 4 + 2 x 4 x 6 + 2 x 6 x 20

= 448cm^2

Volume of a rectangular prism = length × width × height

= 20 x 4 x 6 = 480cm^3

b)



Solution:

Surface area of a square prism = 6l^2 =6(4)^2 = 96cm^2

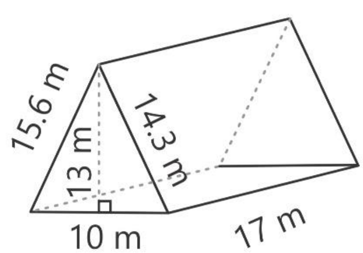
The volume of a square prism (with square vertical faces) = length × width × height

The volume of a square prism (with square vertical faces) =l^3

= (4)^3 = 64cm^3

c)

Solution:



Total surface area = b x h + (s1 + s2 + s3) x L

= 13 x 9 +(13 + 9 + 14.9) x 14

= 633.6cm^2

Volume = Base Area × Length of Prism

= 1/2× b × h × L

= ½ x 13 x 9 x 14 = 814cm^3

5. **Find the total surface area of a notebook whose length is 8 cm, 2 width is 7 cm and height is 5. Express it in m^2.**

Solution:

Length = 8cm

Width = 7cm

Height = 5cm

Total surface area of a notebook = 2(lw +wh+hl)

= 2 x[(8 x 7)+(7 x 5)+(5 x 8)]

= 112 +70+80

= 260cm

We divide area by 10,000 to get answer in m^2

= $0.0262m^2$

6. **A rectangular prism-shaped gift box has a length of 14 cm, a width of 2 cm, and a height of 3 cm, respectively. Find the volume of the gift box.**

Solution:

Length b = 14cm

Width w = 2cm

Height h = 3cm

Volume of the gift box = length × width × height

= 14 x 2 x 3 = $84cm^3$

7. **Find the capacity of a wooden box whose length is 0.4 m, width is 0.3 m, and height is 0.35 m. Convert and express your answer in cubic centimeters.**

Solution

V = l x b x h

Capacity of a wooden box = 0.4 x 0.3 x 0.35

= $0.042m^3$

To convert m^3 to cm^3 we multiply the number by 1,000,000

= $42,000cm^3$

8. Ahad needs to buy cardboard to make a box that is 13 m long, 8 m wide, and 10 m high. How much cardboard does he need to make the box?

Solution:

Surface area of rectangular prism =2lw+2lh +2wh

$= (2 \times 13 \times 8) + (2 \times 13 \times 10) + (2 \times 8 \times 10)$

$= 208 + 260 + 160 = 628m^2$

9. A student paints a wooden box for a school competition with a length of 8 cm, a width of 8 cm, and a height of 6 cm. Find the cost of painting the box at the rate of Rs 25 per cm^2.

Solution:

Surface area of the box = 2lw+2lh +2wh

$= 2 \times 8 \times 8 + 2 \times 8 \times 6 + 2 \times 6 \times 8$

$= 128 + 96 + 96 = 320cm^2$

Dividing the answer by 10,000 we get answer in m^2

$0.032m^2$

cost of painting the box = Surface area of the box x Rate/cm^2

$= 320 \times 25 = 8,000$ Rs

10. The length of a room is 12 m, its width is 5 m and its height is 8 m. Find its total surface area.

Solution:

As we know,

Area of a room = length x width

$= 12 \times 5 = 60m^2$

Area of four vertical faces (two opposite walls,the front and the back wall) is,

Area = 2lh +2hw

$= 2 \times 12 \times 8 + 2 \times 8 \times 5$

$= 192 + 80 = 272m^2$

now we can find the total surface area of the room by adding all six faces

Total surface area = 2 x Area of bottom and top faces + Area of four vertical faces

$= 2 \times 60 + 272 = 120 + 272$

$= 392m^2$

11. Calculate the volume of a triangular prism-shaped roof of a hall if the length of the triangular base is 12 m, the height of the triangular base is 16 m and the length of the roof is 22 m.

Solution:

$V = ½ × b × h × L$

$= 1/2 \times 22 \times 16 \times 12 = 2112m^3$

12. An optics box is triangle prism-shaped having a length 12 cm. The lengths of the sides of the triangular base are 5 cm, 7 cm and 5 cm, respectively. The base length is 7 cm and the height of the base is 2 4 cm. Find the total surface area of the box. Express it in mm^2.

Solution:

Length, b= 12cm

$S_1 = 5cm$

$S_2 = 7cm$

$S_3 = 5cm$

B = 7cm

h = 4cm

total surface area of the box = $b \times h + (S_1+S_2+S_3) \times 7$

$=12 \times 5+(7+5 \times 4) \times 7$

$= 48 + 119 = 167cm^2$

Converting in mm^2 multiply the answer by 100 we get,

$= 16700mm^2$

Learning Check 9.4

1. Find the surface area and volume for the following measurements of cylinders.

Solution:

a) r = 5.3 cm, h = 8 cm

Volume of a cylinder = $\pi r^2 h$

$= 3.14 \times (5.3)^2 \times 8 = 705.62cm^3$

Total surface area of a cylinder = $2\pi r (h + r)$

$= 2 \times 3.14 \times 5.3(8+5.3)$

$= 33.28(13.3) = 442.68cm^2$

b) r = 12 cm, h = 21 cm

Volume of a cylinder = $\pi r^2 h$

$= 3.14 \times (12)^2 \times 21 = 9,495.36cm^3$

Total surface area of a cylinder = $2\pi r (h + r)$

$= 2 \times 3.14 \times 12(21+12)$

$= 925.32cm^2$

c) r = 5.5 m, h = 6.2 m

Volume of a cylinder = $\pi r^2 h$

$= 3.14 \times (5.5)^2 \times 6.2 = 588.907m^3$

Total surface area of a cylinder = $2\pi r (h + r)$

$= 2 \times 3.14 \times 5.5(6.2+5.5)$

$= 196.17m^2$

b) r = 35 mm, h = 55 mm

Volume of a cylinder = $\pi r^2 h$

$= 3.14 \times (35)^2 \times 55 = 211,557.5mm^3$

Total surface area of a cylinder = $2\pi r (h + r)$

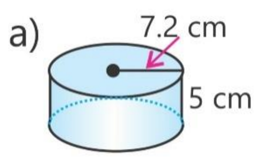
$= 2 \times 3.14 \times 35(55+35)$

$= 7,748mm^2$

2. Find the surface area and volume for the following cylinders.

Solution:

a)



Volume of the cylinder = $\pi r^2 h$

$= 3.14 \times (7.2)^2 \times 5$

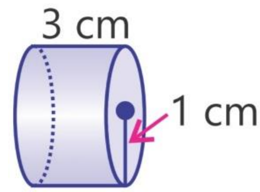
$= 813.89 cm^3$

Surface area of the cylinder = $2\pi r (h + r)$

$= 2 \times 3.14 \times 7.2 (5 + 7.2)$

$= 551.64cm^2$

b)



Volume of the cylinder = πr^2h

$= 3.14x\,(1)^2x\,3$

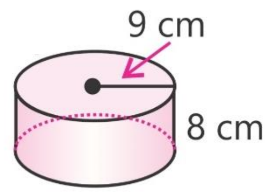
$= 9.42\text{ cm}^3$

Surface area of the cylinder = $2\pi r\,(h + r)$

$= 2x3.14x1\,(3 + 1)$

$=25.12\text{ cm}^2$

c)



Volume of the cylinder = πr^2h

$= 3.14x\,(9)^2x\,8$

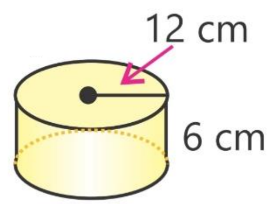
$= 2034.72\text{ cm}^3$

Surface area of the cylinder = $2\pi r\,(h + r)$

$= 2x3.14x9\,(8 + 9)$

$=960.84\text{ cm}^2$

d)



Volume of the cylinder = πr^2h

$= 3.14x\,(12)^2x\,6$

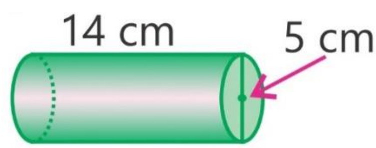
$= 2712.96\text{ cm}^3$

Surface area of the cylinder = $2\pi r\,(h + r)$

$= 2x3.14x12\,(6+ 12)$

$= 1356.48\text{ cm}^2$

e)



$r = d/2$

$r = 5/2$

$r= 2.5$

Volume of the cylinder = πr^2h

$= 3.14x\,(2.5)^2x\,14$

$=274.75\text{ cm}^3$

Surface area of the cylinder = $2\pi r\,(h + r)$

$= 2x3.14x2.5\,(14+2.5)$

$= 259.05\text{ cm}^2$

f)

$r = d/2$

$r = 10/2$

$r= 5$

Volume of the cylinder = πr^2h

$= 3.14 \times (5)^2 \times 20$

$=1570 \text{ cm}^3$

Surface area of the cylinder = $2\pi r (h + r)$

$= 2 \times 3.14 \times 5 (20+ 5)$

$= 785 \text{ cm}^2$

3. **A cylindrical pillar has a radius of 3.5 m and a height of 8.2 m. Find the cost of cementing the curved surface of the pillar at a rate of Rs 850 per m^2.**

Solution:

Radius r = 3.5m

Height h = 8.2m

Rate = $850/m^2$

cost of cementing the curved surface of the pillar =?

Circumference = $2\pi r$

Curved surface area = height x circumference

$= 8.2 \times 2 \times 3.14 \times 3.5 = 182.84 m^2$

cost of cementing the curved surface of the pillar = Curved surface area x rate

$= 182.84 \times 850/m^2$

= 155,414Rs

4. **The circumference of a cylindrical wooden block is 43.96 cm. Find its surface area if its height is 30 cm.**

Solution:

As we know $c = 2\pi r$

So, $r =c/2\pi$

$r = 43.96/2(3.14)$

$r = 43.96/6.28$

$= 7cm^2$

Area $= \pi r^2$

$= 3.14\ (7)^2 = 3.14 \times 49 = 153.94cm^2$

Total surface area of cylinder = Curved surface area + area of both the circular bases

$= 1319.47 + 2 \times 153.94$

$= 1627.35cm^2$

5. **Find the capacity of cylindrical oil tank with a diameter of 50 m and height of 32 m.**

Solution:

capacity of cylindrical oil tank $= \sum r^2h$

as we know, radius = d/2

$r = 50m/2$

$r = 25m$

capacity of cylindrical oil tank $= \pi r^2h$

$= 3.14 \times (25)^2 \times 32 = 62800m^3$

6. **The liquid soap can is a cylinder with a height of 12 cm and a diameter of 8 cm^3. Calculate the volume of the soap can. Express it in mm3**

Solution:

volume of the soap can =?

$= \pi r^2h$

To find r

radius = d/2

radius = 8/2 = 4cm

volume of the soap can $= 3.14 \times 4 \times 4 \times 12 = 602.88cm^3$

expressing in mm^3

$= 602880mm^3$

7. **Each of the 6 cylindrical pillars of a masjid has a circumference of 18 m and a height 3 of 22 m. Find the cost of refilling them with cement at the rate of Rs 1045/m$^{3.}$**

Solution:
Surface area of each cylindrical pillar = 2 πrh + 2 πr

As we know c = 2 ∑r

18 = 2 πr

r = 18/2π

r = 28.26m

Surface Area = 2 πrh + 2 πr^2

= 2 x 3.14 x 28.26 x22 + 2 x 3.14 x (28.26)^2

= 5192.85m^2

cost of refilling them = 1045 x 5192.85 = 5426,528.25Rs

cost of refilling 6 pillars = 5426,528.25 x 6 = 32,559,169.5Rs

8. **How much ice cream can be contained in a cylindrical pail of 15 cm^3 height and 10 cm radius? Express it in m^3.**

Solution:

h = 15cm

r = 10cm

V= πr^{2}h

V = 3.14 x (10)^2 x 15

V = 3.14 x 100 x 15 = 4710cm^3

Dividing by 1000,000 we get volume in m^3

V = 0.00471m^3

Unit Check – 9

1. **Encircle the correct option.**

a) iii

b) iii

c) ii

d) iv

e) ii

2. Fill in the blanks.

a) 6.25cm

b) Squared

c) $1540cm^3$

d) 301.72

e) 21.98

3. Find the area and circumference of the given circles.

Solution:

a) r =28mm

C = 2πr

= 2 x 3.14 x 28 = 175.84mm

Area = πr^2

= $3.14 \times (28)^{2} = 2461.76mm^2$

b) d = 10cm

r = d/2 = 10/2 = 5cm

Area = πr^2

= $3.14 \times (5)^{2} = 78.5cm^2$

C = 2πr

= 2 x 3.14 x 5 = 31.4cm

c) d = 2.5m

r = d/2 = 2.5/2 = 1.25m

Area = πr^2

= $3.14 \times (1.25)^{2} = 4.90m^2$

C = 2πr

= 2 x 3.14 x 1.25 = 7.85m

d) R = 4.2cm

Area = πr^2

= $3.14 \times (4.2)^{2} = 55.39m^2$

C = 2πr

= 2 x 3.14 x 4.2 = 26.38m

4. Find the surface area and volume for the following measurements of cylinders.

Solution:

a) r = 7.3 cm, h = 10 cm

Total surface area of a cylinder = 2πr (h + r)

= 2 x 3.14 x 7.3(10 + 7.3)

= 45.84(10+7.3)

= 458.4 + 334.632

= 793.10cm^2

Volume of cylinder = πr^2 h

= 3.14 x (7.3)^2 x 10

= 1673.306cm^3

b) r = 6 m, h = 11 m

Total surface area of a cylinder = 2πr (h + r)

= 2 x 3.14 x 6(11 + 6)

= 45.84(11+6)

= 504.24 + 275.04

= 779.28m^2

Volume of cylinder = πr^2 h

= 3.14 x (6)^2 x 11

= 1243.44m^3

c) d = 14.5 cm, h = 13 cm

d = 2r, r = d/2, r = 14.5/2 = 7.25cm

Total surface area of a cylinder = 2πr (h + r)

= 2 x 3.14 x 14.5(13 + 7.25)

= 45.84(13+ 7.25)

= 591.89 + 330.09

= 921.98cm^2

Volume of cylinder = πr^2 h

= 3.14 x (7.25)^2 x 13

= 2145.60cm^3

d) d = 24 mm, h = 15 mm

Surface Area of Cylinder = $2\pi r(r + h)$

Volume of Cylinder = $\pi r^2 h$

where r is the radius of the base of the cylinder, h is the height of the cylinder, and π (pi) is a mathematical constant approximately equal to 3.14159.

Given that d = 24 mm, we can first calculate the radius (r) of the cylinder by dividing the diameter by 2:

r = d/2 = 24 mm / 2 = 12 mm

Now we can plug in the values for r and h into the formulas to find the surface area and volume of the cylinder.

Surface Area of Cylinder = $2\pi r(r + h)$

= 2 * 3.14159 * 12 mm * (12 mm + 15 mm)

= 2 * 3.14159 * 12 mm * 27 mm

= 6.28318 * 12 mm * 27 mm

= 2036.168 mm^2 (rounded to three decimal places)

Volume of Cylinder = $\pi r^2 h$

= 3.14159 * $(12\ mm)^2$ * 15 mm

= 3.14159 * 144 mm^2 * 15 mm

= 6782.304 mm^3 (rounded to three decimal places)

So, the surface area of the given cylinder is approximately 2036.168 mm^2 and the volume is approximately 6782.304 mm^3.

e) d = 2r , r = d/2 , r = 24/2 = 12mm

Total surface area of a cylinder = $2\pi r\ (h + r)$

= 2 x 3.14 x 7.3(15 +12)

= 45.84(15+12)

= 1130.4 + 904.32

= 2034.72mm^2

Volume of cylinder = πr^2 h

= 3.14 x $(12)^2$ x 15

= 6782.4mm^3

5. Find the surface area and volume of the given prisms.

a) Solution:

Surface area of a square = $6l^2$

As Length = 4cm

= 6 $(4)^2$ = 96cm^2

Volume = l x lx l

= 4 x 4 x 4 = 64cm^3

b) Solution:

Given l = 25m, w = 20m, h = 42m

Surface area of Rectangle = 2 (lw + wh + hl)

= 2 (25 x 20 +20 x 42 + 42 x 25)

= 1000 + 1680 + 2100

= 4780m^2

Volume of Rectangle = l x w x h

= 25 x 20 x 42

= 21000m^3

c) Solution:

Given b = 10m, h = 13m, L = 17m , S_1 = 13 , S_2 =10 , S_3 = 14.3

Surface area of triangular prism = b x h + (s1 + s2 + s3) x L

= 10 x 13 +(13 +10+14.3) x 17

= 396.1m^2

Volume of Rectangle = ½ x b x h x L

= ½ x 10 x 13 x 17

= 1105m^3

6. Find the area of the following figures.

a) Solution:

Surface area of rectangle A = Length x Width

$= 6 \times 2 = 12cm^2$

Surface area of rectangle B = Length x Width

$= 3 \times 5 = 15\ cm^2$

Area of the composite shape = Surface area of rectangle A+ Surface area of rectangle B

$= 12cm^2 + 15cm^2 = 27cm^2$

b) Solution:

Surface area of rectangle A = Length x Width

$= 8 \times 2 = 16cm^2$

Surface area of rectangle B = Length x Width

$= 5 \times 2 = 10\ cm^2$

Area of the composite shape = Surface area of rectangle A+ Surface area of rectangle B

$= 16cm^2 + 10cm^2 = 26cm^2$

7. A right square prism-shaped ice block with rectangular vertical sides has a height of 20 cm. Find the total surface area of the block if the length of its base is 8 cm.

Solution:

Given data: h = 20cm

Length of base = 8cm

total surface area of the block =?

Total surface area of block = adding the areas of all six faces

Area of Top and bottom faces of square $= 2l^2 = 2 \times (8)^2 = 128cm^2$

Four vertical faces are rectangles so their area is

Area of 4 vertical faces are rectangles = 4 x 8cm x 20cm = $640cm^2$

Adding these areas together we get the total surface area of the block

Total surface area of the block = $128cm^2 + 640cm^2 = 768cm^2$

8. A hall with a length of 8 m is in a triangle prism shape. Find the total surface area of the hall if the sides' lengths are 6 m, 6 m and 4 m, respectively and the total surface area of the bases is $20m^2$.

Solution:

Given data: L = 8m S1= 6cm S2 =6cm S3 =4cm

Total surface area of bases = 20m^2

So each triangular base = 10m^2

Area of first lateral face of hall =6m x 8m =48m^2

Area of second lateral face of hall =6m x 8m =48m^2

Area of third lateral face of hall =4m x 8m =32m^2

Total area of lateral faces = sum of all the lateral faces

= 48 +48+32 =128m^2

As we know total area of triangular bases = 20m^2

Total surface area of hall = Total area of lateral faces+ total area of triangular bases

Total surface area of hall = 128m^2 + 20m^2

= 148m^2

9. **The pillars in a school are cylindrical. Find the cost of filling concrete in a pillar at the rate of Rs 800 per m^3 with radius and height 3.8 m and 9.2 m, respectively.**

Solution:

Given data:

Radius = 3.8m

Height = 9.2m

Cost of filling concrete in a pillar =?

Rate = 800/m^3

As we all know V = $\pi r^2 h$

Volume =3.14 x (3.8)^{2}x 9.2

= 416.754m^3

Cost = Volume x Rate

= 416.754m^3 x 800/m^3

Cost of filling concrete in a pillar = 333403.2Rs

Learning Check 10.1

1. Convert the following units.

a) 25 km to m b) 35 km 600 m to m c) 20 m to cm

d) 6 m 14 cm to cm e) 740 cm to m f) 362 cm to m and cm

g) 12 cm to mm h) 27 cm 8 mm to mm i) 450 mm to cm and mm

j) 55 mm to cm k) 6 cm to mm l) 280 m to cm

m) 512 cm 20 mm to mm n) 41 cm to m o) 1250 m to km

Solution:

a) To convert km to m, we multiply by 1000.

$25 \text{ km} = 25 \times 1000 \text{ m} = 25000 \text{ m}$

b) First, we convert 35 km to meters (as in part a), and then we add the 600 m.

$35 \text{ km } 600 \text{ m} = (35 \times 1000) \text{ m} + 600 \text{ m} = 35600 \text{ m}$

c) To convert m to cm, we multiply by 100.

$20 \text{ m} = 20 \times 100 \text{ cm} = 2000 \text{ cm}$

d) To convert cm to mm, we multiply by 10.

$6 \text{ m } 14 \text{ cm} = (6 \times 100 + 14) \text{ cm} = 614 \text{ cm} = 614 \text{ mm}$

e) To convert cm to m, we divide by 100.

$740 \text{ cm} = 740 \div 100 \text{ m} = 7.4 \text{ m}$

f) To convert cm to m, we divide by 100. The whole part is the number of meters, and the remainder is the number of centimeters.

$362 \text{ cm} = 362 \div 100 \text{ m} = 3.62 \text{ m}$ (and $0.62 \text{ m} \times 100 \text{ cm/m} = 62 \text{ cm}$)

g) To convert cm to mm, we multiply by 10.

$12 \text{ cm} = 12 \times 10 \text{ mm} = 120 \text{ mm}$

h) To convert cm to mm, we multiply by 10. Then we add the millimeters.

$27 \text{ cm } 8 \text{ mm} = (27 \times 10 + 8) \text{ mm} = 278 \text{ mm}$

i) To convert mm to cm, we divide by 10. To convert mm to mm and cm, we divide by 10 and take the whole part as the number of cm, and the remainder as the number of mm.

$450 \text{ mm} = 45 \text{ cm} + 0 \text{ mm} = 45 \text{ cm } 0 \text{ mm}$

j) To convert mm to cm, we divide by 10.

55 mm = 55 ÷ 10 cm = 5.5 cm

k) To convert cm to mm, we multiply by 10.

6 cm = 6 × 10 mm = 60 mm

l) To convert m to cm, we multiply by 100.

280 m = 280 × 100 cm = 28000 cm

m) To convert mm to mm and cm, we divide by 10 and take the whole part as the number of cm, and the remainder as the number of mm.

512 cm 20 mm = 5120 mm + 20 mm = 5140 mm

n) To convert cm to m, we divide by 100.

41 cm = 41 ÷ 100 m = 0.41 m

o) To convert m to km, we divide by 1000.

1250 m = 1250 ÷ 1000 km = 1.25 km

Learning Check 10.2

1. Convert the times to the 24-hour format.

a) 10:00 a.m. b) 3:20 p.m. c) 5:28 p.m. d) 12:00 p.m.

e) 8:32 a.m. f) 12:40 p.m. g) 1:30 a.m. h) 3:45 p.m.

Solution:

a) To convert 10:00 a.m. to 24-hour format, we simply leave the hours as is (since it is before noon) and add a 0 before the minutes.

10:00 a.m. = 10:00

b) To convert 3:20 p.m. to 24-hour format, we add 12 to the hours (since it is in the afternoon) and leave the minutes as is.

3:20 p.m. = 15:20

c) To convert 5:28 p.m. to 24-hour format, we add 12 to the hours (since it is in the afternoon) and leave the minutes as is.

5:28 p.m. = 17:28

d) To convert 12:00 p.m. to 24-hour format, we simply add 12 to the hours (since it is noon).
12:00 p.m. = 12:00

e) To convert 8:32 a.m. to 24-hour format, we leave the hours as is (since it is before noon) and add a 0 before the minutes.
8:32 a.m. = 08:32

f) To convert 12:40 p.m. to 24-hour format, we simply add 12 to the hours (since it is afternoon).
12:40 p.m. = 12:40

g) To convert 1:30 a.m. to 24-hour format, we simply leave the hours as is (since it is before noon) and add a 0 before the minutes.
1:30 a.m. = 01:30

h) To convert 3:45 p.m. to 24-hour format, we add 12 to the hours (since it is in the afternoon) and leave the minutes as is.
3:45 p.m. = 15:45

2. Convert the times to the 12-hour format.

a) 13:37 b) 06:40 c) 11:48 d) 09:15
e) 12:00 f) 00:35 g) 01:00 h) 04:50

Solution:

a) To convert 13:37 to 12-hour format, we subtract 12 from the hours and add "p.m." since it is in the afternoon.
13:37 = 1:37 p.m.

b) To convert 06:40 to 12-hour format, we leave the hours and minutes as is and add "a.m." since it is before noon.
06:40 = 6:40 a.m.

c) To convert 11:48 to 12-hour format, we leave the hours and minutes as is and add "a.m." since it is before noon.
11:48 = 11:48 a.m.

d) To convert 09:15 to 12-hour format, we leave the hours and minutes as is and add "a.m." since it is before noon.
09:15 = 9:15 a.m.

e) To convert 12:00 to 12-hour format, we leave the hours as is and add "p.m." since it is noon.
12:00 = 12:00 p.m.

f) To convert 00:35 to 12-hour format, we add 12 to the hours and add "a.m." since it is in the early morning.
00:35 = 12:35 a.m.

g) To convert 01:00 to 12-hour format, we add 12 to the hours and add "a.m." since it is in the early morning.
01:00 = 1:00 a.m.

h) To convert 04:50 to 12-hour format, we add 12 to the hours and add "p.m." since it is in the afternoon.
04:50 = 4:50 a.m.

3. Convert the following units of time.

a) 3 hours to min b) 4 hours 25 min to min c) 45 minutes to sec

d) 15 min 30 sec to sec e) 1 hour 10 min to min f) 6 hours to min

g) 9 hours 10 min to min h) 50 min 20 sec to sec

Solution:

a) To convert 3 hours to minutes, we multiply by 60 since there are 60 minutes in 1 hour.

3 hours = 3 × 60 = 180 minutes

b) To convert 4 hours 25 minutes to minutes, we multiply the hours by 60 and add the remaining minutes.

4 hours 25 minutes = (4 × 60) + 25 = 265 minutes

c) To convert 45 minutes to seconds, we multiply by 60 since there are 60 seconds in 1 minute.

45 minutes = 45 × 60 = 2700 seconds

d) To convert 15 minutes 30 seconds to seconds, we multiply the minutes by 60 and add the remaining seconds.

15 minutes 30 seconds = (15 × 60) + 30 = 930 seconds

e) To convert 1 hour 10 minutes to minutes, we add the hours (converted to minutes) to the remaining minutes.

1 hour 10 minutes = (1 × 60) + 10 = 70 minutes

f) To convert 6 hours to minutes, we multiply by 60.

6 hours = 6 × 60 = 360 minutes

g) To convert 9 hours 10 minutes to minutes, we multiply the hours by 60 and add the remaining minutes.

9 hours 10 minutes = (9 × 60) + 10 = 550 minutes

h) To convert 50 minutes 20 seconds to seconds, we multiply the minutes by 60 and add the remaining seconds.
50 minutes 20 seconds = (50 × 60) + 20 = 3020 seconds

4. **Madiha completes a science project in 4 hours. In how many minutes does she complete the project?**
To convert hours to minutes, we multiply by 60 since there are 60 minutes in 1 hour.
4 hours = 4 × 60 = 240 minutes
Therefore, Madiha completes the science project in 240 minutes.
5. **Every day, a traffic sergeant works for 540 minutes. How many hours per day does he work?**
To convert minutes to hours, we divide by 60 since there are 60 minutes in 1 hour.
540 minutes = 540 ÷ 60 = 9 hours
Therefore, the traffic sergeant works for 9 hours per day.
6. **Ali finishes reciting surah Fatiha in 22 seconds. How many minutes does he take?**
To convert seconds to minutes, we divide by 60 since there are 60 seconds in 1 minute. 22 seconds = 22 ÷ 60 = 0.37 minutes (rounded to two decimal places)
Therefore, Ali takes approximately 0.37 minutes to finish reciting surah Fatiha.
7. **The bus leaves from City A for City B at 8:40 p.m. It takes 2 hours and 3 minutes to reach City B. What is the arrival time of the bus?**
Solution:
Departure time= 8:40 p.m. Journey time = 2 hours 3 min Arrival time=?
First, add 2 hours to 8:40 p.m.. It is 10:40 p.m. Now add 3 minutes to it. Its 10:43 pm.
So, the arrival time is 10:43 pm.
8. **Find the arrival time of a cyclist at his destination if the departure time is 10:50 a.m. and the journey time is 2 hours and 12 minutes.**
Solution:
Departure time= 10:50 a.m. Journey time = 2 hours 12 min Arrival time=?

First, add 2 hours to 10:50 a.m. It is 12:50 p.m. Next add 12 min to it. It becomes 1:02 p.m.

So, the arrival time is 1:02 p.m.

9. **Find the departure time if the arrival time of a train is 6:30 a.m. and the journey time is 7 hours and 10 minutes.**

   Solution: Arrival time= 6:30 a.m. Journey time = 7 hours 10 min Departure time=?

   First, count 7 hours backward from 6:30 a.m. Its 11:30 p.m. (the previous day). Now subtract 10 min from 11:30 p.m. Its 11:20 pm.

   So, the departure time is 11:20 p.m

10. **Asim began his journey from city A to city B at 8:25 a.m. on Monday and arrived at 6:08 p.m. on Tuesday. How much time did he spend travelling?**

    Solution:

    Departure time= 8:25 a.m. on Monday Arrival time= 6:08 p.m. on Tuesday Total travel time=?

    First, count the time from 8:25 a.m. Monday to 8:25 a.m. Tuesday. Its **24 hours**.

    Now count from 8:25 a.m. to 5: 25 p.m. Its **9 hours**.

    Now count from 5: 25 to 6 pm. Its **35 min**.

    Finally count from 6 pm to 6:08 pm. Its **8 mins**.

    Add all the durations: 24 hrs + 9 hrs + 35 min + 8 min = 33 hrs 43 min.

    Asim spent 33 hours and 43 minutes travelling.

11. **Maheen went from Peshawar to Jhelum in 5 hours and 5 minutes. Find her arrival time if she started her journey at 9:30 a.m.**

    Maheen's journey time is 5 hours and 5 minutes. We need to add this time to her departure time of 9:30 a.m.

    Add 5 hours to the departure time: 09:30 + 5 hrs = 2: 30 p.m.

    Next add 5 min to it: 2:30 p.m. + 5 min = 2:35 p.m.

    Therefore, Maheen arrived in Jhelum at 2:35 p.m.

12. **If the departure time is 9:10 p.m. and the arrival time is 2:40 a.m., what is the journey time**?

The departure time is 9:10 p.m. and the arrival time is 2:40 a.m. We need to find the journey time between these two times.

Calculate time from 9:10 p.m. to 2:10 a.m.(next day). Its **5 hours**.

Next count from 2:10 a.m. to 2: 40 a.m. Its **30 min**.

Therefore, the journey time is 5 hours and 30 minutes.

Learning Check 10.3

1. **Complete the following table.**

Sr. No	Total distance covered	Total time	Average speed
a)	115 km	4 hours	
b)	590 km		105 km/hr
c)		5.5 hours	120 km/hr

a) Total distance covered = 115 km

Total time = 4 hours

Average speed = ?

To find the average speed, divide the total distance by the total time:

Average speed = Total distance / Total time

Average speed = 115 km / 4 hours

Average speed = 28.75 km/hour

Therefore, the average speed is 28.75 km/hour.

b) Total distance covered = 590 km

Total time = ?

Average speed = 105 km /hr

To find the total time, divide the total distance by the average speed:

Total time = Total distance / Average speed

Total time = 590 km / 105 km/hour

Total time = 5.619 hours

Therefore, the total time taken to cover 590 km at an average speed of 105 km/hour is approximately 5.619 hours.

c) Total distance covered = ?

Total time = 5.5 hours

Average speed = 120 km/hr

To find the total distance covered, multiply the average speed by the total time:

Total distance = Average speed x Total time

Total distance = 120 km/hour x 5.5 hours

Total distance = 660 km

Therefore, the total distance covered at an average speed of 120 km/hour for 5.5 hours is 660 km.

2. Convert the units of speed.

a) 8 km/hr to m/sec b) 160 m/sec to km/hr c) 120 m/sec to km/hr

d) 200 km/hr to m/sec e) 800 m/sec to km/hr f) 570 km/hour to m/sec

Solution:

a) 30 m/sec to km/hr:

Speed in km/hr = (30/1000) x 3600 = 108 km/hr

b) 75 m/sec to km/hr:

Speed in km/hr = (75/1000) x 3600 = 270 km/hr

c) 120 m/sec to km/hr:

Speed in km/hr = (120/1000) x 3600 = 432 km/hr

d) 15 m/sec to km/hr:

Speed in km/hr = (15/1000) x 3600 = 54 km/hr

e) 250 m/sec to km/hr:

Speed in km/hr = (250/1000) x 3600 = 900 km/hr

f) 570 m/sec to km/hr:

Speed in km/hr = (570/1000) x 3600 = 2052 km/hr

3. **Muneeb goes to work daily by car. He covered a distance of 40 km in 30 minutes. Find the average speed of the car.**

Average speed = total distance / total time

Total distance covered = 40 km

Total time taken = 30 minutes = 0.5 hours

Average speed = 40 km / 0.5 hours = 80 km/hour

Therefore, the average speed of the car is 80 km/hour.

4. **Fatima travels from Makkah to Madinah by car. She covers a distance of 450.4 km in 4 hours and 12 minutes. What is the average speed of the car in m/sec?**

Convert the time to minutes.

4 hours and 12 minutes = (4 x 60) + 12 = 252 minutes

Convert the time to seconds.

252 minutes = 252 x 60 sec = 15120 sec

Convert the distance to meters.

450.4 km = 450400 m

Therefore, the average speed of Fatima's car can be calculated as:

Speed = Distance / Time

=450400 ÷ 15120

= 29.8 m/s.

So, the average speed of Fatima's car from Makkah to Madinah is 29.8 meters per second.

5. **Ali covered a distance of 130 km by local bus at an average speed of 65 km/hour. Find the total time taken by Ali to cover the distance.**

Average speed = total distance / total time

Total distance covered = 130 km

Average speed = 65 km/hour

Total time taken = total distance / average speed

Total time taken = 130 km / 65 km/hour

Total time taken = 2 hours

Therefore, the total time taken by Ali to cover the distance is 2 hours.

6. **A train's average speed is 80 km/hr. Find the total distance covered by the train in 5 hours.**

Total distance covered = average speed x time taken

Average speed = 80 km/hour

Time taken = 5 hours

Total distance covered = 80 km/hour x 5 hours = 400 km

Therefore, the total distance covered by the train in 5 hours is 400 km.

7. **Usama goes for a prayer, and the distance between his home and the masjid is 1200 m.Find his average speed (m/sec) in 20 minutes.**

Average speed = total distance / total time

Total distance covered = 1200 m

Total time taken = 20 minutes = 20 x 60 seconds = 1200 seconds

Average speed = 1200 m / 1200 seconds = 1 m/sec

Therefore, Usama's average speed is 1 m/sec.

8. **Adnan covered a distance of 4200 m from Point A to Point B in 30 minutes. Find his average speed in m/sec.**

Average speed = total distance / total time

Total distance covered = 4200 m

Total time taken = 30 minutes = 30 x 60 seconds = 1800 seconds

Average speed = 4200 m / 1800 seconds = 2.33 m/sec (approx)

Therefore, Adnan's average speed is 2.33 m/sec.

UNIT CHECK

1. Encircle the correct option.

a) 10 cm =

i) 10 mm ii) $\frac{1}{10}$ mm iii) 100 mm iv) $\frac{1}{100}$ mm

b) 180 km/hr =

i) 25 m/sec ii) 50 m/sec iii) 100 m/sec iv) 150 m/sec

c) 30 min =

i) 0.5 sec ii) 60 sec iii) 1200 sec iv) 1800 sec

d) Faiz starts his journey from Lahore to Peshawar at 3:00 a.m. What is his arrival time in Peshawar if his journey time is 5 hours and 45 minutes?

i) 8:30 a.m. ii) 8:45 a.m. iii) 9:30 a.m. iv) 9:45 p.m.

e) What will be the total distance covered by a car in 5 hours if the speed of the car is 120 km/hr?

i) 24 km ii) 120 km iii) 600 km iv) 6000 m

2. Fill in the blanks.

a) 500 m = __0.5__ km

b) There are __1200__ centimetres in 12 metres.

c) 20 minutes and 30 seconds equals __1230__ seconds.

d) If the arrival time is 10:20 a.m. and the journey time is 2 hours and 45 minutes, then the departure time is __7:35 a.m.__.

e) The average speed of a bike is 10 m/sec. The time taken by riding a bike is __100 sec__ if the total distance is 1000 m.

3. Convert the following units of distance.

a) 24 km to m b) 105 km and 60 m to m c) 500 m to km

d) 200 m to cm e) 800 cm to m f) 15 m and 30 cm to cm

g) 35 cm to mm h) 20 cm and 100 mm to mm i) 15 mm to cm

Solution:

a) 24 km to m:

1 km = 1000 m

Therefore, 24 km = 24 x 1000 = 24000 m.

b) 105 km and 60 m to m:

First, we convert 105 km to m:

1 km = 1000 m

Therefore, 105 km = 105 x 1000 = 105000 m.

Then, we add the additional 60 m:

105000 m + 60 m = 105060 m.

c) 500 m to km:

1 km = 1000 m

Therefore, 500 m = 500 / 1000 = 0.5 km.

d) 200 m to cm:

1 m = 100 cm

Therefore, 200 m = 200 x 100 = 20000 cm.

e) 800 cm to m:

1 m = 100 cm

Therefore, 800 cm = 800 / 100 = 8 m.

f) 15 m and 30 cm to cm:

15 m = 1500 cm (as 1 m = 100 cm)

30 cm = 30 cm

Therefore, 15 m and 30 cm = 1500 cm + 30 cm = 1530 cm.

g) 35 cm to mm:

1 cm = 10 mm

Therefore, 35 cm = 35 x 10 = 350 mm.

h) 20 cm and 100 mm to mm:

20 cm = 200 mm (as 1 cm = 10 mm)

100 mm = 100 mm

Therefore, 20 cm and 100 mm = 200 mm + 100 mm = 300 mm.

i) 15 mm to cm:

1 cm = 10 mm

Therefore, 15 mm = 15 / 10 = 1.5 cm.

4. Convert the units of time.

a) 3 hours to min b) 10 hours and 40 min to min c) 120 min to hours

d) 30 min to sec e) 45 min and 30 sec to sec f) 240 sec to min

Solution:

a) 3 hours to min:

1 hour = 60 min

Therefore, 3 hours = 3 x 60 = 180 min

b) 10 hours and 40 min to min:

10 hours = 10 x 60 = 600 min

Therefore, 10 hours and 40 min = 600 + 40 = 640 min

c) 120 min to hours:

1 hour = 60 min

Therefore, 120 min = 120 / 60 = 2 hours

d) 30 min to sec:

1 min = 60 sec

Therefore, 30 min = 30 x 60 = 1800 sec

e) 45 min and 30 sec to sec:

45 min = 45 x 60 = 2700 sec

Therefore, 45 min and 30 sec = 2700 + 30 = 2730 sec

f) 240 sec to min:

1 min = 60 sec

Therefore, 240 sec = 240 / 60 = 4 min

5. Convert the given units of speed.

a) 270 km/hr to m/sec b) 130 m/sec to km/hr c) 120 m/sec to km/hr

Solution:

a) To convert km/hr to m/sec, we divide by 3.6 (since 1 km/hr = 1000 m/3600 sec).

So, 270 km/hr = (270 * 1000) / 3600 m/sec = 75 m/sec.

Therefore, 270 km/hr is equal to 75 m/sec.

b) To convert m/sec to km/hr, we multiply by 3.6 (since 1 km/hr = 1000 m/3600 sec).

So, 130 m/sec = (130 * 3600) / 1000 km/hr = 468 km/hr.

Therefore, 130 m/sec is equal to 468 km/hr.

c) To convert m/sec to km/hr, we multiply by 3.6 (since 1 km/hr = 1000 m/3600 sec).
So, 120 m/sec = (120 * 3600) / 1000 km/hr = 432 km/hr.
Therefore, 120 m/sec is equal to 432 km/hr.

6. **The distance between Shahdara and Gajjumata is 46 km. Find the average speed of the metro bus (in m/sec) if it covers this distance in 49 minutes.**
Solution:
Find the average speed of the metro bus in m/sec, we need to convert the distance from kilometers to meters and the time from minutes to seconds.

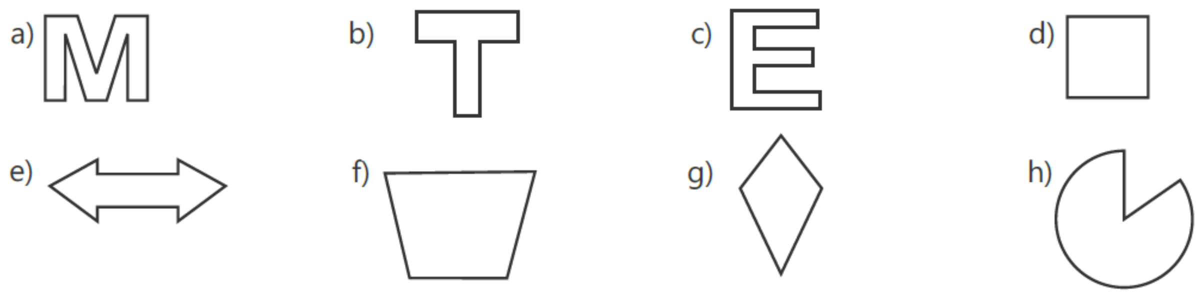
Distance between Shahdara and Gajjumata = 46 km = 46,000 meters
Time taken by metro bus to cover this distance = 49 minutes = 2940 seconds

Average speed of the metro bus = Distance / Time
= 46,000 meters / 2940 seconds
= 15.65 m/s
Therefore, the average speed of the metro bus (in m/sec) is 15.65 m/s.

7. **Ali travels from Faisalabad to Murree by bus with an average speed of 90 km/hr. Find the time taken if the total distance is 278 km.**
We can use the formula time = distance / speed to find the time taken by Ali to travel from Faisalabad to Murree.
Substituting the distance and speed values, we get:
time = 278 km / 90 km/hr
Simplifying the equation, we get:
time = 3.09 hours
Therefore, it took Ali approximately 3.09 hours to travel from Faisalabad to Murree by bus at an average speed of 90 km/hr.

Learning check 11.1

1. Draw the lines(s) of symmetry for the following.

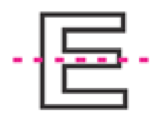


Solution:

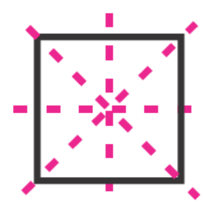
a) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

b) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

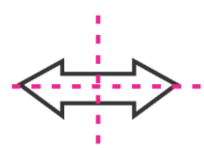
c) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:



d) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:



e) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

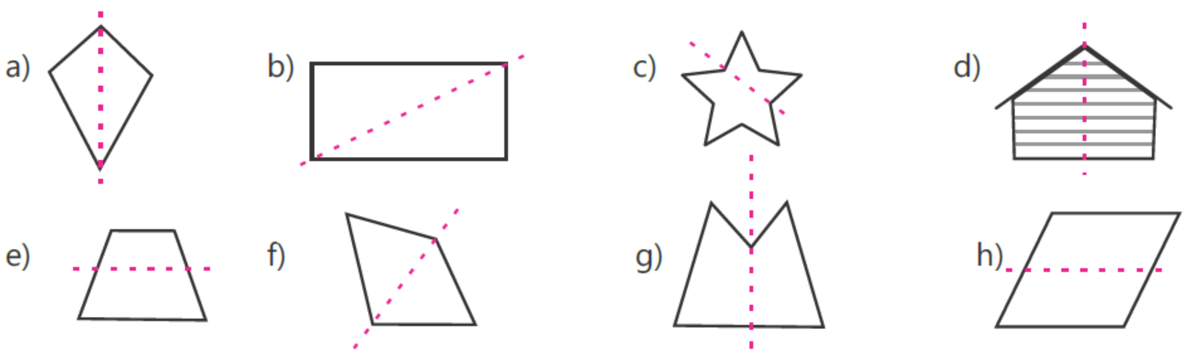


f) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

g) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

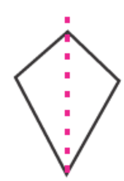
h) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

2. Determine if the dotted line in the following figures is a line of symmetry.

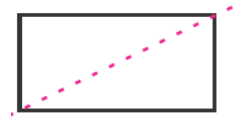


Solution:

a) As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.

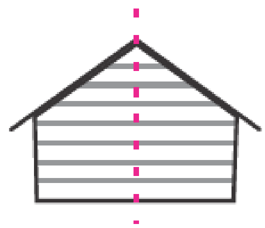


b) As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.

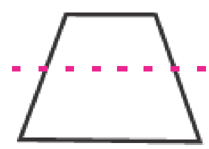


c) As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.

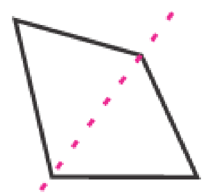
d) As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.



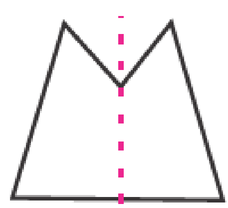
e) As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.



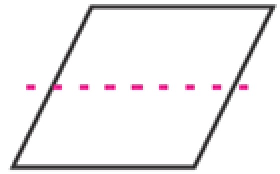
f) As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.



g) As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.

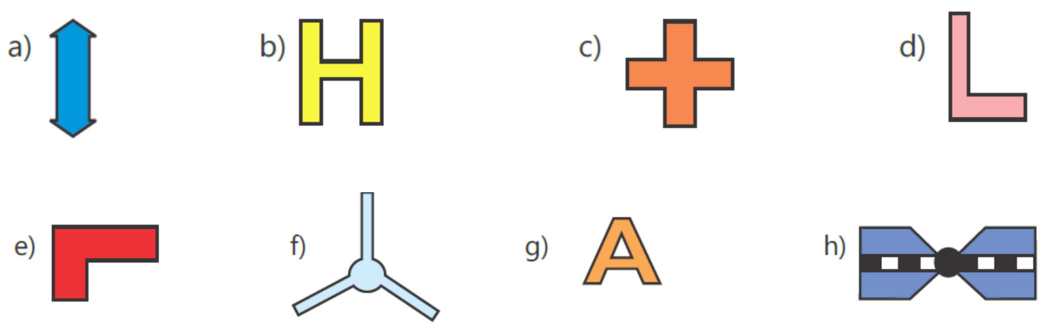


h) As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.

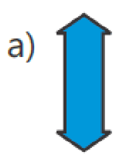


Learning check 11.2

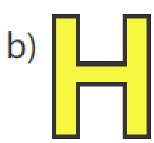
1. Write the order of symmetry for each shape that has rotational symmetry.



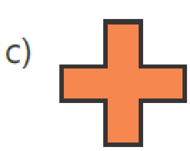
Solution:



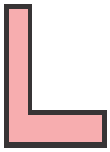
As we know that when a figure retains its original appearance at least twice after a complete 360° rotation around the centre point, this phenomenon is referred to as rotational symmetry. In the given figure it looks exactly the same twice in complete rotation so it has symmetry of order 2



In the given figure it looks exactly the same twice in complete rotation so it has symmetry of order 2



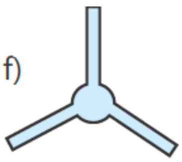
In the given figure it looks exactly the same 4 times in complete rotation so it has symmetry of order 4.

d) 

As we know that when a figure retains its original appearance at least twice after a complete 360° rotation around the centre point, this phenomenon is referred to as rotational symmetry. In the given figure in complete rotation it looks not exactly the same two time so it has 0 order of rotation.

e) 

As we know that when a figure retains its original appearance at least twice after a complete 360° rotation around the centre point, this phenomenon is referred to as rotational symmetry. In the given figure in complete rotation it looks not exactly the same two time so it has 0 order of rotation.

f) 

In the given figure it looks exactly the same 3 times in complete rotation so it has symmetry of order 3.

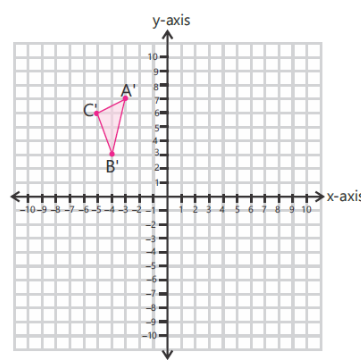
g) 

As we know that when a figure retains its original appearance at least twice after a complete 360° rotation around the centre point, this phenomenon is referred to as rotational symmetry. In the given figure in complete rotation it looks not exactly the same two time so it has 0 order of rotation.

h) 

In the given figure it looks exactly the same twice in complete rotation so it has symmetry of order 2

2. **Copy the given images onto Cartesian graph paper. Then rotate the images about their origin.**
   a)



Through 270° anticlockwise

Solution:

To rotate the triangle A’B’C’ about its origin 270°anticlockwise, we would follow the rule (x, y) → (y, –x), where the y-value of the original point becomes the new

x-value and the x-value of the original point becomes the new y-value with opposite sign.

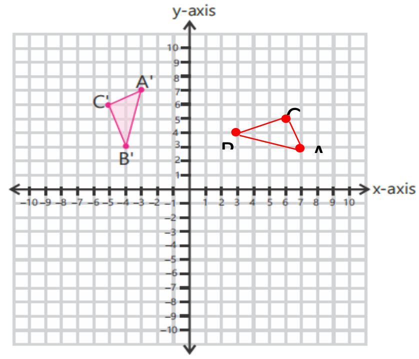
Let's apply the rule to the vertices to create

the new triangle ABC.

A' (–3, 7) becomes A (7, 3)

B' (–4, 3) becomes B (3, 4)

C' (–5, 6) becomes C (6, 5)

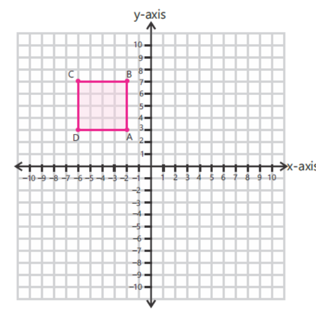


Mark the coordinates that will be the vertices of the reflected object/image. Join the vertices. The triangle ABC is the required rotated image of triangle A'B'C'.

b)

Solution:

To rotate the square ABCD about its origin 90° anticlockwise, we would follow the rule (x,y) → (-y,x), where the y-value of the original point becomes the new x-value with opposite sign and the x-value of the original point becomes the new y-value. Let's apply the rule to the vertices to create the new square A'B'C'D':



Through 90° anticlockwise

A (-2, 3) becomes A' (-3, -2)

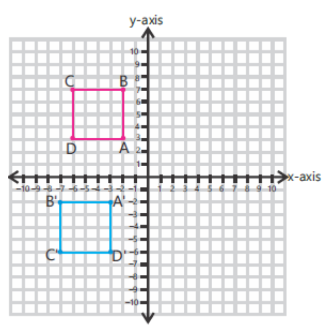
B (-2, 7) becomes B' (-7, -2)

C (-6, 7) becomes C' (-7, -6)

D (-6, 3) becomes D' (-3, -6)

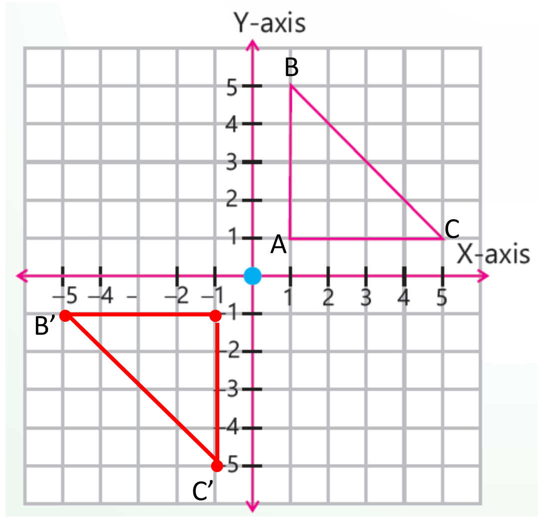
Mark the coordinates that will be the vertices of the reflected object/image.

Join the vertices. The square A'B'C'D' is the required rotated image of square ABCD.



3. **Draw a triangle on a coordinate plane. Then rotate it through 180° anticlockwise about its origin.**

Solution:



To rotate the triangle ABC about its origin 180°anticlockwise, we would follow the rule (x, y) → (–x, –y), where the y-value of the original point becomes the new x-value and the x-value of the original point becomes the new y-value with opposite sign.
Let's apply the rule to the vertices to create the new triangle A'B'C':
A' (1, 1) becomes A (–1, –1)

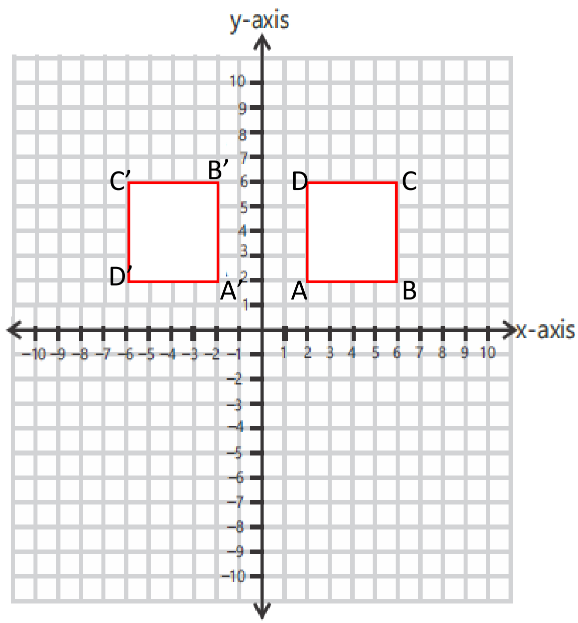
B' (1, 5) becomes B (–1, 5)

C' (5, 1) becomes C (–5, –1)

Mark the coordinates that will be the vertices of the reflected object/image. Join the vertices. The triangle A’B’C’ is the required rotated image of triangle ABC.

4. **Draw a square on a coordinate plane. Then rotate it through 270° clockwise about its origin.**

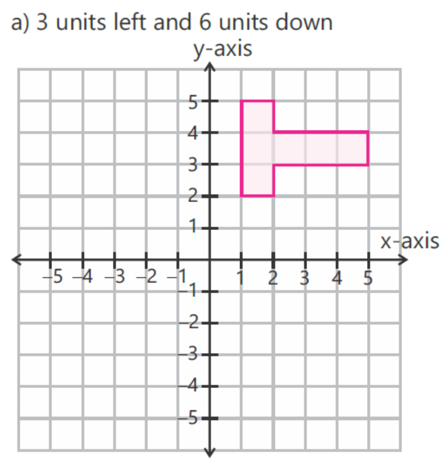
Solution:
To rotate the square ABCD about its origin 270° clockwise, we would follow the rule (x, y) → (–y, x), where the y-value of the original point becomes the new x-value with opposite sign and the x-value of the original point becomes the new y-value.
Let's apply the rule to the vertices to create the new square A'B'C'D':
A (2, 2) becomes A' (-2, -2)
B (6, 2) becomes B' (–2, 6)
C (6, 6) becomes C' (–6, 6)
D (2, 6) becomes D' (–6, 2)
Mark the coordinates that will be the vertices of the reflected object/image. Join the vertices. The square A'B'C'D' is the required rotated image of square ABCD.



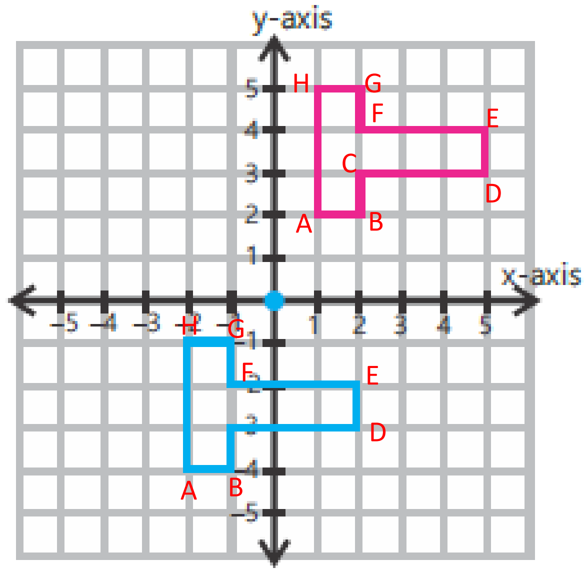
5. **Find the values of *x* and y for P' if an image having point P (-3, 8) is rotated:**
   a) 90° clockwise
   b) 180° anticlockwise
   c) 270° clockwise
   Solution:
   a) To rotate point P (-3, 8) 90º clock wise change the sign of *x* and then switch *x*, y. So the new point P'(8, 3) and the value of x = 8, value of y= 3.
   b) To rotate point P (-3, 8) 180º anticlockwise change the sign of *x* and y. So the new point P'(3, –8) and the value of x = 3, value of y= 8
   c) To rotate point P (-3, 8) 270º clockwise change the sign of *y* and then switch *x*, y. So the new point P'(-8, -3) and the value of x = -8, value of y= -3
6. **Find the values of *x* and y for P' if an image having point P (-5, -2) is rotated:**
   a) 90° anticlockwise
   b) 180° clockwise
   c) 270° anticlockwise
   Solution:
   a) To rotate point P (-5, -2) 90º anticlockwise change the sign of *y* and then switch *x*, y. So the new point P'(2, -5) and the value of x = 2, value of y= -5.
   b) To rotate point P (-5, -2) 180º anticlockwise change the sign of *x* and y. So the new point P'(5, 2) and the value of x = 5, value of y = 2
   c) To rotate point P (-5, -2) 270º anticlockwise change the sign of *x* and then switch *x*, y. So the new point P'(-2, 5) and the value of x = -2, value of y = 5.

Learning check 11.3

1. **Copy the given images onto Cartesian graph paper. Then translate the images according to the given description.**
   Solution:
   We can write it as: (*x*, y) → (*x* – 3, y – 6)
   First, find the coordinates of the pre-image. Here th



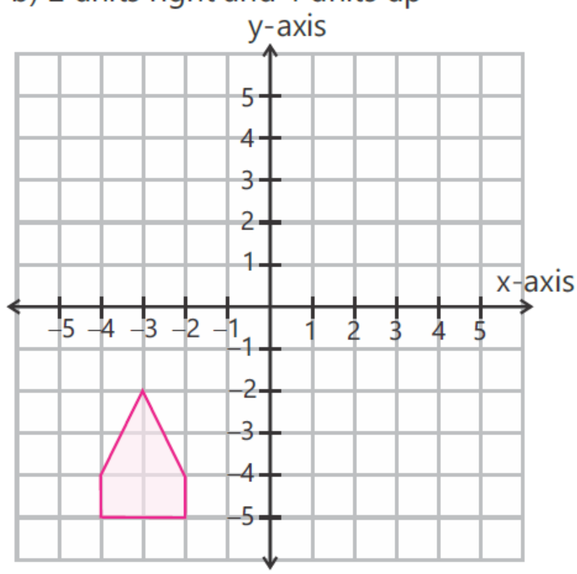
C (2, 3), D(5, 3), E(5, 4), F(2, 4), G(2, 5) and H(1, 5).
According to given description of the translation, the coordinates of the points in the translated image will be:
A (1, 2) → A'(1 – 3 , 2 – 6) or A'(–2, – 4)
B (2, 2) → B'(2 – 3, 2 – 6) or B'(–1, –4)
C (2, 3) → C'(2 – 3, 3 – 6) or C'(–1, –3)
D (5, 3) → C'(5 – 3, 3 – 6) or D'(2, –3)
E (5, 4) → C'(5 – 3, 4 – 6) or E'(2, –2)
F (2, 4) → C'(2 – 3, 4 – 6) or F'(–1, –2)
G (2, 5) → C'(2 – 3, 5 – 6) or G'(–1, –1)
H (1, 5) → C'(1 – 3, 5 – 6) or H'(–2, –1)



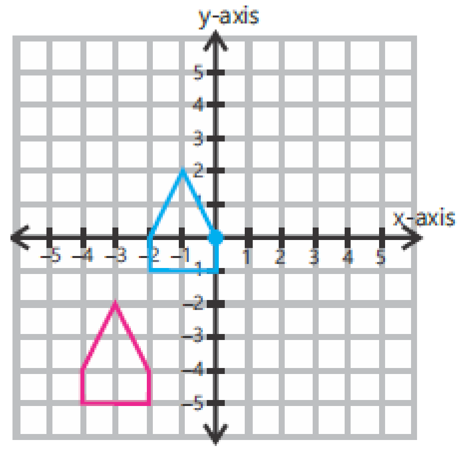
Plot the new coordinates for the translated image. This is the required image.

Solution:
We can write it as: (x, y) → (x + 2, y + 4)
First, find the coordinates of the pre-image. Here the points are: A (–3, –2), B (–2, –4),
C (–2, –5), D(–4, –5) and E(–4, –4).
According to given description of the translation, the coordinates of the points in the translated image will be:
A (–3, –2) → A'(–3 + 2, –2 + 4) or A'(–1, +2)
B (–2, –4) → B'(–2 + 2, –4 + 4) or B'(0, 0)
C (–2, –5) → C'(–2 + 2, –5 + 4) or C'(0, –1)
D(–4, –5) → C'(–4 + 2, –5 + 4) or D'(–2, –1)
E(–4, –4) → C'(–4 + 2, –4 + 4) or E'(–2, 0)

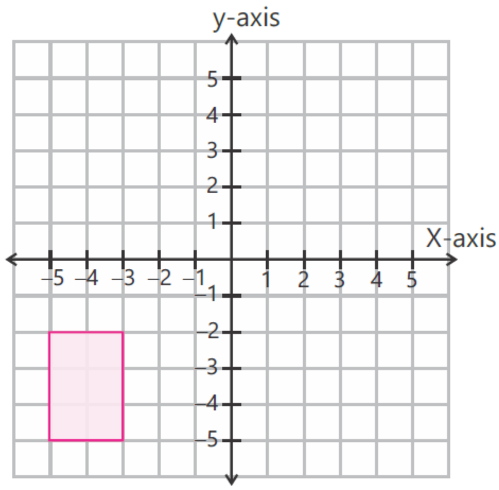
b) 2 units right and 4 units up



Plot the new coordinates for the translated image. This is the required image.

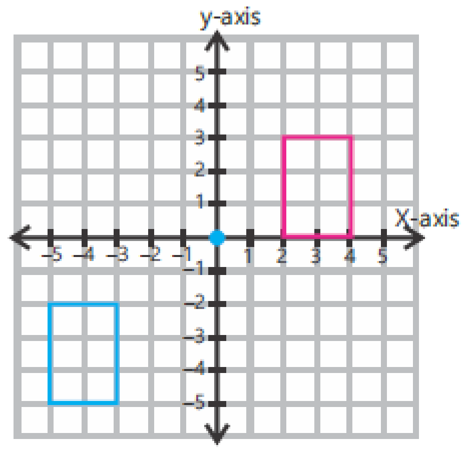


c) 7 units right and 5 units up



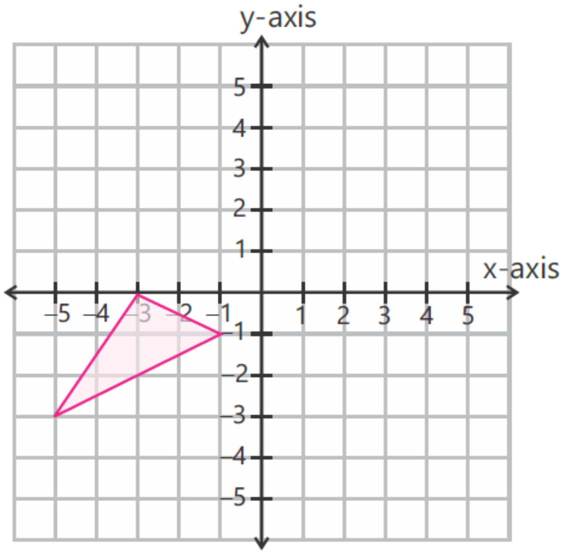
Solution:
We can write it as: (*x*, y) → (*x* + 7, y + 5)
First, find the coordinates of the pre-image. Here the points are: A (–5, –2), B (–3, –2), C (–3, –5), and D(–5, –5).
According to given description of the translation, the coordinates of the points in the translated image will be:

A (–5, –2) → A'(–5 + 7, –2 + 5) or A'(2, 3)
B (–3, –2) → B'(–3 + 7, –2 + 5) or B'(4, 3)
C (–3, –5) → C'(–3 + 7, –5 + 5) or C'(4, 0)
D(–5, –5) → C'(–5 + 7, –5 + 5) or D'(2, 0)



Plot the new coordinates for the translated image. This is the required image.

d) (x, y) → (x + 5, y + 1)



Solution:

We can write it as: (x, y) → (x + 5, y + 1)

First, find the coordinates of the pre-image.
Here the points are: A (–3, 0), B (–1, –1),

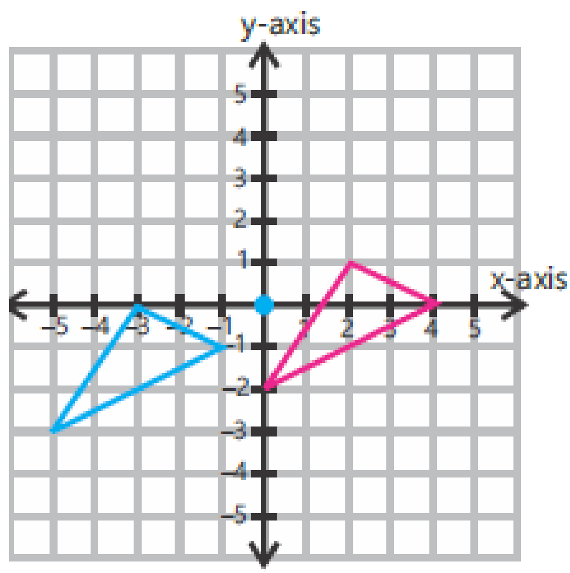
and C(–5, –3).

According to given description of the translation, the coordinates of the points in the translated image will be:

A (–3, 0) → A'(–3 + 5, 0 + 1) or A'(2, 1)

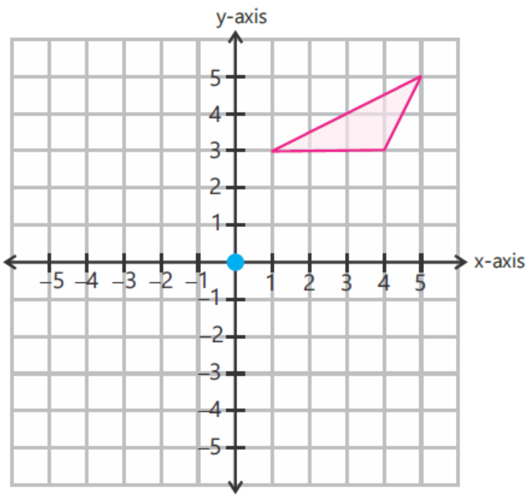
B (–1, –1) → B'(–1 + 5, –1 + 1) or B'(4, 0)

C (–5, –3) → C'(–5 + 5, –3 + 1) or C'(0, –2)



Plot the new coordinates for the translated image. This is the required image.

e) (x, y) → (x – 3, y – 5)



Solution:

We can write it as: (*x*, y) → (*x* –3, y –5)

First, find the coordinates of the pre-image. Here the points are: A (1, 3), B (4, 3) and C (5, 5)

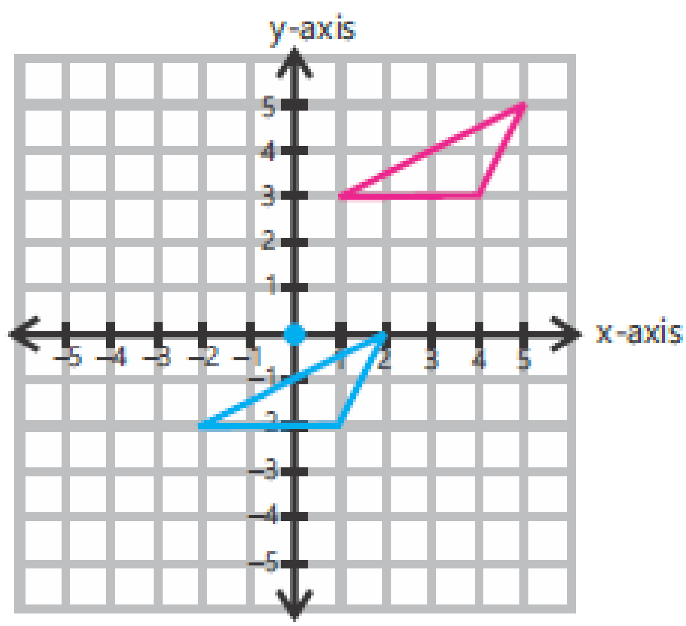
According to given description of the translation, the coordinates of the points in the translated image will be:

A (1, 3) → A'(1 –3, 3 – 5) or A'(–2, –2)

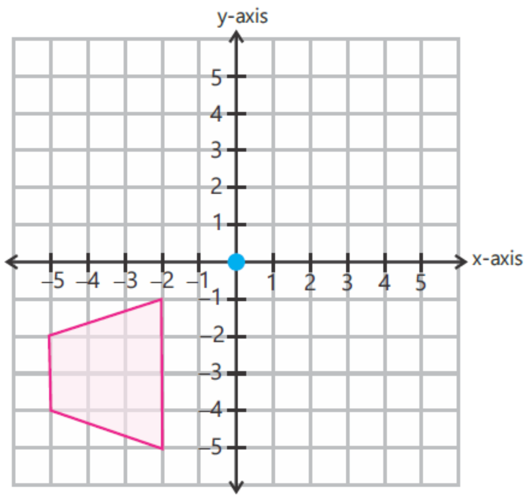
B (4, 3) → B'(4 –3, 3 – 5) or B'(1, –2)

C (5, 5) → C'(5 –3, 5 – 5) or C'(2, 0)

Plot the new coordinates for the translated image. This is the required image.



f) (x, y) → (x + 5, y + 3)

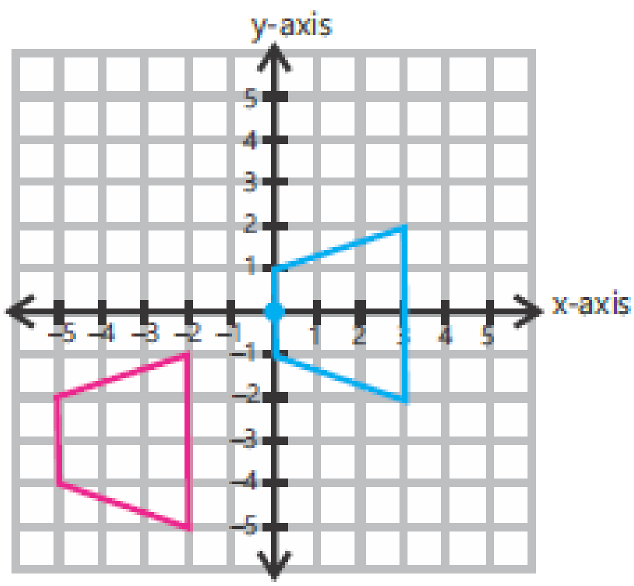


Solution:

We can write it as: (*x*, y) → (*x* + 5, y + 3)

First, find the coordinates of the pre-image. Here the points are: A (–5, –2), B (–2, –1),

C (–2, –5) and D(–4, –5).



According to given description of the translation, the coordinates of the points in the translated image will be:

A (–5, –2) → A'(–5 + 5, –2 + 3) or A'(0, 1)

B (–2, –1) → B'(–2 + 5, –1 + 3) or B'(3, 2)

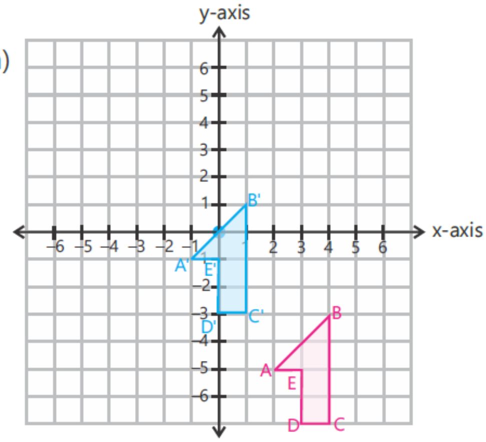
C (–2, –5) → C'(–2 + 5, –5 + 3) or C'(3, –2)

D(–5, –4) → C'(–5 + 5, –4 + 3) or D'(0, –1)

Plot the new coordinates for the translated image. This is the required image.

2. Describe the translation of the given images.

a)



Solution:

To describe this translation of the given image, follow these steps.

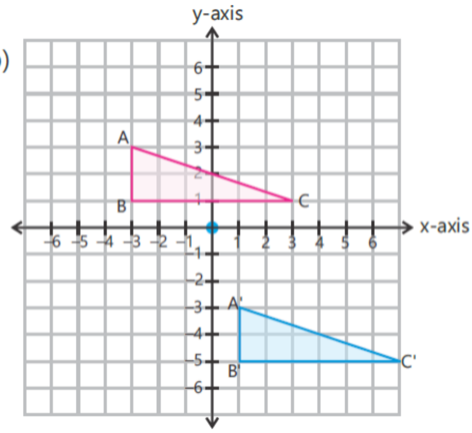
Step I: Choose any of the points of the pre-image.

Step II: Here take point A, its coordinates are (2, –5).

Step III: Now look for its corresponding point on the translated image. A' has the coordinates (–1, –1), so the object has moved 3 units to the left (negative horizontal direction) as 2 –3 = –1.

Next, as –5 + 4 = –1, the object has moved 4 units up (positive vertical direction). Similarly, all other points must be moved by the same number of units. As a result, this translation can be described as: (x, y) → (x –3, y + 4) or in words as: "3 units left and 4 units up."

b)



Solution:

To describe this translation of the given image, follow these steps.

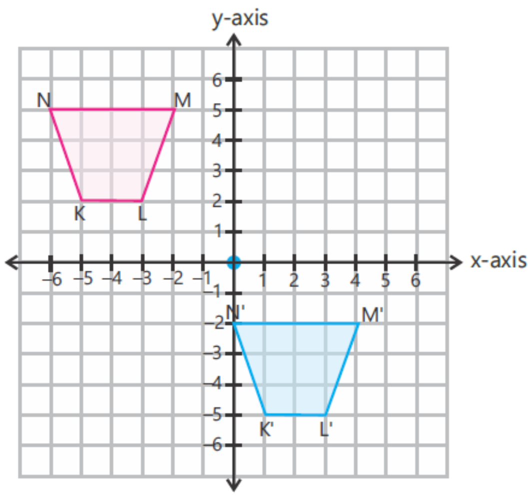
Step I: Choose any of the points of the pre-image.

Step II: Here take point A, its coordinates are (–3, 3).

Step III: Now look for its corresponding point on the translated image. A' has the coordinates (1, –3), so the object has moved 4 units to the right (positive horizontal direction) as –3 + 4 = +1.

Next, as 3 – 6= –3, the object has moved 6 units down (negative vertical direction). Similarly, all other points must be moved by the same number of units. As a result, this translation can be described as: (x, y) → (x + 4, y – 6) or in words as: "4 units left and 6 units down."

c)



Solution:

To describe this translation of the given image, follow these steps.

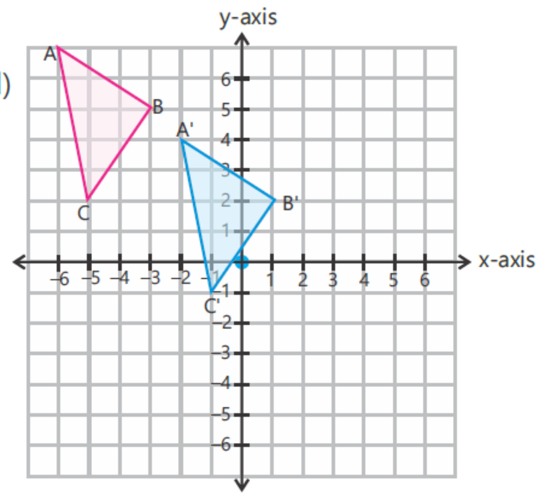
Step I: Choose any of the points of the pre-image.

Step II: Here take point K, its coordinates are (–5, 2).

Step III: Now look for its corresponding point on the translated image. K' has the coordinates (1, –5), so the object has moved 6 units to the right (positive horizontal direction) as –5 + 6 = 1.

Next, as 2 – 7 = –5, the object has moved 5 units down (negative vertical direction). Similarly, all other points must be moved by the same number of units. As a result, this translation can be described as: (x, y) → (x + 6, y – 7) or in words as: "6 units left and 7 units down."

d)



Solution:

To describe this translation of the given image, follow these steps.

Step I: Choose any of the points of the pre-image.

Step II: Here take point C, its coordinates are (–5, 2).

Step III: Now look for its corresponding point on the translated image. C' has the coordinates (–1, –1),so the object has moved 4 units to the right (positive horizontal direction) as –5 + 4 = –1.

Next, as 2 – 3 = –1, the object has moved 3 units down (negative vertical direction). Similarly, all other points must be moved by the same number of units. As a result, this translation can be described as: (x, y) → (x + 4, y – 3) or in words as: "4 units right and 3 units down."

Learning check 11.4

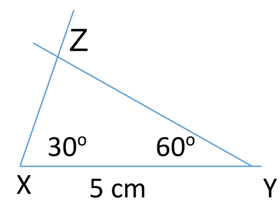
1. Construct a scalene triangle XYZ in which ∠X = 30°, ∠Y = 60° and XY = 5 cm.

Solution:

Steps of Construction:

Step I: Draw a line segment XY = 5 cm with a ruler.

Step II: Place the protractor at XY such that the centre of the protractor is exactly at point X.



Step III: Read the angle 30° on the protractor and mark point V.

Step IV: Again, place the protractor at XY such that the centre of the protractor is exactly at point Y.

Step V: Read the angle 60° on the protractor.

Step VI: Name the point where they intersect as Z. Join X to Z and Y to Z and

Step VII: ∆XYZ is the required scalene triangle.

2. Construct an isosceles triangle ABC in which AB = 6 cm and ∠A = ∠B = 45°.

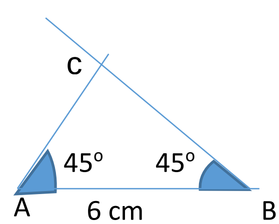
Solution:

Steps of Construction:

Step I: Draw a line segment AB = 6 cm with a ruler.

Step II: Place the protractor at AB such that the centre of the protractor is exactly at point A.

Step III: Read the angle 45° on the protractor.



Step IV: Again, place the protractor at AB such that the centre of the protractor is exactly at point B.

Step V: Read the angle 45° on the protractor.

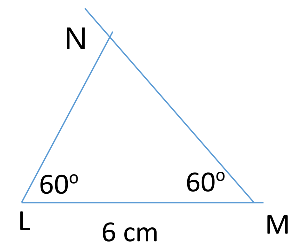
Step VI: Name and name the point where they intersect as C. Join A to C and B to C.

Step VII: ∆ABC is the required isosceles triangle.

3. Construct an equilateral triangle LMN in which ∠L = ∠M = 60° and LM = 6 cm.

Solution:

Steps of Construction:



Step I: Draw a line segment LM = 6 cm with a ruler.

Step II: Place the protractor at LM such that the centre of the protractor is exactly at point L.

Step III: Read the angle 60° on the protractor and mark point J.

Step IV: Again, place the protractor at LM such that the centre of the protractor is exactly at point M.

Step V: Read the angle 60° on the protractor and mark point K.

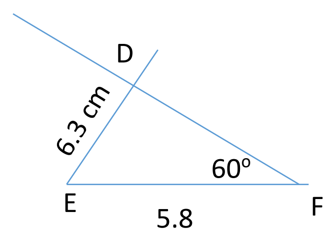
Step VI: Join L to J and M to K and name the point where they intersect as N.

Step VII: ∆LMN is the required equilateral triangle.

4. Construct an acute-angled triangle DEF in which DE = 6.3 cm, EF = 5.8 cm and ∠F = 60°.

Solution:

Steps of Construction:



Step I: Draw a line segment EF = 5.8 cm with a ruler.

Step II: Place the protractor at EF such that the centre of the protractor is exactly at point F.

Step III: Read the angle 60° on the protractor

Step IV: With E as a centre, use a compass to draw an arc DE of radius 6.3.

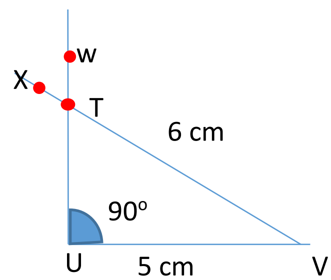
Step VI: Name the point where they intersect as D. Join D to F and E to F.

Step VII: ∆DEF is the required acute-angled triangle.

5. Construct a right-angled triangle TUV in which UV = 5 cm, VT = 6 cm and ∠U = 90°.

Solution:

Steps of Construction:



Step I: Draw a line segment UV = 5 cm with a ruler.

Step II: Place the protractor at UV such that the centre of the protractor is exactly at point U.

Step III: Read the angle 90° on the protractor and mark point W.

Step IV: With V as a centre, use a compass to draw an arc VT of radius 6 cm and mark point X.

Step VI: Join V to T and U to W and name the point where they intersect as T.

Step VII: ∆TUV is the required right-angled triangle.

6. Construct an obtuse-angled triangle ABC in which AB = 4.1 cm, BC = 4.5 cm and ∠A = 110°

Solution:

Steps of Construction:

Step I: Draw a line segment AB = 4.1 cm with a ruler.

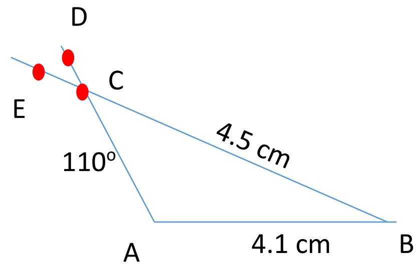
Step II: Place the protractor at AB such that the centre of the protractor is exactly at point A.

Step III: Read the angle 110° on the protractor and mark point D.

Step IV: With B as a centre, use a compass to draw an arc BC of radius 4.5 cm and mark point E.

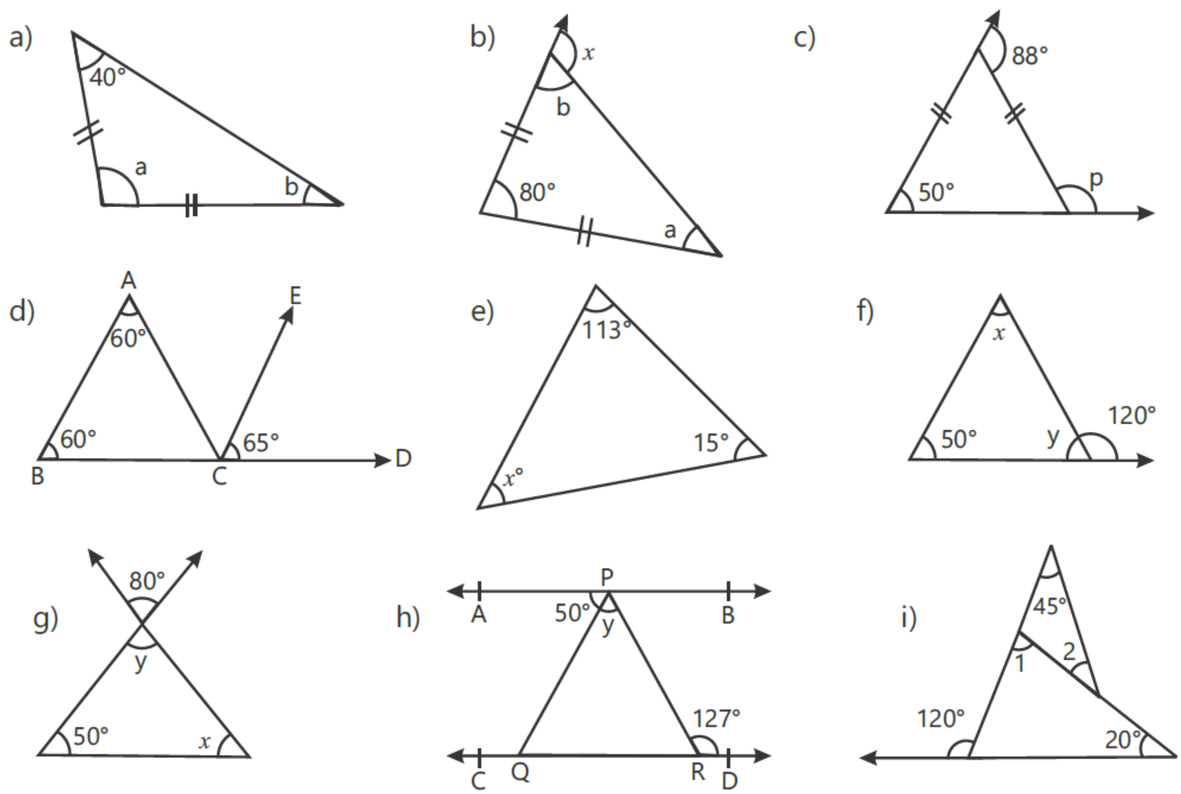
Step VI: Join A to D and B to E and name the point where they intersect as C.

Step VII: ∆ABC is the required obtuse-angled triangle.



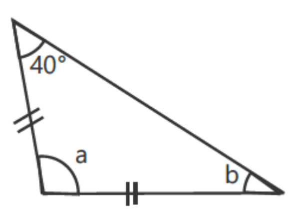
Learning check 11.5

1. **Find the unknown angle of the following triangles.**



Solution:

a) As the two sides of the given triangle are equal so, angle of both sides are also equal.



$\angle a = \angle b$

As the sum of the angles of the triangles is 180° so,

$40^{o} + \angle a + \angle b = 180^{o}$

$40^{o} + \angle a + \angle a = 180^{o}$

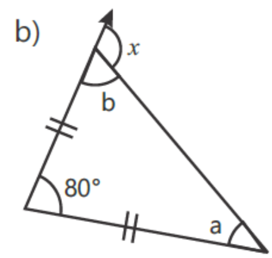
$2\angle a = 180^{o} - 40^{o}$

$2\angle a = 140^{o}$

$\angle a = 70^{o}$

So, $\angle a = \angle b = 70^{o}$

b) As the two sides of the given triangle are equal so, angle of both sides are also equal.



$\angle a = \angle b$

As the sum of the angles of the triangles is 180^{o} so,

$80^{o} + \angle a + \angle b = 180^{o}$

$80^{o} + \angle a + \angle a = 180^{o}$

$2\angle a = 180^{o} - 80^{o}$

$2\angle a = 100^{o}$

$\angle a = 50^{o}$

So, $\angle a = \angle b = 50^{o}$

c) As the two sides of the given triangle are equal so, angle of both sides are also equal.

$\angle a = 50^{o}$

As the sum of the angles of the triangles is 180^{o} so,

$50^{o} + \angle a + \angle b = 180^{o}$

$50^{o} + 50^{o} + \angle b = 180^{o}$

$\angle b = 180^{o} - 100^{o}$

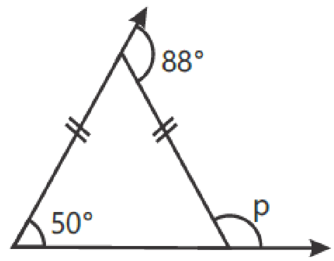
$\angle b = 80^o$

As, $\angle a + \angle p = 180^o$

$50^o + \angle p = 180^o$

$\angle p = 180^o - 100^o$

$\angle p = 80^o$



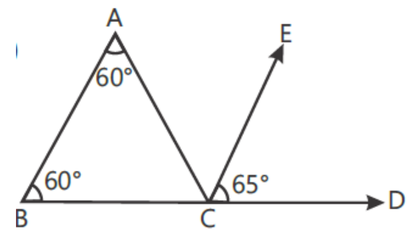
d) As the sum of the angles of the triangles is 180o so,

$60^o + 60^o + \angle b = 180^o$

$50^o + 50^o + \angle b = 180^o$

$\angle b = 180^o - 100^o$

$\angle b = 80^o$



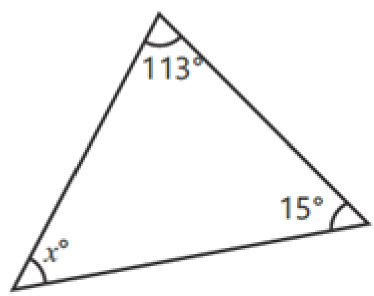
e) As the sum of the angles of the triangles is 180o so,

$113^o + 15^o + \angle b = 180^o$

$128 + \angle b = 180^o$

$\angle b = 180^o - 128^o$

$\angle b = 52^o$



f) As the angle of the straight line is 180o so,

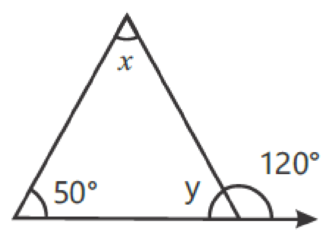
$y + 120^o = 180^o$

$y = 180^o - 120^o$

$y = 60^o$

As the sum of the angles of the triangles is 180^o so,

$50^o + \angle x + \angle y = 180^o$

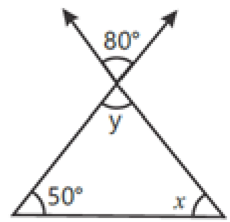


$50^o + \angle x + 60^o = 180^o$

$\angle x = 180^o - 110^o$

$\angle x = 70^o$

g) As vertically opposite angles are equal so,



$y = 80^o$

As the sum of the angles of the triangles is 180^o so,

$50^o + \angle x + \angle y = 180^o$

$50^o + \angle x + 80^o = 180^o$

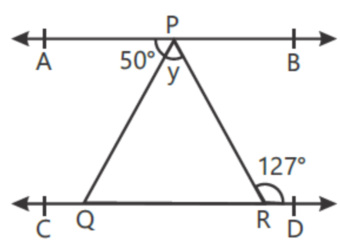
$\angle x = 180^o - 130^o$

$\angle x = 50^o$

h) As alternate angles are equal so,

$y + 50^o = 127^o$

$y = 127^o - 50^o = 77^o$



We know that the angle of a straight angle is 180^o.

$127^o + x = 180^o$

$x = 180^o - 127^o$

$x = 53^o$

As the sum of the angles of the triangles is 180^o so,

$\angle x + \angle y + \angle z = 180^o$

$53^o + 77^o + \angle z = 180^o$

$\angle z = 180^o - 130^o$

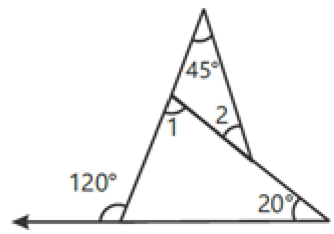
$\angle z = 50^o$

i) We know that the angle of a straight angle is 180o.

$120^o + x = 180^o$

$x = 180^o - 120^o$

$x = 60^o$



As the sum of the angles of the triangles is 180º

$60^o + \angle 1 + 20^o = 180^o$

$\angle 1 + 80^o = 180^o$

$\angle 1 = 180^o - 80^o$

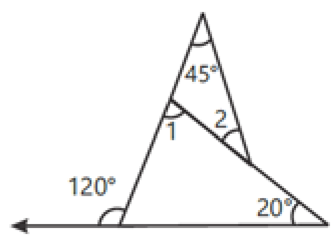
$\angle 1 = 100^o$

We know that the angle of a straight angle is 180º.

$100^o + y = 180^o$

$y = 180^o - 100^o$

$y = 80^o$



As the sum of the angles of the triangles is 180º so,

$45^o + \angle 2 + 80^o = 180^o$

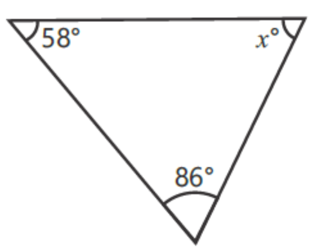
$\angle 2 + 125^o = 180^o$

$\angle 2 = 180^o - 125^o$

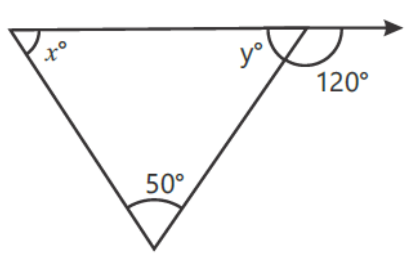
$\angle 2 = 55^o$

2. Find the value of *x* and y.

a)



b)



Solution:

a) As the sum of the angles of a triangle is 180^o. So,

$58^o + 86^o + x = 180^o$

$x = 180^o - 58^o - 86^o$

$x = 180^o - 58^o - 86^o$

$x = 36^o$

b) As the angle of the straight line is 180^o so,

$y + 120^o = 180^o$

$y = 180^o - 120^o$

$y = 60^o$

As the sum of the angles of a triangle is 180^o. So,

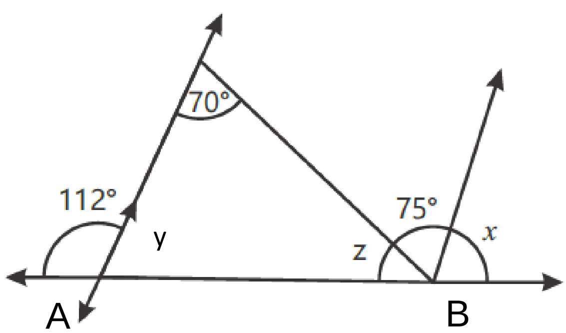
$50^o + 60^o + x = 180^o$

$x = 180^o - 50^o - 60^o$

$x = 180^o - 110^o$

$x = 70^o$

3. Find the value of *x* and z.



Solution:

As AB is a straight line.

As the angle of the straight line is 180^o.

$112^o + y = 180^o \Rightarrow y = 180^o - 112^o = 68^o$

As we know the sum of the angles of the triangle is 180^o.

$70^o + 68^o + z = 180^o$

$z = 180^o - 68^o - 70^o$

$z = 42^o$

As the angle of the straight line is 180^o.

$x + z + 75 = 180^o$

$x + 42^o + 75^o = 180^o$

$x = 180^o - 42^o - 75^o$

$x = 63^o$

Unit Check 11

1. **Encircle the correct option.**

a) What is the number of lines of symmetry in a rectangle?

i) 1 ii) 2 iii) 3 iv) 4

b) What is the number of lines of symmetry in the letter H?

i) 1 ii) 2 iii) 3 iv) 4

c) Which of the following has zero lines of symmetry?

i) isosceles triangle ii) scalene triangle iii) square iv) rectangle

d) What will be the value p' (x, y) if an image having point p (–2, 5) is rotated through 90°clockwise?

i) (2, -5)   ii) (-5, 2)   iii) (5, -2)   iv) (5, 2)

e) The order of rotational symmetry of an equilateral triangle is:

i) 0   ii) 1   iii) 2   iv) 3

f) Translating an object does not affect its:

i) location   ii) position   iii) size   iv) placement

g) In an acute-angled triangle, the sum of three angles equals:

i) 90°   ii) 180°   iii) 270°   iv) 360°

2. Fill in the blanks.

a) When an object or image is rotated 180° clockwise, the point (3, 2) becomes (-3, -2).
b) The line of symmetry divides the symmetric shape into ____two____ equal parts.
c) Turning a shape around a fixed point is called rotation_.
d) Translating an object or image doesn't affect its __size__ or shape.
e) In a triangle, the exterior angle and the adjacent interior angle add up to_180°__.

3. Draw the lines of symmetry for the shapes (if any).

a) 

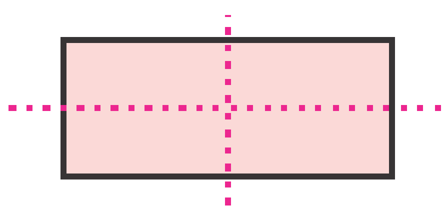
b) 

c) 

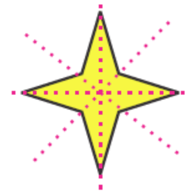
d) 

Solution:

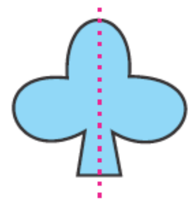
a) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:



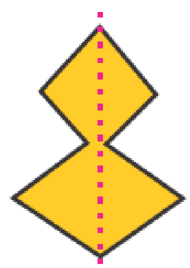
b) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:



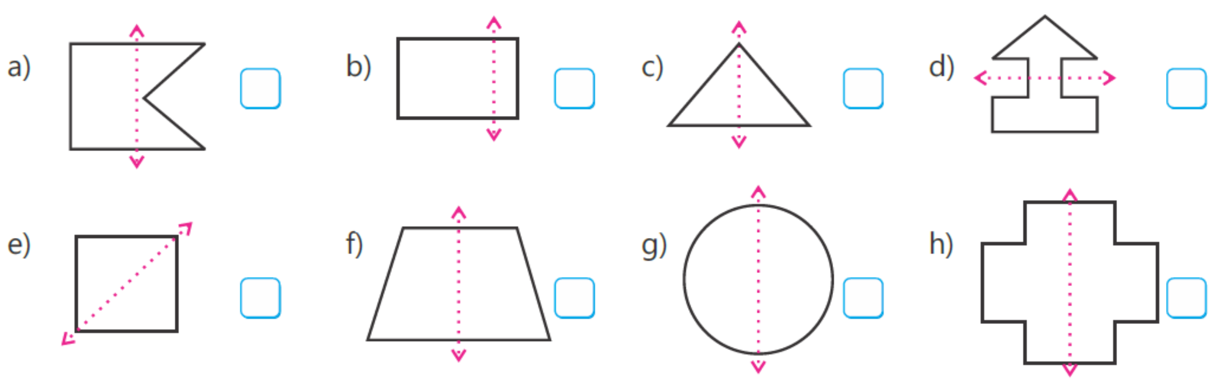
c) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:



d) As the line of symmetry divide the shape into two equal halves so, the line of symmetry for the given shape is:

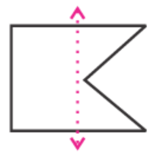


4. Does the dotted line represent the line of symmetry for the following figures? Write "yes or no".



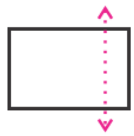
Solution:

a)



As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.

b)

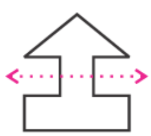


As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.

c)

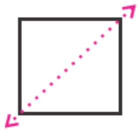
As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry

d)



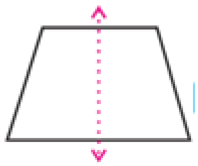
As the line of symmetry divide the given shape exactly into two equal halves. In the shape the dotted line does not divide the shape into two equal halves that it become the mirror image so the dotted line is not the line of symmetry.

e)



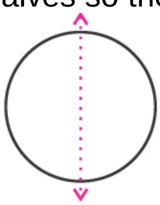
As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.

f)



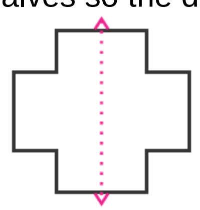
As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.

g)



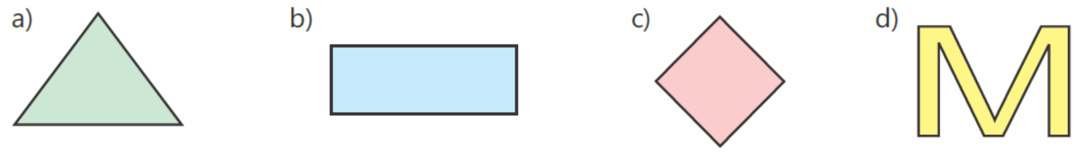
As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.

h)

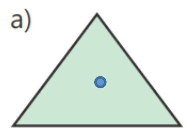


As the line of symmetry divide the given shape exactly into two equal halves. As the dotted line divide the given shape exactly into two equal halves so the dotted line is the line of symmetry.

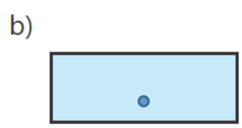
5. Tick (✓) the figures that have rotational symmetry. Write the order of rotation and mark the point of rotation for the symmetric figures.



Solution:

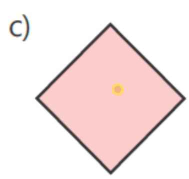


As we know that when a figure retains its original appearance at least twice after a complete 360° rotation around the centre point, this phenomenon is referred to as rotational symmetry. In the given figure it looks exactly the same 3 times in complete rotation so it has symmetry of order 3.



As we know

In the given figure it looks exactly the same twice in complete rotation so it has symmetry of order 2.

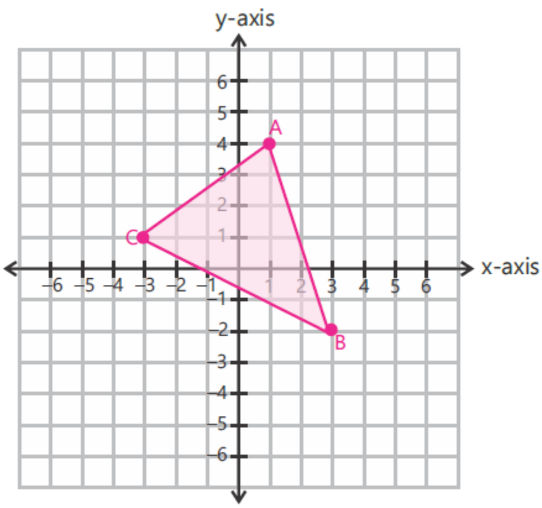


In the given figure it looks exactly the same 4 times in complete rotation so it has symmetry of order 4

As we know that when a figure retains its original appearance at least twice after a complete 360° rotation around the centre point, this phenomenon is referred to as rotational symmetry. In the given figure in complete rotation it looks not exactly the same two time so it has 0 order of rotation.

6. Copy the given images onto Cartesian graph paper. Then rotate the images about their origin.

a) through 180° anticlockwise



Solution:

To rotate the triangle ABC about its origin 180°anticlockwise, we would follow the rule (x, y) → (–x, –y), where the y-value of the original point becomes the new x-value and the x-value of the original point becomes the new y-value with opposite sign.

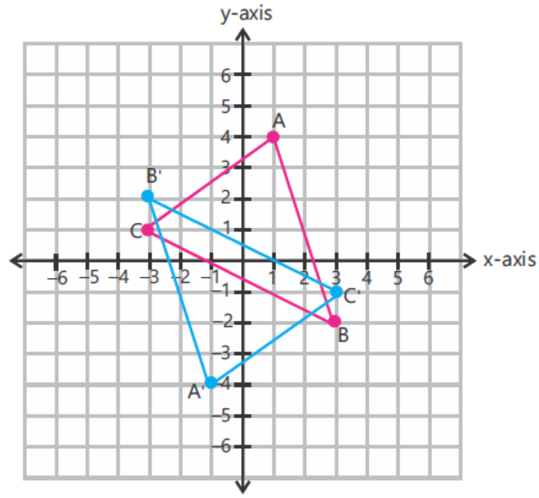
Let's apply the rule to the vertices to create the new triangle A'B'C':

A (1, 4) becomes A' (–1, –4)

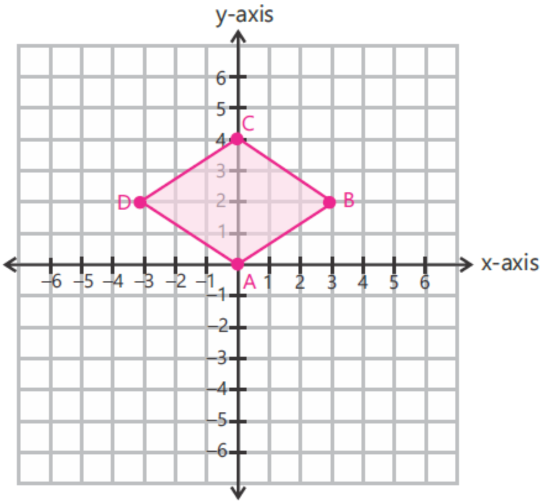
B (3, –2) becomes B' (–3, 2)

C (3, –1) becomes C' (–3, 1)

Mark the coordinates that will be the vertices of the reflected object/image. Join the vertices. The triangle A'B'C' is the required rotated image of triangle ABC.



b) through 270° anticlockwise



Solution:

To rotate the square ABCD about its origin 270°anticlockwise, we would follow the rule (x, y) → (y, –x), where the y-value of the original point becomes the new

x-value and the x-value of the original point becomes the new y-value with opposite sign.

Let's apply the rule to the vertices to create

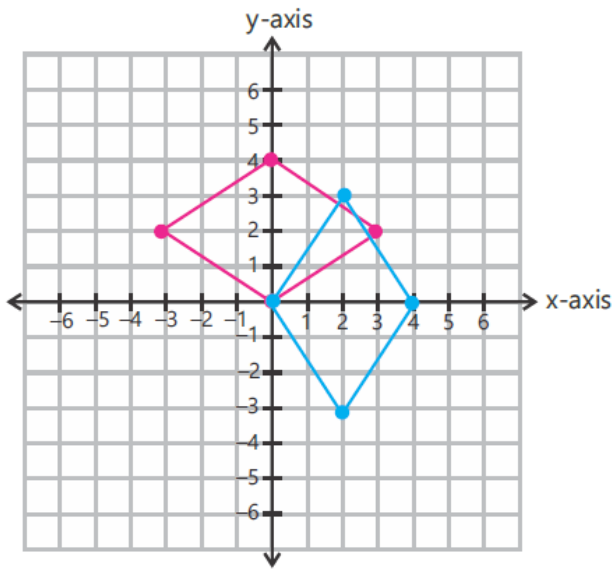
the new triangle ABC.

A' (0, 0) becomes A (0, 0)

B' (3, 2) becomes B (2, –3)

C' (0, 4) becomes C (4, 0)

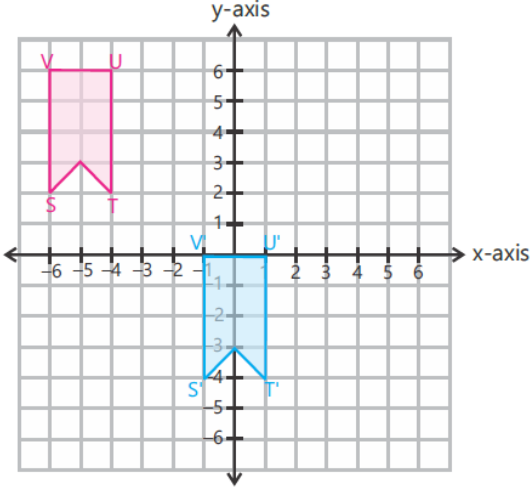
D' (–3, 2) becomes C (2, 3)



Mark the coordinates that will be the
vertices of the reflected object/image. Join the vertices. The square A'B'C'D' is the required rotated image of square ABCD

7. Describe the translation of the given images.

a)



Solution:

To describe this translation of the given image, follow these steps.

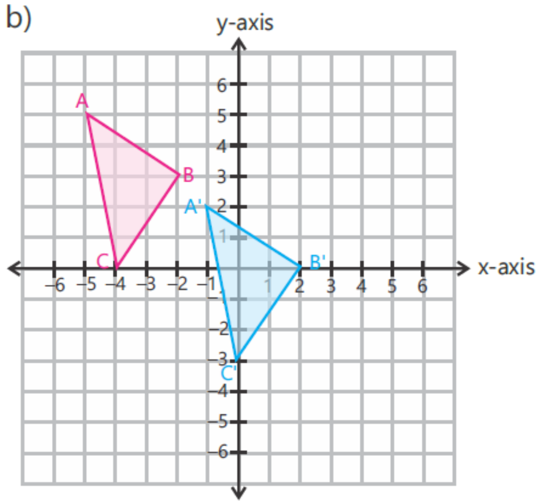
Step I: Choose any of the points of the pre-image.

Step II: Here take point S, its coordinates are (–6, 2).

Step III: Now look for its corresponding point on the translated image. S' has the coordinates (–1, –4), so the object has moved 5 units to the right (positive horizontal direction) as –6 + 5 = –1.

Next, as 2 – 6 = –4, the object has moved 4 units down (negative vertical direction). Similarly, all other points must be moved by the same number of units. As a result, this translation can be described as: (x, y) → (x + 5, y – 6) or in words as: "5 units right and 6 units down."

b)



Solution:

To describe this translation of the given image, follow these steps.

Step I: Choose any of the points of the pre-image.

Step II: Here take point A, its coordinates are (–5, 5).

Step III: Now look for its corresponding point on the translated image. S' has the coordinates (–1, 2), so the object has moved 4 units to the right (positive horizontal direction) as –5 + 4 = –1.

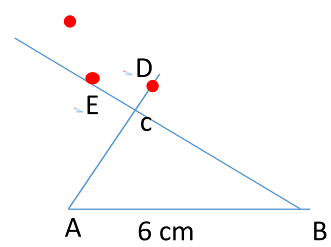
Next, as 5 – 3 = 2 the object has moved 3 units down (negative vertical direction). Similarly, all other points must be moved by the same number of units. As a result, this translation can be described as: (x, y) → (x + 4, y – 3) or in words as: "4 units right and 3 units down."

8. Construct an equilateral triangle ABC in which ∠A = ∠B = 60° and AB = 6 cm.

Solution:

Steps of Construction:

Step I: Draw a line segment AB = 6 cm with a ruler.



Step II: Place the protractor at AB such that the centre of the protractor is exactly at point A.

Step III: Read the angle 60° on the protractor and mark point D.

Step IV: Again, place the protractor at AB such that the centre of the protractor is exactly at point B.

Step V: Read the angle 60° on the protractor and mark point E.

Step VI: Join A to D and B to E and name the point where they intersect as C.

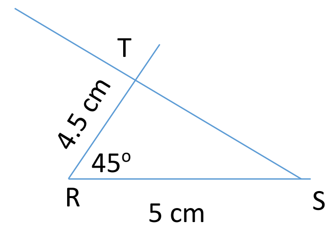
Step VII: ∆ABC is the required equilateral triangle.

9. Construct an acute-angled triangle RST in which RS = 5 cm, ST = 4.5 cm and ∠R = 30°.

Solution:

Steps of Construction:

Step I: Draw a line segment RS = 5 cm with a ruler.



Step II: Place the protractor at RS such that the centre of the protractor is exactly at point R.

Step III: Read the angle 30° on the protractor.

Step IV: With S as a centre, use a compass to draw an arc ST of radius 4.5 cm.

Step VI: Name the point where they intersect as Join S to T and R to T

Step VII: ∆RST is the required acute-angled triangle.

10. Construct a right-angled triangle XYZ in which XY = 7 cm, YZ = 6 cm and ∠X = 90°.

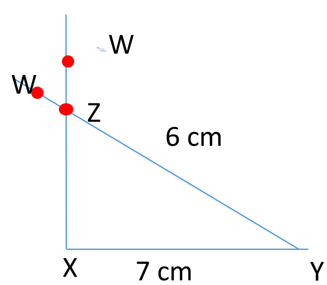
Solution:

Steps of Construction:

Step I: Draw a line segment XY = 7 cm with a ruler.

Step II: Place the protractor at XY such that the centre of the protractor is exactly at point X.

Step III: Read the angle 90° on the protractor and mark point W.

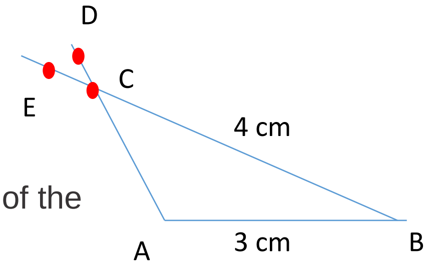


Step IV: With Y as a centre, use a compass to draw an arc YZ of radius 6 cm and mark point BX.

Step VI: Join V to X and U to W and name the point where they intersect as U.

Step VII: ∆XYZ is the required right-angled triangle.

11. Construct an obtuse-angled triangle ABC in which AB = 3 cm, BC = 4 cm and $\angle$A = 105°.



Steps of Construction:

Step I: Draw a line segment AB = 3 cm with a ruler.

Step II: Place the protractor at AB such that the centre of the protractor is exactly at point A.

Step III: Read the angle 105° on the protractor and mark point D.

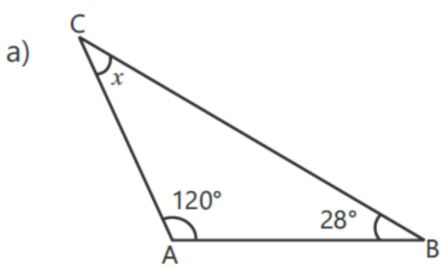
Step IV: With B as a centre, use a compass to draw an arc BC of radius 4 cm and mark point E.

Step VI: Join A to D and B to E and name the point where they intersect as C.

Step VII: ∆ABC is the required obtuse-angled triangle.

12. Find the unknown angles in the following triangles.

a)



Solution:

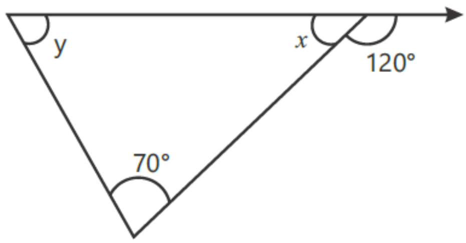
As we know the sum of the angles of the triangle is 180^o.

$120^o + 28^o + x = 180^o$

$x = 180^o – 28^o – 120^o$

$x = 32^o$

b)



As the angle of the straight line is 180^o so,

$x + 120^o = 180^o$

$x = 180^o - 120^o$

$x = 60^o$

As the sum of the angles of a triangle is 180^o. So,

$70^o + 60^o + y = 180^o$

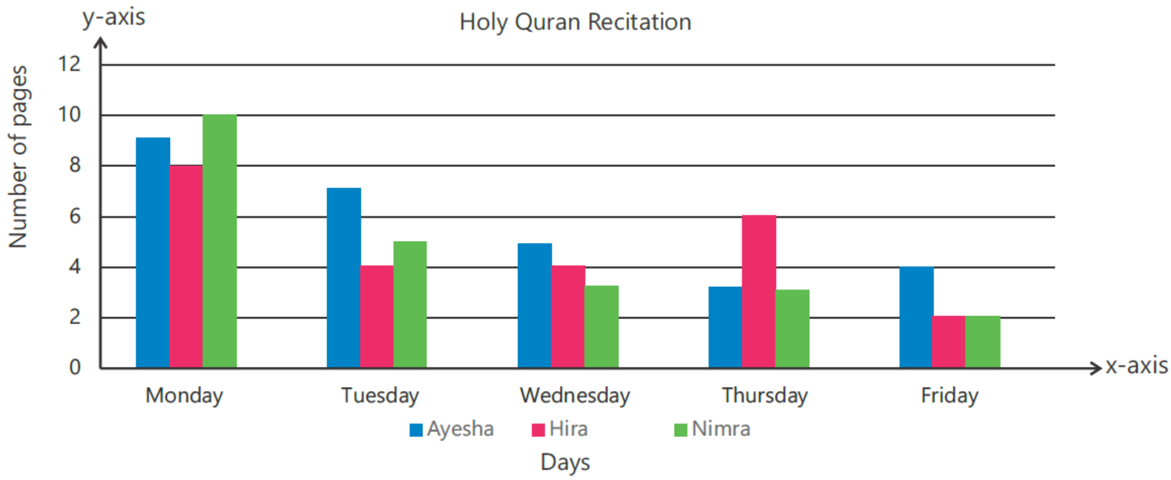
$y = 180^o - 70^o - 60^o$

$x = 180^o - 130^o$

$x = 50^o$

Learning Check 12.1

1. The given multiple bar graph in vertical form shows the number of pages of the Quran read by Ayesha, Hira, and Nimra during 5 days of a week. Study the given graph and answer the following questions:



According to the scale of this graph, the distance between two marks on y-axis is 2.

a) How many pages of the Quran were read on Tuesday?

To tell the number of pages read on Tuesday, we will count the pages read by each person separately and then add them:

Pages read by Ayesha(blue bar)= 7

Pages read by Hira(pink bar)= 4

Pages read by Nimra(green bar)= 5

Total pages read on Tuesday= 7 + 5 + 4 = 16 Pages

b) How many pages of the Quran were read on Thursday?

Pages read by Ayesha(blue bar)= 3

Pages read by Hira(pink bar)= 6

Pages read by Nimra(green bar)= 3

Total pages read on Tuesday= 3 + 6 + 3 = 12 Pages

c) How many pages of the Quran were read by Nimra?

To find the total pages read by Nimra, we add all the pages read by her from Monday to Friday.

Pages read by Nimra(green bar)=10 + 5 + 3 + 3 + 2 = 23 pages

d) On which day were the fewest number of pages of the Quran read?

To find the day when the fewest number of pages of the Quran read, we need to find total pages on each day.

Pages read on Monday= 9+8+10=27

Pages read on Tuesday=7+4+5=16

Pages read on Wednesday=5+4+3=12

Pages read on Thursday=3+6+3=12

Pages read on Friday=4+2+2=8

So, the fewest number of pages were read on Friday i.e., 8 pages.

e) On which day were the most pages of the Quran read?

From the calculations in previous part, we can see that the most pages were read on Monday i.e., 27 pages.

f) How many pages were read by Ayesha on Monday and Friday altogether?

Pages read by Ayesha on Monday= 9

Pages read by Ayesha on Friday= 4

Total pages read by Ayesha on Monday and Friday altogether= 9 + 4 = 13 pages.

2. The given table shows the sales of toys in two different categories over the course of five weeks. Draw a vertical multiple bar graph for this data using an appropriate scale.

Weeks	1st week	2nd week	3rd week	4th week	5th week
Category A	10	20	25	20	35
Category B	15	10	35	40	35

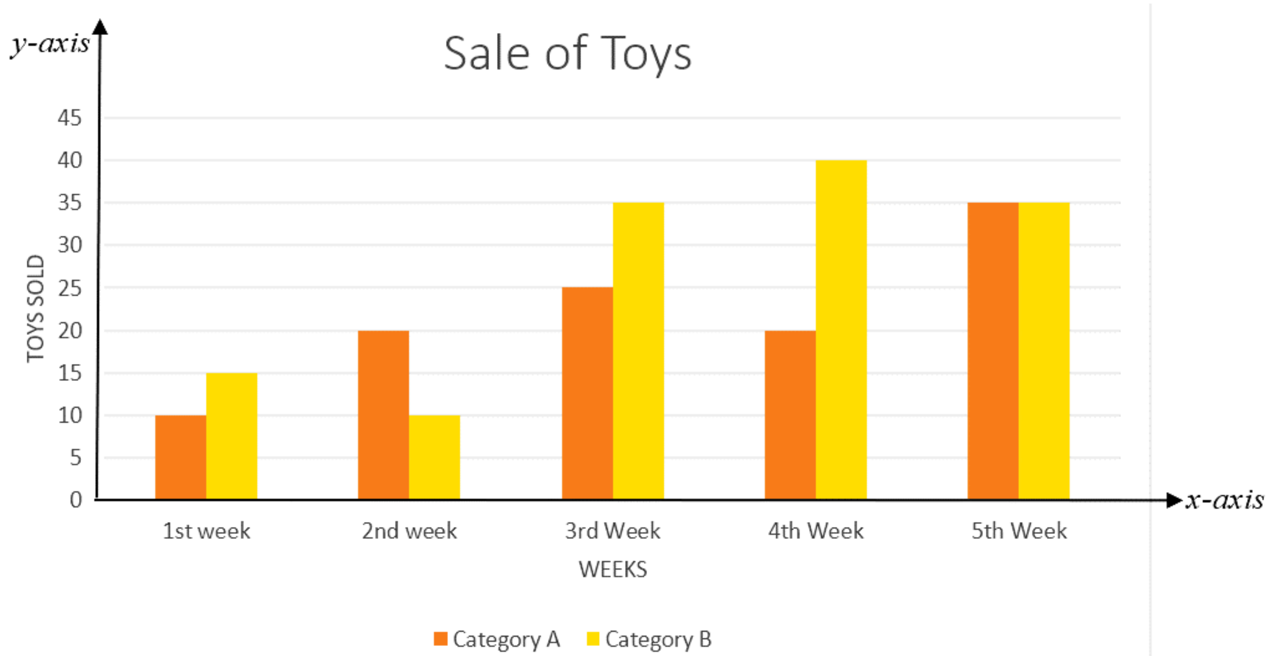
Solution:

Step I: Draw the x-axis (horizontal line) and y-axis (vertical line) perpendicular to each other on graph paper.

Step II: Mention the values along each axis. Here x-axis represents Weeks and y-axis represents the toys sold.

Step III: Choose an appropriate scale. Here one mark along y-axis represents 5 toys.

Step IV: Colour the correct numbers of values for each week and category and draw the multiple bars.



This is the required multiple bar graph, drawn vertically.

3. **On a certain day, the number of absentees in a school from grades 1 to 8 is given in the table. Draw a line graph for the given data.**

Grade	1	2	3	4	5	6	7	8
Number of Students	13	8	5	2	18	10	12	6

Solution:

Step I: Draw the x-axis and y-axis on graph paper.

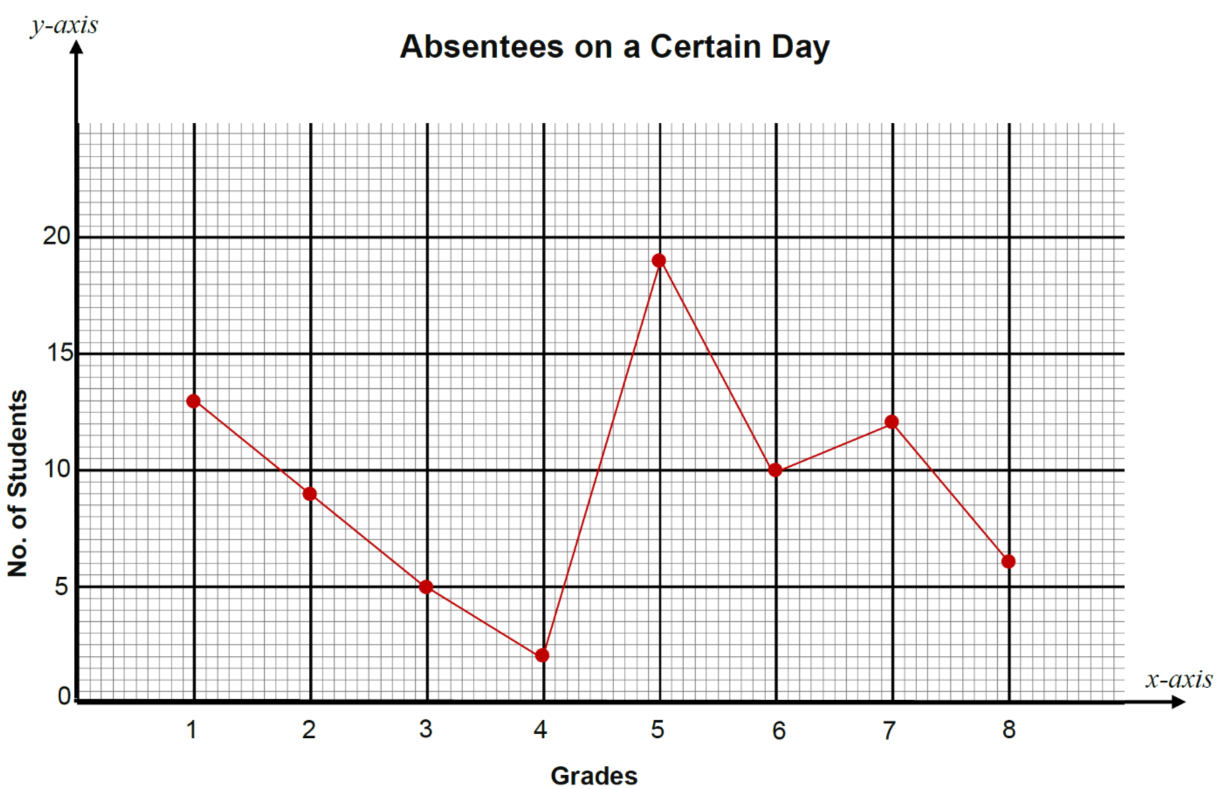
Step II: Mark information "Grades" along the x-axis and "No. of Students" along the y-axis.

Step III: Choose a suitable scale to mark the values. In this case, 2 small squares represent 1 student.

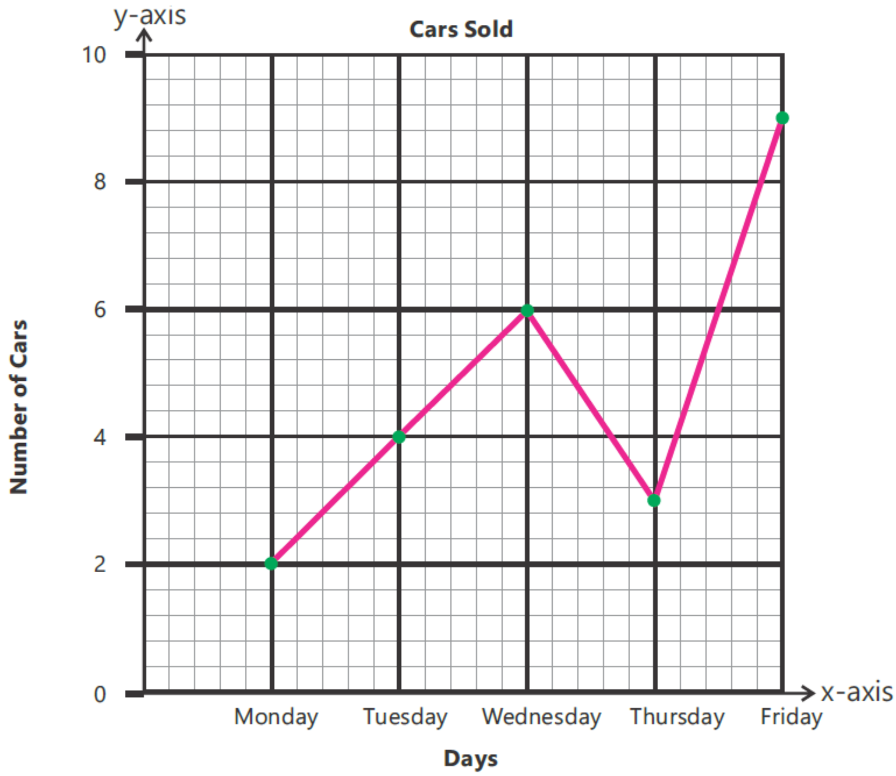
Step IV: Mark the points according to the scale.

Step V: Join the points with a line.

This is the required line graph.



4. **The following line graph shows the number of cars sold in a car showroom during five days of the week.**



Read the graph carefully. Then answer the questions given below.

Solution:

a) How many cars were sold altogether during the five days of the week?

To find the number of cars sold altogether during the five days of the week, find and add the cars sold each day.

Total cars sold= 2(Mon) + 4(Tue) + 6(Wed) + 3(Thu) + 9(Fri) = 24 cars

b) How many cars were sold on Tuesday?

We can see that the point on Tuesday is aligned with mark 4 on y-axis. So, 4 cars were sold on Tuesday.

c) How many cars were sold on Friday?

We can see that the point on Friday is exactly halfway between the mark 8 and 10 on y-axis. So, 9 cars were sold on Friday.

d) How many cars were sold on Monday?

We can see that the point on Monday is aligned with mark 2 on y-axis. So, 2 cars were sold on Monday.

e) On which day were the fewest number of cars sold?

We can see from the graph that the lowest value is 2. So, the fewest number of cars sold on Monday.

f) On which day were the most cars sold?

We can see from the graph that the highest value is 9. So, the most number of cars sold on Friday.

5. The following table shows the number of computer parts in stock. Draw a pie graph for the following data.

Stock	Motherboard	Hard disk	Processor	Keyboard	Monitor
No. of parts	30	40	20	80	50

Solution:

Step I: Find the total number of parts:

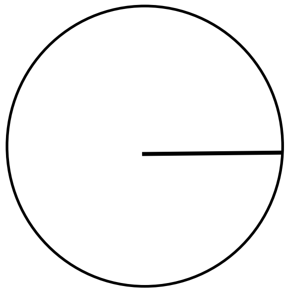
30 + 40 + 20 + 80 + 50 = 220

Step II: Find the measurement of the central angles for each part by using the formula:

$$\textbf{Angle of each sector} = \frac{\textbf{Frequency of each observation}}{\textbf{Total number of observations}} \times 360^0$$

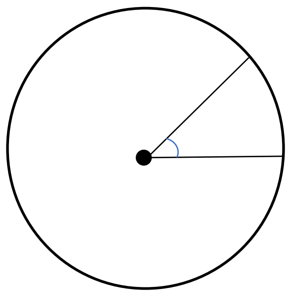
Stock	No. of parts	Measurement of the central angle for each part
Motherboard	30	$\frac{\text{No.of parts}}{\text{Total Parts}}\times360^o=\frac{30}{220}\times360^o=49^o$
Hard disk	40	$\frac{\text{No.of parts}}{\text{Total Parts}}\times360^o=\frac{40}{220}\times360^o=65^o$
Processor	20	$\frac{\text{No.of parts}}{\text{Total Parts}}\times360^o=\frac{20}{220}\times360^o=33^o$
Keyboard	80	$\frac{\text{No.of parts}}{\text{Total Parts}}\times360^o=\frac{80}{220}\times360^o=131^o$
Monitor	50	$\frac{\text{No.of parts}}{\text{Total Parts}}\times360^o=\frac{50}{220}\times360^o=82^o$

Step III: Draw a circle of any radius and draw the radial segment.

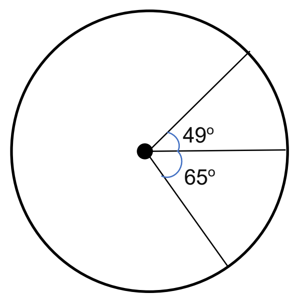


Step IV: Take a protractor and place it on the centre of the circle.

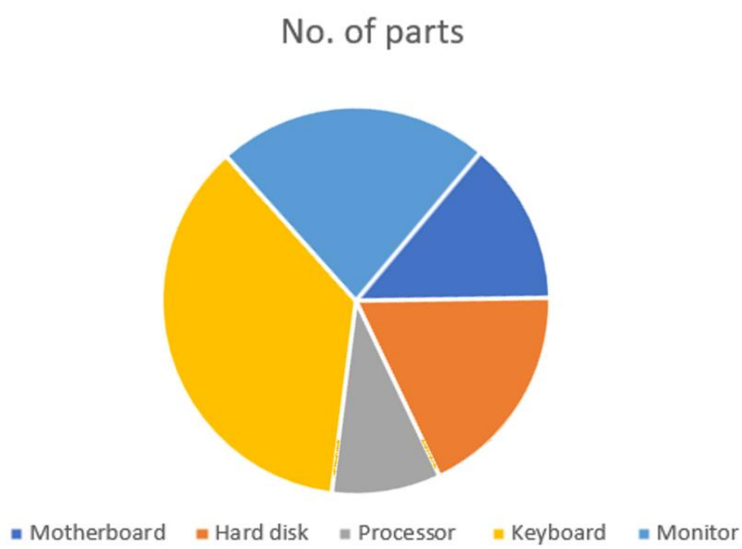
Draw an angle of 49º for the first part, motherboard.



Step V: Again, place a protractor and align it on the centre of the circle aligned with the radial segment (the arm of 49º). Draw an angle of 65º for the second part, hard disk.



Continue the procedure and make all the required angles of each part.



Learning Check 12.2

1. **Construct a frequency distribution table for the different colours of beads in a jar, which were counted as follows:**
   green, black, green, green, blue, yellow, yellow, red, blue, red, green, red, yellow, green.red, green, red, black, green, blue.
   Solution:
   To construct a frequency table, we proceed as follows:
   Step I: Construct a table with three columns.
   Step II: Write the values (colours) in the first column in order.
   Step III: Now count the number of times each value occurred and put the corresponding tally marks against it.
   Step IV: After all the tally marks are tabulated, count them and write the number in the third column.

Color	Tally	Frequency
Green	~~////~~ //	7
Black	//	2
Blue	///	3
Yellow	///	3
Red	////	4
Total		19

   This is the required frequency table.

2. **Construct a grouped frequency distribution table for the following data showing the number of tree saplings planted by 50 schools during a planting campaign.**
   28, 32, 65, 65, 69, 33, 42, 32, 38, 42, 40, 69, 62, 37, 65, 63, 42, 65, 49, 52, 64, 68, 52, 59, 44, 62, 31, 36, 38, 42, 39, 58, 56, 23, 35, 30, 68, 69, 43, 45, 39, 66, 27, 42, 53, 69, 55, 66, 49, 52
   Solution:
   Step I: First arrange the data in order:

23, 27, 28, 30, 31, 32, 32, 33, 35, 36, 37, 38, 38, 39, 39, 40, 42, 42, 42, 42, 42, 43, 44, 45, 49, 49, 52, 52, 52, 53, 55, 56, 58, 59, 62, 62, 63, 64, 65, 65, 65, 65, 66, 66, 68, 68, 69, 69, 69, 69

Here the highest value is 69 and lowest value is 23.

So, Range = Highest data value - Lowest data value = 69 - 23 = 46.

Step II: Decide how many classes you need to divide the data into. We chose 10 classes.

Step III: Find the size of the class interval:

Size of class interval = Range ÷ No. of classes = 46 ÷ 10 = 4.6 ≈ 5

Step IV: Find the class boundaries for each class. For this, first subtract the upper-class limit of the first class from the lower-class limit of the second class. Divide this result by 2. Subtract the result of the division (0.5) from the lower-class limit of each class. Similarly, add the result of the division (0.5) to the upper-class limit of each class. In this way, we will find the class boundaries.

Step V: Write the classes in the first column, the class boundaries in the second column, the tallies in the third column and the frequency in the fourth column

Class	Class Boundaries	Tally marks	Frequency
21 - 25	20.5 - 25.5	|	1
26 - 30	25.5 - 30.5	|||	3
31 - 35	30.5 - 35.5	|||||	5
36 - 40	35.5 - 40.5	||||| ||	7
41 - 45	40.5 - 45.5	||||| |||	8
46 - 50	45.5 - 50.5	||	2
51 - 55	50.5 - 55.5	|||||	5
56 - 60	55.5 - 60.5	|||	3
61 - 65	60.5 - 65.5	||||| |||	8
66 - 70	65.5 - 70.5	||||| |||	8
Total			50

This is the required grouped frequency distribution table.

3. **There are 30 letters of the English alphabet arranged in a table. Represent this data in a frequency distribution table.**

A, B, D, A, F, E, C, C, A, D, B, B, B, E, F, F, C, A, C, D, D, B, B, A, A, F, C, E, A, D

Solution:

To construct a frequency table, we proceed as follows:

Step I: Construct a table with three columns.

Step II: Write the values (Alphabets) in the first column in order.

Step III: Now count the number of times each value occurred and put the corresponding tally marks against it.

Step IV: After all the tally marks are tabulated, count them and write the number in the third column

Data Value	Tally	Frequency
A	~~||||~~ ||	7
B	~~||||~~ |	6
C	~~||||~~	5
D	~~||||~~	5
E	|||	3
F	||||	4
Total		30

This is the required grouped frequency distribution table.

4. Make a grouped frequency distribution table of 30 students who got marks in a mathematics test out of 100, as given below.

92, 40, 50, 50, 56, 60, 70, 60, 60, 88, 10, 20, 36, 92, 95, 40, 50, 56, 60, 70, 40, 36, 40, 36, 70, 72, 70, 80, 88, 92

Solution:

Solution:

Step I: First arrange the data in order:

10, 20, 36, 36, 36, 40, 40, 40, 40, 50, 50, 50, 56, 56, 60, 60, 60, 60, 70, 70, 70, 70, 72, 80, 88, 88, 92, 92, 92, 95.

Here the highest value is 95 and lowest value is 10.

So, Range = Highest data value - Lowest data value = 95 - 10 = 85.

Step II: Decide how many classes you need to divide the data into. We chose 11 classes.

Step III: Find the size of the class interval:

Size of class interval = Range ÷ No. of classes = 85 ÷ 11 = 7.72 ≈ 8

Step IV: Find the class boundaries for each class. For this, first subtract the upper-class limit of the first class from the lower-class limit of the second class. Divide this result by 2. Subtract the result of the division (0.5) from the lower-class limit of each class. Similarly, add the result of the division (0.5) to the upper-class limit of each class. In this way, we will find the class boundaries.

Step V: Write the classes in the first column, the class boundaries in the second column, the tallies in the third column and the frequency in the fourth column.

Class	Class Boundaries	Tally marks	Frequency
9 - 16	8.5 - 16.5	|	1
17 - 24	16.5 - 24.5	|	1
25 - 32	24.5 - 32.5		0
33 - 40	32.5 - 40.5	||||| ||	7
41 - 48	40.5 - 48.5		0
49 - 56	48.5 - 56.5	|||||	5
57 - 64	56.5 - 64.5	||||	4
65 - 72	64.5 - 72.5	|||||	5
73 - 80	72.5 - 80.5	|	1
81 - 88	80.5 - 88.5	||	2
89 - 96	88.5 - 96.5	||||	4
Total			30

This is the required grouped frequency distribution table.

5. The following data shows the scores of 40 students in the 2020 Division Achievement Test. Represent the given data in a grouped frequency distribution table and answer the following questions.

18, 20, 22, 36, 22, 20, 14, 39, 38, 22, 15, 17, 21, 43, 28, 16, 35, 16, 20, 18, 25, 22, 33, 18, 17, 20, 20, 37, 39, 23, 16, 25, 15, 33, 36, 24, 24, 28, 38, 22

a) What is the range of the given data? b) What is the size of the class?

c) Which class has the lowest frequency? d) Which class has the highest frequency?

Solution:

Step I: First arrange the data in order:

14, 15, 15, 16, 16, 16, 17, 17, 18, 18, 18, 20, 20, 20, 20, 20, 21, 22, 22, 22, 22, 22, 23, 24, 24, 25, 25, 28, 28, 33, 33, 35, 36, 36, 37, 38, 38, 39, 39, 43

Here the highest value is 43 and lowest value is 14.

So, Range = Highest data value - Lowest data value = 43 - 14 = 29.

Step II: Decide how many classes you need to divide the data into. We chose 7 classes.

Step III: Find the size of the class interval:

Size of class interval = Range ÷ No. of classes = 29 ÷ 7 = 4.14 ≈ 5 (round up as we need 7 classes)

Step IV: Find the class boundaries for each class. For this, first subtract the upper-class limit of the first class from the lower-class limit of the second class. Divide this result by 2. Subtract the result of the division (0.5) from the lower-class limit of each class. Similarly, add the result of the division (0.5) to the upper-class limit of each class. In this way, we will find the class boundaries.

Step V: Write the classes in the first column, the class boundaries in the second column, the tallies in the third column and the frequency in the fourth column.

Class	Class Boundaries	Tally marks	Frequency
11 - 15	10.5 - 15.5	|||	3
16 - 20	15.5 - 20.5	||||| ||||| |||	13
21 - 25	20.5 - 25.5	||||| ||||| |	11
26 - 30	25.5 - 30.5	||	2
31 - 35	30.5 - 35.5	|||	3
36 - 40	35.5 - 40.5	||||| ||	7
41 - 45	40.5 - 45.5	|	1
Total			40

This is the required grouped frequency distribution table. Lets answer the given questions:

a) What is the range of the given data?

The range of the data is:

Range = Highest data value - Lowest data value = 43 - 14 = 29.

b) What is the size of the class?

The size of the class is 5.

c) Which class has the lowest frequency?

The class 41- 45 has the lowest frequency i.e., 1.

d) Which class has the highest frequency?

The class 16 - 20 has the highest frequency i.e., 13.

Learning Check 12.3

1. The following data shows the number of people who visited the park daily during a month. Construct a histogram for this data.

42, 64, 36, 62, 41, 47, 87, 69, 65, 55, 83, 79, 34, 68, 25, 28, 46, 51, 74, 27, 29, 44, 59, 96, 40, 20, 50, 88, 86, 93

Solution:

Step I: First arrange it in ascending order for convenience and construct a grouped frequency distribution.

20, 25, 27, 28, 29, 34, 36, 40, 41, 42, 44, 46, 47, 50, 51, 55, 59, 62, 64, 65, 68, 69, 74, 79, 83, 86, 87, 88, 93, 96

Class	Class Boundaries	Tally marks	Frequency
17 - 24	16.5 - 24.5	|	1
25 - 32	24.5 - 32.5	||||	4
33 - 40	32.5 - 40.5	|||	3
41 - 48	40.5 - 48.5	|||||	5
49 - 56	48.5 - 56.5	|||	3
57 - 64	56.5 - 64.5	|||	3
65 - 72	64.5 - 72.5	|||	3
73 - 80	72.5 - 80.5	||	2
81 - 88	80.5 - 88.5	||||	4
89 - 96	88.5 - 96.5	||	2
Total			30

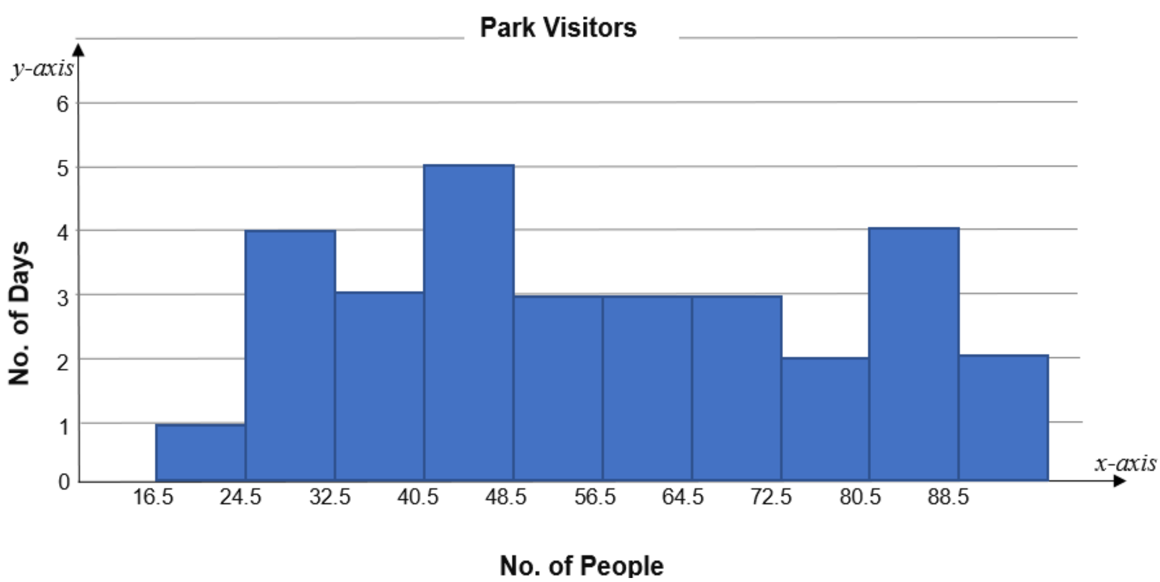
Step II: Draw the x and y axis.

Step III: Label the x-axis as "Number of people" and the y-axis as the "number of days".

Step IV: On the horizontal axis, label the lower boundary of each interval.

Step V: Draw a bar extending from the lower value of each interval to the lower value of the next interval.

The height of each bar should be equal to its corresponding frequency.



This is the required histogram.

2. **The following data are the weights (in grams) of different kinds of candies. Draw a histogram for this continuous data.**
60, 60.5, 61, 61, 61.5, 63.5, 63.5, 63.5, 64, 64, 64, 64, 64, 64, 64, 64.5, 64.5, 64.5, 64.5,64.5, 64.5, 64.5, 64.5, 66, 66, 66, 66, 66, 66, 66, 66, 66, 66, 66.5, 66.5, 66.5, 66.5, 66.5,66.5, 66.5, 66.5, 66.5, 66.5, 66.5, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67, 67.5, 67.5,67.5, 67.5, 67.5, 67.5, 67.5, 68, 68, 69, 69, 69, 69, 69, 69, 69, 69, 69, 69, 69.5, 69.5, 69.5,69.5, 69.5, 70, 70, 70, 70, 70, 70, 70.5, 70.5, 70.5, 71, 71, 71

Step I: First a grouped frequency distribution.

Class	Class Boundaries	Tally marks	Frequency
57 - 60	56.5 - 60.5	||	2
61 - 64	60.5 - 64.5	||||| ||||| ||||| ||||| |	21
65 - 68	64.5 - 68.5	||||| ||||| ||||| ||||| ||||| ||||| ||||| ||||| ||	42
69 - 72	68.5 - 72.5	||||| ||||| ||||| ||||| ||||| ||	27
Total			92

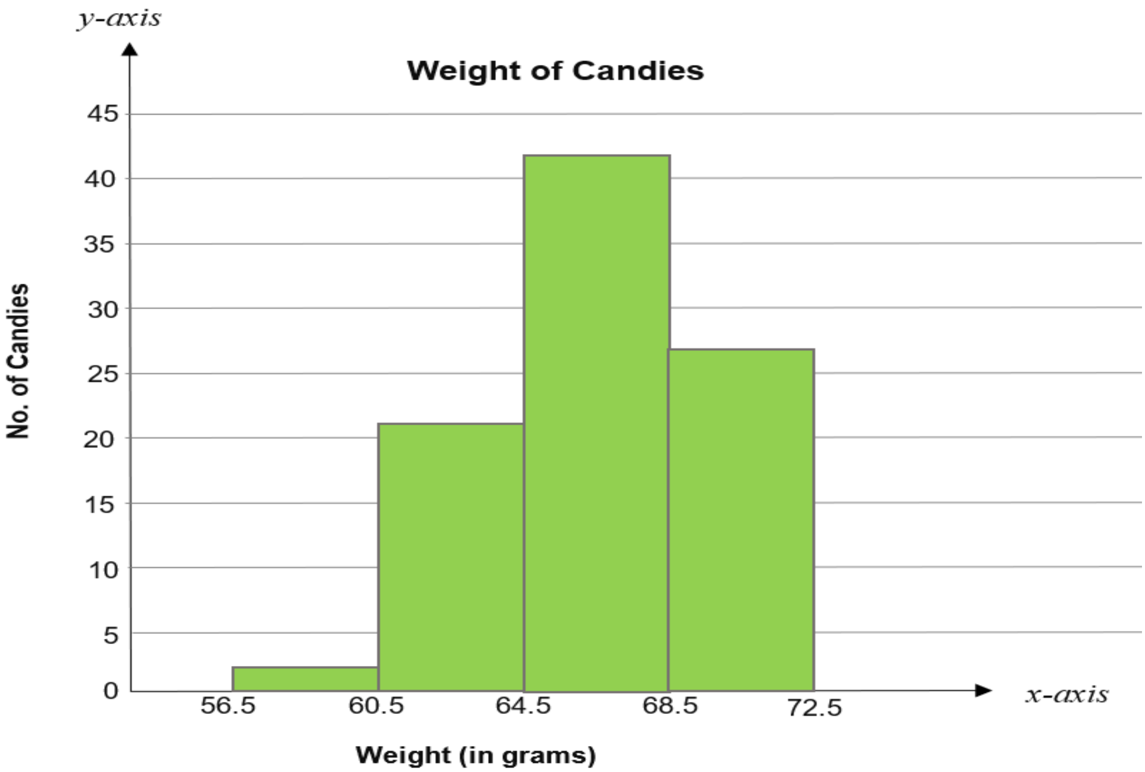
Step II: Draw the x and y axis.

Step III: Label the x-axis as "Eight (in grams)" and the y-axis as the "number of candies".

Step IV: On the horizontal axis, label the lower boundary of each interval.

Step V: Draw a bar extending from the lower value of each interval to the lower value of the next interval.

The height of each bar should be equal to its corresponding frequency.



This is the required histogram.

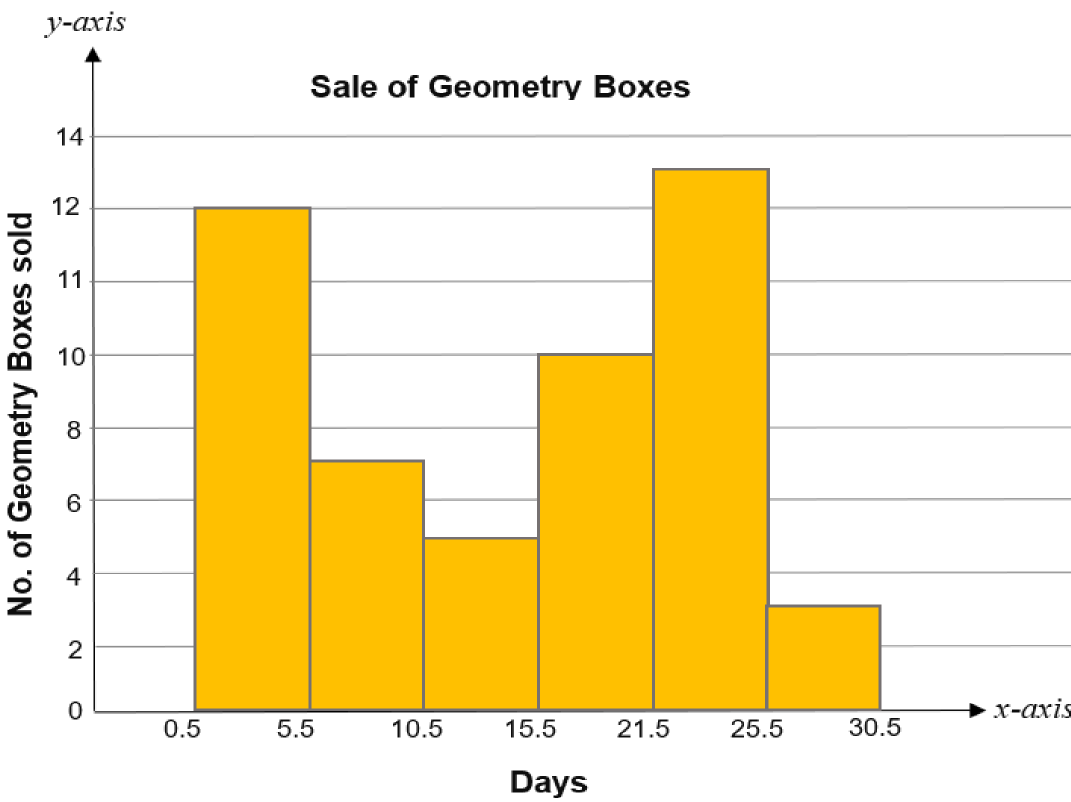
3. Construct a histogram for the following frequency distribution table below shows the sales of geometry boxes in a stationery shop for 30 days..

Class Interval	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30
Frequency	12	7	5	10	13	3

First create frequency distribution table.

Class Interval	Class Boundaries	Tally	Frequency
1 – 5	0.5 – 5.5	𝍸 𝍸 //	12
6 – 10	5.5 – 10.5	𝍸 //	7
11 – 15	10.5 – 15.5	𝍸	5
16 – 20	15.5 – 20.5	𝍸 𝍸	10
21 – 25	21.5 – 25.5	𝍸 𝍸 ///	13
26 – 30	25.5. – 30.5	///	3
Total			50

Draw axis and label them. Then draw bars according to the frequencies.



This is the required histogram.

4. Construct a histogram for the following data.

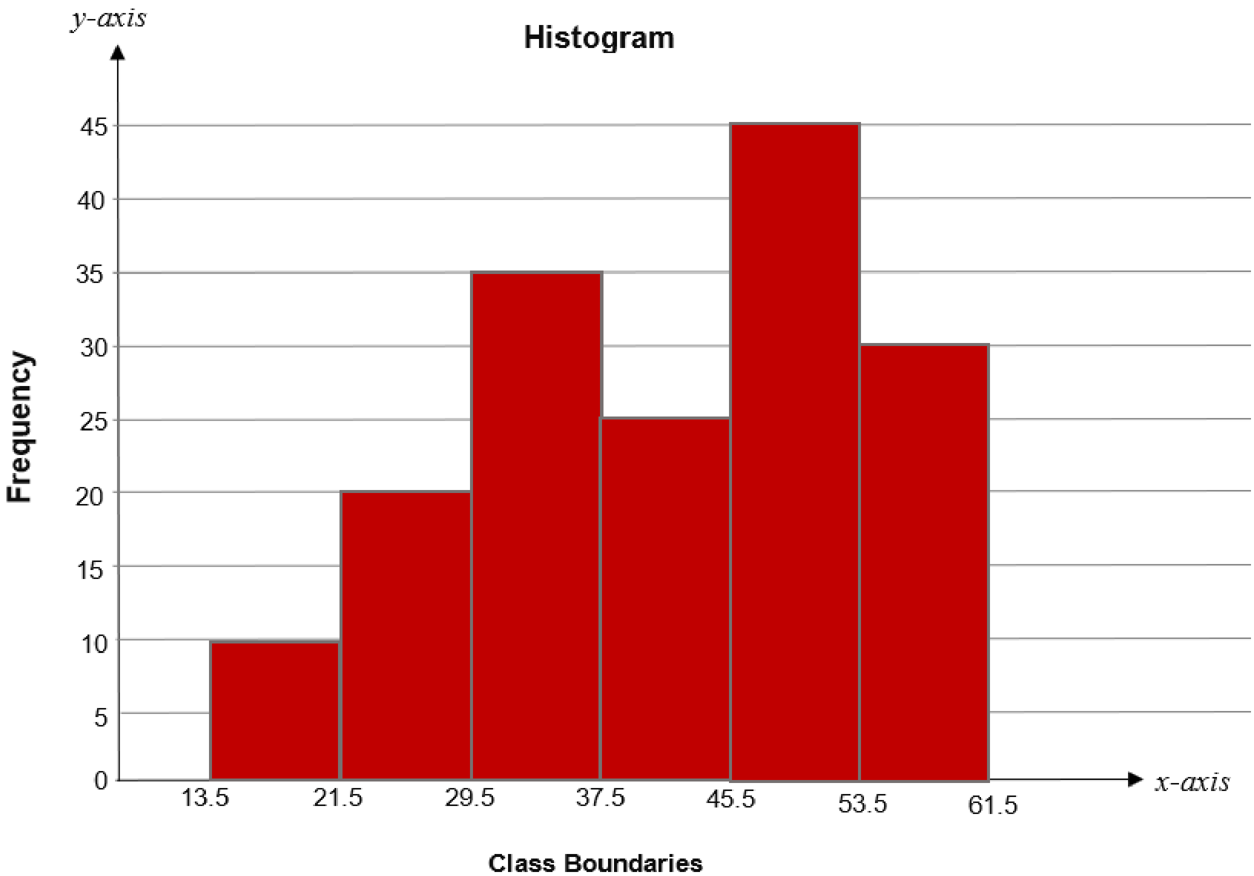
Class Interval	14 – 21	22 – 29	30 – 37	38 – 45	46 – 53	54 – 61

Frequency	10	20	35	25	45	30

First create frequency distribution table.

Class Interval	Class Boundaries	Tally	Frequency
14 – 21	13.5 – 21.5	𝍸 𝍸	10
22 – 29	21.5 – 29.5	𝍸 𝍸 𝍸 𝍸	20
30 – 37	29.5 – 37.5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	35
38 – 45	37.5 – 45.5	𝍸 𝍸 𝍸 𝍸 𝍸	25
46 – 53	45.5 – 53.5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	45
54 – 61	53.5 – 61.5	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸	30
Total			165

Draw axis and label them. Then draw bars according to the frequencies.



This is the required histogram.

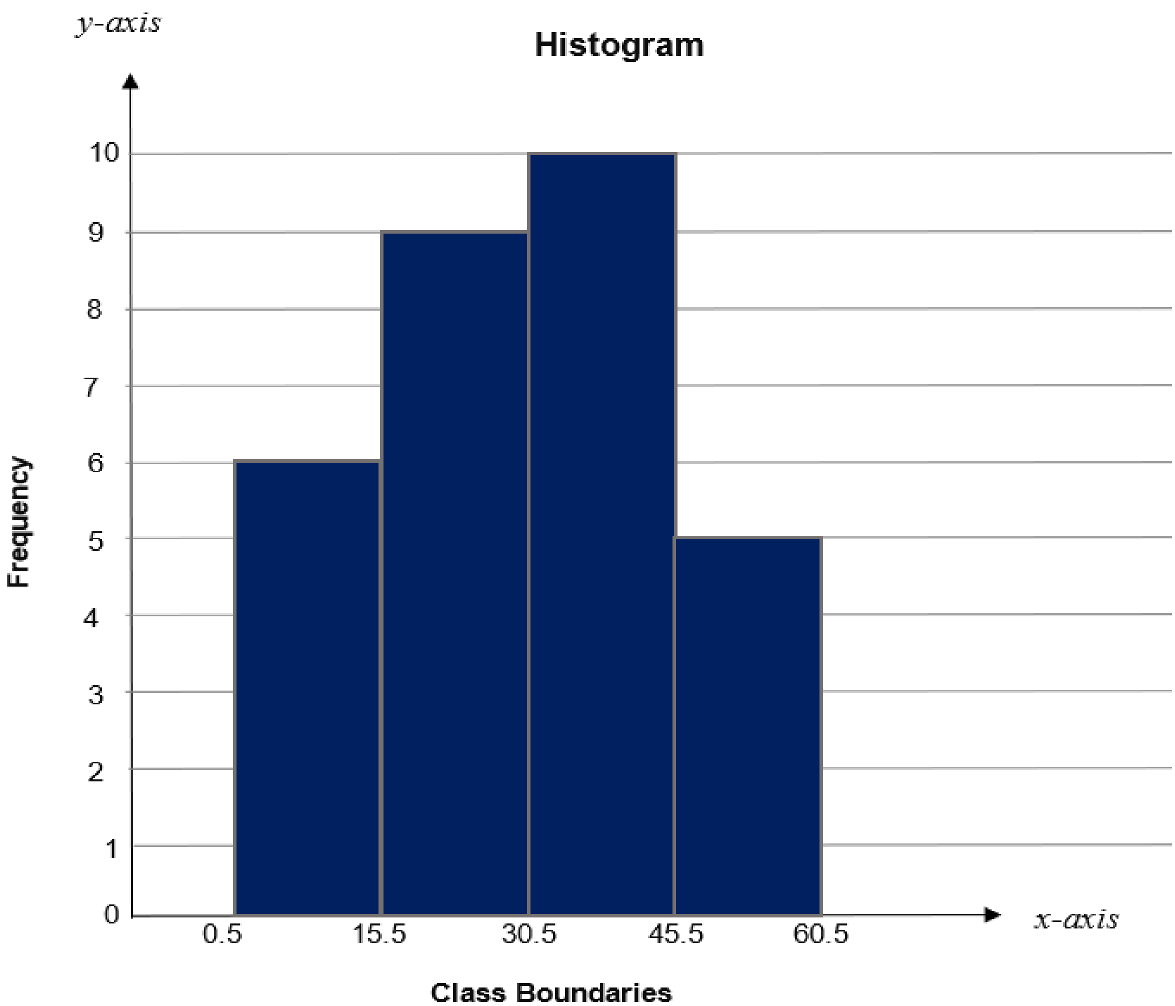
5. Construct a histogram for the following data.

Class Interval	1 – 15	16 – 30	31 – 45	46 – 60
Frequency	6	9	10	5

First create frequency distribution table.

Class Interval	Class Boundaries	Tally	Frequency
1 – 15	0.5-15.5	~~||||~~ /	6
16 – 30	15.5-30.5	~~||||~~ ////	9
31 – 45	30.5-45.5	~~||||~~ ~~||||~~	10
46 – 60	45.5-60.5	~~||||~~	5
			30

Draw axis and label them. Then draw bars according to the frequencies.



This is the required histogram.

Learning Check 12.4

1. **Find the mean of the following data:**

a) 13, 44, 65, 73, 95 b) 37, 53, 67, 72, 85, 94

c) 123, 137, 148, 152, 175 d) 240, 355, 420, 590, 760, 947

Solution:

a) 13, 44, 65, 73, 95

Mean $=\bar{x}=\sum x/n$

= (13 + 44 + 65 + 73 + 95) / 5

= 290 / 5

= 58

b) 37, 53, 67, 72, 85, 94

Mean $=\bar{x}=\sum x/n$

= (37 + 53 + 67 + 72 + 85 + 94) / 6

= 408 / 6

= 68

c) 123, 137, 148, 152, 175

Mean $=\bar{x}=\sum x/n$

= (123 + 137 + 148 + 152 + 175) / 5

= 735 / 5

= 147

d) 240, 355, 420, 590, 760, 947

Mean $= \bar{x}=\sum x/n$

=(240 + 355 + 420 + 590 + 760 + 947) / 6

= 3312 / 6

= 552

2. The weights (in kilograms) of students in grade VII are given below. Calculate their mean weight.

24, 29, 37, 42, 46, 48, 52, 56, 55, 45, 43, 39

Solution:

Mean weight = $\bar{x} = \sum x / n$

= (24 + 29 + 37 + 42 + 46 + 48 + 52 + 56 + 55 + 45 + 43 + 39) / 12

= 516 / 12

= 43

Therefore, the mean weight of the students is 43 kilograms.

3. The following table shows the grouped data for the number of bulbs in 40 houses.

Calculate the mean of the grouped data.

No. of bulbs	1 - 4	5 - 8	9 - 12	13 - 16	17 - 20
No. of houses	12	13	6	4	3

Solution:

First, we need to find the mid-points of the given data by using the formula.

$$\textbf{Mid-point = X = } \frac{\text{Lower class limit + Upper class limit}}{2}$$

Class Interval	Lower Class	Upper class	Mid-points (X)
1 – 4	1	4	(1+ 4)/2 = 2.5
5 – 8	5	8	(5+8)/2 = 6.5
9 – 12	9	12	(9+12)/2 = 10.5
13 – 16	13	16	(13+16)/2 = 14.5
17 – 20	17	20	(17+20)/2 = 18.5

Now, by multiplying the mid-point (X) with frequency (f), we get (fX).

Class Interval	Lower Class	Upper class	Mid-points (X)	Frequency (f)	fX
1 – 4	1	4	2.5	12	30
5 – 8	5	8	6.5	13	84.5
9 – 12	9	12	10.5	6	63

13 – 16	13	16	14.5	4	58
17 - 20	17	20	18.5	3	55.5
Total				$\sum f = 38$	$\sum fX$ =291

Using the formula of arithmetic mean for grouped data:

Arithmetic Mean = $\bar{X} = \frac{\sum fX}{\sum f}$

$$\bar{X} = \frac{291}{38} = 7.68 \approx 8$$

Hence, the average bulbs = $\bar{X} = 8\ bulbs$

4. The following table shows the grouped data for the rainfall in a certain city for a sample of 20 years. Calculate the mean of the grouped data.

Rainfall (inches)	21 - 25	26 - 30	31 - 35	36 - 40	41 - 45	46 - 50
No. of years	2	5	3	9	0	1

Solution:

First, we need to find the mid-points of the given data by using the formula.

Mid-point = X = $\frac{\textbf{Lower class limit + Upper class limit}}{\textbf{2}}$

Class Interval	Lower Class	Upper class	Mid-points (X)
21 – 25	21	25	(21+25)/2=23
26 – 30	26	30	(26+30)/2=28
31 – 35	31	35	(31+35)/2=33
36 – 40	36	40	(36+40)/2=38
41 – 45	41	45	(41+45)/2=43
46 – 50	46	50	(46+50)/2=48

Now, by multiplying the mid-point (X) with frequency (f), we get (fX).

Class Interval	Lower Class	Upper class	Mid-points (X)	Frequency (f)	fX
21 – 25	21	25	23	2	46
26 – 30	26	30	28	5	140

31 – 35	31	35	33	3	99
36 – 40	36	40	38	9	342
41 – 45	41	45	43	0	0
46 – 50	46	50	48	1	48
Total				$\sum f$ =20	$\sum fX$ =675

Using the formula of arithmetic mean for grouped data:

Arithmetic Mean = $\bar{X}$ = $\frac{\sum fX}{\sum f}$

$$\bar{X} = \frac{675}{20} = 33.75 \approx 34$$

Hence, the average rainfall = $\bar{X} = 34\ inches$

5. The marks of students in grade VII in a math test are 3, 2, 8, 9, 6, 5, 6, 3, 8, 3. Calculate the median and mode.

For median, arrange the data in ascending order:

2, 3, 3, 3, 5, 6, 6, 8, 8, 9

Number of values = n = 10 (even)

Median = arithmetic mean of $(n/2)^{th}$ and $[(n/2)+1]^{th}$ observation

= arithmetic mean of $(10/2)^{th}$ and $[(10/2)+1]^{th}$ observation

= arithmetic mean of 5th and 6th observation

2, 3, 3, 3, **5**, **6**, 6, 8, 8, 9

5th observation=5,

6th observation= 6

Arithmetic mean of 5th and 6th observation= (5+6)/2 = 11/2= 5.5

So, the median of the given data values is 5.5.

To find the mode of the data, we need to find the value that appears most frequently.

Here, the value 3 appears three times, which is more than any other value.

So, the mode is 3.

6. **Find the median of the following data:**

   a) 72, 68, 91, 87, 52 **b) 250, 657, 530, 810, 425, 590**

   Solution:

   a) Arrange the given data in ascending order:

   52, 68, 72, 87, 91

   Number of values = n =5 (odd)

   Median = $\left(\frac{n+1}{2}\right) th\ observation$

   Median = $\left(\frac{5+1}{2}\right) th\ observation$ = $\left(\frac{6}{2}\right) th\ observation$ = 3rd observation

   52, 68, **72**, 87, 91

   Median = 72

   b) 250, 657, 530, 810, 425, 590

   Arrange the given data in ascending order:

   250, 425, 530, 590, 657, 810

   Number of values = n = 6 (even)

   Median = arithmetic mean of $(n/2)^{th}$ and $[(n/2)+1]^{th}$ observation

   = arithmetic mean of $(6/2)^{th}$ and $[(6/2)+1]^{th}$ observation

   = arithmetic mean of 3rd and 4th observation

   250, 425, **530**, **590**, 657, 810

   5th observation=530

   6th observation= 590

   Arithmetic mean of 5th and 6th observation= (530+590)/2 = 1120/2= 560

   So, the median of the given data values is 560.

7. **Find the mode of the following data:**

   a) 3, 6, 2, 8, 9, 2, 2, 5 b) 35, 42, 50, 32, 67, 72, 63

   Solution:

a) To find the mode of the data, we need to find the value that appears most frequently. Here, the value 2 appears three times, which is more than any other value. So, the mode is 2.

b) To find the mode of the data, we need to find the value that appears most frequently. Here, none of the values appear more than once. So, there is no mode for this data.

Learning Check 12.5

1. Marwa rolled a single six-sided dice. Find the probability of the given events. Tell if there are any certain or impossible events.

a) rolling a factor of 10

b) rolling an even prime number

c) rolling a prime number

d) rolling a number

Solution:

a) rolling a factor of 10

Possible outcomes = 1, 2, 3, 4, 5, 6

Favourable Outcomes: 1, 2, 5

Number of favourable outcomes = 3

Total number of possible outcomes = 6

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{3}{6}$

$= 1/2$

b) rolling an even prime number

Possible outcomes = 1, 2, 3, 4, 5, 6

Favourable Outcomes: 2

Number of favourable outcomes = 1

Total number of possible outcomes = 6

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{1}{6}$

c) rolling a prime number

Possible outcomes = 1, 2, 3, 4, 5, 6

Favourable Outcomes: 2, 3, 5

Number of favourable outcomes= 3

Total number of possible outcomes = 6

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{3}{6}$

$= 1/2$

d) rolling a number

Possible outcomes = 1, 2, 3, 4, 5, 6

Favourable Outcomes: 1, 2, 3, 4, 5, 6

Number of favourable outcomes= 6

Total number of possible outcomes = 6

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{6}{6}$

= 1

As the probability of a certain event is 1, so this is a certain event.

2. Ali chose a ticket at random. If the numbers on it ranged from 1 to 50, find the probability of the given events. Tell if there are any certain or impossible events. Also find the probability of the complement of these events.

a) picking the number 20 b) picking a number less than 15

c) picking a number greater than 25 d) picking a number

e) picking an even number

f) picking a number between 31 and 37

g) picking a multiple of 4

h) not picking a number

Solution:

a) picking the number 20

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{1}{50}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{1}{50}$

$= \frac{50-1}{50}$

$= \frac{49}{50}$

b) picking a number less than 15

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{14}{50}$

$= \frac{7}{25}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{7}{25}$

$= \frac{25-7}{25}$

$$= \frac{18}{25}$$

c) picking a number greater than 25

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$$= \frac{25}{50}$$

$$= \frac{1}{2}$$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$$= 1 - \frac{1}{2}$$

$$= \frac{2-1}{2}$$

$$= \frac{1}{2}$$

d) picking a number

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$$= \frac{50}{50}$$

$$=1$$

As the probability of a certain event is 1, so this is a certain event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$$= 1 - 1$$

$$= 0$$

So, the compliment of this event is impossible.

e) picking an even number

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{25}{50}$

=1/2

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{1}{2}$

$= \frac{2-1}{2}$

$= \frac{1}{2}$

f) picking a number between 31 and 37

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{5}{50}$

=1/10

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{1}{10}$

$$= \frac{10-1}{10}$$

$$= \frac{9}{10}$$

g) picking a multiple of 4

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{12}{50}$

=6/25

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{6}{25}$

$= \frac{25-6}{25}$

$= \frac{19}{25}$

h) not picking a number

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{0}{50}$

=0

As the probability of an impossible event is 0, so this is an impossible event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 0$

$= 1$

So, the compliment of given event is certain.

3. Huma purchased 40 prayer mats of various materials, including 13 wool mats, 16 cotton mats, and the remaining are plastic. She picked up a mat randomly from the packing box. Find the probability of the following and tell if any of them is certain or impossible. Also find the probability of the complement of these events.

a) P (wool mats) **b) P (a mat)** **c) P (not a mat)**

Solution:

a) P (wool mats)

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{13}{40}$$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$$= 1 - \frac{13}{40}$$

$$= \frac{40-13}{40}$$

$$= \frac{27}{40}$$

b) P (a mat)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{40}{40}$

$=1$

As the probability of a certain event is 1, so this is a certain event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 1$

$= 0$

So, the compliment of given event is impossible.

c) P (not a mat)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{0}{50}$

$=0$

As the probability of an impossible event is 0, so this is an impossible event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 0$

$= 1$

So, the compliment of given event is certain.

4. A fair coin is tossed. Find the probability of the following events and tell if any of them is certain or impossible. Also find the probability of the complement of these events.

a) P (head) **b) P (tail)** **c) P (number 3)**

Solution:

a) P (head)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{1}{2}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{1}{2}$

$= \frac{2-1}{2}$

$= \frac{1}{2}$

b) P (tail)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{1}{2}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{1}{2}$

$= \frac{2-1}{2}$

$$= \frac{1}{2}$$

c) P (number 3)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$$= \frac{0}{2}$$

$$=0$$

As the probability of an impossible event is 0, so this is an impossible event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$$= 1 - 0$$

$$= 1$$

So, the compliment of given event is certain.

5. Arham chooses a random block from a bag that contains 5 yellow, 7 grey, 10 red, and 8 white blocks. Find the probability of the following events and tell if any of them is certain or impossible. Also find the probability of the complement of these events.

a) P (white) b) P (yellow) c) P (a block) d) P (not a block)

Solution:

a) P (white)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$$= \frac{8}{30} = \frac{4}{15}$$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{4}{15}$

$= \frac{15-4}{15}$

$= \frac{11}{15}$

b) P (yellow)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{5}{30} \quad = \frac{1}{6}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{1}{6}$

$= \frac{6-1}{6} \quad = \frac{5}{6}$

c) P (a block)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{30}{30} \quad = 1$

As the probability of a certain event is 1, so this is a certain event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 1$

$= 0$

So, the compliment of given event is impossible.

d) P (not a block)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{0}{30} \qquad = 0$

As the probability of an impossible event is 0, so this is an impossible event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 0 \qquad = \ 1$

So, the compliment of given event is certain.

Unit Check

1. Encircle the correct option.

a) When a graph is drawn using only lines, instead of pictures or rectangular bars, it is called a:

i) pie graph ii) line graph iii) bar graph iv) multiple bar graph

b) The term "discrete" means distinct or:

i) straight ii) grouped iii) separate iv) combined

c) The value of the data that is repeated the maximum number of times is called the:

i) mean ii) median iii) mode iv) frequency

d) The median of 13, 14, 18, 25, 27, 28, 35, 39 is:

i) 23 ii) 24 iii) 25 iv) 26

e) The arithmetic mean of the values: 2, 8, 5, 7, 3 is:

i) 3 ii) 5 iii) 6 iv) 7

2. Fill in the blanks.

a) If data is represented in sectors of a circle, then that graph is called a pie graph.

b) The values or quantities that can be presented in fractions or decimals fall under continuous data.

c) The mode of 2, 4, 5, 7, 7, 7, 7, 9 is 7.

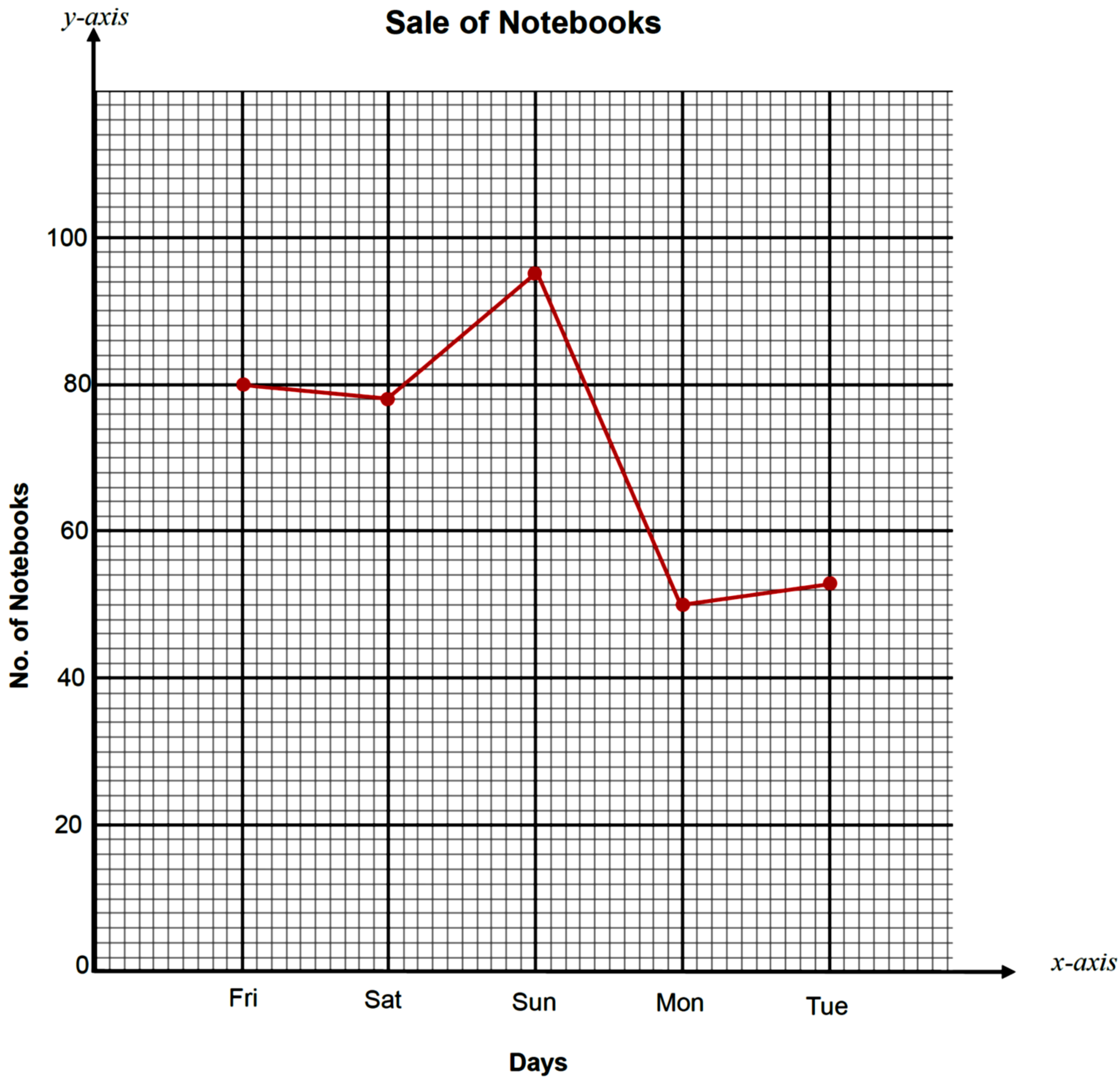
d) The median of data 21, 47, 32, 65, 43, 76, 52, 45 is 46 .

e) If a box has 5 green lights, 7 blue lights and 5 red lights, the probability of the complement of picking a red light is 12/17.

3. **A shopkeeper sold 80 notebooks on Friday, 78 notebooks on Saturday, 95 notebooks on Sunday, 50 notebooks on Monday and 53 notebooks on Tuesday. Draw a line graph for the given data.**

Solution:

Step I: Draw the x-axis and y-axis on graph paper.
Step II: Mark information "Days" along the x-axis and "No. of Notebooks" along the y-axis.
Step III: Choose a suitable scale to mark the values. In this case, 1 small square represents 2 notebooks.
Step IV: Mark the points according to the scale.
Step V: Join the points with a line.

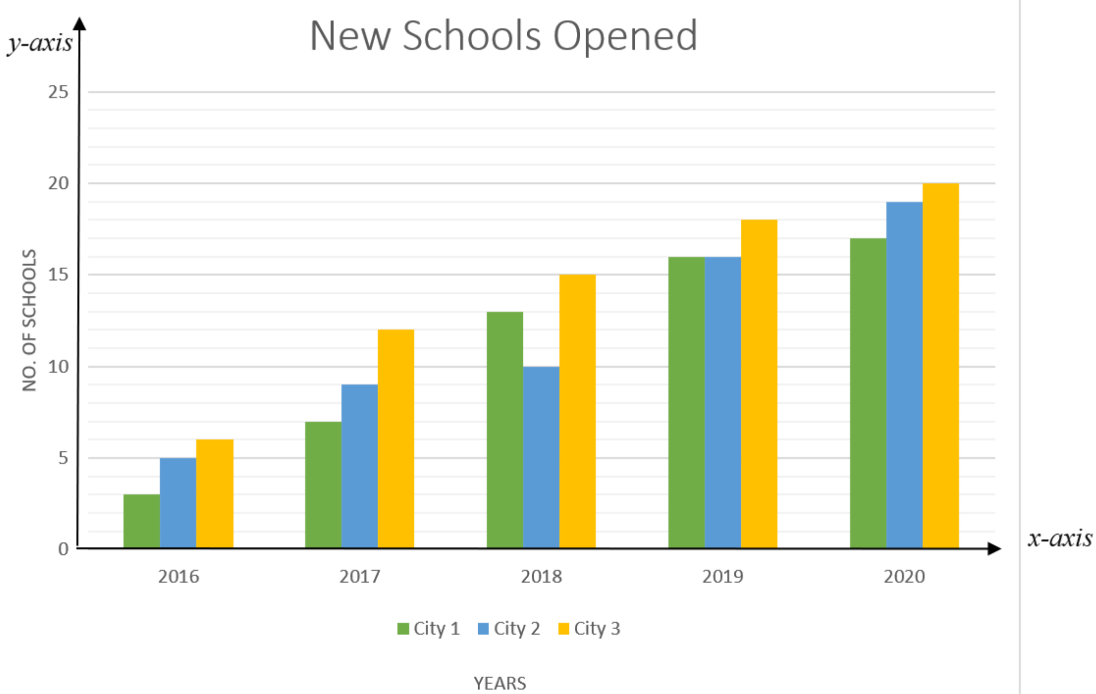


This is the required line graph.

4. The following data shows the number of new schools in three different cities over five successive years. Draw a vertical multiple bar graph for this data using an appropriate scale.

Years	2016	2017	2018	2019	2020
City 1	3	7	13	16	17
City 2	5	9	10	16	19
City 3	6	12	15	18	20

Solution:
Step I: Draw the x-axis (horizontal line) and y-axis (vertical line) perpendicular to each other on graph paper.
Step II: Mention the values along each axis. Here x-axis represents years and y-axis represents the no. of schools.
Step III: Choose an appropriate scale. Here one mark along y-axis represents 5 schools.
Step IV: Colour the correct numbers of values for each year and category and draw the multiple bars.



This is the required multiple bar graph, drawn vertically.

5. **The following data shows the mass (in tons) of various grains stored in a certain warehouse. Construct a pie graph of the given data below:**

Grains	**Wheat**	**Rice**	**Barley**	**Corn**	**Oats**
Mass (in tons)	350	400	150	100	90

Solution:

Step I: Find the total mass:

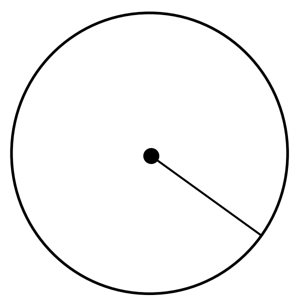
350 + 400 + 150 + 100 + 90 = 1090 ton

Step II: Find the measurement of the central angles for each part by using the formula:

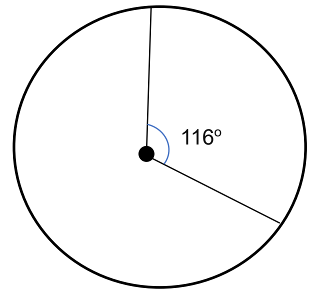
$$\textbf{Angle of each sector} = \frac{\textbf{Frequency of each observation}}{\textbf{Total number of observations}} \times 360^0$$

Grains	Mass(In Tons)	Measurement of the central angle
Wheat	350	$\frac{Mass\ of\ Grain}{\text{Total Mass}}\times 360^o = \frac{350}{1090}\times 360^o = 116^o$
Rice	400	$\frac{Mass\ of\ Grain}{\text{Total Mass}}\times 360^o = \frac{350}{1090}\times 360^o = 132^o$
Barley	150	$\frac{Mass\ of\ Grain}{\text{Total Mass}}\times 360^o = \frac{350}{1090}\times 360^o = 50^o$
Corn	100	$\frac{Mass\ of\ Grain}{\text{Total Mass}}\times 360^o = \frac{350}{1090}\times 360^o = 33^o$
Oats	90	$\frac{Mass\ of\ Grain}{\text{Total Mass}}\times 360^o = \frac{350}{1090}\times 360^o = 29^o$

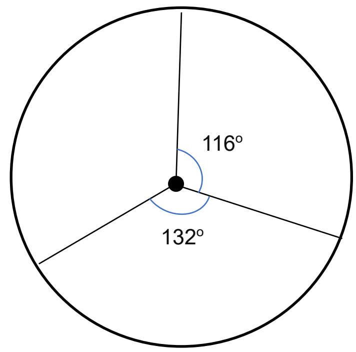
Step III: Draw a circle of any radius and draw the radial segment.



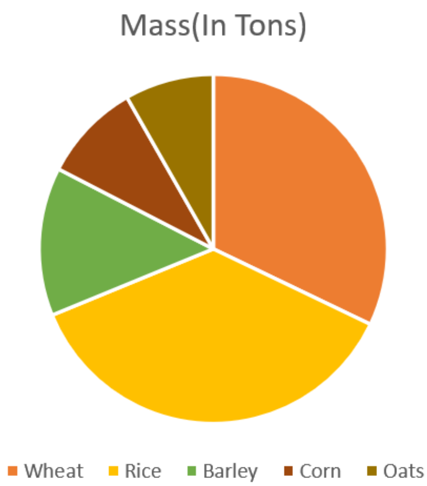
Step IV: Take a protractor and place it on the centre of the circle.
Draw an angle of 115^o for wheat.



Step V: Again, place a protractor and align it on the centre of the circle aligned with the radial segment (the arm of 116^o).
Draw an angle of 132^o for rice.



Continue the procedure and make all the required angles of each part.



6. **The following data shows the duration of conversations for calls in minutes. Construct the frequency distribution table for this data.**
   5, 8, 9, 21, 23, 20, 16, 19, 18, 26, 27, 11, 12, 14, 10, 12, 18, 16, 16, 19
   Solution:
   Step I: First arrange the data in order:
   5, 8, 9, 10, 11, 12, 12, 14, 16, 16, 16, 18, 18, 19, 19, 20, 21, 23, 26, 27
   Here the highest value is 27 and lowest value is 5.
   So, Range = Highest data value - Lowest data value = 27 - 5 = 22.

Step II: Decide how many classes you need to divide the data into. We chose 6 classes.

Step III: Find the size of the class interval:

Size of class interval = Range ÷ No. of classes = 22 ÷ 6 = 3.6 ≈ 4

Step IV: Find the class boundaries for each class. For this, first subtract the upper-class limit of the first class from the lower-class limit of the second class. Divide this result by 2. Subtract the result of the division (0.5) from the lower-class limit of each class. Similarly, add the result of the division (0.5) to the upper-class limit of each class. In this way, we will find the class boundaries.

Step V: Write the classes in the first column, the class boundaries in the second column, the tallies in the third column and the frequency in the fourth column.

Class	Class Boundaries	Tally marks	Frequency
5 - 8	4.5 - 8.5	||	2
9 - 12	8.5 - 12.5	|||||	5
13 - 16	12.5 - 16.5	||||	4
17 - 20	16.5 - 20.5	|||||	5
21 - 24	20.5 - 24.5	||	2
25 - 28	24.5 - 28.5	||	2
Total			20

This is the required grouped frequency distribution table.

7. **The following data shows the monthly income of the employees in a factory. Construct a histogram for this data.**

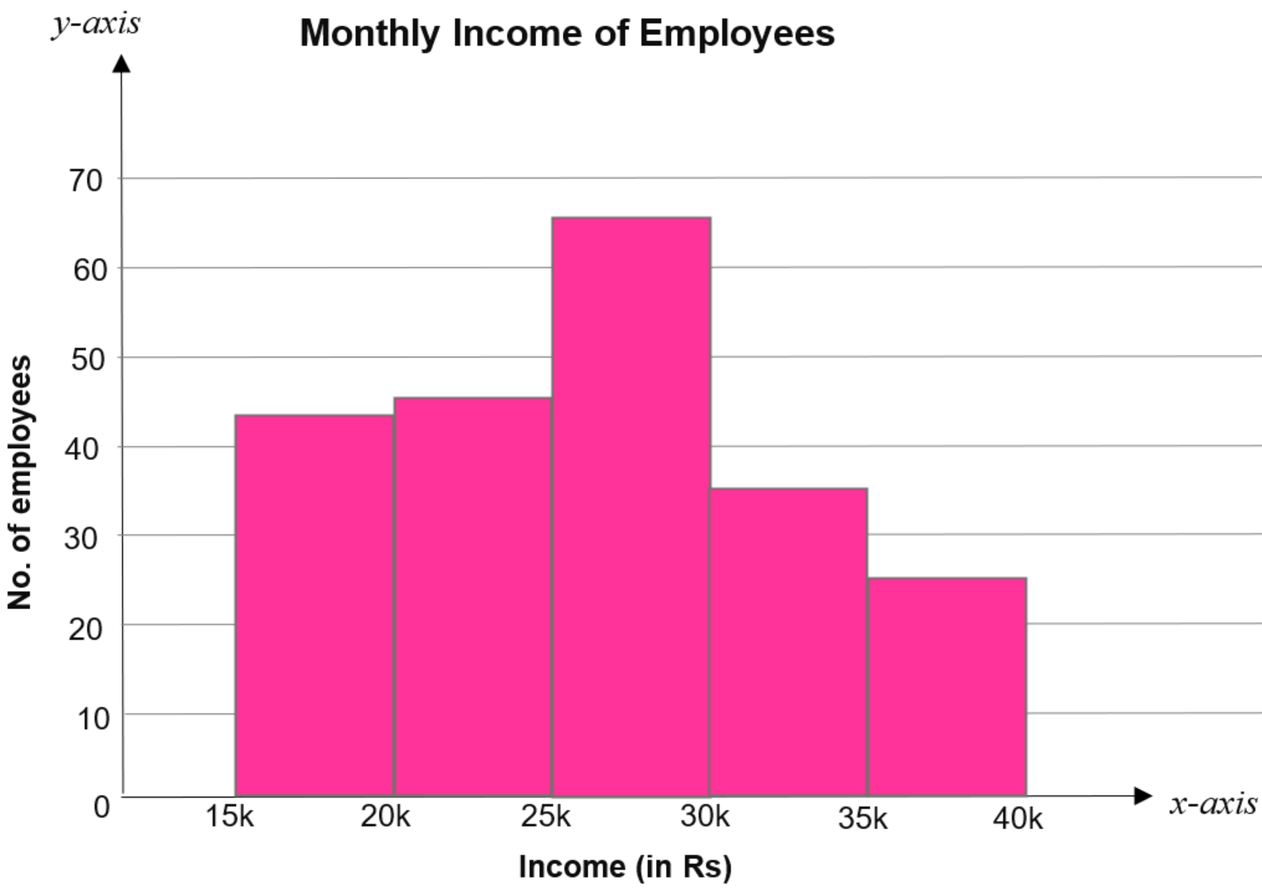
Monthly income (Rs)	15k – 20k	20k – 25k	25k – 30k	30k – 35k	35k – 40k
No. of employees	44	45	66	35	25

Solution:

First create frequency distribution table. In the given question, the monthly income ranges are already given as intervals. Therefore, it is not necessary to calculate the class boundaries again as they are already defined.

Monthly income (in Rs)	**Tally marks**	**No. of employees**
15k – 20k	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ////	44
20k – 25k	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	45
25k – 30k	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ /	66
30k – 35k	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	35
35k – 40k	~~////~~ ~~////~~ ~~////~~ ~~////~~ ~~////~~	25
		215

Draw axis and label them. Then draw bars according to the frequencies.



This is the required histogram.

8. Calculate the mean of the grouped data.

Classes	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25	26 – 30
Frequency	4	7	9	8	7	5

Solution:

First, we need to find the mid-points of the given data by using the formula.

Mid-point = X = $\frac{\textbf{Lower class limit + Upper class limit}}{\textbf{2}}$

Class Interval	Lower Class	Upper class	Mid-points (X)
1 − 5	1	5	(1+5)/2=3
6 − 10	6	10	(6+10)/2=8
11 − 15	11	15	(11+15)/2=13
16 − 20	16	20	(16+20)/2=18
21 − 25	21	25	(21+25)/2=23
26 − 30	26	30	(26+30)/2=28

Now, by multiplying the mid-point (X) with frequency (f), we get (fX).

Class Interval	Lower Class	Upper class	Mid-points (X)	Frequency (f)	fX
1 − 5	1	5	3	4	12
6 − 10	6	10	8	7	56
11 − 15	11	15	13	9	117
16 − 20	16	20	18	8	144
21 − 25	21	25	23	7	161
26 − 30	26	30	28	5	140
Total				$\sum f = 40$	$\sum fX$ =630

Using the formula of arithmetic mean for grouped data:

Arithmetic Mean = $\bar{X}$ = $\frac{\sum fX}{\sum f}$

$$\bar{X} = \frac{630}{40} = 15.75$$

9. Find the mean, median, and mode for the following data.

a) 2, 6, 3, 9, 2, 2, 5, 7, 8, 6, 2, 8

b) 15, 35, 75, 35, 45, 98, 90, 71

c) 128, 324, 459, 249, 779, 541, 915, 893

Solution:

a) 2, 6, 3, 9, 2, 2, 5, 7, 8, 6, 2, 8

Mean:

Mean $=\bar{x}=\sum x/n$

$= (2 + 6 + 3 + 9 + 2 + 2 + 5 + 7 + 8 + 6 + 2 + 8) / 12$

$= 60 / 12$

$= 5$

Median

Arrange the given data in ascending order:

2, 2, 2, 2, 3, 5, 6, 6, 7, 8, 8, 9.

Number of values = n = 12 (even)

Median = arithmetic mean of $(n/2)^{th}$ and $[(n/2)+1]^{th}$ observation

= arithmetic mean of $(12/2)^{th}$ and $[(12/2)+1]^{th}$ observation

= arithmetic mean of 6th and 7th observation

2, 2, 2, 2, 3, **5**, **6**, 6, 7, 8, 8, 9.

6th observation=5

7th observation= 6

Arithmetic mean of 6th and 7th observation= (5+6)/2 = 11/2= 5.5

So, the median of the given data values is 5.5.

Mode:

To find the mode, we look for the number that occurs most frequently. In this case, 2 occurs most frequently, so the mode is 2.

b) 15, 35, 75, 35, 45, 98, 90, 71

Mean:

Mean $=\bar{x}=\sum x/n$

= (15 + 35 + 75 + 35 + 45 + 98 + 90 + 71) / 8

= 464 / 8

= 58

Median:

Arrange the given data in ascending order:

15, 35, 35, 45, 71, 75, 90, 98.

Number of values = n = 8 (even)

Median = arithmetic mean of $(n/2)^{th}$ and $[(n/2)+1]^{th}$ observation

= arithmetic mean of $(8/2)^{th}$ and $[(8/2)+1]^{th}$ observation

= arithmetic mean of 4th and 5th observation

15, 35, 35, **45**, **71**, 75, 90, 98.

4th observation=45

5th observation= 71

Arithmetic mean of 4th and 5th observation= (45+71)/2 = 116/2= 58

So, the median of the given data values is 58.

Mode:

To find the mode, we look for the number that occurs most frequently. In this case, 35 occurs twice, so the mode is 35.

c) 128, 324, 459, 249, 779, 541, 915, 893

Mean:

Mean $=\bar{x}=\sum x/n$

= (128 + 324 + 459 + 249 + 779 + 541 + 915 + 893) / 8

= 4288 / 8

= 536

Median:

Arrange the given data in ascending order:

28, 249, 324, 459, 541, 779, 893, 915

Number of values = n = 8 (even)

Median = arithmetic mean of $(n/2)^{th}$ and $[(n/2)+1]^{th}$ observation

= arithmetic mean of $(8/2)^{th}$ and $[(8/2)+1]^{th}$ observation

= arithmetic mean of 4th and 5th observation

28, 249, 324, **459**, **541**, 779, 893, 915

4th observation=459

5th observation= 541

Arithmetic mean of 4th and 5th observation= (459+541)/2 = 1000/2= 500

So, the median of the given data values is 500.

Mode:

To find the mode, we look for the number that occurs most frequently. In this case, none of the numbers occur more than once, so there is no mode.

10. There are 20 marbles in a jar, including 4 red, 8 green, 5 yellow, and 3 black marbles. Sidra takes one marble without looking at it. Find the probability and the probability of the complements of these events. Also tell if the events are certain or impossible (if any).

a) P (picking a green marble) **b) P (picking a blue marble)**

c) P (not picking yellow marble) **d) P (picking a black marble)**

e) P (not picking a marble) **f) P (picking a marble)**

Solution:

a) P (picking a green marble)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{8}{20}$

$= \frac{2}{5}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{2}{5}$

$= \frac{5-2}{5}$

$= \frac{3}{5}$

b) P (picking a blue marble)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{0}{20}$ (favourable outcomes=0 as there are no blue marbles)

$= 0$

As the probability is 0, so this is an impossible event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 0$

$= 1$

So, the complement of the given event is certain.

c) P (not picking yellow marble)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{15}{20}$ (here favourable outcomes are all colours except yellow)

$= \frac{3}{4}$

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{3}{4}$

$= \frac{4-3}{4}$

$= \frac{1}{4}$

d) P (picking a black marble)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{3}{20}$ (here favourable outcomes are all colours except yellow)

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - \frac{3}{20}$

$= \frac{20-3}{20}$

$= \frac{17}{20}$

e) P (not picking a marble)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{0}{20}$ (favourable outcomes=0 as outcomes will have only marbles)

$= 0$

As the probability is 0, so this is an impossible event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 0$

$= 1$

So, the complement of the given event is certain.

f) P (picking a marble)

Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$

$= \frac{20}{20}$ (favourable outcomes=20 as all outcomes will have marbles)

$= 1$

As the probability is 1, so this is a certain event.

If this event is represented by A, then the probability of its complement, say A^c, is calculated by:

$P(A^c) = 1 - P(A)$

$= 1 - 1$

$= 0$

So, the complement of the given event is impossible.