بِسۡمِ اللّٰہِ الرَّحۡمٰنِ الرَّحِیۡمِ

اللہ کے نام سے شروع جو بڑا مہربان نہایت رحم فرمانے والا ہے۔

AFAQ SUN SERIES

# KEYBOOK

# Mathematics

# 8

ASSOCIATION FOR ACADEMIC QUALITY

## Table of Content

## Learning Check 1.1

1. **Convert the given fractions to decimal form, and then write the type of decimal.**

| Fractions | $\frac{2}{3}$ | $\frac{7}{11}$ | $\frac{8}{15}$ | $\frac{14}{23}$ | $\frac{17}{20}$ | $\frac{21}{26}$ | $\frac{2}{15}$ | $\frac{3}{8}$ | $\frac{20}{27}$ |
|---|---|---|---|---|---|---|---|---|---|
| Decimals | $\mathbf{0.\overline{6}}$ | $\mathbf{0.\overline{63}}$ | $\mathbf{0.5\overline{3}}$ | 0.60869… | 0.85 | 0.80769… | $\mathbf{0.1\overline{3}}$ | 0.375 | $\mathbf{0.\overline{740}}$ |
| Type of decimal | Recurring | Recurring | Recurring | Irrational | Terminating | Irrational | Recurring | Terminating | Recurring |

2. **Identify the terminating and non-terminating decimal numbers.**

a) $0.\overline{3}$ b) $0.\dot{4}\dot{1}$ c) $2.\overline{5}$ d) $3.\overline{26}$ e) 0.126
f) 1.31 g) $60.\overline{9}$ h) 21.16 i) 4.718 j) $31.\dot{6}$
k) $1.\overline{165}$ l) $2.\dot{1}\dot{7}\dot{1}$ m) $0.61\overline{9}$ n) 1.021 o) $8.\overline{215}$

**Solution:**

a) $0.\overline{3}$ is a non-terminating decimal because the digit 3 repeats infinitely.
b) $0.\dot{4}\dot{1}$ is a non-terminating decimal because the block of digits 41 repeats infinitely.
c) $2.\overline{5}$ is a non-terminating decimal because the digit 5 repeats infinitely.
d) $3.\overline{26}$ is a non-terminating decimal because the block of digits 26 repeats infinitely.
e) 0.126 is a terminating decimal because it ends after the last digit.
f) 1.31 is a terminating decimal because it ends after the last digit.
g) $60.\overline{9}$ is a non-terminating decimal because the digit 9 repeats infinitely.
h) 21.16 is a terminating decimal because it ends after the last digit.
i) 4.718 is a terminating decimal because it ends after the last digit.
j) $31.\dot{6}$ is a non-terminating decimal because the digit 6 repeats infinitely.
k) $1.\overline{165}$ is a non-terminating decimal because the block of digits 165 repeats infinitely.
l) $2.\dot{1}\dot{7}\dot{1}$ is a non-terminating decimal because the block of digits 171 repeats infinitely.
m) $0.61\overline{9}$ is a non-terminating decimal because the digit 9 repeats infinitely.
n) 1.021 is a terminating decimal because it ends after the last digit.
o) $8.\overline{215}$ is a non-terminating decimal because the block of digits 215 repeats infinitely.

3. **Identify the recurring and non-recurring decimal numbers.**

a) 0.316316… b) 0.327163… c) 0.555… d) 1.36219232…
e) 3.2444… f) 0.9342867 g) 3.141593 h) 2.718281828…
i) 6.8333… j) 5.0714285… k) 6.17640588… l) 0.732732…

**Solution:**

a) 0.316316…
This is a recurring decimal because the pattern 316 infinitely repeated.

b) 0.327163…
This is a non-recurring decimal because it does not have a repeating pattern of digits.
c) 0.555…
This is a recurring decimal because the digit 5 infinitely repeated.
d) 1.36219232…
This is a non-recurring decimal because it does not have a repeating pattern of digits.
e) 3.2444…
This is a recurring decimal because the digit 4 infinitely repeated.
f) 0.9342867
This is a non-recurring decimal because it does not have a repeating pattern of digits.
g) 3.141593
This is a non-recurring decimal because it does not have a repeating pattern of digits.
h) 2.718281828…
This is a non-recurring decimal because it does not have a repeating pattern of digits.
i) 6.8333…
This is a recurring decimal because the digit 3 infinitely repeated.
j) 5.0714285…
This is a non-recurring decimal because it does not have a repeating pattern of digits.
k) 6.17640588…
This is a non-recurring decimal because it does not have a repeating pattern of digits.
l) 0.732732…
This is a recurring decimal because the pattern 732 infinitely repeated.

**4. Write the following numbers in short recurring form.**

a) 0.777… b) 0.232323… c) 1.216216… d) 0.467467…
e) 2.123333… f) 0.2888… g) 1.010101… h) 0.115115…

**Solution:**

a) 0.777…
The digit 7 repeats indefinitely. Therefore, we can write this number in short recurring form as $0.\overline{7}$.
b) 0.232323…
The digits 23 repeat indefinitely. Therefore, we can write this number in short recurring form as $0.\overline{23}$.
c) 1.216216…
The digits 216 repeat indefinitely. Therefore, we can write this number in short recurring form as $1.\overline{216}$.
d) 0.467467…
The digits 467 repeat indefinitely. Therefore, we can write this number in short recurring form as $0.\overline{467}$.

e) 2.123333…
The digit 3 repeats indefinitely. Therefore, we can write this number in short recurring form as $2.12\overline{3}$.

f) 0.2888…
The digit 8 repeats indefinitely. Therefore, we can write this number in short recurring form as $0.2\overline{8}$.

g) 1.010101…
The digits 01 repeat indefinitely. Therefore, we can write this number in short recurring form as$1.\overline{01}$.

h) 0.115115…
The digits 115 repeat indefinitely. Therefore, we can write this number in short recurring form as $0.\overline{115}$.

5. **There are 560 students in a school, with one-quarter of the students being boys. How many girls are there in the school?**
**Solution:**
Total number of students in a school = 560
Number of boys = $\frac{1}{4}$
Number of girls = ?
Number of boys = $\frac{1}{4} \times 560 = 140$
Number of girls = total number of students – number of boys
= 560 – 140 = 420
Therefore, there are 420 girls in the school.

6. **Each day, Mahad walks $3\frac{1}{4}$ km in the morning and $\frac{11}{2}$ km in the evening. How much does he walk daily? Change the answer into decimals, and then find out what type of decimal it is.**
**Solution:**
Mahad walks in the morning = $3\frac{1}{4}$ km
Mahad walks in the evening = $\frac{11}{2}$ km
Mahad daily walks = $3\frac{1}{4} + \frac{11}{2}$
$= \frac{13}{4} + \frac{11}{2}$
$= \frac{13 + 22}{4} = \frac{35}{4} = 8\frac{3}{4}$ km = 8.75 km
So Mahad walks 8.75 km daily.

7. **In a high jump competition, Haris jumped 2.25 m. Another competitor, Jawad jumped 3.5 m. Which one jumped higher? Find the answer in decimals.**
**Solution:**
Haris jumped = 2.25 m
Jawad jumped = 3.5 m
Difference between their jumps = 3.5 – 2.25 = 1.25 m
Therefore, Jawad jumped 1.25 meters higher than Haris.

8. **A vessel contains $1\frac{2}{3}$ litre of milk. Farhan drinks $\frac{5}{6}$ litre of milk. How much milk is left in the vessel? Find the answer in decimal and then tell what type of decimal it is.**
**Solution:**
Total milk in a vessel = $1\frac{2}{3}$ litre
Farhan drinks milk = $\frac{5}{6}$ litre
Left milk in the vessel = $1\frac{2}{3} - \frac{5}{6}$
$= \frac{5}{3} - \frac{5}{6}$
$= \frac{10-5}{6} = \frac{5}{6} = 0.83$ litre
So, there is 0.83 litres of milk left in the vessel.

9. **Ali purchased 4.5 kg of sugar and 3.25 kg of rice. What is the total quantity of sugar and rice he bought in decimals?**
**Solution:**
Quantity of sugar = 4.5 kg
Quantity of rice = 3.25 kg
Total quantity of both = 4.5 + 3.25 = 7.75 kg
Therefore, Ali bought a total of 7.75 kg of sugar and rice combined.

## Learning Check 1.2

1. **Identify and separate the rational and irrational numbers.**

a) $\sqrt{3}$ b) $-\frac{5}{8}$ c) $\frac{2}{3}$ d) $-\frac{3}{7}$ e) $\sqrt{289}$ f) 1.82
g) 0.7373... h) $\sqrt{12}$ i) 0.625 j) $1\frac{8}{23}$ k) $\frac{13}{4}$

**Solution:**

a) The $\sqrt{3}$ cannot be expressed as a simple fraction, as it is a non-repeating decimal. Therefore, it is an irrational number.
b) -5/8 is a rational number, because it can be expressed as a ratio of two integers (-5 and 8).
c) 2/3 is a rational number, because it can also be expressed as a ratio of two integers (2 and 3).
d) -3/7 is also a rational number, because it can be expressed as a ratio of two integers (-3 and 7).
e) $\sqrt{289} = 17$, which is a rational number since it can be expressed as a ratio of two integers (17/1). Therefore, the number $\sqrt{289}$ is a rational number.
f) 1.82
This number is a rational number, because it can be written as a fraction of two integers.
g) 0.7373...
This number is an irrational number, because it is a non-terminating, non-repeating decimal.
h) The $\sqrt{12}$ cannot be expressed as a simple fraction, as it is a non-repeating decimal. Therefore, it is an irrational number.

i) 0.625
This number is a rational number, because it can be expressed as a fraction of two integers.

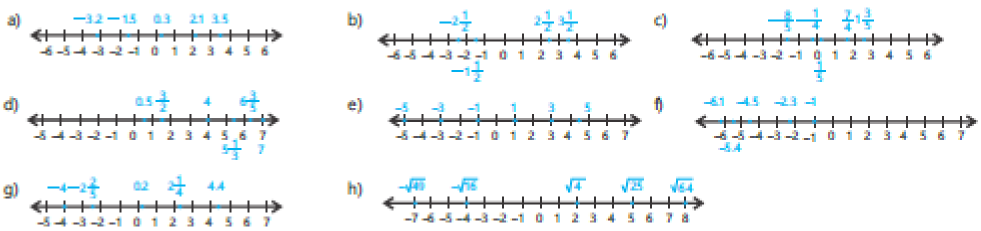
j) 31/23
This number is already in the form of a fraction, so it is a rational number. It can be expressed as the ratio of two integers (31 and 23).

k) 13/4
This number is also a rational number, because it is already in the form of a fraction. It can be expressed as the ratio of two integers (13 and 4).

**2. Show the given real numbers on a number line.**

a) $-3.2, -1.5, 0.3, 2.1, 3.5$
b) $-2\frac{1}{2}, -1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$
c) $-\frac{8}{5}, -\frac{1}{4}, \frac{1}{5}, \frac{7}{4}, \frac{13}{5}$
d) $0.5, \frac{3}{2}, 4, 5\frac{1}{3}, 7, 6\frac{3}{5}$
e) $-5, -3, -1, 1, 3, 5$
f) $-5.4, -6.1, -4.5, -2.3, -1$
g) $-4, -2\frac{2}{5}, 0.2, 2\frac{1}{4}, 4.4$
h) $-\sqrt{49}, -\sqrt{16}, \sqrt{4}, \sqrt{25}, \sqrt{64}$

**Solution:**



**3. Find the absolute value of the following real numbers.**

a) $-\sqrt{5}$ and $\sqrt{5}$
b) $-8.5$ and $8.5$
c) $-2\frac{1}{4}$ and $2\frac{1}{4}$
d) $-\frac{6}{7}$ and $\frac{6}{7}$
e) $-3.5$ and $3.5$
f) $-\frac{1}{2}$ and $\frac{1}{2}$
g) $-5$ and $+5$
h) $-\sqrt{10}$ and $\sqrt{10}$

**Solution:**

a) Both $-\sqrt{5}$ and $\sqrt{5}$ are same distance away from the origin on a number line. Therefore,
$|-\sqrt{5}| = \sqrt{5}$ and $|\sqrt{5}| = \sqrt{5}$

b) Both –8.5 and 8.5 are same distance away from the origin on a number line. Therefore,
$|-8.5| = 8.5$ and $|8.5| = 8.5$

c) Both $-2\frac{1}{4}$ and $2\frac{1}{4}$ are same distance away from the origin on a number line. Therefore,
$\left|-2\frac{1}{4}\right| = 2\frac{1}{4}$ and $\left|2\frac{1}{4}\right| = 2\frac{1}{4}$

d) Both $-\frac{6}{7}$ and $\frac{6}{7}$ are same distance away from the origin on a number line. Therefore,
$\left|-\frac{6}{7}\right| = \frac{6}{7}$ and $\left|\frac{6}{7}\right| = \frac{6}{7}$

e) Both –3.5 and 3.5 are same distance away from the origin on a number line. Therefore,
$|-3.5| = 3.5$ and $|3.5| = 3.5$

f) Both $-\frac{1}{2}$ and $\frac{1}{2}$ are same distance away from the origin on a number line. Therefore,
$\left|-\frac{1}{2}\right| = \frac{1}{2}$ and $\left|\frac{1}{2}\right| = \frac{1}{2}$

g) Both –5 and 5 are same distance away from the origin on a number line. Therefore,
$|-5| = 5$ and $|5| = 5$

h) Both $-\sqrt{10}$ and $\sqrt{10}$ are same distance away from the origin on a number line. Therefore,
$\left|-\sqrt{10}\right| = \sqrt{10}$ and $\left|\sqrt{10}\right| = \sqrt{10}$

## Learning Check 1.3

**1. What are the additive identity and multiplicative identity of real numbers?**

**Solution:**
The additive identity of a real number is "0" such that a + 0 = a.
The multiplicative identity of a real number is "1" such that a × 1 = a.

**2. Identify the properties used in the following.**

a) $\sqrt{3} + 0 = \sqrt{3}$ b) $\frac{2}{3} + \frac{1}{6} = \frac{1}{6} + \frac{2}{3}$ c) $(2 + 4) + 7 = 2 + (4 + 7)$

d) $\sqrt{2} \times \frac{1}{\sqrt{2}} = 1$ e) $\sqrt{7} \times \sqrt{7} = 7$ f) $\frac{2}{3} \times \frac{1}{4} = \frac{1}{4} \times \frac{2}{3}$

g) $(2 + \sqrt{5}) + 7 = 2 + (\sqrt{5} + 7)$ h) $4 + (-4) = 0$ i) $0 + 100 = 100$ j) $\frac{1}{\sqrt{2}} \times 1 = \frac{1}{\sqrt{2}}$

**Solution:**

a) Additive identity
b) Commutative property w.r.t "+"
c) Associative property w.r.t "+"
d) Multiplicative identity
e) Closure property w.r.t "×"
f) Commutative property w.r.t "×"
g) Associative property w.r.t "+"
h) Additive inverse
i) Additive identity
j) Multiplicative identity

**3. Find the additive inverse of each of the following.**

a) $-5$ b) $-\frac{1}{\sqrt{6}}$ c) $\sqrt{2}$ d) $\frac{3}{7}$ e) $-2.7$

f) 8 g) $-\frac{\sqrt{2}}{\sqrt{3}}$ h) $-\frac{7}{10}$ i) 1.2 j) 0

**Solution:**
Since the additive inverse can be found by changing the opposite sign of a given number

a) The additive inverse of $-5$ is 5.
b) The additive inverse of $-\frac{1}{\sqrt{6}}$ is $\frac{1}{\sqrt{6}}$.
c) The additive inverse of $\sqrt{2}$ is $-\sqrt{2}$.
d) The additive inverse of $\frac{3}{7}$ is $-\frac{3}{7}$.

e) The additive inverse of – 2.7 is 2.7.
f) The additive inverse of 8 is – 8.
g) The additive inverse of $-\frac{\sqrt{2}}{\sqrt{3}}$ is $\frac{\sqrt{2}}{\sqrt{3}}$.
h) The additive inverse of $-\frac{7}{10}$ is $\frac{7}{10}$.
i) The additive inverse of 1.2 is – 1.2.
j) The additive inverse of 0 is 0.

**4. Find the multiplicative inverse of each of the following.**

a) $-\frac{1}{7}$ b) 9 c) – 6.8 d) $\frac{\sqrt{2}}{2}$ e) $-\frac{1}{\sqrt{7}}$
f) – 5.3 g) $\frac{\sqrt{6}}{\sqrt{11}}$ h) $\frac{1}{0}$ i) – 10 j) $-\frac{15}{26}$

**Solution:**
Since the multiplicative inverse can be found by the reciprocal of a given number.

a) The multiplicative inverse of $-\frac{1}{7}$ is – 7.
b) The multiplicative inverse of 9 is $\frac{1}{9}$.
c) The multiplicative inverse of – 6.8is $-\frac{1}{6.8}$.
d) The multiplicative inverse of $\frac{\sqrt{2}}{2}$ is $\frac{2}{\sqrt{2}}$.
e) The multiplicative inverse of $-\frac{1}{\sqrt{7}}$ is $-\sqrt{7}$.
f) The multiplicative inverse of –5.3 is $-\frac{1}{5.3}$.
g) The multiplicative inverse of $\frac{\sqrt{6}}{\sqrt{11}}$ is $\frac{\sqrt{11}}{\sqrt{6}}$.
h) The multiplicative inverse of $\frac{1}{0}$ is 0.
i) The multiplicative inverse of –10 is $-\frac{1}{10}$.
j) The multiplicative inverse of $-\frac{15}{26}$ is $-\frac{26}{15}$.

**5. Verify the commutative property under addition and multiplication for the given real numbers.**

a) – 2 and 5 b) $\frac{2}{3}$ and $\frac{5}{11}$ c) 3.4 and $\sqrt{5}$
b) 4 and $\frac{4}{7}$ e) 6.1 and $\sqrt{2.1}$ f) $\frac{6}{11}$ and $\frac{7}{11}$

**Solution:**
a) – 2 and 5:
For addition:
-2 + 5 = 3, 5 + (-2) = 3
So, the commutative property holds w.r.t "+".
For multiplication:
(-2) * 5 = -10, 5 * (-2) = -10
So, the commutative property holds w.r.t "×".
b) 2/3 and 5/11:
For addition:
2/3 + 5/11 = $\frac{22 + 15}{33} = \frac{37}{33}$

5/11 + 2/3 = = $\frac{15 + 22}{33} = \frac{37}{33}$
So, the commutative property holds w.r.t "+".
For multiplication:
(2/3) * (5/11) = 10/33
(5/11) * (2/3) = 10/33
So, the commutative property holds w.r.t "×".
c) 3.4 and $\sqrt{5}$:
Take $\sqrt{5} \approx 2.24$
For addition:
3.4 + $\sqrt{5}$ = 3.4 + 2.24 = 5.64
$\sqrt{5}$+ 3.4 = 2.24 + 3.4 = 5.64
So, the commutative property holds w.r.t "+".
For multiplication:
3.4 × $\sqrt{5}$= 3.4 × 2.24 = 7.62
$\sqrt{5}$ × 3.4 = 2.24 × 3.4 = 7.62
So the commutative property holds w.r.t "×".
d) 4 and 4/7:
For addition:
4 + 4/7 = $\frac{28 + 4}{7} = \frac{32}{7} = 4\frac{4}{7}$
4/7 + 4 = $\frac{4 + 28}{7} = \frac{32}{7} = 4\frac{4}{7}$
So, the commutative property holds w.r.t "+".
For multiplication:
4 * 4/7 = 16/7 = $2\frac{2}{7}$
4/7 * 4 = 16/7 = $2\frac{2}{7}$
So, the commutative property holds w.r.t "×".
e) 6.1 and $\sqrt{2.1}$ :
Take $\sqrt{2.1} \approx 1.45$
For addition:
6.1 + $\sqrt{2.1}$ = 6.1 + 1.45 = 7.55
$\sqrt{2.1}$ + 6.1 = 1.45 + 6.1 = 7.55
So, the commutative property holds w.r.t "+".
For multiplication:
6.1 × $\sqrt{2.1}$ = 6.1 × 1.45 = 8.85
$\sqrt{2.1}$ × 6.1 = 1.45 × 6.1 = 8.85
So, the commutative property holds w.r.t "×".
f) 6/11 and 7/11:
For addition:
6/11 + 7/11 = $\frac{6 + 7}{11}$ = 13/11
7/11 + 6/11 = $\frac{7 + 6}{11}$13/11
So, the commutative property holds w.r.t "+".
For multiplication:
(6/11) × (7/11) = 42/121

(7/11) × (6/11) = 42/121

So, the commutative property holds w.r.t "×".

6. **Verify the associative property of addition and multiplication for the given real numbers.**

a) $\frac{7}{2}$, 4.2 and 2.3 b) 4.8, 1.9 and – 7.2 c) $\sqrt{4}\,, 2\sqrt{5}$ and 5

d) 7.6, –6, 5.7 e) 8, – 4, $\frac{11}{3}$ f) 5, 4.2 and 6.3

**Solution:**

a) $\frac{7}{2}$, 4.2 and 2.3

Addition:

(7/2 + 4.2) + 2.3 = ($\frac{7 + 8.4}{2}$) + 2.3 = $\frac{15.4}{2}$ + 2.3 = 7.7 + 2.3 = 10

7/2 + (4.2 + 2.3) = 7/2 + 6.5 = $\frac{7 + 13}{2}$ = $\frac{20}{2}$ = 10

Both sides have the same result, so addition is associative.

Multiplication:

(7/2 × 4.2) × 2.3 = 14.70 × 2.3 = 33.81

7/2 × (4.2 × 2.3) = $\frac{7}{2}$ × 9.66 = 33.81

Both sides have the same result, so multiplication is associative.

b) 4.8, 1.9 and – 7.2

Addition:

(4.8 + 1.9) + (–7.2) = 6.7 – 7.2 = – 0.5

4.8 + (1.9 + (–7.2)) = 4.8 – 5.30 = – 0.5

Both sides have the same result, so addition is associative.

Multiplication:

(4.8 × 1.9) × (–7.2) = 9.12 × (– 7.2) = – 65.66

4.8 × (1.9 × (–7.2)) = 4.8 × (– 13.68) = – 65.66

Both sides have the same result, so multiplication is associative.

c) $\sqrt{4}\,, 2\sqrt{5}$ and 5

Addition:

($\sqrt{4}$ + $2\sqrt{5}$) + 5 = 2 + $2\sqrt{5}$ + 5 = 7 + $2\sqrt{5}$

$\sqrt{4}$ + ($2\sqrt{5}$+ 5) = 2 + $2\sqrt{5}$ + 5 = 7 + $2\sqrt{5}$

Both sides have the same result, so addition is associative.

Multiplication:

($\sqrt{4}$ × $2\sqrt{5}$) × 5 = 2 × $2\sqrt{5} \times 5 = 20\sqrt{5}$

$\sqrt{4}$ × ($2\sqrt{5}$ × 5) = 2 × $2\sqrt{5} \times 5 = 20\sqrt{5}$

Both sides have the same result, so multiplication is associative.

d) 7.6, –6, 5.7

Addition:

(7.6 + (-6)) + 5.7 = (7.6 – 6) + 5.7 = 1.6 + 5.7 = 7.3

7.6 + ((-6) + 5.7) = 7.6 + (- 6 + 5.7) = 7.6 – 0.3 = 7.3

Both sides have the same result, so addition is associative.

Multiplication:

(7.6 × (-6)) × 5.7 = (-45.60) × 5.7 = -259.92

7.6 × ((-6) × 5.7) = 7.6 × (-34.2) = -259.92
Both sides have the same result, so multiplication is associative.
e) $8, -4, \frac{11}{3}$
Addition:
(8 + (-4)) + (11/3) = (8 – 4) + $\frac{11}{3}$ = 4 + $\frac{11}{3}$ = $\frac{12+11}{3}$ = $\frac{23}{3}$ = $7\frac{2}{3}$
8 + ((-4) + 11/3) = 8 + (– 4 + $\frac{11}{3}$ )= 8 + $\frac{-12+11}{3}$ =8 – $\frac{1}{3}$ = $\frac{24-1}{3}$ = $\frac{23}{3}$= $7\frac{2}{3}$
Both sides have the same result, so addition is associative.
Multiplication:
(8 × (-4)) × (11/3) = (– 32) × $\frac{11}{3}$ = $\frac{-352}{3}$ = $-117\frac{1}{3}$
8 × ((-4) × 11/3) = 8 × ($\frac{-44}{3}$) = $\frac{-352}{3}$ = $-117\frac{1}{3}$
Both sides have the same result, so multiplication is associative.
f) 5, 4.2 and 6.3
Addition:
(5 + 4.2) + 6.3 = 9.2 + 6.3 = 15.5
5 + (4.2 + 6.3) = 5 + 10.5 = 15.5
Both sides have the same result, so addition is associative.
Multiplication:
(5 × 4.2) × 6.3 = 21 × 6.3 = 132.3
5 × (4.2 × 6.3) = 5 × 26.46 = 132.3
Both sides have the same result, so multiplication is associative.

7. **Verify the distributive property of multiplication over addition and subtraction for the given real numbers.**

a) $\frac{1}{5}$, 9.3 and 4.4 b) 7.1, 11 and 17 c) $\sqrt{7}, \sqrt{2}$ and - 1.3

**Solution:**
a) Multiplication over addition:
(1/5) × (9.3 + 4.4) = $\frac{1}{5}$ × 13.7 = 2.74
(1/5 × 9.3) + (1/5 × 4.4) = 1.86 + 0.88 = 2.74
Both sides have the same result, so multiplication is distributive over addition.
Multiplication over subtraction:
(1/5) × (9.3 – 4.4) = $\frac{1}{5}$ × 4.9 = 0.98
(1/5 × 9.3) – (1/5 × 4.4) = 1.86 – 0.88 = 0.98
Both sides have the same result, so multiplication is distributive over subtraction.
b) Multiplication over addition:
7.1 × (11 + 17) = 7.1 × 28 = 198.8
(7.1 ×11) + (7.1 × 17) = 78.1 + 120.7 = 198.8
Both sides have the same result, so multiplication is distributive over addition.
Multiplication over subtraction:
7.1 × (11 – 17) = 7.1 × (– 6) = – 42.6
(7.1 × 11) – (7.1 × 17) = 78.1 – 120.7 = – 42.6
Both sides have the same result, so multiplication is distributive over subtraction.

c) Multiplication over addition:

$\sqrt{7} \times (\sqrt{2} + (-1.3)) = \sqrt{7} \times (\sqrt{2} - 1.3) = \sqrt{7} \times \sqrt{2} + \sqrt{7}\,(-1.3) = \sqrt{14} - 1.3\sqrt{7}$

$(\sqrt{7} \times \sqrt{2}) + (\sqrt{7} \times (-1.3)) = \sqrt{14} + (-1.3\sqrt{7}) = \sqrt{14} - 1.3\sqrt{7}$

Both sides have the same result, so multiplication is distributive over addition.

Multiplication over subtraction:

$\sqrt{7} \times (\sqrt{2} - (-1.3)) = \sqrt{7} \times (\sqrt{2} + 1.3) = \sqrt{7} \times \sqrt{2} + \sqrt{7}\,(+1.3) = \sqrt{14} + 1.3\sqrt{7}$

$(\sqrt{7} \times \sqrt{2}) -- (\sqrt{7} \times (-1.3)) = \sqrt{14} - (-1.3\sqrt{7}) = \sqrt{14} + 1.3\sqrt{7}$

Both sides have the same result, so multiplication is distributive over subtraction.

## Learning Check 1.4

**1. Round off the following numbers to 3 significant figures.**

a) 13563 b) 21952 c) 1274 d) 2010

e) 10899 f) 71250 g) 16.691 h) 1.9252

i) 681.732 j) 1463.017 k) 1.90523 l) 23.1805

**Solution:**

a) 13563

Here we consider the three leftmost digits, which are 135. The digit next to the digit 5 is 6, which is greater than 5. So, we will round up as 136 and replace the next digits with zeros. Thus, 13563 rounded to 3-significant figures is 13600.

b) 21952

Here we consider the three leftmost digits, which are 219. The digit next to the digit 9 is 5, which is greater than 5. So, we will round up as 220 and replace the next digits with zeros. Thus, 21952 rounded to 3-significant figures is 22000.

c) 1274

Here we consider the three leftmost digits, which are 127. The digit next to the digit 7 is 4, which is less than 5. So, we will round down. Keep 127 as it is and replace the next digits with zeros. Thus, 1274 rounded to 3-significant figures is 1270.

d) 2010

Here we consider the three leftmost digits, which are 201. The digit next to the digit 1 is 0, which is less than 5. So, we will round down. Keep 201 as it is and replace the next digits with zeros. Thus, 2010 rounded to 3-significant figures is 2010.

e) 10899

Here we consider the three leftmost digits, which are 108. The digit next to the digit 8 is 9, which is greater than 5. So, we will round up as 109 and replace the next digits with zeros. Thus, 10899 rounded to 3-significant figures is 10900.

f) 71250

Here we consider the three leftmost digits, which are 712. The digit next to the digit 2 is 5, which is greater than or equal to 5. So, we will round up as 713 and replace the next digits with zeros. Thus, 71250 rounded to 3-significant figures is 71300.

g) 16.691

Here we consider the three leftmost digits, which are 16.6. The digit next to the digit 6 is 9, which is greater than 5. So, we will round up as 16.7. Thus, 16.691 rounded to 3-significant figures is 16.7.

h) 1.9252

Here we consider the three leftmost digits, which are 1.92. The digit next to the digit 2 is 5, which is greater than 5. So, we will round up as 1.93. Thus, 1.9252 rounded to 3-significant figures is 1.93.

i) 681.732

Here we consider the three leftmost digits, which are 681. The digit next to the digit 1 is 7, which is greater than 5. So, we will round up as 682. Thus, 681.732 rounded to 3-significant figures is 682.

j) 1463.017

Here we consider the three leftmost digits, which are 146. The digit next to the digit 6 is 3, which is less than 5. So, we will round down. Keep 146. Thus, 1446.017 rounded to 3-significant figures is 1460.

k) 1.90523

Here we consider the three leftmost digits, which are 1.90. The digit next to the digit 0 is 5, which is greater than or equal to 5. So, we will round up as 1.91. Thus, 1.90523 rounded to 3-significant figures is 1.91.

l) 23.1805

Here we consider the three leftmost digits, which are 23.1. The digit next to the digit 1 is 8, which is greater than 5. So, we will round up as 23.2. Thus, 23.1805 rounded to 3-significant figures is 23.2.

**2. Round off the following numbers to 4 significant figures.**

| | | | |
|---|---|---|---|
| a) 15269 | b) 21036 | c) 20687 | d) 4463298 |
| e) 714538 | f) 134504 | g) 5.23617 | h) 32.8645 |
| i) 432.6592 | j) 23.9052 | k) 4317.912 | l) 432.95201 |

**Solution:**

a) 15269

Here we consider the four leftmost digits, which are 1526. The digit next to the digit 6 is 9, which is greater than 5. So, we will round up as 1527 and replace the next digits with zeros. Thus, 15269 rounded to 4-significant figures is 15270.

b) 21036

Here we consider the four leftmost digits, which are 2103. The digit next to the digit 3 is 6, which is greater than 5. So, we will round up as 2104 and replace the next digits with zeros. Thus, 21036 rounded to 4-significant figures is 21040.

c) 20687

Here we consider the four leftmost digits, which are 2068. The digit next to the digit 8 is 7, which is greater than 5. So, we will round up as 2069 and replace the next digits with zeros. Thus, 20687 rounded to 4-significant figures is 20690.

d) 4463298

Here we consider the four leftmost digits, which are 4463. The digit next to the digit 3 is 2, which is less than 5. So, we will round down. Keep 4463 as it is and replace the next digits with zeros. Thus, 4463298 rounded to 4-significant figures is 4463000.

e) 714538

Here we consider the four leftmost digits, which are 7145. The digit next to the digit 5 is 3, which is less than 5. So, we will round down. Keep 7145 as it is and replace the next digits with zeros. Thus, 714538 rounded to 4-significant figures is 714500.

f) 134504

Here we consider the four leftmost digits, which are 1345. The digit next to the digit 5 is 0, which is less than 5. So, we will round down. Keep 1345 as it is and replace the next digits with zeros. Thus, 134504 rounded to 4-significant figures is 134500.

g) 5.23617

Here we consider the four leftmost digits, which are 5.236. The digit next to the digit 6 is 1, which is less than 5. So, we will round down. Keep 5.236 as it is. Thus, 5.23617 rounded to 4-significant figures is 5.236.

h) 32.8645

Here we consider the four leftmost digits, which are 32.86. The digit next to the digit 6 is 4, which is less than 5. So, we will round down as 32.86. Thus, 32.8645 rounded to 4-significant figures is 32.86.

i) 432.6592

Here we consider the four leftmost digits, which are 432.6. The digit next to the digit 6 is 5, which is greater than or equal to 5. So, we will round up as 432.7. Thus, 432.6592 rounded to 4-significant figures is 432.7.

j) 23.9052

Here we consider the four leftmost digits, which are 23.90. The digit next to the digit 0 is 5, which is greater than or equal to 5. So, we will round up as 23.91. Thus, 23.9052 rounded to 4-significant figures is 23.91.

k) 4317.912

Here we consider the four leftmost digits, which are 4317. The digit next to the digit 7 is 9, which is greater than 5. So, we will round up as 4318. Thus, 4317.912 rounded to 4-significant figures is 4318.

l) 432.95201

Here we consider the four leftmost digits, which are 432.9. The digit next to the digit 9 is 5, which is greater than or equal to 5. So, we will round up as 433.0. Thus, 432.95201 rounded to 4-significant figures is 433.0.

**3. Round off the following numbers to 5 significant figures.**

a) 453190 b) 3274523 c) 8723862 d) 46298913
e) 149129041 f) 8.9871439 g) 536.87551 h) 39.87642
i) 3190.8527 j) 12569.324 k) 673.0782 l) 1256.3985

**Solution:**

a) 453190

Here we consider the five leftmost digits, which are 45319. The digit next to the digit 9 is 0, which is less than 5. So, we will round down. Keep 45319 as it is and replace the next digits with zeros. Thus, 453190 rounded to 5-significant figures is 453190.

b) 3274523

Here we consider the five leftmost digits, which are 32745. The digit next to the digit 5 is 2, which is less than 5. So, we will round down. Keep 32745 as it is and replace the next digits with zeros. Thus, 3274523 rounded to 5-significant figures is 3274500.

c) 8723862

Here we consider the five leftmost digits, which are 87238. The digit next to the digit 8 is 6, which is greater than 5. So, we will round up as 87239 and replace the next digits with zeros. Thus, 8723862 rounded to 5-significant figures is 8723900.

d) 46298913

Here we consider the five leftmost digits, which are 46298. The digit next to the digit 8 is 9, which is greater than 5. So, we will round up as 46299 and replace the next digits with zeros. Thus, 46298913 rounded to 5-significant figures is 46299000.

e) 149129041

Here we consider the five leftmost digits, which are 14912. The digit next to the digit 2 is 9, which is greater than 5. So, we will round up as 14913 and replace the next digits with zeros. Thus, 149129041 rounded to 5-significant figures is 149130000.

f) 8.9871439

Here we consider the five leftmost digits, which are 8.9871. The digit next to the digit 1 is 4, which is less than 5. So, we will round down. Keep 8.9871 as it is. Thus, 8.9871439 rounded to 5-significant figures is 8.9871.

g) 536.87551

Here we consider the five leftmost digits, which are 536.87. The digit next to the digit 7 is 5, which is greater than or equal to 5. So, we will round up as 536.88. Thus, 536.87551 rounded to 5-significant figures is 536.88.

h) 39.87642

Here we consider the five leftmost digits, which are 39.876. The digit next to the digit 6 is 4, which is less than 5. So, we will round down. Keep 39.876as it is. Thus, 39.87642rounded to 5-significant figures is 39.876.

i) 3190.8527

Here we consider the five leftmost digits, which are 3190.8. The digit next to the digit 8 is 5, which is greater than or equal to 5. So, we will round up as 3190.9. Thus, 3190.8527 rounded to 5-significant figures is 3190.9.

j) 12569.324

Here we consider the five leftmost digits, which are 12569. The digit next to the digit 9 is 3, which is less than 5. So, we will round down. Keep 12569 as it is. Thus, 12569.324 rounded to 5-significant figures is 12569.

k) 673.0782

Here we consider the five leftmost digits, which are 673.07. The digit next to the digit 7 is 8, which is greater than 5. So, we will round up as 673.08 Thus, 673.0782 rounded to 5-significant figures is 673.08.

l) 1256.3985

Here we consider the five leftmost digits, which are 1256.3. The digit next to the digit 3 is 9, which is greater than 5. So, we will round up as 1256.4. Thus, 1256.3985 rounded to 5-significant figures is 1256.4.

4. **Usman sold his home for Rs 6,0457,450. Round off this amount to 5 significant figures.**
   **Solution:**
   Here we consider the five leftmost digits, which are 60457. The digit next to the digit 7 is 4, which is less than 5. So, we will round down. Keep 60457 as it is and replace the next digits with zeros. Thus, 60457450 rounded to 5-significant figures is 60457000.
5. **Anum purchased a laptop for Rs 45378. Round off this amount to 3 significant figures.**
   **Solution:**
   Here we consider the five leftmost digits, which are 453. The digit next to the digit 3 is 9, which is greater than 5. So, we will round up as 454and replace the next digits with zeros. Thus, 45378 rounded to 5-significant figures is 45400.
6. **Nida measured the length of the rope and found it to be 12 m (correct to the nearest m). What could be the actual possible length of the rope?**
   **Solution:**
   12 m, correct to the nearest metre, means the actual length could be up to 0.5 metre less or more than the measured value.
   12 + 0.5 = 12.5 m or
   12 – 0.5 = 11.5 m
   So, the actual length of the rope could be anywhere between 11.5 m and 12.5 m.

## Unit Check 1

1. **Encircle the correct option.**
   a) The difference between exact and approximated value is called:
   i) exact value **ii) approximation error**
   iii) significant figure iv) rounding off
   b) Which one has 4 significant digits?
   i) 0.0012 ii) 0.0125 **iii) 63.27** iv) 0.356
   c) Which fraction shows the terminating decimal?
   **i)** $\mathbf{\frac{6}{5}}$ ii) $\frac{7}{13}$ iii) $\frac{5}{3}$ iv) $\frac{5}{9}$
   d) The union of rational and irrational numbers is called:

i) integers ii) whole numbers **iii) real numbers** iv) natural numbers

e) The recurring decimal is

**i) 3.4444…** ii) 2.34178… iii) 0.23142… iv) 3.0014…

f) Multiplicative identity for the set of real numbers is:

i) 0 **ii) 1** iii) $-1$ iv) $\frac{1}{2}$

g) The additive inverse of is:

i) $-\frac{8}{3}$ ii) $-\frac{3}{8}$ **iii)** $\mathbf{\frac{3}{8}}$ iv) 0

h) Which of these is an irrational number?

**i)** $\sqrt{\mathbf{5}}$ ii) $\sqrt{9}$ iii) $\sqrt{4}$ iv) $\sqrt{36}$

2. **Fill in the blanks.**

a) There are **83476** significant figures in the number 0.083476.

b) The absolute value of – 20 is **20**.

c) Decimal numbers that have an infinite number of digits after the decimal point are called **non-terminating decimal**.

d) The multiplicative inverse of $\frac{2}{11}$ is $\mathbf{\frac{11}{2}}$.

e) $4 + \sqrt{2} = \sqrt{2} + 4$ is the **commutative property w.r.t "+"** property of real numbers.

f) If $a < b \wedge c < d \Rightarrow a + c < b + d$.

3. **Identify the terminating and non-terminating decimal numbers.**

a) 4.16 b) 0.175… c) 1.376364… d) 2.901
e) 0.172 f) 16.0001 g) 9.113… h) 14.761…

**Solution:**

a) 4.16 is a terminating decimal number, since there are only two digits after the decimal point.

b) 0.175… is a non-terminating decimal number, since it has an infinite number of digits after the decimal point.

c) 1.376364… is a non-terminating decimal number, since it has an infinite number of digits after the decimal point that do not repeat in a pattern.

d) 2.901 is a terminating decimal number, since there are no digits after the decimal point.

e) 0.172 is a terminating decimal number, since there are only three digits after the decimal point.

f) 16.0001 is a terminating decimal number, since there are four digits after the decimal point.

g) 9.113… is a non-terminating decimal number, since it has an infinite number of digits after the decimal point.

h) 14.761… is a non-terminating decimal number, since it has an infinite number of digits after the decimal point.

4. **Identify the recurring and non-recurring decimal numbers.**

a) 2.6721… b) 41.2222… c) 1.2121… d) 1.31616…
e) 3.116565… f) 11.362 g) 6677.333… h) 6.73782

**Solution:**

a) 2.6721… is a non-recurring decimal number, since there is no repeating pattern of digits.
b) 41.2222… is a recurring decimal number, since the digit 2 repeats.
c) 1.2121… is a recurring decimal number, since the digits 1 and 2 repeat in a pattern.
d) 1.31616… is a recurring decimal number, since the digits 1 and 6 repeat in a pattern.
e) 3.116565… is a recurring decimal number, since the digits 5 and 6 repeat in a pattern.
f) 11.362 is a non-recurring decimal number, since there is no repeating pattern of digits.
g) 6677.333… is a recurring decimal number, since the digit 3 repeats.
h) 6.73782 is a non-recurring decimal number, since there is no repeating pattern of digits.

**5. Separate rational and irrational numbers and justify your answer.**

a) $-4\frac{1}{2}$ b) $-\frac{5}{2}$ c) $\sqrt{13}$ d) $\frac{\pi}{3}$ e) $\sqrt{23}$
f) 3.1414 g) $\frac{16}{5}$ h) $\sqrt{81}$ i) 4.23 j) 3.6277348

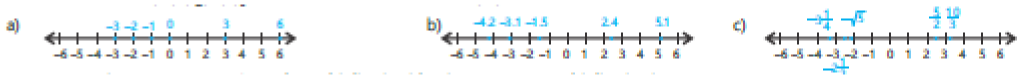
**Solution:**

a) $-4\frac{1}{2} = -\frac{9}{2}$ is a rational number because it can be expressed as the ratio of two integers.
b) $-\frac{5}{2}$ is a rational number because it can be expressed as the ratio of two integers.
c) $\sqrt{13}$ is an irrational number because it cannot be expressed as the ratio of two integers. It is a non-repeating and non-terminating decimal.
d) $\frac{\pi}{3}$ is an irrational number because it cannot be expressed as the ratio of two integers. It is a non-repeating and non-terminating decimal.
e) $\sqrt{23}$ is an irrational number because it cannot be expressed as the ratio of two integers. It is a non-repeating and non-terminating decimal.
f) 3.1414 is a rational number because it can be expressed as the ratio of two integers.
g) $\frac{16}{5}$ is a rational number because it can be expressed as the ratio of two integers.
h) $\sqrt{81} = 9$ is a rational number because it can be expressed as the ratio of two integers.
i) 4.23 is a rational number because it can be expressed as the ratio of two integers.
j) 3.6277348 is an irrational number because it cannot be expressed as the ratio of two integers. It is a non-repeating and non-terminating decimal.

**6. Represent the following real numbers on a number line.**

a) $-3, -2, -1, 0, 3, 6$ b) $-4.2, -3.1, -1.5, 2.4, 5.1$ c) $-3\frac{1}{4}, -2\frac{1}{2}, -\sqrt{3}, \frac{5}{2}, \frac{10}{3}$

**Solution:**



**7. Identify and write the name of the properties used in the following:**

a) $41 \times 7 = 7 \times 41$ b) $-35 \times 1 = -35$ c) $\frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{2} = 1$

d) $9.11 + (-9.11) = 0$ e) $(4 + 11.20) + 9 = 4 + (11.20 + 9)$

f) $\frac{3}{5} + \frac{4}{15} = \frac{4}{15} + \frac{3}{5}$ g) $\frac{\sqrt{3}}{5} \times 1 = \frac{\sqrt{3}}{5}$

**Solution:**

a) Commutative property w.r.t "×" b) Multiplicative identity
c) Multiplicative inverse d) Additive inverse
e) Associative property w.r.t "+" f) Commutative property w.r.t "+"
g) Multiplicative identity

**8. Find the additive inverse and the multiplicative inverse of each number.**

a) $-2$ b) $-\frac{6}{13}$ c) $\sqrt{15}$ d) $\frac{\sqrt{4}}{9}$ e) 37

**Solution:**

a) The additive inverse of -2 is 2, since -2 + 2 = 0.
The multiplicative inverse of -2 is -1/2, since (-2) x (-1/2) = 1.

b) The additive inverse of -6/13 is 6/13, since -6/13 + 6/13 = 0.
The multiplicative inverse of -6/13 is -13/6, since (-6/13) x (-13/6) = 1.

c) The additive inverse of $\sqrt{15}$ is - $\sqrt{15}$, since $\sqrt{15}$ + (-$\sqrt{15}$) = 0.
The multiplicative inverse of $\sqrt{15}$ is 1/$\sqrt{15}$, since $\sqrt{15}$ x 1/$\sqrt{15}$= 1.

d) The additive inverse of $\frac{\sqrt{4}}{9}$ is - $\frac{\sqrt{4}}{9}$= -2/9, since $\frac{\sqrt{4}}{9}$ + (- $\frac{\sqrt{4}}{9}$) = 0.
The multiplicative inverse of $\frac{\sqrt{4}}{9}$ is $\frac{9}{\sqrt{4}}$, which can be simplified as 9/2, since $\frac{2}{9} \times \frac{9}{2} = 1$

e) The additive inverse of 37 is -37, since 37 + (-37) = 0.
The multiplicative inverse of 37 is 1/37, since 37 x (1/37) = 1.

**9. Round off the following to the specified number of significant figures.**

a) 34850 to 3 significant figures b) 12.6096 to 2 significant figures
c) 1.01069327 to 5 significant figures d) 12.896735 to 4 significant figures
e) 0.3743926 to 3 significant figures

**Solution:**

a) 34850 to 3 significant figures

Here we consider the three leftmost digits, which are 348. The digit next to the digit 8 is 5, which is greater than or equal to 5. So, we will round up as 349 and replace the next digits with zeros. Thus, 34850 rounded to 3-significant figures is 34900.

b) 12.6096 to 2 significant figures

Here we consider the two leftmost digits, which are 12. The digit next to the digit 2 is 6, which is greater than 5. So, we will round up as 13 and. Thus, 12.6096 rounded to 2-significant figures is 13.

c) 1.01069327 to 5 significant figures

Here we consider the five leftmost digits, which are 1.0106. The digit next to the digit 6 is 9, which is greater than 5. So, we will round up as 1.0107 and. Thus, 1.01069327 rounded to 5-significant figures is 1.0107.

d) 12.896735 to 4 significant figures

Here we consider the four leftmost digits, which are 12.89. The digit next to the digit 9 is 6, which is greater than 5. So, we will round up as 12.90 and. Thus, 12.896735 rounded to 4-significant figures is 12.90.

e) 0.3743926 to 3 significant figures

Here we consider the four leftmost digits, which are 0.374. The digit next to the digit 4 is 3, which is less than 5. So, we will round down as 0.374. Thus, 0.3743926 rounded to 4-significant figures is 0.374.

**10. Irha measured the mass of a pack of rice and found it to be 12 kg (correct to the nearest kg). What could be the actual possible mass of the packet of rice?**

**Solution:**

12 kg, correct to the nearest kg, means the actual mass of a pack of rice could be up to 0.5 kg less or more than the measured value.
12 + 0.5 = 12.5 kg or
12 – 0.5 = 11.5 kg
So, the actual length of the rope could be anywhere between 11.5 kg and 12.5 kg.

## Learning Check 2.1

**1. Write the base, index and value of each of the following expressions:**

a) $3^4$ b) $4^3$ c) $6^3$ d) $2^5$ e) $8^2$ f) $9^3$
g) $12^2$ h) 152

**Solution:**

a) $3^4$
Base: 3, Index: 4, Value: $3^4 = 3 \times 3 \times 3 \times 3 = 81$

b) $4^3$
Base = 4, Index = 3, Value: $4^3 = 4 \times 4 \times 4 = 64$

c) $6^3$
Base = 6, Index = 3, Value: $6^3 = 6 \times 6 \times 6 = 216$

d) $2^5$
Base = 2, Index = 5, Value: $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$

e) $8^2$
Base = 8, Index = 2, Value: $8^2 = 8 \times 8 = 64$

f) $9^3$
Base = 9, Index = 3, Value: $9^3 = 9 \times 9 \times 9 = 729$

g) $12^2$
Base = 12, Index = 2, Value: $12^2 = 12 \times 12 = 144$

h) $15^2$
Base = 15, Index = 2, Value: $15^2 = 15 \times 15 = 225$

**2. Write the following products in index notation form and find the value.**

a) $3 \times 3 \times 3 \times 3$ b) $a \times a \times a \times b \times b$ c) $2 \times 2 \times 5 \times 5 \times 5 \times 6$
d) $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$ e) $\frac{2}{5} \times \frac{2}{5} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9}$ f) $y \times y \times y \times y \times y$

**Solution:**

a) $3 \times 3 \times 3 \times 3$
As we can write the same bases in index form as:
$3 \times 3 \times 3 \times 3 = 3^4 = 81$

b) $a \times a \times a \times b \times b$
As we can write the same bases in index form as:
$a \times a \times a \times b \times b = a^3 \times b^2 = a^3b^2$

c) $2 \times 2 \times 5 \times 5 \times 5 \times 6$
As we can write the same bases in index form as:
$2 \times 2 \times 5 \times 5 \times 5 \times 6 = 2^2 \times 5^3 \times 6^1 = 4 \times 125 \times 6 = 3000$

d) $\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}$
As we can write the same bases in index form as:
$\sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2} = (\sqrt{2})^4 = 4$

e) $\frac{2}{5} \times \frac{2}{5} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9}$
As we can write the same bases in index form as:
$\frac{2}{5} \times \frac{2}{5} \times \frac{7}{9} \times \frac{7}{9} \times \frac{7}{9} = (\frac{2}{5})^2 \times (\frac{7}{9})^3 = \frac{1372}{18225}$

f) $y \times y \times y \times y \times y$
As we can write the same bases in index form as:
$y \times y \times y \times y \times y = y^5$

**2. Apply the product law of exponents to simplify the following expressions:**

a) $x^4 \times x^6$ b) $a^2 \times a^4$ c) $2^5 \times 2^2$ d) $5^3 \times 5^4$
e) $y^2 \times y^3$ f) $(\frac{1}{2a})^3 \times (\frac{1}{2a})^{2n}$ g) $b^m \times b^n$ h) $(\frac{a^m}{5})^2 \times (\frac{a^m}{5})^2$
i) $3^2 \times 2^2$ j) $(\sqrt{3})^6 \times (\sqrt{2})^6$ k) $3^5 \times 4^5$ l) $7^3 \times 7^2$

**Solution:**

a) $x^4 \times x^6$
As the bases are the same, we can add the exponents by product law.
$x^4 \times x^6 = x^{4+6} = x^{10}$

b) $a^2 \times a^4$
As the bases are the same, we can add the exponents by product law.
$a^2 \times a^4 = a^{2+4} = a^6$

c) $2^5 \times 2^2$
As the bases are the same, we can add the exponents by product law.
$2^5 \times 2^2 = 2^{5+2} = 2^7 = 128$

d) $5^3 \times 5^4$
As the bases are the same, we can add the exponents by product law.
$5^3 \times 5^4 = 5^{3+4} = 5^7 = 78125$

e) $y^2 \times y^3$
As the bases are the same, we can add the exponents by product law.
$y^2 \times y^3 = y^{2+3} = y^5$

f) $(\frac{1}{2a})^3 \times (\frac{1}{2a})^{2n}$
As the bases are the same, we can add the exponents by product law.
$(\frac{1}{2a})^3 \times (\frac{1}{2a})^{2n} = (\frac{1}{2a})^{3+2n} = (\frac{1}{2a})^{2n+3}$

g) $b^m \times b^n$
As the bases are the same, we can add the exponents by product law.
$b^m \times b^n = b^{m+n}$

h) $(\frac{a^m}{5})^2 \times (\frac{a^m}{5})^2$
As the bases are the same, we can add the exponents by product law.
$(\frac{a^m}{5})^2 \times (\frac{a^m}{5})^2 = (\frac{a^m}{5})^{2+2} = (\frac{a^m}{5})^4 = \frac{(a^m)^4}{5^4} = \frac{a^{4m}}{625}$

i) $3^2 \times 2^2$
As the bases are different, we cannot add the exponents by product law. So
$3^2 \times 2^2 = 3 \times 3 \times 2 \times 2 = 36$

j) $(\sqrt{3})^6 \times (\sqrt{2})^6$
As the bases are different, we cannot add the exponents by product law. So
$(\sqrt{3})^6 \times (\sqrt{2})^6 = (\sqrt{3})^{2\times3} \times (\sqrt{2})^{2\times3} = (3)^3 \times (2)^3 = 27 \times 8 = 216$

k) $3^5 \times 4^5$
As the bases are different, we cannot add the exponents by product law. So
$3^5 \times 4^5 = 3 \times 3 \times 3 \times 3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4 = 248832$

l) $7^3 \times 7^2$
As the bases are the same, we can add the exponents by product law.
$7^3 \times 7^2 = 7^{3+2} = 7^5 = 16807$

**3. Apply the quotient law to simplify the following expressions:**

a) $a^6 \div a^2$ b) $3^8 \div 3^6$ c) $b^{25} \div b^{20}$ d) $2^5 \div 2^{10}$
e) $2^{12} \div 2^7$ f) $(15)^{11} \div (15)^9$ g) $(3x)^7 \div (3x)^4$ h) $7^3 \div 2^3$
i) $5^5 \div 7^5$ j) $11^3 \div 12^3$ k) $5^4 \div 5^4$

**Solution:**

a) $a^6 \div a^2$
$a^6 \div a^2 = \frac{a \times a \times a \times a \times a \times a}{a \times a}$
$= a \times a \times a \times a = a^4$
We can observe that the exponent 4 of the product $a^4$ is the difference of the exponents 6 and 2 of $a^6$ and $a^2$. So, we can write $a^6 \div a^2 = a^{6-2} = a^4$

b) $3^8 \div 3^6$
$3^8 \div 3^6 = \frac{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}{3 \times 3 \times 3 \times 3 \times 3 \times 3}$
$= 3 \times 3 = 3^2 = 9$
We can observe that the exponent 2 of the product $3^2$ is the difference of the exponents 8 and 6 of $3^8$ and $3^6$. So, we can write $3^8 \div 3^6 = 3^{8-6} = 3^2 = 9$

c) $b^{25} \div b^{20}$
We can observe that the exponent 5 of the product $b^2$ is the difference of the exponents 25 and 20 of $b^{25}$ and $b^{20}$. So, we can write as:
$b^{25} \div b^{20} = 3^{25-20} = b^5$

d) $2^5 \div 2^{10}$
$2^5 \div 2^{10} = \frac{2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= \frac{1}{2 \times 2 \times 2 \times 2 \times 2}$
$= \frac{1}{2^5}$ ………. (A)

But according to quotient law:

$2^5 \div 2^{10} = 3^{5-10} = 3^{-5}$ ---------- **(B)**
From (A) and (B), we can observe that $3^{-5} = \frac{1}{2^5} = \frac{1}{32}$
So, in a rational number, if the numerator with having a negative exponent move to the denominator, its exponent number becomes positive.

e) $2^{12} \div 2^7$
$2^{12} \div 2^7 = \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$
$= 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$
We can observe that the exponent 5 of the product $2^5$ is the difference of the exponents 12 and 7 of $2^{12}$ and $2^7$. So, we can write as:

$2^{12} \div 2^7 = 2^{12-7} = 2^5 = 32$

f) $(15)^{11} \div (15)^9$

$15^{11} \div 15^9 = \frac{15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15}{15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15 \times 15}$

$= 15 \times 15 = 15^2 = 225$

We can observe that the exponent 2 of the product $15^2$ is the difference of the exponents 11 and 9 of $15^{11}$ and $15^9$. So, we can write as:

$15^{11} \div 15^9 = 15^{11-9} = 15^2 = 225$

g) $(3x)^7 \div (3x)^4$

To apply the quotient law, we need to subtract the exponents of the same base. In this case, the base is "3x". The exponent of the numerator is 7 and the exponent of the denominator is 4.

$(3x)^7 \div (3x)^4 = (3x)^{7-4} = (3x)^3 = 3^3 \times x^3 = 27x^3$

h) $7^3 \div 2^3$

$7^3 \div 2^3 = \frac{7 \times 7 \times 7}{2 \times 2 \times 2}$

$= \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$

$= \frac{7^3}{2^3}$

$= (\frac{7}{2})^3 = \frac{343}{8}$

We can observe that the quotient of $7^3$ and $2^3$ can be written as the quotient of 7 and 2 with the same exponent 3.

i) $5^5 \div 7^5$

$7^3 \div 2^3 = \frac{5 \times 5 \times 5 \times 5 \times 5}{7 \times 7 \times 7 \times 7 \times 7}$

$= \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7} \times \frac{5}{7}$

$= \frac{5^5}{7^5}$

$= (\frac{5}{7})^5 = \frac{3125}{16807}$

We can observe that the quotient of $5^5$ and $7^5$ can be written as the quotient of 5 and 7 with the same exponent 5.

j) $11^3 \div 12^3$

$7^3 \div 2^3 = \frac{11 \times 11 \times 11}{12 \times 12 \times 12}$

$= \frac{11}{12} \times \frac{11}{12} \times \frac{11}{12}$

$= \frac{11^3}{12^3}$

$= (\frac{11}{12})^3 = \frac{1331}{1728}$

We can observe that the quotient of $11^3$ and $12^3$ can be written as the quotient of 11 and 12 with the same exponent 3.

k) $5^4 \div 5^4 = \frac{5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5}$
$= 1$

We can observe that the exponent 5 of the product $5^0$ is the difference of the exponents 5 and 5 of $5^4$ and $5^4$. So, we can write as:

$5^4 \div 5^4 = 5^{5-5} = 15^0 = 1$

Because $a^0 = 1$, $a \neq 0$, where a is any positive number.

**4. Apply the power law to simplify the following expressions:**

a) $(3a^2)^4$ b) $(100^3)^0$ c) $(y^5)^2$ d) $(6p^2q)^2$ e) $(\sqrt{2x})^2$ f) $(\frac{10^2x^3}{11})^2$

**Solution:**

a) First, we consider $(3 \times a^2)$ as the base with exponent 4.

$(3 \times a^2)^4 = 3 \times 3 \times 3 \times 3 \times a^2 \times a^2 \times a^2 \times a^2$

$(3 \times a^2)^4 = 3^{1+1+1+1} \times a^{2+2+2+2}$ (applying product law)

$(3 \times a^2)^4 = 3^4 \times a^8$

$(3 \times a^2)^4 = 81 \times a^8 = 81a^8$

We can note that $(3 \times a^2)^4 = 81 \times a^8 = 81a^8$

It shows that if a number has two or more exponents (powers), we multiply the powers together while keeping the same base.

b) First, we consider $(100^3)$ as the base with exponent 0.
$(100^3)^0 = 100^{3 \times 0} = 100^0 = 1$
Because $a^0 = 1$, $a \neq 0$, where a is any positive number.

c) First, we consider $(y^5)$ as the base with exponent 2.
$(y^5)^2 = y^5 \times y^5$
$(y^5)^2 = y^{5+5}$ (applying product law)
$(y^5)^2 = y^{10}$
We can note that $(y^5)^2 = y^{10}$
It shows that if a number has two or more exponents (powers), we multiply the powers together while keeping the same base.

d) First, we consider $(6p^2q)$ as the base with exponent 2.
$(6p^2q)^2 = 6p^2q \times 6p^2q$
$(6p^2q)^2 = 6 \times 6 \times p^2 \times p^2 \times q \times q$
$(6p^2q)^2 = 6^{1+1} \times p^{2+2} \times q^{1+1}$ (applying product law)
$(6p^2q)^2 = 6^2 \times p^4 \times q^2$
$(6p^2q)^2 = 36p^4q^2$
We can note that $(6p^2q)^2 = 36p^4q^2$
It shows that if a number has two or more exponents (powers), we multiply the powers together while keeping the same base.

e) First, we consider ($\sqrt{2x}$) as the base with exponent 2.
$(\sqrt{2x})^2 = \sqrt{2x} \times \sqrt{2x}$
$(\sqrt{2x})^2 = \sqrt{2x} \times \sqrt{2x}$ (applying product law)
$(\sqrt{2x})^2 = (\sqrt{2x})^{1+1} = (\sqrt{2x})^2$
We can note that $(\sqrt{2x})^2 = (\sqrt{2x})^2 = 2x^2$
It shows that if a number has two or more exponents (powers), we multiply the powers together while keeping the same base.

f) First, we consider $(\frac{10^2x^3}{11})$ as the base with exponent 2.
$(\frac{10^2x^3}{11})^2 = \frac{10^2x^3}{11} \times \frac{10^2x^3}{11}$
$(\frac{10^2x^3}{11})^2 = \frac{10^{2+2} \times x^{3+3}}{11^{1+1}}$ (applying product law)
$(\frac{10^2x^3}{11})^2 = \frac{10^4 \times x^6}{11^2}$
$(\frac{10^2x^3}{11})^2 = \frac{10000}{121}x^6$

We can note that $(\frac{10^2x^3}{11})^2 = \frac{10000}{121}x^6$
It shows that if a number has two or more exponents (powers), we multiply the powers together while keeping the same base.

### Learning Check 2.2

**1. Find the square root of the following numbers by factorization method.**

a) 36  b) 169  c) 1369  d) 13225  e) 293764
e) $\frac{49}{576}$  f) $\frac{324}{1681}$  g) $\frac{1296}{2025}$  h) $\frac{2601}{3844}$  i) $5\frac{19}{25}$
j) $8\frac{776}{961}$  k) 0.81  l) 0.000144  m) 12.25  n) 45.1584

**Solution:**

a) $\sqrt{36}$

| | |
|---|---|
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

$\sqrt{36} = \sqrt{2 \times 2 \times 3 \times 3}$

$= \sqrt{2^2 \times 3^2}$

$= 2 \times 3$

$= 6$

Therefore, $\sqrt{36} = 6$

b) $\sqrt{169}$

| | |
|---|---|
| 13 | 169 |
| 13 | 13 |
| | 1 |

$\sqrt{169} = \sqrt{13 \times 13}$

$= \sqrt{13^2}$

$= 13$

Therefore, $\sqrt{169} = 13$

c) $\sqrt{1369}$

| | |
|---|---|
| 37 | 1369 |
| 37 | 37 |
| | 1 |

$\sqrt{1369} = \sqrt{37 \times 37}$

$= \sqrt{37^2}$

$= 37$

Therefore, $\sqrt{1369} = 37$

d) $\sqrt{13225}$

| | |
|---|---|
| 5 | 13225 |
| 5 | 2645 |
| 23 | 529 |
| 23 | 23 |
| | 1 |

$\sqrt{13225} = \sqrt{5 \times 5 \times 23 \times 23}$

$= \sqrt{5^2 \times 23^2}$

$= 5 \times 23$

$= 115$

Therefore, $\sqrt{13225} = 115$

e) $\sqrt{293764}$

| | |
|---|---|
| 2 | 293764 |
| 2 | 146882 |
| 271 | 73441 |
| 271 | 271 |
| | 1 |

$\sqrt{293764} = \sqrt{2 \times 2 \times 271 \times 271}$

$= \sqrt{2^2 \times 271^2}$

$= 2 \times 271$

$= 542$

Therefore, $\sqrt{293764} = 542$

f) $\sqrt{\frac{49}{576}}$

| | |
|---|---|
| 2 | 576 |
| 2 | 288 |
| 2 | 144 |
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 7 | 49 |
| 7 | 7 |
| | 1 |

$\sqrt{576} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3}$
$= \sqrt{2^2 \times 2^2 \times 2^2 \times 3^2}$
$= 2 \times 2 \times 2 \times 3$
$= 24$

$\sqrt{49} = \sqrt{7 \times 7}$
$= \sqrt{7^2}$
$= 7$

Therefore, $\sqrt{\frac{49}{576}} = \frac{7}{24}$

g) $\sqrt{\frac{324}{1681}}$

| | |
|---|---|
| 2 | 324 |
| 2 | 162 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 41 | 1681 |
| 41 | 41 |
| | 1 |

$\sqrt{324} = \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3}$
$= \sqrt{2^2 \times 3^2 \times 3^2}$
$= 2 \times 3 \times 3$
$= 18$

$\sqrt{1681} = \sqrt{41 \times 41}$
$= \sqrt{41^2}$
$= 41$

Therefore, $\sqrt{\frac{324}{1681}} = \frac{18}{41}$

h) $\sqrt{\frac{1296}{2025}}$

| | |
|---|---|
| 2 | 1296 |
| 2 | 648 |
| 2 | 324 |
| 2 | 162 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 3 | 2025 |
| 3 | 675 |
| 3 | 225 |
| 3 | 75 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

$\sqrt{1296} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$
$= \sqrt{2^2 \times 2^2 \times 3^2 \times 3^2}$

$= 2 \times 2 \times 3 \times 3$
$= 36$

$\sqrt{2025} = \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 5}$
$= \sqrt{3^2 \times 3^2 \times 5^2}$
$= 3 \times 3 \times 5$
$= 45$

Therefore, $\sqrt{\frac{1296}{2025}} = \frac{36}{45} = \frac{4}{5}$

i) $\sqrt{\frac{2601}{3844}}$

| | |
|---|---|
| 3 | 2601 |
| 3 | 867 |
| 17 | 289 |
| 17 | 17 |
| | 1 |

| | |
|---|---|
| 2 | 3844 |
| 2 | 1922 |
| 31 | 961 |
| 31 | 31 |
| | 1 |

$\sqrt{2601} = \sqrt{3 \times 3 \times 17 \times 17}$
$= \sqrt{3^2 \times 17^2}$
$= 3 \times 17$
$= 51$

$\sqrt{3844} = \sqrt{2 \times 2 \times 31 \times 31}$
$= \sqrt{2^2 \times 31^2}$
$= 2 \times 31$
$= 62$

Therefore, $\sqrt{\frac{2601}{3844}} = \frac{51}{62}$

j) $\sqrt{5\,\frac{19}{25}} = \sqrt{\frac{144}{25}}$

| | |
|---|---|
| 2 | 144 |
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 5 | 25 |
| 5 | 5 |
| | 1 |

$\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$
$= \sqrt{2^2 \times 2^2 \times 3^2}$
$= 2 \times 2 \times 3$
$= 12$

$\sqrt{3844} = \sqrt{5 \times 5}$
$= \sqrt{5^2}$
$= 5$

Therefore, $\sqrt{\frac{144}{25}} = \frac{12}{5}$

k) $\sqrt{8\frac{776}{961}} = \sqrt{\frac{8464}{961}}$

| | |
|---|---|
| 2 | 8464 |
| 2 | 4232 |
| 2 | 2116 |
| 2 | 1058 |
| 23 | 529 |
| 23 | 23 |
| | 1 |

| | |
|---|---|
| 31 | 961 |
| 31 | 31 |
| | 1 |

$\sqrt{8464} = \sqrt{2 \times 2 \times 2 \times 2 \times 23 \times 23}$

$= \sqrt{2^2 \times 2^2 \times 23^2}$

$= 2 \times 2 \times 23$

$= 92$

$\sqrt{961} = \sqrt{31 \times 31}$

$= \sqrt{31^2}$

$= 31$

Therefore, $\sqrt{\frac{8464}{961}} = \frac{92}{31}$

l) $\sqrt{0.81} = \sqrt{\frac{81}{100}}$

| | |
|---|---|
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 2 | 100 |
| 2 | 50 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

$\sqrt{81} = \sqrt{3 \times 3 \times 3 \times 3}$

$= \sqrt{3^2 \times 3^2}$

$= 3 \times 3$

$= 9$

$\sqrt{100} = \sqrt{2 \times 2 \times 5 \times 5}$

$= \sqrt{2^2 \times 5^2}$

$= 2 \times 5$

$= 10$

Therefore, $\sqrt{\frac{81}{100}} = \frac{9}{10} = 0.9$

m) $\sqrt{0.000144} = \sqrt{\frac{144}{1000000}}$

| | |
|---|---|
| 2 | 144 |
| 2 | 72 |
| 2 | 36 |
| 2 | 18 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 2 | 1000000 |
| 2 | 500000 |
| 2 | 250000 |
| 2 | 125000 |
| 2 | 62500 |
| 2 | 31250 |
| 5 | 15625 |
| 5 | 3125 |
| 5 | 625 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

$\sqrt{144} = \sqrt{2 \times 2 \times 2 \times 2 \times 3 \times 3}$

$= \sqrt{2^2 \times 2^2 \times 3^2}$

$= 2 \times 2 \times 3$

$= 12$

$\sqrt{1000000} = \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}$

$= \sqrt{2^2 \times 2^2 \times 2^2 \times 5^2 \times 5^2 \times 5^2}$

$= 2 \times 2 \times 2 \times 5 \times 5 \times 5$

$= 1000$

Therefore, $\sqrt{\frac{144}{1000000}} = \frac{12}{1000} = \frac{3}{250} = 0.012$

n) $\sqrt{12.25} = \sqrt{\frac{1225}{100}}$

| | |
|---|---|
| 5 | 1225 |
| 5 | 245 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

| | |
|---|---|
| 2 | 100 |
| 2 | 50 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

$\sqrt{1225} = \sqrt{5 \times 5 \times 7 \times 7}$

$= \sqrt{5^2 \times 7^2}$

$= 5 \times 7$

$= 35$

$\sqrt{100} = \sqrt{2 \times 2 \times 5 \times 5}$

$= \sqrt{2^2 \times 5^2}$

$= 2 \times 5$

$= 10$

Therefore, $\sqrt{\frac{1225}{100}} = \frac{35}{10} = \frac{7}{2} = 3.5$

o) $\sqrt{45.1584} = \sqrt{\frac{451584}{10000}}$

| | |
|---|---|
| 2 | 451584 |
| 2 | 225792 |
| 2 | 112896 |
| 2 | 56448 |
| 2 | 28224 |
| 2 | 14112 |
| 2 | 7056 |
| 2 | 3528 |
| 2 | 1764 |
| 2 | 882 |
| 3 | 441 |
| 3 | 147 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

| | |
|---|---|
| 2 | 10000 |
| 2 | 5000 |
| 2 | 2500 |
| 2 | 1250 |
| 5 | 625 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

$\sqrt{451584} = \sqrt{2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times7\times7}$

$= \sqrt{2^2\times2^2\times2^2\times2^2\times2^2\times3^2\times7^2}$

$= 2\times2\times2\times2\times2\times3\times7$

$= 672$

$\sqrt{10000} = \sqrt{2\times2\times2\times2\times5\times5\times5\times5}$

$= \sqrt{2^2\times2^2\times5^2\times5^2}$

$= 2\times2\times5\times5$

$= 100$

Therefore, $\sqrt{\frac{144}{1000000}} = \frac{672}{100} = \frac{168}{25} = 6.72$

**2. Find the square root of the following natural numbers by the division method.**

a) 196 b) 1444 c) 2401 d) 3844 e) 10404 f) 12321 g) 422500

**Solution:**

a) $\sqrt{196}$

| | |
|---|---|
| | 1 4 |
| 1 | $\overline{1}$ $\overline{96}$ |
| | 1 |
| 24 | 96 |
| | 96 |
| 28 | 0 |

The square root of 196 = 14.

b) $\sqrt{1444}$

| | |
|---|---|
| | 3 8 |
| 3 | $\overline{14}$ $\overline{44}$ |
| | 9 |
| 68 | 5 44 |
| | 5 44 |
| 76 | 0 |

The square root of 1444 = 38.

c) $\sqrt{2401}$

$$
\begin{array}{r|r}
 & 4 \quad 9 \\
4 & \overline{24}\ \overline{01} \\
 & 16 \quad\ \ \\ \hline
89 & 8\ 01 \\
 & 8\ 01 \\ \hline
98 & 0
\end{array}
$$

The square root of 2401 = 49.

d) $\sqrt{3844}$

$$
\begin{array}{r|r}
 & 6 \quad 2 \\
6 & \overline{38}\ \overline{44} \\
 & 36 \quad\ \ \\ \hline
122 & 2\ 44 \\
 & 2\ 44 \\ \hline
124 & 0
\end{array}
$$

The square root of 3844 = 62.

e) $\sqrt{10404}$

$$
\begin{array}{r|r}
 & 1 \quad 0 \quad 2 \\
1 & \overline{1}\ \overline{04}\ \overline{04} \\
 & 1 \qquad\qquad \\ \hline
20 & 4 \qquad \\
 & 0 \qquad \\ \hline
202 & 4\ 04 \\
 & 4\ 04 \\ \hline
204 & 0
\end{array}
$$

The square root of 10404 = 102.

f) $\sqrt{12321}$

$$
\begin{array}{r|r}
 & 1 \quad 1 \quad 1 \\
1 & \overline{1}\ \overline{23}\ \overline{21} \\
 & 1 \qquad\qquad \\ \hline
21 & 23 \qquad \\
 & 21 \qquad \\ \hline
221 & 2\ 21 \\
 & 2\ 21 \\ \hline
222 & 0
\end{array}
$$

The square root of 12321 = 111.

g) $\sqrt{422500}$

$$\begin{array}{r|l}
 & \;\;6\;\;\;5\;\;\;0 \\
6 & \overline{42}\;\overline{25}\;\overline{00} \\
 & 36 \\
\hline
125 & \;\;6\;25 \\
 & \;\;6\;25 \\
\hline
1300 & \qquad\;\;0 \\
 & \qquad\;\;0 \\
\hline
1300 & \qquad\;\;0
\end{array}$$

The square root of 422500 = 650.

**3. Find the square root of the following fractions by the division method.**

a) $1\frac{24}{25}$ b) $32\frac{196}{225}$ c) $\frac{2809}{5184}$ d) $\frac{5625}{8649}$ e) $\frac{9801}{11025}$ f) $\frac{2209}{4096}$ g) $\frac{484}{43264}$

**Solution:**

a) $1\frac{24}{25} = \frac{49}{25}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.

$49 = \overline{49}$

$25 = \overline{25}$

$$\begin{array}{r|l}
 & \;7 \\
7 & \overline{49} \\
 & 49 \\
\hline
14 & \;0
\end{array}
\qquad
\begin{array}{r|l}
 & \;5 \\
5 & \overline{25} \\
 & 25 \\
\hline
10 & \;0
\end{array}$$

The square root of $\frac{49}{25} = \frac{7}{5} = 1\frac{2}{5}$.

b) $32\frac{196}{225} = \frac{7396}{225}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.

$7396 = \overline{73}\;\overline{96}$

$225 = \overline{2}\;\overline{25}$

$$\begin{array}{r|l}
 & \;\;8\;\;\;6 \\
8 & \overline{73}\;\overline{96} \\
 & 64 \\
\hline
166 & \;\;9\;96 \\
 & \;\;9\;96 \\
\hline
172 & \qquad 0
\end{array}
\qquad
\begin{array}{r|l}
 & 1\;\;\;5 \\
1 & \overline{2}\;\overline{25} \\
 & 1 \\
\hline
25 & 1\;25 \\
 & 1\;25 \\
\hline
30 & \quad\;0
\end{array}$$

The square root of $\frac{7396}{225} = \frac{86}{15} = 5\frac{11}{15}$.

c) $\frac{2809}{5184}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.
2809 = $\overline{28}\ \overline{09}$
5184 = $\overline{51}\ \overline{84}$

| | |
|---|---|
| | 5 3 |
| 5 | $\overline{28}\ \overline{09}$ |
| | 25 |
| 103 | 3 09 |
| | 3 09 |
| 106 | 0 |

| | |
|---|---|
| | 7 2 |
| 7 | $\overline{51}\ \overline{84}$ |
| | 49 |
| 142 | 2 84 |
| | 2 84 |
| 144 | 0 |

The square root of $\frac{2809}{5184} = \frac{53}{72}$

d) $\frac{5625}{8649}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.
5625 = $\overline{56}\ \overline{25}$
8649 = $\overline{86}\ \overline{49}$

| | |
|---|---|
| | 7 5 |
| 7 | $\overline{56}\ \overline{25}$ |
| | 49 |
| 145 | 7 25 |
| | 7 25 |
| 150 | 0 |

| | |
|---|---|
| | 9 3 |
| 9 | $\overline{86}\ \overline{49}$ |
| | 81 |
| 183 | 5 49 |
| | 5 49 |
| 186 | 0 |

The square root of $\frac{5625}{8649} = \frac{75}{93} = \frac{25}{31}$

e) $\frac{9801}{11025}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.
9801 = $\overline{98}\ \overline{01}$
11025 = $\overline{1}\ \overline{10}\ \ \overline{25}$

| | |
|---|---|
| | 9 9 |
| 9 | $\overline{98}\ \overline{01}$ |
| | 81 |
| 189 | 17 01 |
| | 17 01 |
| 198 | 0 |

| | |
|---|---|
| | 1 0 5 |
| 1 | $\overline{1}\ \overline{10}\ \overline{25}$ |
| | 1 |
| 20 | 10 |
| | 0 |
| 205 | 10 25 |
| | 10 25 |
| 210 | 0 |

The square root of $\frac{9801}{11025} = \frac{99}{105}$

f) $\frac{2209}{4096}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.
2209 = $\overline{22}\ \overline{09}$
4096 = $\overline{40}\ \overline{96}$

$$\begin{array}{r|r} & 4\quad 7 \\ \hline 4 & \overline{22}\ \overline{09} \\ & 16\phantom{\ 09} \\ \hline 87 & 6\ 09 \\ & 6\ 09 \\ \hline 94 & 0 \end{array} \qquad \begin{array}{r|r} & 6\quad 4 \\ \hline 6 & \overline{40}\ \overline{96} \\ & 36\phantom{\ 96} \\ \hline 124 & 4\ 96 \\ & 4\ 96 \\ \hline 128 & 0 \end{array}$$

The square root of $\frac{2209}{4096} = \frac{47}{64}$

g) $\frac{484}{43264}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.
484 = $\overline{4}\ \overline{84}$
43264 = $\overline{4}\ \overline{32}\ \overline{64}$

$$\begin{array}{r|r} & 2\quad 2 \\ \hline 2 & \overline{4}\ \overline{84} \\ & 4\phantom{\ 84} \\ \hline 42 & 84 \\ & 84 \\ \hline 44 & 0 \end{array} \qquad \begin{array}{r|r} & 2\quad 0\quad 8 \\ \hline 2 & \overline{4}\ \overline{32}\ \overline{64} \\ & 4\phantom{\ 32\ 64} \\ \hline 40 & 32\phantom{\ 64} \\ & 0\phantom{\ 64} \\ \hline 408 & 32\ 64 \\ & 32\ 64 \\ \hline 416 & 0 \end{array}$$

The square root of $\frac{484}{43264} = \frac{22}{208} = \frac{11}{104}$

**4. Find the square root of the following decimal numbers by the division method.**

a) 1.21 b) 8.41 c) 11.56 d) 94.09 e) 1.2544 f) 146.41 g) 4382.44

**Solution:**

a) 1.21
For finding the square root of 1.21 using the division method, we proceed as follows:

| | |
|---|---|
| | 1 .1 |
| 1 | $\overline{1}$ .$\overline{21}$ |
| | 1 |
| 21 | 21 |
| | 21 |
| 22 | 0 |

Hence, the square root of 1.21 is 1.1.

b) 8.41

For finding the square root of 8.41 using the division method, we proceed as follows:

| | |
|---|---|
| | 2 .9 |
| 2 | $\overline{8}$ .$\overline{41}$ |
| | 4 |
| 49 | 4 41 |
| | 4 41 |
| 58 | 0 |

Hence, the square root of 8.41 is 2.9.

c) 11.56

For finding the square root of 11.56 using the division method, we proceed as follows:

| | |
|---|---|
| | 3 .4 |
| 3 | $\overline{11}$ .$\overline{56}$ |
| | 9 |
| 64 | 2 56 |
| | 2 56 |
| 68 | 0 |

Hence, the square root of 11.56 is 3.4.

d) 94.09

For finding the square root of 94.09 using the division method, we proceed as follows:

| | |
|---|---|
| | 9 .7 |
| 9 | $\overline{94}$ .$\overline{09}$ |
| | 81 |
| 187 | 13 09 |
| | 13 09 |
| 194 | 0 |

Hence, the square root of 94.09 is 9.7.

e) 1.2544
For finding the square root of 1.2544 using the division method, we proceed as follows:

| | |
|---|---|
| | 1 .1 2 |
| 1 | $\overline{1}$ .$\overline{25}$ $\overline{44}$ |
| | 1 |
| 21 | 25 |
| | 21 |
| 222 | 4 44 |
| | 4 44 |
| 224 | 0 |

Hence, the square root of 1.2544 is 1.12.

f) 146.41
For finding the square root of 146.41 using the division method, we proceed as follows:

| | |
|---|---|
| | 1 2 .1 |
| 1 | $\overline{1}$ $\overline{46}$ .$\overline{41}$ |
| | 1 |
| 22 | 46 |
| | 44 |
| 241 | 2 41 |
| | 2 41 |
| 242 | 0 |

Hence, the square root of 146.41 is 12.1.

g) 4382.44
For finding the square root of 4382.44 using the division method, we proceed as follows:

| | |
|---|---|
| | 6 6 .2 |
| 6 | $\overline{43}$ $\overline{82}$ .$\overline{44}$ |
| | 36 |
| 126 | 7 82 |
| | 7 56 |
| 1322 | 26 44 |
| | 26 44 |
| 1324 | 0 |

Hence, the square root of 4382.44 is 66.2.

5. **The area of a square-shaped plot is 3364 m$^2$. Find the length of the plot.**
**Solution:**

| | |
|---|---|
| | 5 8 |
| 5 | $\overline{33}\ \overline{64}$ |
| | 25 |
| 108 | 8 64 |
| | 8 64 |
| 116 | 0 |

As the length of the four sides of a square is equal, so:
Area of a square = length × length = $l^2$
3364 m$^2$ = $l^2$
taking square root on both sides
$\sqrt{l^2} = \sqrt{3364}$
l = 58
Therefore, length of a square-shaped plot is 58 m.

6. **The area of a rectangular football ground is equal to the square-shaped ground. Find the length of a square-shaped ground if the length and width of the rectangular football ground are 1700 m and 1306 m, respectively.**
**Solution:**

| | |
|---|---|
| | 1 4 9 0 .0 3 3 |
| 1 | $\overline{2}\ \overline{22}\ \overline{02}\ \overline{00}\ .\overline{00}\ \overline{00}\ \overline{00}$ |
| | 1 |
| 24 | 1 22 |
| | 96 |
| 289 | 26 02 |
| | 26 01 |
| 2980 | 1 00 |
| | 0 |
| 29800 | 1 00 00 |
| | 0 |
| 298003 | 1 00 00 00 |
| | 89 40 09 |
| 2980063 | 10 59 91 00 |
| | 8 94 01 89 |
| 2980066 | 1 65 89 11 |

Length of rectangular football ground = 1700 m
Width of rectangular football ground = 1306 m
Area of rectangular football ground = length × width
= 1700 × 1306
= 2220200 m$^2$

Since, Area of square-shaped ground = Area of rectangular ground

So, $(\text{length})^2 = 2220200 \text{ m}^2$

Taking square root on both sides

$$\sqrt{(\text{length})2} = \sqrt{2220200}$$

$$\text{length} = 1490.033$$

Therefore, length of square-shaped ground is 1490.033 m.

7. **The area of a square-shaped courtyard of a Masjid is 102400 square metres. Find the length of the square-shaped courtyard.**

**Solution:**

| | |
|---|---|
| | 3 2 0 |
| 3 | $\overline{10}\ \overline{24}\ \overline{00}$ |
| | 9 |
| 62 | 1 24 |
| | 1 24 |
| 640 | 0 |
| | 0 |
| 640 | 0 |

As the length of the four sides of a square is equal, so:

Area of a square-shaped courtyard = length × length = $l^2$

$102400 \text{ m}^2 = l^2$

taking square root on both sides

$\sqrt{l^2} = \sqrt{102400}$

l = 320

Therefore, length of a square-shaped courtyard of a Masjid is 320 m.

8. **The length and width of a rectangular park are 1445 m and 605 m, respectively. If a square-shaped park has the same area as a rectangular park, then find the cost of fencing the square park at the rate of Rs.340 per metre.**

**Solution:**

| | |
|---|---|
| | 9 3 5 |
| 9 | $\overline{87}\ \overline{42}\ \overline{25}$ |
| | 81 |
| 183 | 6 42 |
| | 5 49 |
| 1865 | 93 25 |
| | 93 25 |
| 1870 | 0 |

Length of a rectangular park = 1445 m

Width of a rectangular park = 605 m

Area of rectangular park = length x width

= 1445 m x 605 m

= $874225 \text{ m}^2$

Since,

Area of a square-shaped park = Area of a rectangular park

$(\text{length})^2 = 874225$

Taking square root on both sides

$$\sqrt{(length)^2} = \sqrt{874225}$$

Length = 935 m

Therefore, the side length of the square park is 935 m.

The perimeter of the square-shaped park is given by:

Perimeter = 4 x side length = 4 x 935 m = 3740 m

The cost of fencing the square park at the rate of Rs.340 per metre is:

Cost = Perimeter x Rate

= 3740 m x Rs.340/m

= Rs.1,271,600

Therefore, the cost of fencing the square-shaped park at the given rate is Rs.1,271,600.

9. **The area of a square-shaped garage is 12544 square metre. Find the perimeter of a square-shaped garage.**

**Solution:**

| | 1 1 2 |
|---|---|
| 1 | $\overline{1}\ \overline{25}\ \overline{44}$ |
| | 1 |
| 21 | 25 |
| | 21 |
| 222 | 4 44 |
| | 4 44 |
| 224 | 0 |

As the length of the four sides of a square is equal, so:

Area of a square-shaped garage = length × length = $l^2$

$12544\ m^2 = l^2$

taking square root on both sides

$\sqrt{l^2} = \sqrt{12544m^2}$

l = 112 m

Therefore, length of a square-shaped garage is 112 m.

Perimeter of a square-shaped garage = 4 × $l$

= 4 × 112 m

= 448 m

Therefore, the perimeter of the square-shaped garage is 448 metres.

**10. A garden has 195364 plants in such a way that the number of plants in each row equals the number of rows. Find the number of rows.**

**Solution:**

| | 4 4 2 |
|---|---|
| 4 | $\overline{19}\ \overline{53}\ \overline{64}$ |
| | 16 |
| 84 | 3 53 |
| | 3 36 |
| 882 | 17 64 |
| | 17 64 |
| 884 | 0 |

As the length and width are equal, as the plants are equal to number of rows.

To find the number of plants in each row, we have to find the square root of 195364.

$\sqrt{195364}$ = 442 rows

So, there are 442 rows in the garden.

**11. The area of a square-shaped window is 1444 square metre. Find the cost of a wood border at the rate of Rs. 450 per centimetre.**

**Solution:**

| | 3 8 |
|---|---|
| 3 | $\overline{14}\ \overline{44}$ |
| | 9 |
| 68 | 5 44 |
| | 5 44 |
| 76 | 0 |

As the length of the four sides of a square is equal, so:

Area of a square-shaped window = length × length = $l^2$

1444 $m^2$ = $l^2$

taking square root on both sides

$\sqrt{l^2} = \sqrt{1444m^2}$

l = 38 m

Therefore, length of a square-shaped window is 38 m.

Perimeter of a square-shaped window = 4 × $l$

= 4 × 38 m

= 152 m

We need to convert this to centimeters since the rate is given in Rs. 450 per centimeter:

152 metres = 15200 centimeters

The cost of a wood border is the product of the total length of wood needed and the rate per centimeter:

Cost = Length x Rate
Cost = 15200 x Rs. 450
Cost = Rs. 6,840,000
Therefore, the cost of a wood border at the rate of Rs. 450 per centimeter for the square-shaped window is Rs. 6,840,000.

### Learning Check 2.3

**1. Find the cubes of the following numbers.**

a) 7 b) 12 c) 15 d) 21 e) 27 f) 32
g) 35 h) 41 i) 68 j) 70 k) 84 l) 99

**Solution:**

a) We multiply 7 by itself three times to get the cube as:
$7^3 = 7 \times 7 \times 7 = 343$
b) We multiply 12 by itself three times to get the cube as:
$12^3 = 12 \times 12 \times 12 = 1728$
c) We multiply 15 by itself three times to get the cube as:
$15^3 = 15 \times 15 \times 15 = 3375$
d) We multiply 21 by itself three times to get the cube as:
$21^3 = 21 \times 21 \times 21 = 9261$
e) We multiply 27 by itself three times to get the cube as:
$27^3 = 27 \times 27 \times 27 = 19683$
f) We multiply 32 by itself three times to get the cube as:
$32^3 = 32 \times 32 \times 32 = 32768$
g) We multiply 35 by itself three times to get the cube as:
$35^3 = 35 \times 35 \times 35 = 42875$
h) We multiply 41 by itself three times to get the cube as:
$41^3 = 41 \times 41 \times 41 = 68921$
i) We multiply 68 by itself three times to get the cube as:
$68^3 = 68 \times 68 \times 68 = 314432$
j) We multiply 70 by itself three times to get the cube as:
$70^3 = 70 \times 70 \times 70 = 343000$
k) We multiply 84 by itself three times to get the cube as:
$84^3 = 84 \times 84 \times 84 = 592704$
l) We multiply 99 by itself three times to get the cube as:
$99^3 = 99 \times 99 \times 99 = 970299$

**2. Find the cube root of the following if possible.**

a) 27 b) 64 c) 216 d) 512 e) 1000 f) 1728
g) 9261 h) 46656 i) 74088 j) 4913 k) 2744 l) 12167
m) $\frac{8}{343}$ n) $\frac{729}{2197}$ o) 59319 r) 39.304 s) 91.125 t) 6.859

**Solution:**

a) 27

| | |
|---|---|
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 27 = $\sqrt[3]{27}$

$= \sqrt[3]{3 \times 3 \times 3}$

$= \sqrt[3]{3^3}$

$= 3$

So, cube root of 27 is 3.

b) 64

| | |
|---|---|
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

Cube root of 64 = $\sqrt[3]{64}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt[3]{2^3 \times 2^3}$

$= 2 \times 2 = 4$

So, cube root of 64 is 4.

c) 216

| | |
|---|---|
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 216 = $\sqrt[3]{216}$

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 3^3}$

$= 2 \times 3 = 6$

So, cube root of 216 is 6.

d) 512

| | |
|---|---|
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

Cube root of 512 = $\sqrt[3]{512}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3}$

$= 2 \times 2 \times 2 = 8$

So, cube root of 512 is 8.

e) 1000

| | |
|---|---|
| 2 | 1000 |
| 2 | 500 |
| 2 | 250 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

$= 2 \times 5 = 10$

So, cube root of 1000 is 10.

f) 1728

| | |
|---|---|
| 2 | 1728 |
| 2 | 864 |
| 2 | 432 |
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 1728 = $\sqrt[3]{1728}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 2^3 \times 3^3}$

$= 2 \times 2 \times 3 = 12$

So, cube root of 1728 is 12.

g) 9261

| | |
|---|---|
| 3 | 9261 |
| 3 | 3087 |
| 3 | 1029 |
| 7 | 343 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

Cube root of 9261 = $\sqrt[3]{9261}$

$= \sqrt[3]{3 \times 3 \times 3 \times 7 \times 7 \times 7}$

$= \sqrt[3]{3^3 \times 7^3}$

= 3 × 7 = 21

So, cube root of 9261 is 21.

h) 46656

| | |
|---|---|
| 2 | 46656 |
| 2 | 23328 |
| 2 | 11664 |
| 2 | 5832 |
| 2 | 2916 |
| 2 | 1458 |
| 3 | 729 |
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 46656 = $\sqrt[3]{46656}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 2^3 \times 3^3 \times 3^3}$

= 2 × 2 × 3 × 3= 36

So, cube root of 46656 is 36.

i) 74088

| | |
|---|---|
| 2 | 74088 |
| 2 | 37044 |
| 2 | 18522 |
| 3 | 9261 |
| 3 | 3087 |
| 3 | 1029 |
| 7 | 343 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

Cube root of 74088 = $\sqrt[3]{74088}$

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 \times 7 \times 7}$

$= \sqrt[3]{2^3 \times 3^3 \times 7^3}$

= 2 × 3 × 7 = 42

So, cube root of 74088 is 42.

j) 4913

| | |
|---|---|
| 17 | 4913 |
| 17 | 289 |
| 17 | 17 |
| | 1 |

Cube root of 74088 = $\sqrt[3]{74088}$

$= \sqrt[3]{17 \times 17 \times 17}$

$= \sqrt[3]{17^3}$

= 17

So, cube root of 4913 is 17.

k) 2744

| | |
|---|---|
| 2 | 2744 |
| 2 | 1372 |
| 2 | 686 |
| 7 | 343 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

Cube root of 2744 = $\sqrt[3]{2744}$

$= \sqrt[3]{2 \times 2 \times 2 \times 7 \times 7 \times 7}$

$= \sqrt[3]{2^3 \times 7^3}$

= 2 × 7 = 14

So, cube root of 2744 is 14.

l) 12167

| | |
|---|---|
| 23 | 12167 |
| 23 | 529 |
| 23 | 23 |
| | 1 |

Cube root of 12167 = $\sqrt[3]{12167}$

$= \sqrt[3]{23 \times 23 \times 23}$

$= \sqrt[3]{23^3}$

= 23

So, cube root of 12167 is 23.

m) $\frac{8}{343}$

| | |
|---|---|
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

| | |
|---|---|
| 7 | 343 |
| 7 | 49 |
| 7 | 7 |
| | 1 |

Cube root of 8 = $\sqrt[3]{8}$

$= \sqrt[3]{2 \times 2 \times 2}$

$= \sqrt[3]{2^3}$

= 2

Cube root of 343 = $\sqrt[3]{343}$

= $\sqrt[3]{7 \times 7 \times 7}$

= $\sqrt[3]{7^3}$

= 7

So, cube root of $\frac{8}{343}$ is $\frac{2}{7}$.

n) $\frac{729}{2197}$

| | |
|---|---|
| 3 | 729 |
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 13 | 2197 |
| 13 | 169 |
| 13 | 13 |
| | 1 |

Cube root of 729 = $\sqrt[3]{729}$

= $\sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3}$

= $\sqrt[3]{3^3 \times 3^3}$

= 3 × 3 = 9

Cube root of 2197 = $\sqrt[3]{2197}$

= $\sqrt[3]{13 \times 13 \times 13}$

= $\sqrt[3]{13^3}$

= 13

So, cube root of $\frac{729}{2197}$ is $\frac{9}{13}$.

o) 59319

| | |
|---|---|
| 3 | 59319 |
| 3 | 19773 |
| 3 | 6591 |
| 13 | 2197 |
| 13 | 169 |
| 13 | 13 |
| | 1 |

Cube root of 59319 = $\sqrt[3]{59319}$

= $\sqrt[3]{3 \times 3 \times 3 \times 13 \times 13 \times 13}$

= $\sqrt[3]{3^3 \times 13^3}$

= 3 × 13 = 39

So, cube root of 59319 is 39.

p) 39.304 = $\frac{39304}{1000}$

| | | | | |
|---|---|---|---|---|
| 2 | 39304 | | 2 | 1000 |
| 2 | 19652 | | 2 | 500 |
| 2 | 9826 | | 2 | 250 |
| 17 | 4913 | | 5 | 125 |
| 17 | 289 | | 5 | 25 |
| 17 | 17 | | 5 | 5 |
| | 1 | | | 1 |

Cube root of 39304 = $\sqrt[3]{39304}$

$= \sqrt[3]{2 \times 2 \times 2 \times 17 \times 17 \times 17}$

$= \sqrt[3]{2^3 \times 17^3}$

= 2 × 17 = 34

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

= 2 × 5 = 10

So, cube root of $\frac{39304}{1000}$ is $\frac{34}{10}$ = 3.4.

q) 91.125 = $\frac{91125}{1000}$

| | | | | |
|---|---|---|---|---|
| 3 | 91125 | | 2 | 1000 |
| 3 | 30375 | | 2 | 500 |
| 3 | 10125 | | 2 | 250 |
| 3 | 3375 | | 5 | 125 |
| 3 | 1125 | | 5 | 25 |
| 3 | 375 | | 5 | 5 |
| 5 | 125 | | | 1 |
| 5 | 25 | | | |
| 5 | 5 | | | |
| | 1 | | | |

Cube root of 91125 = $\sqrt[3]{91125}$

$= \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5}$

$= \sqrt[3]{3^3 \times 3^3 \times 5^3}$

= 3 × 3 × 5 = 45

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

= 2 × 5 = 10

So, cube root of $\frac{91125}{1000}$ is $\frac{45}{10}$ = 4.5.

r) 6.859

| | |
|---|---|
| 19 | 6859 |
| 19 | 361 |
| 19 | 19 |
| | 1 |

| | |
|---|---|
| 2 | 1000 |
| 2 | 500 |
| 2 | 250 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

Cube root of 6859 = $\sqrt[3]{6859}$

$= \sqrt[3]{19 \times 19 \times 19}$

$= \sqrt[3]{19^3}$

= 19

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

= 2 × 5 = 10

So, cube root of $\frac{6859}{1000}$ is $\frac{19}{10}$ = 1.9.

**3. The volume of a cube shaped wooden box is 12.167 cubic metres. What is the length of the box.**

**Solution:**

The volume of cube = length × length × length

To find the length, we need to find the cube root of its volume, i.e., 12.167 $m^3 = \frac{12167}{1000} m^3$

| | |
|---|---|
| 23 | 12167 |
| 23 | 529 |
| 23 | 23 |
| | 1 |

| | |
|---|---|
| 2 | 1000 |
| 2 | 500 |
| 2 | 250 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

Cube root of 12167 = $\sqrt[3]{12167}$

$= \sqrt[3]{23 \times 23 \times 23}$

$= \sqrt[3]{23^3}$

= 23

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

= 2 × 5 = 10

As, cube root of $\frac{12167}{1000}$ is $\frac{23}{10}$ = 2.3.

So, the length of the wooden box = 2.3 m

4. **The volume of a cube-shaped water tank is 91125 cubic metres. What is the length of one side of the water tank?**
   **Solution:**
   The volume of cube = length × length × length

   To find the length, we need to find the cube root of its volume, i.e., 91125

   | | |
   |---|---|
   | 3 | 91125 |
   | 3 | 30375 |
   | 3 | 10125 |
   | 3 | 3375 |
   | 3 | 1125 |
   | 3 | 375 |
   | 5 | 125 |
   | 5 | 25 |
   | 5 | 5 |
   | | 1 |

   Cube root of 91125 = $\sqrt[3]{91125}$

   $= \sqrt[3]{3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 5}$
   $= \sqrt[3]{3^3 \times 3^3 \times 5^3}$
   = 3 × 3 × 5 = 45

   So, the length of one side of the water tank is 45 m.

5. **The length of a cubic-shaped ice block is 12 cm. What is the volume of an ice block?**
   **Solution:**
   Length of ice block = 12 cm
   Volume of an ice block = ?
   We know that
   Volume of an ice block = length × length × length
   = 12 cm × 12 cm × 12 cm
   = 1728 $cm^3$
   So, the volume of an ice block is 1728 $cm^3$.

6. **The length of a cube-shaped prism is 17 cm. What is the volume of a prism?**
   **Solution:**
   Length of a cube-shaped prism = 17 cm
   Volume of a cube-shaped prism = ?
   We know that
   Volume of an ice block = length × length × length
   = 17 cm × 17 cm × 17 cm
   = 4913 $cm^3$
   So, the volume of a cube-shaped prism is 4913 $cm^3$.

## Unit Check 2

1. **Encircle the correct option.**
   a) The square of any integer is always __________.
      i) 0 **ii) positive** iii) negative iv) none of these
   b) The square root of 1681 is:
      i) 31 **ii) 41** iii) 45 iv) 51
   c) The index number of $\sqrt[3]{1728}$ is:
      i) 21 ii) 3 **iii) 12** iv) 1728
   d) The cube of a number is that number raised to the power.
      i) 1 ii) 2 **iii) 3** iv) 6
   e) 69 is the square root of:
      **i) 328509** ii) 343000 iii) 357911 iv) 373248
2. **Fill in the blanks.**
   a) The square of a proper fraction is **smaller** than itself.
   b) Square root is the **inverse** process of squaring the number.
   c) In $\sqrt[n]{6}$ , radicand is **6**.
   d) $(18.2)^3$ = **6128.49**.
   e) Square root of 18769 is **137**.
3. **Find the square root of the following numbers.**
   a) 81 b) 900 c) 1225 d) $\frac{2209}{961}$ e) $\frac{1444}{3364}$ f) $\frac{3721}{2704}$
   g) 1.3225 h) 0.0841 i) 11.0224 j) 156.25 k) 1.1881 l) $\frac{361}{961}$

   **Solution**

   Note: Usually, we solve square roots by the factorization or division method, but if the method is not mentioned in the question, then we can use any method for the solution. Here, we will use the division method.

   a) 81

| | |
|---|---|
| | 9 |
| 9 | 81 |
| | 81 |
| 18 | 0 |

   The square root of 81 = 9.

   b) 900

| | | |
|---|---|---|
| | 3 | 0 |
| 3 | 9 | 00 |
| | 9 | |
| 60 | | 0 |
| | | 0 |
| 60 | | 0 |

   The square root of 900 = 30.

c) 1225

$$\begin{array}{r|r} & 3\ \ \ 5 \\ 3 & \overline{12}\ \overline{25} \\ & 9\ \ \ \ \ \\ \hline 65 & 3\ 25 \\ & 3\ 25 \\ \hline 70 & 0 \end{array}$$

The square root of 1225 = 35

d) $\frac{2209}{961}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.

2209 = $\overline{22}\ \ \overline{09}$

961 = $\overline{9}\ \ \overline{61}$

$$\begin{array}{r|r} & 4\ \ \ 7 \\ 4 & \overline{22}\ \overline{09} \\ & 16\ \ \ \ \ \\ \hline 87 & 6\ 09 \\ & 6\ 09 \\ \hline 94 & 0 \end{array} \qquad \begin{array}{r|r} & 3\ \ \ 1 \\ 3 & \overline{9}\ \overline{61} \\ & 9\ \ \ \ \ \\ \hline 61 & 61 \\ & 61 \\ \hline 62 & 0 \end{array}$$

The square root of $\frac{2209}{961} = \frac{47}{31}$

e) $\frac{1444}{3364}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.

1444 = $\overline{14}\ \ \overline{44}$

3364 = $\overline{33}\ \ \overline{64}$

$$\begin{array}{r|r} & 3\ \ \ 8 \\ 3 & \overline{14}\ \overline{44} \\ & 9\ \ \ \ \ \\ \hline 68 & 5\ 44 \\ & 5\ 44 \\ \hline 76 & 0 \end{array} \qquad \begin{array}{r|r} & 5\ \ \ 8 \\ 5 & \overline{33}\ \overline{64} \\ & 25\ \ \ \ \ \\ \hline 108 & 8\ 64 \\ & 8\ 64 \\ \hline 116 & 0 \end{array}$$

The square root of $\frac{1444}{3364} = \frac{38}{58} = \frac{19}{29}$

f) $\frac{3721}{2704}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.

3721 = $\overline{37}\ \ \overline{21}$

2704 = $\overline{27}\ \ \overline{04}$

| | |
|---|---|
| | 6 1 |
| 6 | $\overline{37}$ $\overline{21}$ |
| | 36 |
| 121 | 1 21 |
| | 1 21 |
| 122 | 0 |

| | |
|---|---|
| | 5 2 |
| 5 | $\overline{27}$ $\overline{04}$ |
| | 25 |
| 102 | 2 04 |
| | 2 04 |
| 104 | 0 |

The square root of $\frac{3721}{2704} = \frac{61}{52}$

g) 1.3225

| | |
|---|---|
| | 1 .1 5 |
| 1 | $\overline{1}$ .$\overline{32}$ $\overline{25}$ |
| | 1 |
| 21 | 32 |
| | 21 |
| 225 | 11 25 |
| | 11 25 |
| 230 | 0 |

The square root of 1.3225 = 1.15.

h) 0.0841

| | |
|---|---|
| | 0 .2 9 |
| 0 | $\overline{0}$ .$\overline{08}$ $\overline{41}$ |
| | 0 |
| 2 | 8 |
| | 4 |
| 49 | 4 41 |
| | 4 41 |
| 58 | 0 |

The square root of 0.0841 = 0.29.

i) 11.0224

| | |
|---|---|
| | 3 .3 2 |
| 3 | $\overline{11}$ .$\overline{02}$ $\overline{24}$ |
| | 9 |
| 63 | 2 02 |
| | 1 89 |
| 662 | 13 24 |
| | 13 24 |
| 664 | 0 |

The square root of 11.0224 = 3.32.

j) 156.25

$$\begin{array}{r|r} & 1\quad 2\quad .5 \\ \hline 1 & \overline{1}\ \overline{56}\ .\overline{25} \\ & 1\qquad\quad \\ \hline 22 & 56\quad\ \ \\ & 44\quad\ \ \\ \hline 245 & 12\ \ 25 \\ & 12\ \ 25 \\ \hline 250 & 0 \end{array}$$

The square root of 156.25 = 12.5.

k) 1.1881

$$\begin{array}{r|r} & 1\quad .0\quad 9 \\ \hline 1 & \overline{1}\ .\overline{18}\ \overline{81} \\ & 1\qquad\quad \\ \hline 20 & 18\quad\ \ \\ & 0\quad\ \ \\ \hline 209 & 18\ 81 \\ & 18\ 81 \\ \hline 218 & 0 \end{array}$$

The square root of 1.1881 = 1.09.

l) $\frac{361}{961}$

To find the square root of the given fraction by the division method, we find the square roots of the numerator and denominator separately.

361 = $\overline{3}\ \ \overline{61}$

961 = $\overline{9}\ \ \overline{61}$

$$\begin{array}{r|r} & 1\quad 9 \\ \hline 1 & \overline{3}\ \overline{61} \\ & 1\quad\ \ \\ \hline 29 & 2\ 61 \\ & 2\ 61 \\ \hline 38 & 0 \end{array} \qquad \begin{array}{r|r} & 3\quad 1 \\ \hline 3 & \overline{9}\ \overline{61} \\ & 9\quad\ \ \\ \hline 61 & 61 \\ & 61 \\ \hline 62 & 0 \end{array}$$

The square root of $\frac{361}{961} = \frac{19}{31}$.

**4. Find the square root of the following natural numbers.**

a) 5776 b) 4225 c) 2916 d) 529 e) 1521

f) 6241 g) 17956 h) 24649

**Solution:**

Note: Usually, we solve square roots by the factorization or division method, but if the method is not mentioned in the question, then we can use any method for the solution. Here, we will use the factorization method.

a) 5776

$$\begin{array}{r|r} 2 & 5776 \\ \hline 2 & 2888 \\ \hline 2 & 1444 \\ \hline 2 & 722 \\ \hline 19 & 361 \\ \hline 19 & 19 \\ \hline & 1 \end{array}$$

$\sqrt{5776} = \sqrt{2 \times 2 \times 2 \times 2 \times 19 \times 19}$

$= \sqrt{2^2 \times 2^2 \times 19^2}$

$= 2 \times 2 \times 19$

$= 76$

Therefore, $\sqrt{5776} = 76$

b) 4225

$$\begin{array}{r|r} 5 & 4225 \\ \hline 5 & 845 \\ \hline 13 & 169 \\ \hline 13 & 13 \\ \hline & 1 \end{array}$$

$\sqrt{4225} = \sqrt{5 \times 5 \times 13 \times 13}$

$= \sqrt{5^2 \times 13^2}$

$= 5 \times 13$

$= 65$

Therefore, $\sqrt{4225} = 65$

c) 2916

$$\begin{array}{r|r} 2 & 2916 \\ \hline 2 & 1458 \\ \hline 3 & 729 \\ \hline 3 & 243 \\ \hline 3 & 81 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$\sqrt{2916} = \sqrt{2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt{2^2 \times 3^2 \times 3^2 \times 3^2}$

$= 2 \times 3 \times 3 \times 3$

$= 54$

Therefore, $\sqrt{2916} = 54$

d) 529

| | |
|---|---|
| 23 | 529 |
| 23 | 23 |
| | 1 |

$\sqrt{529} = \sqrt{23 \times 23}$

$= \sqrt{23^2}$

$= 23$

Therefore, $\sqrt{529} = 23$

e) 1521

| | |
|---|---|
| 3 | 1521 |
| 3 | 507 |
| 13 | 169 |
| 13 | 13 |
| | 1 |

$\sqrt{1521} = \sqrt{3 \times 3 \times 13 \times 13}$

$= \sqrt{3^2 \times 13^2}$

$= 3 \times 13$

$= 39$

Therefore, $\sqrt{1521} = 39$

f) 6241

| | |
|---|---|
| 79 | 6241 |
| 79 | 79 |
| | 1 |

$\sqrt{6241} = \sqrt{79 \times 79}$

$= \sqrt{79^2}$

$= 79$

Therefore, $\sqrt{6241} = 79$

g) 17956

| | |
|---|---|
| 2 | 17956 |
| 2 | 8978 |
| 67 | 4489 |
| 67 | 67 |
| | 1 |

$\sqrt{17956} = \sqrt{2 \times 2 \times 67 \times 67}$

$= \sqrt{2^2 \times 67^2}$

$= 2 \times 67$

$= 134$

Therefore, $\sqrt{17956} = 134$

h) 24649

| | |
|---|---|
| 157 | 24649 |
| 157 | 157 |
| | 1 |

$\sqrt{24649} = \sqrt{157 \times 157}$
$= \sqrt{157^2}$
$= 157$

Therefore, $\sqrt{24649} = 157$

**5. Find the cube of the following numbers.**

a) 6 b) 11 c) 14 d) 27 e) 32 f) 63
g) 58 h) 74

**Solution:**

a) 6
We multiply 6 by itself three times to get the cube as:
$6^3 = 6 \times 6 \times 6 = 216$

b) 11
We multiply 11 by itself three times to get the cube as:
$11^3 = 11 \times 11 \times 11 = 1331$

c) 14
We multiply 14 by itself three times to get the cube as:
$14^3 = 14 \times 14 \times 14 = 2744$

d) 27
We multiply 27 by itself three times to get the cube as:
$27^3 = 27 \times 27 \times 27 = 19683$

e) 32
We multiply 32 by itself three times to get the cube as:
$32^3 = 32 \times 32 \times 32 = 32768$

f) 63
We multiply 63 by itself three times to get the cube as:
$63^3 = 63 \times 63 \times 63 = 250047$

g) 58
We multiply 58 by itself three times to get the cube as:
$58^3 = 58 \times 58 \times 58 = 195112$

h) 74
We multiply 74 by itself three times to get the cube as:
$74^3 = 74 \times 74 \times 74 = 405224$

**6. Find the cube root of the following.**

a) 512 b) 1728 c) 2197 d) $\frac{216}{5832}$ e) $9\frac{305}{512}$ f) $\frac{64000}{3375}$
g) 17.576 h) 1.728

**Solution:**

a) 512

| | |
|---|---|
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

Cube root of 512 = $\sqrt[3]{512}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3}$

$= 2 \times 2 \times 2 = 8$

So, cube root of 512 is 8.

b) 1728

| | |
|---|---|
| 2 | 1728 |
| 2 | 864 |
| 2 | 432 |
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 1728 = $\sqrt[3]{1728}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 2^3 \times 3^3}$

$= 2 \times 2 \times 3 = 12$

So, cube root of 1728 is 12.

c) 2197

| | |
|---|---|
| 13 | 2197 |
| 13 | 169 |
| 13 | 13 |
| | 1 |

Cube root of 2197 = $\sqrt[3]{2197}$

$= \sqrt[3]{13 \times 13 \times 13}$

$= \sqrt[3]{13^3}$

= 13

So, cube root of 2197 is 13.

d) $\frac{216}{5832}$

| | |
|---|---|
| 2 | 216 |
| 2 | 108 |
| 2 | 54 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

| | |
|---|---|
| 2 | 5832 |
| 2 | 2916 |
| 2 | 1458 |
| 3 | 729 |
| 3 | 243 |
| 3 | 81 |
| 3 | 27 |
| 3 | 9 |
| 3 | 3 |
| | 1 |

Cube root of 216 = $\sqrt[3]{216}$

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 3^3}$

= 2 × 3 = 6

Cube root of 5832 = $\sqrt[3]{5832}$

$= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 3^3 \times 3^3}$

= 2 × 3 × 3 = 18

Cube root of $\frac{216}{5832} = \sqrt[3]{\frac{216}{5832}}$

$= \frac{\sqrt[3]{216}}{\sqrt[3]{5832}}$

$= \frac{6}{18}$

$= \frac{1}{3}$

So, cube root of $\frac{216}{5832}$ is $\frac{1}{3}$.

e) $9\frac{305}{512}$

| | |
|---|---|
| 17 | 4913 |
| 17 | 289 |
| 17 | 17 |
| | 1 |

| | |
|---|---|
| 2 | 512 |
| 2 | 256 |
| 2 | 128 |
| 2 | 64 |
| 2 | 32 |
| 2 | 16 |
| 2 | 8 |
| 2 | 4 |
| 2 | 2 |
| | 1 |

Cube root of 4913 = $\sqrt[3]{4913}$

$= \sqrt[3]{17 \times 17 \times 17}$

$= \sqrt[3]{17^3}$

= 17

Cube root of 512 = $\sqrt[3]{512}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3}$

= 2 × 2 × 2 = 8

Cube root of $9\frac{305}{512} = \sqrt[3]{\frac{4913}{512}}$

$= \frac{\sqrt[3]{4913}}{\sqrt[3]{512}}$

$= \frac{17}{8}$

$= 2\frac{1}{8}$

So, cube root of $9\frac{305}{512}$ is $2\frac{1}{8}$.

f) $\frac{64000}{3375}$

| | |
|---|---|
| 2 | 64000 |
| 2 | 32000 |
| 2 | 16000 |
| 2 | 8000 |
| 2 | 4000 |
| 2 | 2000 |
| 2 | 1000 |
| 2 | 500 |
| 2 | 250 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

| | |
|---|---|
| 3 | 3375 |
| 3 | 1125 |
| 3 | 375 |
| 5 | 125 |
| 5 | 25 |
| 5 | 5 |
| | 1 |

Cube root of 64000 = $\sqrt[3]{64000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 2^3 \times 2^3 \times 5^3}$

$= 2 \times 2 \times 2 \times 5 = 40$

Cube root of 3375 = $\sqrt[3]{3375}$

$= \sqrt[3]{3 \times 3 \times 3 \times 5 \times 5 \times 5}$

$= \sqrt[3]{3^3 \times 5^3}$

$= 3 \times 5 = 15$

Cube root of $\frac{64000}{3375} = \sqrt[3]{\frac{64000}{3375}}$

$= \frac{\sqrt[3]{64000}}{\sqrt[3]{3375}}$

$= \frac{40}{15}$

$= \frac{8}{3}$

So, cube root of $\frac{64000}{3375}$ is $\frac{8}{3}$.

g) $17.576 = \frac{17576}{1000}$

| | | | | |
|---|---|---|---|---|
| 2 | 17576 | | 2 | 1000 |
| 2 | 8788 | | 2 | 500 |
| 2 | 4394 | | 2 | 250 |
| 13 | 2197 | | 5 | 125 |
| 13 | 169 | | 5 | 25 |
| 13 | 13 | | 5 | 5 |
| | 1 | | | 1 |

Cube root of 17576 = $\sqrt[3]{17576}$

$= \sqrt[3]{2 \times 2 \times 2 \times 13 \times 13 \times 13}$

$= \sqrt[3]{2^3 \times 13^3}$

$= 2 \times 13 = 26$

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

$= 2 \times 5 = 10$

So, cube root of $\frac{17576}{1000}$ is $\frac{26}{10} = 2.6$.

h) $1.728 = \frac{1728}{1000}$

| | | | | |
|---|---|---|---|---|
| 2 | 1728 | | 2 | 1000 |
| 2 | 864 | | 2 | 500 |
| 2 | 432 | | 2 | 250 |
| 2 | 216 | | 5 | 125 |
| 2 | 108 | | 5 | 25 |
| 2 | 54 | | 5 | 5 |
| 3 | 27 | | | 1 |
| 3 | 9 | | | |
| 3 | 3 | | | |
| | 1 | | | |

Cube root of 1728 = $\sqrt[3]{1728}$

$= \sqrt[3]{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3}$

$= \sqrt[3]{2^3 \times 2^3 \times 3^3}$

$= 2 \times 2 \times 3 = 12$

Cube root of 1000 = $\sqrt[3]{1000}$

$= \sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}$

$= \sqrt[3]{2^3 \times 5^3}$

$= 2 \times 5 = 10$

So, cube root of $\frac{1728}{1000}$ is $\frac{12}{10}$ = 1.2.

7. **2304 blocks have been arranged in such a way that the number of rows and the number of blocks in each row is equal. Find the number of blocks in each row.**

**Solution:**

| | 4 8 |
|---|---|
| 4 | $\overline{23}\ \overline{04}$ |
| | 16 |
| 88 | 7 04 |
| | 7 04 |
| 96 | 0 |

As the length and width are equal, as the number of blocks are equal to number of rows.

To find the number of blocks in each row, we have to find the square root of 2304.

$\sqrt{2304}$ = 48 rows

So, there are 442 rows in the garden.

8. **The capacity of a cube-shaped swimming pool is 4913 m$^3$. What is the cost of tiling its base floor at the rate of Rs.1250 per square metre?**

**Solution:**

| | |
|---|---|
| 17 | 4913 |
| 17 | 289 |
| 17 | 17 |
| | 1 |

Cube root of 4913 = $\sqrt[3]{4913}$

$= \sqrt[3]{17 \times 17 \times 17}$

$= \sqrt[3]{17^3}$

$= 17$

The surface area of the base floor is given by the formula:

Area = length x width

Since the pool is square-shaped, the length and width are both equal to the length of one side, which is 17 meters. Therefore:

Area = 17 x 17 = 289 m$^2$

Now we can calculate the cost of tiling the base floor at the rate of Rs.1250 per square metre:

Cost = Area x Rate

Cost = 289 x 1250

Cost = Rs. 361,250

Therefore, the cost of tiling the base floor of the swimming pool at the rate of Rs.1250 per square metre is Rs.361,250.

9. **The length of a cube-shaped plastic container is 25 centimetre. Find the volume of the plastic container.**

**Solution:**

Length of a cube-shaped container = 25 cm

Volume of a cube-shaped container = ?

We know that

Volume of a plastic container = length × length × length

$= 25 \text{ cm} \times 25 \text{ cm} \times 25 \text{ cm}$

$= 15625 \text{ cm}^3$

So, the volume of a cube-shaped plastic container is $15625 \text{ cm}^3$.

## Exercise 3.1

1. **8 men can paint a wall in three hours. How long will it take 6 men to paint a wall?**
**Solution:**
Let x be the number of hours.

We know that the number of men is inversely proportional to time. So,

| Time (In hours) | Men |
|---|---|
| 3 | 8 |
| x | 6 |

$$\frac{x}{3} = \frac{8}{6}$$

Multiplying 3 on both sides, we get

x = $\frac{8}{6} \times 3$

x = 4

Therefore, it would take 6 men 18 hours to paint the same wall that 8 men can paint in 4 hours.

2. **The price of a 12-cup coffee pack is Rs.350. How much would 30 cups of coffee cost?**
**Solution:**
Let x be the price of a cup of coffee.

We know that cups of coffee are directly proportional to their price. So,

| Price (In rupees) | Cups (coffee) |
|---|---|
| 350 | 12 |
| x | 30 |

$$\frac{x}{350} = \frac{30}{12}$$

12x = 30 × 350

Dividing 12 on both sides

$\frac{12x}{12} = \frac{30 \times 350}{12}$

x = 875

Therefore, 30 cups of coffee would cost Rs. 875.

3. **There are 5 pipes, and it takes 56 minutes to water the fields. How much will it take to water the field with 14 pipes?**

**Solution:**
Let x be the time in minutes.

We know that the number of pages is directly proportional to time. So,

| Time | pipes |
|---|---|
| 56 ↑ | 5 ↓ |
| x | 14 |

$$\frac{x}{56} = \frac{5}{14}$$

14x = 56 × 5

Dividing 14 on both sides

$$\frac{14x}{14} = \frac{56 \times 5}{14}$$

x = 20 min.

Therefore, it would take 6 men 18 hours to paint the same wall that 8 men can paint in 4 hours.

4. **Six gardeners grassing the park completed the task in 12 days. How many days will it take if there are 8 gardeners?**
**Solution:**
Let x be the number of days.

We know that the number of men is inversely proportional to time. So,

| Days | Gardeners |
|---|---|
| 12 ↑ | 6 ↓ |
| x | 8 |

$$\frac{x}{12} = \frac{6}{8}$$

Multiplying 6 on both sides, we get

8x = 6 × 12

Dividing 8 on both sides

$$\frac{8x}{8} = \frac{6 \times 12}{8}$$

x = 9

Therefore, it will take 9 days for 8 gardeners to complete the same job.

5. **Saad read 4 pages in 8 days. How many pages will he read in 18 days?**
   **Solution:**
   Let x be the number of pages.

   We know that the number of pages is directly proportional to days. So,

| Pages | Days |
|---|---|
| 4 ↑ | 8 ↑ |
| x | 18 |

$$\frac{x}{4} = \frac{18}{8}$$

Multiplying 6 on both sides, we get

8x = 18 × 4

Dividing 8 on both sides

$$\frac{8x}{8} = \frac{18 \times 4}{8}$$

x = 9

Therefore, Saad will read 9 pages in 18 days.

6. **The average speed of a train is 120 km/hr and it completes a journey in 3 hours and 30 minutes. How long will it take to cover the same distance at a speed of 80 km/hr?**
   **Solution:**
   Let x be the time taken.

   We know that the speed of train is inversely proportional to the time taken. So,

| Time (In hours) | Speed (km) |
|---|---|
| 3 hours 30 minutes ↑ | 120 ↓ |
| X | 80 |

$$\frac{x}{3.5} = \frac{120}{80}$$

80x = 120 × 3.5

Dividing 80 on both sides

$$\frac{80x}{80} = \frac{120 \times 3.5}{80}$$

x = 5.25

Therefore, it will take 5 hours and 15 minutes to cover the same distance at a speed of 80 km/hr.

7. **A baker baked 45 cupcakes in 30 minutes. In how many minutes will he bake 90 cupcakes?**
**Solution:**
Let x be the time taken.

We know that the number of cupcakes is directly proportional to time. So,

| Time | Cupcakes |
|---|---|
| 15 ↑ | 45 ↑ |
| x | 90 |

$$\frac{x}{15} = \frac{90}{45}$$

45x = 90 × 15

Dividing 45 on both sides

$$\frac{45x}{45} = \frac{90 \times 15}{45}$$

x = 30

Therefore, the baker will take 30 minutes to bake 90 cupcakes.

8. **Two people arranged 200 books in 15 minutes. If one more person is added, then how much time will it take to arrange the books?**
**Solution:**
Let x be the time taken.

We know that the number of people is inversely proportional to time. So,

| Time | People |
|---|---|
| 15 ↑ | 2 |
| x | 3 ↓ |

$$\frac{x}{15} = \frac{2}{3}$$

3x = 2 × 15

Dividing 3 on both sides

$$\frac{3x}{3} = \frac{2 \times 15}{3}$$

x = 10

Therefore, it will take 10 minutes for three people to arrange 200 books.

9. **Three machines can produce 150 bottles in 20 minutes. How many bottles will they make in 20 minutes if the number of machines is increased by 5?**
**Solution:**
Let x be the number of bottles.

We know that the number of bottles is directly proportional to machines. So,

| Bottles | Machines |
|---|---|
| 150 ↑ | 3 ↑ |
| x | 8 |

$$\frac{x}{150} = \frac{8}{3}$$

3x = 8 × 150

Dividing 3 on both sides

$$\frac{3x}{3} = \frac{8 \times 150}{3}$$

x = 400

Therefore, with 8 machines working together, they will produce 400 bottles in 20 minutes.

10. **A medical camp has enough food for 105 people for 26 days. How long will the food last if 25 more people join them?**
**Solution:**
Let x be the number of days.

We know that the number of men is inversely proportional to the number of days. So,

| Days | People |
|---|---|
| 26 ↑ | 105 ↓ |
| x | 130 |

$$\frac{x}{26} = \frac{105}{130}$$

130x = 105 × 26

Dividing 130 on both sides

$$\frac{130x}{130} = \frac{105 \times 26}{130}$$

x = 21

Therefore, the food will last for 21 days if 25 more people join the medical camp.

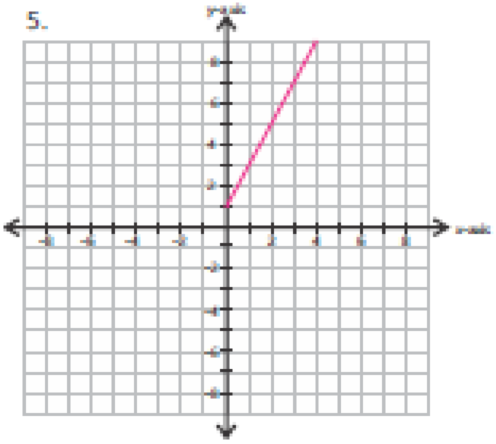
**11. Draw graphs for the following and identify if the quantities are in direct proportion or inverse proportion?**

| | | | | | |
|---|---|---|---|---|---|
| **X** | **0** | **1** | **2** | **3** | **4** |
| **y** | **0** | **4** | **8** | **12** | **16** |

| | | | | | | |
|---|---|---|---|---|---|---|
| **X** | **2** | **3** | **4** | **6** | **8** | **12** |
| **y** | **12** | **8** | **6** | **4** | **3** | **2** |

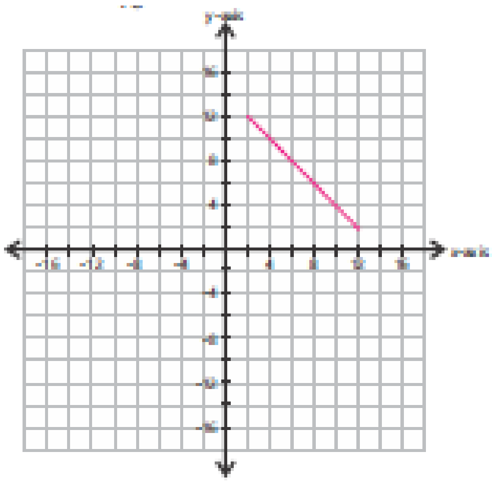
**Solution:**

a)



From the graph, we can see that the points lie on a straight line, and as the value of the first coordinate (x-value) increases, the value of the second coordinate (y-value) also increases. Therefore, the quantities are in direct proportion.

b)



From the graph, we can see that the points form a roughly straight line, but it does not pass through the origin., and as the value of the first coordinate (x-value) decreases, the value of the second coordinate (y-value) increases and the x-coordinate increases, the value of the t-coordinate decreases. Therefore, the quantities are in inverse proportion.

## Exercise 3.2

1. **8 carpenters made 15 tables in 10 days. How many tables can be made in 8 days by 12 carpenters?**
   **Solution:**
   Let x be the number of tables.

| table | carpenter | day |
|---|---|---|
| 15 | 8 | 10 |
| x | 12 | 8 |

$$\frac{x}{15} = \frac{12}{8} \times \frac{8}{10}$$

x = $\frac{12}{8} \times \frac{8}{10} \times 15$

x = 18 tables

Therefore, 12 carpenters can make 18 tables in 8 days.

2. **28 workers built 4 rooms in 9 days. How many days will it take 18 workers to build 12 rooms?**
   **Solution:**
   Let x be the number of days.

| days | workers | rooms |
|---|---|---|
| 9 | 28 | 4 |
| x | 18 | 12 |

$$\frac{x}{9} = \frac{28}{18} \times \frac{12}{4}$$

x = $\frac{28}{18} \times \frac{12}{4} \times 9$

x = 42 days

Therefore, it will take 18 workers 42 days to build 12 rooms.

3. **If 12 people typed 436 pages in 2 days, how many pages could be typed by 18 people in 5 days?**
   **Solution:**
   Let x be the number of pages.

| pages | people | days |
|---|---|---|
| 436 | 12 | 2 |
| x | 18 | 5 |

$$\frac{x}{436} = \frac{18}{12} \times \frac{5}{2}$$

x = $\frac{18}{12} \times \frac{5}{2} \times 436$

x = 1635 pages

Therefore, 18 people can type 1,635 pages in 5 days.

4. **It will cost Rs 58320 for 25 men to reap a wheat crop in one week. How long will it take for 35 men to earn Rs 93,312?**
**Solution:**
Let x be the number of days.

| days | cost | Men |
|---|---|---|
| 7 | 58320 | 25 |
| x | 93312 | 35 |

$$\frac{x}{7} = \frac{93312}{58320} \times \frac{25}{35}$$

x = $\frac{93312}{58320} \times \frac{25}{35} \times 7$

x = 8 days

Therefore, 35 men can earn Rs 93,312 in 8 days.

5. **Naveed can complete 12 paintings in 24 days if he works for 8 hours a day. How many paintings will be complete in 32 days if he works 10 hours a day?**
**Solution:**
Let x be the number of paintings.

| paintings | days | working |
|---|---|---|
| 12 | 24 | 8 |
| x | 32 | 10 |

$$\frac{x}{12} = \frac{32}{24} \times \frac{10}{8}$$

x = $\frac{32}{24} \times \frac{10}{8} \times 12$

x = 20 paintings

Therefore, Naveed can complete 20 paintings in 32 days if he works 10 hours a day.

6. **If the fare for 21 kg of luggage for a distance of 12 km is Rs 650, how much will the fare be for 14 kg of luggage for a distance of 36 km?**
**Solution:**
Let x be the fare price of the luggage.

| fare | luggage | distance |
|---|---|---|
| 650 | 21 | 12 |
| x | 14 | 36 |

$$\frac{x}{650} = \frac{14}{21} \times \frac{36}{12}$$

x = $\frac{14}{21} \times \frac{36}{12} \times 650$

x = Rs. 1,300

Therefore, the fare for 14 kg of luggage for a distance of 36 km is Rs 1,300.

## Exercise 3.3

1. **Roshan purchased a computer for Rs. 36,000 and sold it for Rs. 42,000. Find his profit or loss percentage.**
   **Solution:**
   Cost price = Rs 36,000
   Sale price = Rs 42,000
   Profit = SP – CP
   = Rs 42,000 – Rs 36,000 = Rs 6,000
   Profit % = $\frac{profit}{CP} \times 100$
   = $\frac{6000}{36000} \times 100$ = 16.67 %
   Therefore, computer profit percentage is 16.67 %.

2. **Sadaf purchased a plot for Rs.1,590,000 and sold it to gain a profit of 5%. Find the sale price of the plot.**
   **Solution:**
   Cost price = Rs 1,590,000
   Profit % = 5 %
   Sale price = ?
   We know that
   Profit % = $\frac{profit}{CP} \times 100$
   5 = $\frac{profit}{1{,}590{,}000} \times 100$
   5 × 15,900 = profit
   ⟹ profit = Rs 79,500
   Sale price = CP + profit
   = Rs 1,590,000 + Rs 79500
   = Rs 1,669,500
   Therefore, sale price of the plot is Rs 1,669,500
3. **Tehreem bought a cell phone for Rs.52,000 and sold it for Rs.44,000. Find her profit or loss percentage.**
   **Solution:**
   Cost price = Rs 52,000
   Sale price = Rs 44,000
   Loss = CP – SP
   = Rs 52,000 – Rs 44,000 = Rs 8,000
   Loss % = $\frac{loss}{CP} \times 100$
   = $\frac{8000}{52000} \times 100$ = 15.38 %
   Therefore, loss percentage of a cell phone is 15.38 %.
4. **Alina sold a carpet for Rs.6500 and has a loss of 6%. Find out the cost price of a carpet.**
   **Solution:**
   Sale price = Rs 6,500
   Loss % = 6 %
   Cost price = ?

We know that
Loss = cost price – sale price
Sale price = CP – loss
Rs 6,500 = CP – 6% of CP
Rs 6,500 = CP - $\frac{6}{100}$ CP
6500 = CP (1 – 0.06)
6500 = 0.94 CP
Dividing 0.94 on both sides
$\frac{6500}{0.94} = \frac{0.94}{0.94}$ CP
⇒ CP = 6914.89
Therefore, the cost price of the carpet is Rs. 6914.89.

5. **A shopkeeper has a 3% loss after selling an oven at Rs.25,600. What price should it sell at to get 5% profit?**
**Solution:**
Sale price = Rs 25,600
Loss % = 3 %
Cost price = ?
We know that
Loss = cost price – sale price
Sale price = CP – loss
Rs 25,600 = CP – 3% of CP
Rs 25,600 = CP - $\frac{3}{100}$ CP
25600 = CP (1 – 0.03)
25600 = 0.97 CP
Dividing 0.97 on both sides
$\frac{25600}{0.97} = \frac{0.97}{0.97}$ CP
⇒ CP = 26391.75
The selling price in order to get 5 % profit.
Profit = SP – CP
SP = CP + profit
SP = CP + 5 % of CP
SP = 26391.75 + $\frac{5}{100}$ × 26391.75
SP = 27711.34
Therefore, the selling price is Rs.27711.34.

6. **Saad bought a study table worth Rs.3200. He received a 17% discount on his purchase. Find the sale price of the table.**
**Solution:**
Actual price = Rs. 3,200
Discount % = 17 %
Sale price of a table = ?
We know that
Discount price = discount % × actual price
= 17 % × 3200
= $\frac{17}{100}$ × 3200 = Rs. 544

Sale price of the table = actual price – discount price
= 3200 – 544 = Rs. 2,656
Therefore, the sale price of a study is Rs. 2,656.

7. **The marked price of a shirt is Rs.2800. The sale price of the shirt is Rs.2380. Find the percentage discount.**
**Solution:**
Discount = Marked Price - Sale Price
Discount = Rs. 2800 - Rs. 2380
Discount = Rs. 420
Now, we can calculate the percentage discount as:
Percentage Discount = $\frac{Discount}{M.P}$ x 100
Percentage Discount = $\frac{420}{2800}$ x 100
Percentage Discount = 15%
Therefore, the percentage discount on the shirt is 15%.

8. **Find the marked price of a calculator if it is sold for Rs.650 after a 10% discount.**
**Solution:**
Selling price = Marked price - Discount
Selling price = M.P - 10% of M.P
Rs. 650 = M.P – $\frac{10}{100}$ M.P
Rs. 650 = M.P – 0.10 M. P
Rs. 650 = M.P (1 – 0.10)
Rs. 650 = 0.90 M. P
Dividing 0.90 on both sides
$\frac{Rs.650}{0.90} = \frac{0.90}{0.90}$ M.P
Rs. 722.22 = M.P
Therefore, the marked price of the calculator is Rs. 722.22

9. **The cost price of a fan is Rs.4000. The shopkeeper writes that the market price of the fan is 12% more than the sale price. Find the discount given to the customer.**
**Solution:**
Sale price of fan = Rs. 4,000
Market price = SP + 12 % of SP
= SP + $\frac{12}{100}$ SP
= SP + 0.12 SP
= SP (1+ 0.12)
= 1.12 SP
= 1.12 (4000)
= Rs. 4,480
Discount = M.P – S.P
= Rs.4,480 – Rs.4,000 = Rs.480
Therefore, the discount given to the customer is Rs. 480.

10. **A shopkeeper sold a pair of shoes for Rs 4500 after a 16% discount. What is the marked price of the pair of shoes?**
    **Solution:**
    Sale price = SP = Rs. 4,500
    Discount % = 16 %
    Market price = ?
    We know that
    SP = Marked Price (MP) - Discount
    4500 = MP - (16/100) MP
    4500 = MP - 0.16MP
    4500 = MP (1 – 0.16)
    4500 = 0.84x
    Dividing 0.84 on both sides
    x = 4500/0.84
    x = 5357.14
    Therefore, the marked price of the pair of shoes is Rs 5357.14.

## Exercise 3.4

1. **Find the principle amount invested by Bilal if he receives a profit of Rs. 5,500 in 4 years at the rate of 8% per year.**
   **Solution:**
   Profit = I = Rs. 5,500
   Rate = R = 8% per year
   Time = T = 4 years
   Principle amount = P = ?
   We know that
   I = $\frac{P \times R \times T}{100}$
   5500 = $\frac{P \times 8 \times 4}{100}$
   100 × 5500 = 32 P
   Dividing 32 on both sides
   $\frac{100 \times 5500}{32} = \frac{32\,P}{32}$
   ⟹ P = Rs. 17,187.50
   Therefore, the principal amount invested by Bilal is Rs. 17,187.50.
2. **Find the 5-year profit of Rs 85,000 at the rate of 6%.**
   **Solution:**
   Principal = P = Rs. 85,000
   Rate = R = 6% per year
   Time = T = 5 years
   Profit = I = ?
   We know that
   I = $\frac{P \times R \times T}{100}$
   I = $\frac{85000 \times 6 \times 5}{100}$
   I = Rs. 25,500

Therefore, the 5-year profit at the rate of 6% for a principal of Rs. 85,000 is Rs. 25,500.

3. **Rabia invested some amount in a business. She receives a profit of Rs 65,000 at the rate of 10% per year for 4 years. Find her original investment from it at.**
   **Solution:**
   Profit = Rs. 65,000
   Rate = 10% per year
   Time = 4 years
   Principle amount = P = ?
   We know that
   $I = \frac{P \times R \times T}{100}$
   $65000 = \frac{P \times 10 \times 4}{100}$
   $100 \times 65000 = 40\ P$
   Dividing 40 on both sides
   $\frac{100 \times 65000}{40} = \frac{40\ P}{40}$
   $\Rightarrow P = \text{Rs. } 162{,}500$
   Therefore, Rabia's original investment in the business was Rs. 162,500.
4. **Find the markup of the bank if Hamza borrowed Rs 125,000 from bank at the rate of 18% per year for 5 years.**
   **Solution:**
   Principal amount = P = Rs. 125,000
   Rate = R =18% per year
   Time = T = 5 years
   Markup = ?
   We know that
   $I = \frac{P \times R \times T}{100}$
   $\text{Markup} = \frac{125000 \times 18 \times 5}{100}$
   $= \text{Rs.} 112{,}500$
   Therefore, the markup (interest rate charged by the bank) on Hamza's loan is Rs.112,500.
5. **Find the rate of profit when Saud lent Rs 240,000 for 3 years and received Rs 32,000 extra.**
   **Solution:**
   Principal (P) = Rs. 240,000
   Time (t) = 3 years
   Extra amount received = Rs. 32,000
   Total amount received = Principal + Extra amount
   Total amount received = P + 32,000
   Also
   Total amount = P + I
   Comparing
   P + I = P + 32000
   I = P – P + 32000

I = 32000
We know that
I = $\frac{P \times R \times T}{100}$
$\frac{P \times R \times T}{100}$ = 32000
$\frac{240{,}000 \times R \times 3}{100}$ = 32000
$240{,}000 \times R \times 3$ = 32000 × 100
R = $\frac{32000 \times 100}{240000 \times 3}$
R = 4.44%
Therefore, the rate of profit when Saud lent Rs. 240,000 for 3 years and received Rs. 32,000 extra is 4.44%.

6. **In 7 years, at what annual percentage rate would Rs 70,000 become Rs 82,000?**
**Solution:**
Profit = I = 82000 – 70,000 = Rs. 12,000
Principle = P = Rs. 70,000
Tome T = 7 years
Rate = R = ?
We know that
I = $\frac{P \times R \times T}{100}$
R = $\frac{100 \times I}{P \times T}$
R = $\frac{100 \times 12000}{70000 \times 7}$
R = 2.45 %
Therefore, the annual percentage rate required for Rs 70,000 to become Rs 82,000 in 7 years is 2.45%.

7. **How long would Rs 25,000 have to be invested at a markup rate of 8% per year to gain Rs 4,500?**
**Solution:**
Principle amount = P = Rs. 25,000
Markup = I = 4500
Rate = R = 8 %
Time = T = ?
We know that
I = $\frac{P \times R \times T}{100}$
T = $\frac{100 \times I}{P \times T}$
T = $\frac{100 \times 4500}{25000 \times 8}$
T = 2.25
T = 2 years 3 months
Therefore, Rs 25,000 would have to be invested for 2 years 3 months at a markup rate of 8% per year to gain Rs 4,500.

8. **How long would Rs 30,000 have to be deposited in the bank at the rate of 15% per year to receive Rs 45,000 back?**
   **Solution:**
   Principle amount = P = Rs. 30,000
   Profit = I = 45000 – 30000 = Rs.15,000
   Rate = R = 15 %
   Time = T = ?
   We know that
   I = $\frac{P \times R \times T}{100}$
   T = $\frac{100 \times I}{P \times T}$
   T = $\frac{100 \times 15000}{30000 \times 15}$
   T = 2.33 years
   T = 2 years 4 months (approx.)
   Therefore, Rs 30,000 would have to be deposited in the bank for 2 years 4 months (approx.) at the rate of 15% per year to receive Rs 45,000 back.

**Exercise 3.5**

1. **Aliza purchased a car for Rs.2,750,000 and insured it for one year at the rate of 12%. Find the annual premium.**
   **Solution:**
   Price of car = Rs. 2,750,000
   Insurance rate = 12 %
   Annual premium = ?
   We know that
   Amount of premium = 12 % × 2,750,000
   = $\frac{12}{100}$ × 2,750,000
   = Rs. 330,000
   So, the amount of annual premium is Rs. 330,000
2. **Ahmad got a life insurance policy for Rs.300,000. Find the first premium he has to pay when the rate of annual premium is 4.5% and the policy fee is 0.25%.**
   **Solution:**
   Amount of insurance policy = Rs. 300,000
   Insurance rate = 4.5 %
   Policy fee = 0.25 %
   First premium = ?
   Policy fee @ 0.25 % = $\frac{0.25}{100} \times$ 300,000
   = $\frac{25}{100 \times 100} \times$ 300,000
   = Rs. 750
   Premium amount @ 4.5 % = $\frac{4.5}{100} \times$ 300,000
   = $\frac{45}{100 \times 10} \times$ 300,000
   = Rs. 13,500
   First premium = Rs.750 + Rs. 13,500 = Rs.14,250

3. **Nabeel got a life insurance policy for Rs. 800,000 at a rate of 15%, and the policy fee is 0.25%. Find the annual premium of Nabeel's life insurance policy.**
   **Solution:**
   Amount of insurance policy = Rs. 800,000
   Insurance rate = 15 %
   Policy fee = 0.25 %
   Annual premium = ?
   Policy fee @ 0.25 % = $\frac{0.25}{100} \times 800{,}000$
   $= \frac{25}{100 \times 100} \times 800{,}000$
   = Rs. 2,000
   First premium @ 15 % = $\frac{15}{100} \times 800{,}000$
   $= \frac{15}{100} \times 800{,}000$
   = Rs. 120,000
   Annual premium = Rs.2,000 + Rs. 120,000 = Rs. 122,000
4. **Ali purchased a car for Rs.1,400,000 and got it insured at a rate of 6.2% annual premium for 4 years. Determine how much of a premium he will pay if the depreciation rate is 15%.**
   **Solution:**
   Price of car = Rs.1,400,000
   Rate of insurance = 6.2 %
   Depreciation rate = 15 %
   Time period = 4 years
   First premium = 6.2 % of 1,400,000
   $= \frac{6.2}{100} \times 1{,}400{,}000$
   $= \frac{62}{100 \times 10} \times 1{,}400{,}000$
   = Rs.86,800
   Depreciation after one year = 15 % of 1,400,000
   $= \frac{15}{100} \times 1{,}400{,}000$
   = Rs.210,000
   Depreciation price after one year = 1,400,000 – 210,000 = Rs. 1,190,000
   $2^{nd}$ premium = 6.2 % of 1,190,000
   $= \frac{6.2}{100} \times 1{,}190{,}000$
   $= \frac{62}{100 \times 10} \times 1{,}190{,}000$
   = Rs.73,780
   Depreciation after two years = 15 % of 1,190,000
   $= \frac{15}{100} \times 1{,}190{,}000$
   = Rs.178,500
   Depreciation price after two years = 1,190,000 – 178,500 = Rs. 1,011,500
   $3^{rd}$ premium = 6.2 % of 1,011,500
   $= \frac{6.2}{100} \times 1{,}011{,}500$

$= \frac{62}{100 \times 10} \times 1{,}011{,}500$
= Rs.62,713

Depreciation after three years = 15 % of 1,011,500
$= \frac{15}{100} \times 1{,}011{,}500$
= Rs.151,725

Depreciation price after three years = 1,011,500 – 151,725 = Rs. 859,775

$4^{th}$ premium = 6.2 % of 859,775
$= \frac{6.2}{100} \times 859{,}775$
$= \frac{62}{100 \times 10} \times 859{,}775$
= Rs.53,306.05

Total amount paid as premium
= Rs.86,800 + Rs.73,780 + Rs.62,713 + Rs.53,306.05 = Rs.276,599.05

5. **Asim and Umar started a business by sharing the amounts of Rs. 110,000 and Rs. 90,000. After 6 months, Haris also joined them with an investment of Rs.130,000. After one year, they made a profit of Rs.70,000. Find the profit share of each person.**

**Solution:**

Asim investment for 12 months = Rs. 110,000
Asim investment for 1 months = 12 × Rs. 110,000 = Rs. 1,320,000
Umar investment for 12 months = Rs. 90,000
Umar investment for 1 months = 12 × Rs. 90,000 = Rs. 1,080,000
Haris investment for 6 months = Rs. 130,000
Haris investment for 1 months = 6 × Rs. 130,000 = Rs. 780,000
The simplest for the capital's share ratio:

| Asim | : | Umar | : | Haris |
|---|---|---|---|---|
| 1,320,000 | : | 1080000 | : | 780,000 |
| 132 | : | 108 | : | 78 |
| 22 | : | 18 | : | 13 |

Sum of ratio = 22 + 18 + 13 = 53
Profit = 70,000
Asim's share in profit = $\frac{22}{53} \times 70000$ = Rs.29,056.60
Umar's share in profit = $\frac{18}{53} \times 70000$ = Rs.23,773.58
Haris's share in profit = $\frac{13}{53} \times 70000$ = Rs.17,169.81

6. **Two partners contributed R.50,000 and Rs. 65,000. First contributed for 8 months and second contributed the amount for 6 months. Divide a profit of Rs.40,000 between the partners.**

**Solution:**

$1^{st}$ investment for 8 months = Rs. 50,000
$1^{st}$ investment for 1 months = 8 × Rs. 50,000 = Rs. 400,000
$2^{nd}$ investment for 6 months = Rs. 65,000
$2^{nd}$ investment for 1 months = 6 × Rs. 65,000 = Rs. 390,000
The simplest for the capital's share ratio:

| | | |
|---|---|---|
| $1^{st}$ | : | $2^{nd}$ |
| 400,000 | : | 390,000 |
| 40 | : | 39 |

Sum of ratio = 40 + 39 = 79

Profit = 40,000

$1^{st}$ share in profit = $\frac{40}{79}$ × 40000 = Rs.20,253.16

$2^{nd}$ share in profit = $\frac{39}{79}$ × 40000 = Rs.19,746.84

7. **Saeed, Ali and Asim started a business with capitals. Rs. 25,000, Rs.32,000 and Rs.40,000 respectively. At the end of the year, they suffered with a loss of Rs.16,000. Find the share of each in this loss.**

**Solution:**

Saeed investment for 12 months = Rs. 25,000

Ali investment for 12 months = Rs. 32,000

Asim investment for 6 months = Rs. 40,000

The simplest for the capital's share ratio:

| | | | | |
|---|---|---|---|---|
| Asim | : | Umar | : | Haris |
| 25,000 | : | 32,000 | : | 40,000 |
| 25 | : | 32 | : | 40 |

Sum of ratio = 25 + 32 + 40 = 97

Profit = 16,000

Saeed's share in profit = $\frac{25}{97}$ × 16,000 = Rs.4,123.71

Ali's share in profit = $\frac{32}{97}$ × 16,000 = Rs.5,278.35

Asim's share in profit = $\frac{40}{97}$ × 16,000 = Rs.6,597.94

8. **Aslam left a wealth of Rs.1,200,000. His heirs are widow, 2 sons, and 3 daughters. Find the share of each if the widow got 1/8 of the total amount.**

**Solution:**

Total amount left = Rs.1,200,000

Widow's share = $\frac{1}{8}$ × 1200,000

= Rs.150,000

Remaining amount = 1,200,000 – 150,000 = Rs. 1,050,000

Now ratios of share

| | | |
|---|---|---|
| son | : | daughter |
| 2 | : | 3 |
| 2 × 2 = 4 | : | 3 × 1 = 3 |

Sum of ratios = 4 + 3 = 7

Share of 2 sons = $\frac{4}{7}$ × 1050,000 = Rs. 600,000

Share of each son = $\frac{600,000}{2}$ = Rs.300,000

Share of 3 daughters = $\frac{3}{7}$ × 1050,000 = Rs.450,000

Share of each daughter = $\frac{450,000}{3}$ = Rs.150,000

9. **Saud died leaving a property of Rs.800,000. He left 3 daughters and a son. Find the share of each in property if the burial expenditure is Rs.50,000.**

   **Solution:**

   Total amount left = Rs. 800,000
   Burial expenditure = Rs.50,000
   Remaining amount = 800,000 – 50,000 = Rs. 750,000
   Now ratios of share

   | | | |
   |---|---|---|
   | son | : | daughter |
   | 1 | : | 3 |
   | 1 × 2 = 2 | : | 3 × 1 = 3 |

   Sum of ratios = 2 + 3 = 5
   Share of a son = $\frac{2}{5}$ × 750,000 = Rs. 300,000
   Share of 3 daughters = $\frac{3}{5}$ × 750,000 = Rs.450,000
   Share of each daughter = $\frac{450,000}{3}$ = Rs.150,000

10. **Wife of Akram died, leaving behind 4 sons and 2 daughters. Akram got ¼ of the inheritance of Rs. 400,000. The remaining amount was distributed among her children.**

    **Solution:**

    Total amount left = Rs. 400,000
    Akram's share = $\frac{1}{4}$ × 400,000
    = Rs.100,000
    Remaining amount = 400,000 – 100,000 = Rs. 300,000
    Now ratios of share

    | | | |
    |---|---|---|
    | son | : | daughter |
    | 4 | : | 2 |
    | 4 × 2 = 8 | : | 2 × 1 = 2 |

    Sum of ratios = 8 + 2 = 10
    Share of 4 sons = $\frac{8}{10}$ × 300,000 = Rs. 240,000
    Share of each son = $\frac{240,000}{4}$ = Rs.60,000
    Share of 3 daughters = $\frac{2}{10}$ × 300,000 = Rs.60,000
    Share of each daughter = $\frac{60,000}{2}$ = Rs.30,000

## Exercise 3.6

1. **Convert the following currencies of PKR. (Use updated currency rates from the internet.)**

   | | | |
   |---|---|---|
   | a) 205 USD | b) 75 CAD | c) 540 GBP |
   | d) 575 USD | e) 500 CAD | f) 1260 GBP |
   | g) 850 USD | h) 685 CAD | i) 1000 GBP |

**Solution:**

a) 205 USD to PKR

1 United States Dollar = Rs 221.60

205 United States Dollars = Rs 221.60 × 205 = Rs 45,428

b) 75 CAD to PKR

1 CAD = 167.12 PKR

75 CAD = 75 × 167.12 PKR = Rs 12,534

c) 540 GBP to PKR

1 GBP = 260.67 PKR

540 GBP = 260.67 PKR × 540 = Rs 140,761.80

d) 575 USD to PKR

1 United States Dollar = Rs 221.60

575 United States Dollars = Rs 221.60 × 575 = Rs 127,420

e) 500 CAD to PKR

1 CAD = 167.12 PKR

500 CAD = 500 × 167.12 PKR = Rs 83,560

f) 1260 GBP to PKR

1 GBP = 260.67 PKR

1260 GBP = 260.67 PKR × 1260 = Rs 328,444.20

g) 850 USD to PKR

1 United States Dollar = Rs 221.60

850 United States Dollars = Rs 221.60 × 850 = Rs 188,360

h) 685 CAD to PKR

1 CAD = 167.12 PKR

685 CAD = 685 × 167.12 PKR = Rs 114,477.20

i) 1000 GBP to PKR

1 GBP = 260.67 PKR

1000 GBP = 260.67 PKR × 1000 = Rs 260,670

**2. Convert the following amounts given in PKR to the mentioned currencies. (Use updated currency rated from the internet.)**

a) 1000 PKR to USD b) 590 PKR to CAD c) 640 PKR to GBP

d) 350 PKR to USD e) 1500 PKR to CAD f) 820 PKR to GBP

g) 935 PKR to USD h) 675 PKR to CAD i) 2000 PKR to GBP

**Solution:**

a) 1000 PKR to USD

1 Pakistani Rupee = 0.0045 USD

1000 Pakistani Rupees = $1000 \times 0.0045$ USD = 4.50 USD

b) 590 PKR to CAD

1 PKR = 0.00598 CAD

590 Pakistani Rupees = $590 \times 0.00598$ CAD = 3.5282 CAD

c) 640 PKR to GBP

1 Pakistani Rupee = 0.0038 GBP

640 Pakistani Rupees = $640 \times 0.0038$ GBP = 2.432 GBP

d) 350 PKR to USD

1 Pakistani Rupee = 0.0045 USD

350 Pakistani Rupees = $350 \times 0.0045$ USD = 1.5750 USD

e) 1500 PKR to CAD

1 PKR = 0.00598 CAD

1500 Pakistani Rupees = $1500 \times 0.00598$ CAD = 8.97 CAD

f) 820 PKR to GBP

1 Pakistani Rupee = 0.0038 GBP

820 Pakistani Rupees = $820 \times 0.0038$ GBP = 3.116 GBP

g) 935 PKR to USD

1 Pakistani Rupee = 0.0045 USD

935 Pakistani Rupees = $935 \times 0.0045$ USD = 4.2075 USD

h) 675 PKR to CAD

1 PKR = 0.00598 CAD

675 Pakistani Rupees = $675 \times 0.00598$ CAD = 4.0365 CAD

i) 2000 PKR to GBP

1 Pakistani Rupee = 0.0038 GBP

2000 Pakistani Rupees = $2000 \times 0.0038$ GBP = 7.6 GBP

## Unit Evaluation 3

1. **Encircle the correct option.**

a) A labourer earns Rs.25,000 in 30 days. How much does he earn in 6 days?

i) 2000 ii) 4000 **iii) 5000** iv) 8333

b) Two quantities that always have a curved graph are called:

i) direct proportion **ii) inverse proportion** iii) ratio iv) none of these

c) If the selling price (SP) of things or goods are less than its cost price (CP), we can say that there is a:
   i) profit **ii) loss** iii) discount iv) principal

d) A person who borrows the money is known as:
   i) creditor **ii) debtor** iii) cashier iv) none of these

e) According to book rates, 25 USD = _____________.
   **i) 4991** ii) 3611.25 iii) 5991.50 iv) 4479.54

**2. Fill in the blanks.**

a) If one quantity increases (decreases) and causes the other quantity to increase (decrease), it is called **direct proportion**.

b) The marked price of a shirt is Rs. 500. It is sold at 30% discount, then actual selling price of shirt is **Rs. 350**.

c) Markup = $\frac{\boldsymbol{P \times R \times T}}{\mathbf{100}}$.

d) The amount of last premium is always equal to **0**.

e) The person who lends the money is called **creditor**.

**3. If the price of 1 dozen is Rs. 204, how many eggs can be bought for Rs.119?**

**4. If 32 men dig 650 cm$^3$ in 6 hours, how much of the earth can 24 men dig in 8 hours?**

**Solution:**

Let x be the earth's digging.

| Earth | Men | Hours |
|---|---|---|
| 650 | 32 | 6 |
| x | 24 | 8 |

$$\frac{x}{650} = \frac{24}{32} \times \frac{8}{6}$$

x = $\frac{24}{32} \times \frac{8}{6} \times 650$

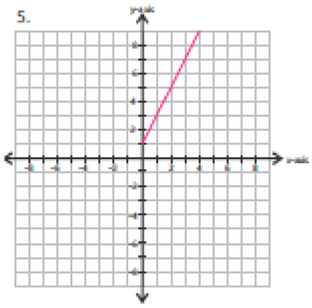
x = 650 cm$^3$

Therefore, 24 men can dig 650 cm$^3$ of earth in 8 hours.

**5. Draw the graphs for the following and identify if the quantities are in direct proportion or inverse proportion.**

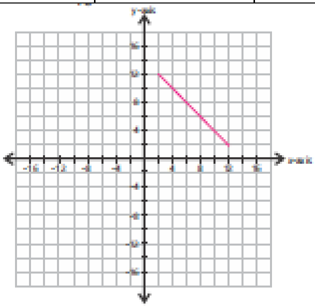
a)

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 1 | 3 | 5 | 7 | 9 |



b)

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 2 | 4 | 6 | 8 | 10 | 12 |
| y | 12 | 10 | 8 | 6 | 4 | 2 |



6. **Find the marked price of an article if it is sold for Rs.1050 after a discount of 30%.**
   **Solution:**
   Sale price of an article = Rs.1,050
   Discount = 30 % of marked price
   We know that
   Sale price = Marked price - Discount
   sale price = M.P - 30% of M.P
   1050 = M.P – $\frac{30}{100}$ M.P
   1050 = M.P – 0.30 M. P
   1050 = M.P (1 – 0.30)
   1050 = 0.70 M. P
   Dividing 0.70 on both sides
   $\frac{1050}{0.70} = \frac{0.70}{0.70}$ M.P
   Rs. 1,500 = M.P
   Therefore, the marked price of an article is Rs. 1,500.
7. **Usama borrowed Rs. 80,000 from the bank for 3 years at 5% per annum. Find the extra amount paid by her when repaying the borrowed amount.**
   **Solution:**
   Principal = P = Rs. 80,000
   Rate = R = 5% per annum
   Time = T = 3 years
   Simple interest = I = ?
   We know that
   I = $\frac{P \times R \times T}{100}$
   I = $\frac{80{,}000 \text{ x } 5 \text{ x } 3}{100}$
   I = Rs.12,000
   Therefore, Usama will have to pay an extra amount of Rs. 12,000 when repaying the borrowed amount.

8. **Azam got his van insured at a rate of 4% for 5 years. The worth of his van is Rs.1,540,000. Find the total amount paid as premium if rate of depreciation is 12% per year.**

**Solution:**

Price of car = Rs.1,540,000
Rate of insurance = 4 %
Depreciation rate = 12 %
Time period = 5 years
First premium = 4 % of 1,540,000
$= \frac{4}{100} \times 1{,}540{,}000$
= Rs.61,600
Depreciation after one year = 12 % of 1,540,000
$= \frac{12}{100} \times 1{,}540{,}000$
= Rs.184,800
Depreciation price after one year = 1,540,000 – 184,800 = Rs. 1,355,200
2nd premium = 4 % of 1,355,200
$= \frac{4}{100} \times 1{,}355{,}200$
= Rs.54,208
Depreciation after two years = 12 % of 1,355,200
$= \frac{12}{100} \times 1{,}355{,}200$
= Rs.162,624
Depreciation price after two years = 1,355,200 – 162,624 = Rs. 1,192,576
3rd premium = 4 % of 1,192,576
$= \frac{4}{100} \times 1{,}192{,}576$
= Rs.47,703.04
Depreciation after three years = 12 % of 1,192,576
$= \frac{12}{100} \times 1{,}192{,}576$
= Rs.143,109.12
Depreciation price after three years = 1,192,576 –143,109.12 = Rs. 1,049,466.88
4th premium = 4 % of 1,049,466.88
$= \frac{4}{100} \times 1{,}049{,}466.88$
= Rs. 41,978.68
Depreciation after four years = 12 % of 1,049,466.88
$= \frac{12}{100} \times 1{,}049{,}466.88$
= Rs.125,936.03
Depreciation price after four years = 1,049,466.88 –125,936.03 = Rs. 923,530.85
5th premium = 4 % of 923,530.85
$= \frac{4}{100} \times 923{,}530.85$
= Rs.36,941.23
Total amount paid as premium
= Rs. 61,600 + Rs. 54,208 + Rs. 47,703.04 + Rs. 41,978.68 + 36,941.23

= Rs.242,430.95

9. **Ramzan, Imran and Raza started a business by sharing Rs. 420,000, Rs. 350,000 and Rs.570,000 respectively. They made a profit of Rs.280,000 at the end of the year. Find the profit share of each.**
**Solution:**
Ramzan investment for 12 months = Rs. 420,000
Imran investment for 12 months = Rs. 350,000
Raza investment for 12 months = Rs. 570,000
The simplest for the capital's share ratio:

| | | | | |
|---|---|---|---|---|
| Asim | : | Umar | : | Haris |
| 420,000 | : | 350,000 | : | 570,000 |
| 42 | : | 35 | : | 57 |

Sum of ratio = 42 + 35 + 57 = 134
Profit = 280,000
Saeed's share in profit = $\frac{42}{134}$ × 280,000 = Rs.87,761.19
Ali's share in profit = $\frac{35}{134}$ × 280,000 = Rs.73,134.33
Asim's share in profit = $\frac{57}{134}$ × 280,000 = Rs.119,104.48

10. **Saad invests Rs. 60,000 at $5\frac{1}{2}$ % per year profit. How much would the amount after 1 year 6 months.**
**Solution:**
Principal = P = Rs. 60,000
Rate = R = 5 1/2% per anum = 5.5 %per annum
Time = T = 1.5 years (1 year and 6 months)
Simple interest = I = ?
We know that
I = $\frac{P \times R \times T}{100}$
I = $\frac{60{,}000 \times 5.5 \times 1.5}{100}$
I = Rs.4,950
Amount = Principal + Total Simple Interest
= Rs. 60,000 + Rs. 4,950 = Rs. 64,950
Therefore, the amount after 1 year and 6 months is Rs. 64,950.

11. **Asad passed away leaving property worth Rs.800,000. Distribute the amount among his heirs: a widow, two sons and 3 daughters, if the widow gets 1/8 of the total amount and the son gets double the amount as compared to each daughter.**
**Solution:**
Total amount left = Rs.800,000
Widow's share = $\frac{1}{8}$ × 800,000
= Rs.100,000

Remaining amount = 800,000 – 100,000 = Rs. 700,000
Now ratios of share

| | | |
|---|---|---|
| son | : | daughter |
| 2 | : | 3 |
| 2 × 2 = 4 | : | 3 × 1 = 3 |

Sum of ratios = 4 + 3 = 7

Share of 2 sons = $\frac{4}{7} \times 700{,}000$ = Rs. 400,000

Share of each son = $\frac{400{,}000}{2}$ = Rs.200,000

Share of 3 daughters = $\frac{3}{7} \times 700{,}000$ = Rs.300,000

Share of each daughter = $\frac{300{,}000}{3}$ = Rs.100,000

**12. Convert the following currencies of PKR. (Use updated currency rates from the internet.)**

a) 200 USD  b) 150 CAD  c) 460 GBP

**Solution:**

a) 200 USD to PKR
1 United States Dollar = Rs 221.60
200 United States Dollars = Rs 221.60 × 200 = Rs 44,320

b) 150 CAD to PKR
1 CAD = 167.12 PKR
150 CAD = 150 × 167.12 PKR = Rs 25,068

c) 460 GBP to PKR
1 GBP = 260.67 PKR
460 GBP = 260.67 PKR × 460 = Rs 119,908.2

**13. Convert the following amounts given in PKR to the mentioned currencies. (Use updated currency rated from the internet.)**

c) 2000 PKR to USD  b) 1550 PKR to CAD  c) 2500 PKR to GBP

**Solution:**

a) 2000 PKR to USD
1 Pakistani Rupee = 0.0045 USD
2000 Pakistani Rupees = 2000 × 0.0045 USD = 9 USD

b) 1550 PKR to CAD
1 PKR = 0.00598 CAD
1550 Pakistani Rupees = 1550 × 0.00598 CAD = 2.27 CAD

c) 2500 PKR to GBP
1 Pakistani Rupee = 0.0038 GBP
2500 Pakistani Rupees = 2500 × 0.0038 GBP = 9.5 GBP

## Exercise 4.1

1. **Express the following sets in tabular form and descriptive form.**

a) {x/x∈ $W$ ∧ x< 15}  b) {x/x∈ $N$ ∧ x≤12}  c) {x/x∈ $P$ ∧ 1 < x < 32}
d) {x/x∈ $O$ ∧ 10 < x < 20}  e) {x/x∈ $E$ ∧ 4≤ $x$ ≤ 16}  f) {x/x∈ $Z$ ∧ -8 < x < 8}
g) {x/x∈ $P$ ∧13 < x < 17}  h) {x/x∈ $O$ ∧ 21 ≤ $x$ ≤25}  i) {x/x∈ $Z$ ∧ -50< x < -40}

**Solution:**

a) {x/x∈ $W$ ∧ x< 15}

Tabular form: {0, 1, 2, …, 14}
Descriptive form: The set of whole numbers less than 15

b) {x/x∈ $N$ ∧ x≤12}

Tabular form: {1, 2, 3, …, 12}
Descriptive form: The set of natural numbers upto 12

c) {x/x∈ $P$ ∧ 1 < x < 32}

Tabular form: {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31}

Descriptive form: The set of prime numbers between 1 and 32

d) {x/x∈ $O$ ∧ 10 < x < 20}

Tabular form: {11, 13, 15, 17, 19}
Descriptive form: The set of odd numbers between 10 and 20

e) {x/x∈ $E$ ∧ 4≤ $x$ ≤ 16}

Tabular form: {4, 6, 8, 10, 12, 14, 16}
Descriptive form: The set of even numbers from 4 upto 16

f) {x/x∈ $Z$ ∧ -8 < x < 8}

Tabular form: {-7, -6, -5, …, 0, 1, 2, …, 7}
Descriptive form: The set of integers between -8 and 8

g) {x/x∈ $P$ ∧13 < x < 17}

Tabular form: {}
Descriptive form: The set of prime numbers between 13 and 17

h) {x/x∈ $O$ ∧ 21 ≤ $x$ ≤25}

Tabular form: {21, 23, 25}
Descriptive form: The set of odd numbers from 21 upto 25

i) {x/x∈ $Z$ ∧ -50< x < -40}

Tabular form: {-49, -48, -47, -46, -45, -44, -43, -42, -41}
Descriptive form: The set of negative integers between – 50 and – 40.

2. **Write the following sets builder form.**

a) A = {1, 3, 5, 7, 9, 11, 13, 15}
b) B = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40}
c) C = The set of odd numbers less than 13.
d) D= The set of even numbers between 6 and 12.
e) E = {-12, -11, -10, …, 0, 1, 2, 3, 4, 5}
f) F = {-100, -99, -98, …, 0}
g) G = The set of natural numbers greater than 50 but less than 65.
h) H = The of odd numbers from 15 to 25.

**Solution:**

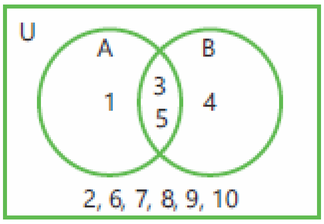
a) A = {1, 3, 5, 7, 9, 11, 13, 15}
   Set builder form: A = {x / x ∈ N ∧ x ≤ 15}
b) B = {4, 8, 12, 16, 20, 24, 28, 32, 36, 40}
   Set builder form: B = {x / x is the set of multiples of 4 upto 40}
c) C = The set of odd numbers less than 13.
   Set builder form: C = {x / x ∈ O ∧ x < 13}
d) D= The set of even numbers between 6 and 12.
   Set builder form: D = {x / x ∈ E ∧ 6 < x < 12}
e) E = {-12, -11, -10, …, 0, 1, 2, 3, 4, 5}
   Set builder form: E = {x / x ∈ Z ∧ - 12 ≤ x ≤ 5}
f) F = {-100, -99, -98, …, 0}
   Set builder form: F = {x / x ∈ Z ∧ - 100 ≤ x ≤ 0}
g) G = The set of natural numbers greater than 50 but less than 65.
   Set builder form: G = {x / x ∈ N ∧ 50 < x < 65}
h) H = The of odd numbers from 15 to 25.
   Set builder form: H = {x / x ∈ O ∧ 15 ≤ x ≤ 25}

**3. Represent the following pairs of sets using a Venn diagram.**

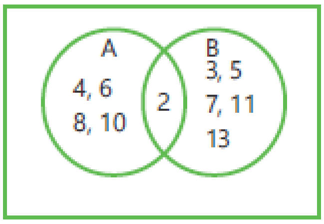
a) U = {1, 2, 3, ..., 10}
   A = {1, 3, 5}, B = {3, 4, 5}
b) A = {2, 4, 6, 8, 10}, B = {2, 3, 5, 7, 11, 13}
c) A = The set of odd numbers less than 10
   B= The set of even numbers less than 15.
d) A = The set of natural numbers upto 10
   B = The set of common factors of 12, 20, 32.

**Solution:**

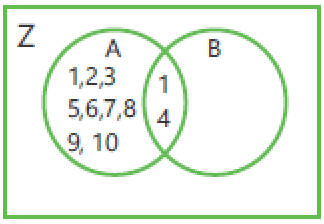
a) U = {1, 2, 3, ..., 10}
   A = {1, 3, 5}, B = {3, 4, 5}



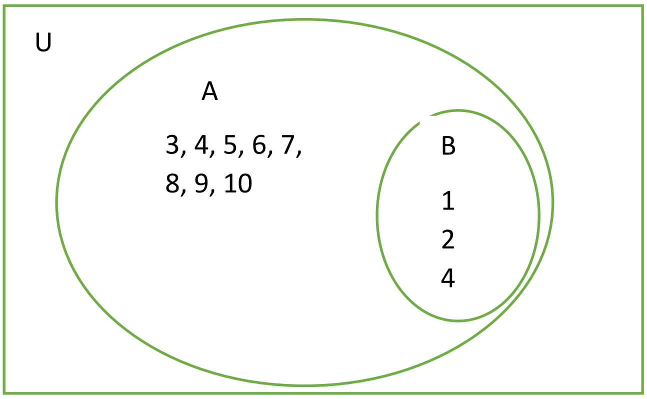

b) A = {2, 4, 6, 8, 10}, B = {2, 3, 5, 7, 11, 13}



c) A = The set of odd numbers less than 10
B= The set of even numbers less than 15.




d) A = The set of natural numbers upto 10
B = The set of common factors of 12, 20, 32.
A = The set of natural numbers up to 10
A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
B = The set of common factors of 12, 20, 32
B = {1, 2, 4}




## Exercise 4.2

1. **How many subsets can be made out of the following sets? Find all the subsets of each of the following sets.**

a) A= { 5 } b) B = {2, 4} c) C = {-1, 0, 1} d) D = {a, b, c, d}
e) E = {x/x$\in$ $P$ $\wedge$1 < x < 8} f) F = {x/x$\in$ $N$ $\wedge$1$\leq x \leq$ 4}
g) G = Set of even numbers between 20 and 22.
h) H = Set of integers between -3 and 1.

**Solution:**

a) A = {5}
Number of elements in set A = 1
Number of subsets in set A = $2^n = 2^1 = 2$
Subsets of set A = $\phi$, {5}

b) B = {2, 4}
Number of elements in set B = 2
Number of subsets in set B = $2^n = 2^2 = 4$
Subsets of set B = $\phi$, {2}, {4}, {2, 4}

c) C = {-1, 0, 1}
Number of elements in set C = 3
Number of subsets in set C = $2^n = 2^3 = 8$
Subsets of set C = ϕ, {-1}, {0}, {1}, {-1, 0}, {-1, 1}, {0, 1}, {-1, 0, 1}

d) D = {a, b, c, d}
Number of elements in set D = 4
Number of subsets in set D = $2^n = 2^4 = 16$
Subsets of set D = ϕ, {a}, {b}, {c}, {d}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {c, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}, {a, b, c, d}

e) E = {x/x∈ $P$ ∧1 < x < 8} = {2, 3, 5, 7}
Number of elements in set E = 4
Number of subsets in set E = $2^n = 2^4 = 16$
Subsets of set E = ϕ, {2}, {3}, {5}, {7}, {2, 3}, {2, 5}, {2, 7}, {3, 5}, {3, 7}, {5, 7}, {2, 3, 5}, {2, 3, 7}, {2, 5, 7}, {3, 5, 7}, {2, 3, 5, 7}

f) F = {x/x∈ $N$ ∧1≤ $x \leq 4$} = {1, 2, 3, 4}
Number of elements in set F = 4
Number of subsets in set F = $2^n = 2^4 = 16$
Subsets of set F = ϕ, {1}, {2}, {3}, {4}, {1, 2}, {1, 3}, {1, 4}, {2, 3}, {2, 4}, {3, 4}, {1, 2, 3}, {1, 2, 4}, {1, 3, 4}, {2, 3, 4}, {1, 2, 3, 4}

g) G = Set of even numbers between 20 and 22 = { }.
Number of elements in set G = 0
Number of subsets in set G = $2^0 = 1$
Subsets of set G = ϕ

h) H = Set of integers between -3 and 1 = {-2, -1, 0}.
Number of elements in set H = 3
Number of subsets in set H = $2^n = 2^3 = 8$
Subsets of set H = ϕ, {-2}, {-1}, {0}, {-2, -1}, {-2, 0}, {-1, 0}, {-2, -1, 0}

**2. How many elements will the power set of these sets have? Find the power sets of the following sets.**

a) A= {1, 3, 5} b) B = {4, 8} c) C = {-2} d) D = {w, x, y, z}
e) E = {x/x∈ $P$ ∧13 < x < 23} f) F = {x/x∈ $N$ ∧18≤ $x \leq 21$}
g) G = Set of odd numbers between 9 and 15.
h) H = Set of whole numbers less than 4.

**Solution:**

a) A = {1, 3, 5}
Number of elements in set A = 3
Number of power sets in set A = $2^n = 2^3 = 8$
P (A) = { ϕ, {1}, {3}, {5}, {1, 3}, {1, 5}, {3, 5}, {1, 3, 5}}

b) B = {4, 8}
Number of elements in set B = 2
Number of power sets in set B = $2^n = 2^2 = 4$
P (B) = {ϕ, {4}, {8}, {4, 8}}

c) C = {-2}
Number of elements in set C = 1
Number of power sets in set C = $2^n = 2^1 = 2$
P (C) = {ϕ, {-2 }}

d) D = {w, x, y, z}
Number of elements in set D = 4
Number of power sets in set D = $2^n = 2^4 = 16$
P (D) = {ϕ,{w}, {x}, {y}, {z}, {w, x}, {w, y}, {w, z}, {x, y}, {x, z}, {y, z}, {w, x, y}, {w, x, z}, {w, y, z}, {x, y, z}, {w, x, y, z}}

e) E = {x/x∈ $P$ ∧13 < x < 23} = {17, 19}
Number of elements in set E = 2
Number of power sets in set E = $2^n = 2^2 = 4$
P (E) = {ϕ, {17}, {19}, {17, 19}}

f) F = {x/x∈ $N$ ∧18$\leq x \leq$ 21} = {18, 19, 20, 21}
Number of elements in set F = 4
Number of power sets in set F = $2^n = 2^4 = 16$
P (F) = {ϕ, {18}, {19}, {20}, {21}, {18, 19}, {18, 20}, {18, 21}, {19, 20}, {19, 21}, {20, 21}, {18, 19, 20}, {18, 19, 21}, {18, 20, 21}, {19, 20, 21}, {18, 19, 20, 21}}

g) G = Set of odd numbers between 9 and 15 = {11, 13}.
Number of elements in set G = 2
Number of power sets in set G = $2^n = 2^2 = 4$
P (G) = {ϕ, {11}, {13}, {11, 13}}

h) H = Set of whole numbers less than 4 = {0, 1, 2, 3, 4}.
Number of elements in set H = 4
Number of power sets in set H = $2^n = 2^4 = 16$
P (H) = {ϕ, {0}, {1}, {2}, {3}, {0, 1}, {0, 2}, {0, 3}, {1, 2}, {1, 3}, {2, 3}, {0, 1, 2}, {0, 1, 3}, {o, 2, 3}, {1, 2, 3}, {0, 1, 2, 3}}

## Exercise 4.3

**1. If A = {3, 5, 7}, B = {2, 4, 7}, C = {1, 6, 8, 9}, then prove the following.**

a) AUB = BUA b) A∩B = B∩A c) (AUB)UC = AU(BUC)
d) (A∩B)∩C = A∩(B∩C) e) AU(B∩C) = (AUB)∩(AUC)
f) A∩(BUC) = (A∩B) U (A∩C)

**Solution:**

a) AUB = BUA
LHS = AUB
AUB = {3, 5, 7} U {2, 4, 7}
= {2, 3, 4, 5, 7}
RHS = BUA
BUA = {2, 4, 7} U {3, 5, 7}
= {2, 3, 4, 5, 7}
Since, LHS = RHS
Hence proved.

b) A∩B = B∩A
LHS = AUB
A∩B = {3, 5, 7} ∩ {2, 4, 7}
= {7}
RHS = BUA
B∩A = {2, 4, 7} ∩ {3, 5, 7}
= {7}
Since, LHS = RHS
Hence proved.

c) (AUB)UC = AU(BUC)
LHS = (AUB)UC
AUB = {3, 5, 7} U {2, 4, 7}
= {2, 3, 4, 5, 7}
(AUB)UC = {2, 3, 4, 5, 7} U {1, 6, 8, 9}
= {1, 2, 3, 4, 5, 6, 7, 8, 9}
RHS = AU(BUC)
BUC = {2, 4, 7} U {1, 6, 8, 9}
= {1, 2, 4, 6, 7, 8, 9}
AU(BUC) = {3, 5, 7} U {1, 2, 4, 6, 7, 8, 9}
= {1, 2, 3, 4, 5, 6, 7, 8, 9}
Since, LHS = RHS
Hence proved.

d) (A∩B)∩C = A∩(B∩C)
LHS = (A∩B)∩C
A∩B = {3, 5, 7} ∩ {2, 4, 7}
= {7}
(A∩B)∩C = {7} ∩ {1, 6, 8, 9}
= { }
RHS = A∩(B∩C)
B∩C = {2, 4, 7} ∩ {1, 6, 8, 9}
= { }
A∩(B∩C) = {3, 5, 7} ∩ { }
= { }
Since, LHS = RHS
Hence proved.

e) AU(B∩C) = (AUB)∩(AUC)
LHS = AU(B∩C)
B∩C = {2, 4, 7} ∩ {1, 6, 8, 9}
= { }
AU(B∩C) = {3, 5, 7} U { }
= {3, 5, 7}
RHS = (AUB)∩(AUC)
AUB = {3, 5, 7} U {2, 4, 7}
= {2, 3, 4, 5, 7}
AUC = {3, 5, 7} U {1, 6, 8, 9}
= {1, 3, 5, 6, 7, 8, 9}
(AUB)∩(AUC) = {2, 3, 4, 5, 7} ∩ {1, 3, 5, 6, 7, 8, 9}
= {3, 5, 7}

Since, LHS = RHS
Hence proved.

f) A∩(BUC) = (A∩B) U (A∩C)
LHS = A∩(BUC)
BUC = {2, 4, 7} U {1, 6, 8, 9}
= {1, 2, 4, 6, 7, 8, 9}
A∩(BUC) = {3, 5, 7} ∩{1, 2, 4, 6, 7, 8, 9}
= {7}
RHS = (A∩B) U (A∩C)
A∩B = {3, 5, 7} ∩ {2, 4, 7}
= {7}
A∩C = {3, 5, 7} ∩{1, 6, 8, 9}
= { }
(A∩B) U (A∩C) = {7} U { }
= {7}
Since, LHS = RHS
Hence proved.

2. **Name the following properties of sets. Then for the sets, A = {2, 5, 7, 9, 12}, B = {1, 4, 6, 9, 12}, and C = {1, 3, 6, 8, 10}, verify these properties.**

a) BUC = CUB  b) A∩B = B∩A  c) (AUB)UC = AU(BUC)
d) (A∩B)∩C = A∩(B∩C)  e) AU(B∩C) = (AUB)∩(AUC)
f) A∩(BUC) = (A∩B) U (A∩C)

**Solution:**

a) BUC = CUB
LHS = BUC
BUC = {1, 4, 6, 9, 12} U {1, 3, 6, 8, 10}
= {1, 3, 4, 6, 8, 9, 10, 12}
RHS = CUB
CUB = {1, 3, 6, 8, 10} U {1, 4, 6, 9, 12}
= {1, 3, 4, 6, 8, 9, 10, 12}
Since, LHS = RHS
Hence proved.

b) A∩B = B∩A
LHS = A∩B
A∩B = {2, 5, 7, 9, 12} ∩ {1, 4, 6, 9, 12}
= {9, 12}
RHS = B∩A
B∩A = {1, 4, 6, 9, 12} ∩ {2, 5, 7, 9, 12}
= {9, 12}
Since, LHS = RHS
Hence proved.

c) (AUB)UC = AU(BUC)
LHS = (AUB)UC
AUB = {2, 5, 7, 9, 12} U {1, 4, 6, 9, 12}
= {1, 2, 4, 5, 6, 7, 9, 12}

(AUB)UC = {1, 2, 4, 5, 6, 7, 9, 12} U {1, 3, 6, 8, 10}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12}
RHS = AU(BUC)
BUC = {1, 4, 6, 9, 12} U {1, 3, 6, 8, 10}
= {1, 3, 4, 6, 8, 9, 10, 12}
AU(BUC) = {2, 5, 7, 9, 12} U {1, 3, 4, 6, 8, 9, 10, 12}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12}
Since, LHS = RHS
Hence proved.

d) (A∩B)∩C = A∩(B∩C)
LHS = (A∩B)∩C
A∩B = {2, 5, 7, 9, 12} ∩ {1, 4, 6, 9, 12}
= {9, 12}
(A∩B)∩C = {9, 12} ∩ {1, 3, 6, 8, 10}
= { }
RHS = A∩(B∩C)
B∩C = {1, 4, 6, 9,12} ∩ {1, 3, 6, 8, 10}
= {1, 6}
A∩(B∩C) = {2, 5, 7, 9, 12} ∩ {1, 6}
= { }
Since, LHS = RHS
Hence proved.

e) AU(B∩C) = (AUB)∩(AUC)
LHS = AU(B∩C)
B∩C = {1, 4, 6, 9,12} ∩ {1, 3, 6, 8, 10}
= {1, 6}
AU(B∩C) = {2, 5, 7, 9, 12} U {1, 6}
= {1, 2, 5, 6, 7, 9, 12}
RHS = (AUB)∩(AUC)
AUB = {2, 5, 7, 9, 12} U {1, 4, 6, 9, 12}
= {1, 2, 4, 5, 6, 7, 9, 12}
AUC = {2, 5, 7, 9, 12} U {1, 3, 6, 8, 10}
= {1, 2, 3, 5, 6, 7, 8, 9, 10, 12}
(AUB)∩(AUC) = {1, 2, 4, 5, 6, 7, 9, 12} ∩ {1, 2, 3, 5, 6, 7, 8, 9, 10, 12}
= {1, 2, 5, 6, 7, 9, 12}
Since, LHS = RHS
Hence proved.

f) A∩(BUC) = (A∩B) U (A∩C)
LHS = A∩(BUC)
BUC = {1, 4, 6, 9, 12} U {1, 3, 6, 8, 10}
= {1, 3, 4, 6, 8, 9, 10, 12}
A∩(BUC) = {2, 5, 7, 9, 12} ∩ {1, 3, 4, 6, 8, 9, 10, 12}
= {9, 12}
RHS = (A∩B) U (A∩C)
A∩B = {2, 5, 7, 9, 12} ∩ {1, 4, 6, 9, 12}
= {9, 12}

$A \cap C = \{2, 5, 7, 9, 12\} \cap \{1, 3, 6, 8, 10\}$
$= \{\ \}$
$(A \cap B) \cup (A \cap C) = \{9, 12\} \cup \{\ \}$
$= \{9, 12\}$
Since, LHS = RHS
Hence proved.

**3. If A = {a, d, g, j, m, p}, B = {b, e, g, k, n}, and C = {b, e, g, h, j, m, n, q, s, t}, then show the following results.**

a) $B \cup C = C \cup B$ b) $A \cap C = C \cap A$ c) $B \cap C = C \cap B$
d) $(A \cup B) \cup C = A \cup (B \cup C)$ e) $(A \cap B) \cap C = A \cap (B \cap C)$

**Solution:**

a) $B \cup C = C \cup B$
LHS = $B \cup C$
$B \cup C = \{b, e, g, k, n\} \cup \{b, e, g, h, j, m, n, q, s, t\}$
$= \{b, e, g, h, j, k, m, n, q, s, t\}$
RHS = $C \cup B$
$C \cup B = \{b, e, g, h, j, m, n, q, s, t\} \cup \{b, e, g, k, n\}$
$= \{b, e, g, h, j, k, m, n, q, s, t\}$
Since, LHS = RHS
Hence proved.

b) $A \cap C = C \cap A$
LHS = $A \cap C$
$A \cap C = \{a, d, g, j, m, p\} \cap \{b, e, g, h, j, m, n, q, s, t\}$
$= \{g, j, m\}$
RHS = $C \cap A$
$C \cap A = \{b, e, g, h, j, m, n, q, s, t\} \cap \{a, d, g, j, m, p\}$
$= \{g, j, m\}$
Since, LHS = RHS
Hence proved.

c) $B \cap C = C \cap B$
LHS = $B \cap C$
$B \cap C = \{b, e, g, k, n\} \cap \{b, e, g, h, j, m, n, q, s, t\}$
$= \{b, e, g, n\}$
RHS = $C \cap B$
$C \cap B = \{b, e, g, h, j, m, n, q, s, t\} \cap \{b, e, g, k, n\}$
$= \{b, e, g, n\}$
Since, LHS = RHS
Hence proved.

d) $(A \cup B) \cup C = A \cup (B \cup C)$
LHS = $(A \cup B) \cup C$
$A \cup B = \{a, d, g, j, m, p\} \cup \{b, e, g, k, n\}$
$= \{a, b, d, e, g, j, k, m, n, p\}$
$(A \cup B) \cup C = \{a, b, d, e, g, j, k, m, n, p\} \cup \{b, e, g, h, j, m, n, q, s, t\}$
$= \{a, b, d, e, g, h, j, k, m, n, p, q, s, t\}$
RHS = $A \cup (B \cup C)$

BUC = {b, e, g, k, n} U {b, e, g, h, j, m, n, q, s, t}
= {b, e, g, h, j, k, m, n, q, s, t}
AU(BUC) = {a, d, g, j, m, p} U {b, e, g, h, j, k, m, n, q, s, t}
= {a, b, d, e, g, h, j, k, m, n, p, q, s, t}
Since, LHS = RHS
Hence proved.

e) (A∩B)∩C = A∩(B∩C)
LHS = (A∩B)∩C
A∩B = {a, d, g, j, m, p} ∩ {b, e, g, k, n}
= {g}
(A∩B)∩C = {g} ∩ {b, e, g, h, j, m, n, q, s, t}
= {g}
RHS = A∩(B∩C)
B∩C = {b, e, g, k, n} ∩ {b, e, g, h, j, m, n, q, s, t}
= {b, e, g, n}
A∩(B∩C) = {a, d, g, j, m, p} ∩ {b, e, g, n}
= {g}
Since, LHS = RHS
Hence proved.

**4. State and verify De-Morgan's laws for the following sets:**

a) U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {3, 4, 5, 7}, B = {1, 5, 8}
b) U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 3, 5, 7}, B = {2, 3, 5, 8}
c) U = {1, 2, 3, …, 15}, A = {2, 3, 5, 7, 11, 13}, B = {3, 6, 9, 12, 15}
d) U = {e, f, g, h, i, j, k, *l*, m, n}, A = {e, h, k, m}, B = {g, h, k, *l*, m}
e) U = The set of whole numbers upto 20,
A = The set of prime numbers less than 12
B = The set of even numbers between 1 and 16.
f) U = {a, b, c, d, e, f, g, h}, A = {a, b, c, d}, B = {a, d, e}

**Solution:**

If two sets are given, we can perform different operations on them, like union, intersection, and complement of sets. Now, to further simplify the sets, we have a law that is called **De Morgan's law**. It has two parts: the first part is called De Morgan's law of union, and the second part is called De-Morgan's law of intersection.

i) $(AUB)^c = A^c \cap B^c$
ii) $(A\cap B)^c = A^c \cup B^c$

a) U = {1, 2, 3, 4, 5, 6, 7, 8}, A = {3, 4, 5, 7}, B = {1, 5, 8}
i) $(AUB)^c = A^c \cap B^c$
LHS = $(AUB)^c$
AUB = {3, 4, 5, 7} U {1, 5, 8}
= {1, 3, 4, 5, 7, 8}
$(AUB)^c$ = U – (AUB)
= {1, 2, 3, 4, 5, 6, 7, 8} – {1, 3, 4, 5, 7, 8}
= {2, 6}
RHS = $A^c \cap B^c$
$A^c$ = U – A

$= \{1, 2, 3, 4, 5, 6, 7, 8\} – \{3, 4, 5, 7\}$
$= \{1, 2, 6, 8\}$
$B^c = U – B$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} – \{1, 5, 8\}$
$= \{2, 3, 4, 6, 7\}$

$A^c \cap B^c = \{1, 2, 6, 8\} \cap \{2, 3, 4, 6, 7\}$
$= \{2, 6\}$
Since, LHS = RHS
Hence proved.
ii) $(A\cap B)^c = A^c \cup B^c$
LHS = $(A\cap B)^c$
$A\cap B = \{3, 4, 5, 7\} \cap \{1, 5, 8\}$
$= \{5\}$
$(A\cap B)^c = U – (A\cap B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} – \{5\}$
$= \{1, 2, 3, 4, 6, 7, 8\}$
RHS = $A^c \cup B^c$

$A^c = U – A$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} – \{3, 4, 5, 7\}$
$= \{1, 2, 6, 8\}$
$B^c = U – B$
$= \{1, 2, 3, 4, 5, 6, 7, 8\} – \{1, 5, 8\}$
$= \{2, 3, 4, 6, 7\}$

$A^c \cap B^c = \{1, 2, 6, 8\}\ U\ \{2, 3, 4, 6, 7\}$
$= \{1, 2, 3, 4, 6, 7, 8\}$
Since, LHS = RHS
Hence proved.

b) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7\}$, $B = \{2, 3, 5, 8\}$
i) $(AUB)^c = A^c \cap B^c$
LHS = $(AUB)^c$
$AUB = \{1, 3, 5, 7\}\ U\ \{2, 3, 5, 8\}$
$= \{1, 2, 3, 5, 7, 8\}$
$(AUB)^c = U – (AUB)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} – \{1, 2, 3, 5, 7, 8\}$
$= \{4, 6, 9, 10\}$
RHS = $A^c \cap B^c$
$A^c = U – A$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} – \{1, 3, 5, 7\}$
$= \{2, 4, 6, 8, 9, 10\}$
$B^c = U – B$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} – \{2, 3, 5, 8\}$
$= \{1, 4, 6, 7, 9, 10\}$

$A^c \cap B^c = \{2, 4, 6, 8, 9, 10\} \cap \{1, 4, 6, 7, 9, 10\}$
$= \{4, 6, 9, 10\}$
Since, LHS = RHS
Hence proved.

ii) $(A \cap B)^c = A^c \cup B^c$
LHS = $(A \cap B)^c$
$A \cap B = \{1, 3, 5, 7\} \cap \{2, 3, 5, 8\}$
$= \{3, 5\}$
$(A \cap B)^c = U - (A \cap B)$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{3, 5\}$
$= \{1, 2, 4, 6, 7, 8, 9, 10\}$
RHS = $A^c \cup B^c$

$A^c = U - A$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{1, 3, 5, 7\}$
$= \{2, 4, 6, 8, 9, 10\}$
$B^c = U - B$
$= \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} - \{2, 3, 5, 8\}$

$= \{1, 4, 6, 7, 9, 10\}$
$A^c U B^c = \{2, 4, 6, 8, 9, 10\} \cup \{1, 4, 6, 7, 9, 10\}$
$= \{1, 2, 4, 6, 7, 8, 9, 10\}$
Since, LHS = RHS
Hence proved.

c) $U = \{1, 2, 3, \ldots, 15\}$, $A = \{2, 3, 5, 7, 11, 13\}$, $B = \{3, 6, 9, 12, 15\}$

i) $(AUB)^c = A^c \cap B^c$
LHS = $(AUB)^c$
$AUB = \{2, 3, 5, 7, 11, 13\} \cup \{3, 6, 9, 12, 15\}$
$= \{2, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$
$(AUB)^c = U - (AUB)$
$= \{1, 2, 3, \ldots, 15\} - \{2, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$
$= \{1, 4, 8, 10, 14\}$
RHS = $A^c \cap B^c$
$A^c = U - A$
$= \{1, 2, 3, \ldots, 15\} - \{2, 3, 5, 7, 11, 13\}$
$= \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$
$B^c = U - B$
$= \{1, 2, 3, \ldots, 15\} - \{3, 6, 9, 12, 15\}$

$= \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$

$A^c \cap B^c = \{1, 4, 6, 8, 9, 10, 12, 14, 15\} \cap \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$
$= \{1, 4, 8, 10, 14\}$
Since, LHS = RHS
Hence proved.

ii) $(A \cap B)^c = A^c \cup B^c$

LHS = $(A \cap B)^c$

$A \cap B = \{2, 3, 5, 7, 11, 13\} \cap \{3, 6, 9, 12, 15\}$
$= \{3\}$
$(A \cap B)^c = U - (A \cap B)$

$= \{1, 2, 3, \ldots, 15\} - \{3\}$

$= \{1, 2, 4, 5, 6, \ldots, 15\}$

RHS = $A^c \cup B^c$

$A^c = U - A$
$= \{1, 2, 3, \ldots, 15\} - \{2, 3, 5, 7, 11, 13\}$
$= \{1, 4, 6, 8, 9, 10, 12, 14, 15\}$
$B^c = U - B$
$= \{1, 2, 3, \ldots, 15\} - \{3, 6, 9, 12, 15\}$

$= \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$

$A^c U B^c = \{1, 4, 6, 8, 9, 10, 12, 14, 15\}\ U\ \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14\}$
$= \{1, 2, 4, 5, 6, \ldots, 15\}$
Since, LHS = RHS
Hence proved.

d) $U = \{e, f, g, h, i, j, k, l, m, n\}$, $A = \{e, h, k, m\}$, $B = \{g, h, k, l, m\}$

i) $(AUB)^c = A^c \cap B^c$
LHS = $(AUB)^c$
$AUB = \{e, h, k, m\}\ U\ \{g, h, k, l, m\}$
$= \{e, g, h, k, l, m\}$
$(AUB)^c = U - (AUB)$
$= \{e, f, g, h, i, j, k, l, m, n\} - \{e, g, h, k, l, m\}$
$= \{f, i, j, n\}$
RHS = $A^c \cap B^c$
$A^c = U - A$
$= \{e, f, g, h, i, j, k, l, m, n\} - \{e, h, k, m\}$
$= \{f, g, i, j, l, n\}$
$B^c = U - B$
$= \{e, f, g, h, i, j, k, l, m, n\} - \{g, h, k, l, m\}$

$= \{e, f, i, j, n\}$

$A^c \cap B^c = \{f, g, i, j, l, n\} \cap \{e, f, i, j, n\}$
$= \{f, i, j, n\}$
Since, LHS = RHS
Hence proved.

ii) $(A \cap B)^c = A^c \cup B^c$

LHS = $(A \cap B)^c$

$A \cap B = \{e, h, k, m\} \cap \{g, h, k, l, m\}$
$= \{h, k, m\}$
$(A \cap B)^c = U - (A \cap B)$
$= \{e, f, g, h, i, j, k, l, m, n\} - \{h, k, m\}$

$= \{e, f, g, i, j, l, n\}$

RHS = $A^c \cup B^c$

$A^c = U - A$
$= \{e, f, g, h, i, j, k, l, m, n\} - \{e, h, k, m\}$

= {f, g, i, j, l, n}
$B^c$ = U – B
= {e, f, g, h, i, j, k, *l*, m, n} – {g, h, k, *l*, m}

= {e, f, i, j, n}

$A^c$U$B^c$ = {f, g, i, j, l, n} U {e, f, i, j, n}
= {e, f, g, i, j, l, n}
Since, LHS = RHS
Hence proved.

e) U = The set of whole numbers upto 20
A = The set of prime numbers less than 12
B = The set of even numbers between 1 and 16.
In tabular form:
U = {0, 1, 2, 3, …, 20}, A = {2, 3, 5, 7, 11}, B = {2, 4, 6, 8, 10, 12, 14}

i) $(AUB)^c = A^c \cap B^c$
LHS = $(AUB)^c$
AUB = {2, 3, 5, 7, 11} U {2, 4, 6, 8, 10, 12, 14}
= {2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14}
$(AUB)^c$ = U – (AUB)
= {0, 1, 2, 3, …, 20} – {2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14}
= {0, 1, 9, 13, 15, 16, 17, 18, 19, 20}
RHS = $A^c \cap B^c$
$A^c$ = U – A
= {0, 1, 2, 3, …, 20} – {2, 3, 5, 7, 11}
= {0, 1, 4, 6, 8, 9, 10, 12, 13, …, 20}
$B^c$ = U – B
= {0, 1, 2, 3, …, 20} – {2, 4, 6, 8, 10, 12, 14}

= {0, 1, 3, 5, 7, 9, 11, 13, 15, 16, 17, 18, 19, 20}

$A^c \cap B^c$ = {0, 1, 4, 6, 8, 9, 10, 12, 13, …, 20} ∩ {0, 1, 3, 5, 7, 9, 11, 13, 15, 16, 17, 18, 19, 20}
= {0, 1, 9, 13, 15, 16, 17, 18, 19, 20}
Since, LHS = RHS
Hence proved.

ii) $(A \cap B)^c = A^c \cup B^c$

LHS = $(A \cap B)^c$

A∩B = {2, 3, 5, 7, 11} ∩ {2, 4, 6, 8, 10, 12, 14}
= {2}
$(A \cap B)^c$ = U – (A∩B)

= {0, 1, 2, 3, …, 20} – {2}

= {0, 1, 3, 4, 5, …, 20}

RHS = $A^c \cup B^c$

$A^c$ = U – A

$= \{0, 1, 2, 3, \ldots, 20\} - \{2, 3, 5, 7, 11\}$
$= \{0, 1, 4, 6, 8, 9, 10, 12, 13, \ldots, 20\}$
$B^c = U - B$
$= \{0, 1, 2, 3, \ldots, 20\} - \{2, 4, 6, 8, 10, 12, 14\}$

$= \{0, 1, 3, 5, 7, 9, 11, 13, 15, 16, 17, 18, 19, 20\}$

$A^c \cup B^c = \{0, 1, 4, 6, 8, 9, 10, 12, 13, \ldots, 20\} \cup \{0, 1, 3, 5, 7, 9, 11, 13, 15, 16, 17, 18, 19, 20\}$
$= \{0, 1, 3, 4, 5, \ldots, 20\}$
Since, LHS = RHS
Hence proved

f) $U = \{a, b, c, d, e, f, g, h\}$, $A = \{a, b, c, d\}$, $B = \{a, d, e\}$

i) $(A \cup B)^c = A^c \cap B^c$
LHS = $(A \cup B)^c$
$A \cup B = \{a, b, c, d\} \cup \{a, d, e\}$
$= \{a, b, c, d, e\}$
$(A \cup B)^c = U - (A \cup B)$
$= \{a, b, c, d, e, f, g, h\} - \{a, b, c, d, e\}$
$= \{f, g, h\}$
RHS = $A^c \cap B^c$
$A^c = U - A$
$= \{a, b, c, d, e, f, g, h\} - \{a, b, c, d\}$
$= \{e, f, g, h\}$
$B^c = U - B$
$= \{a, b, c, d, e, f, g, h\} - \{a, d, e\}$

$= \{b, c, f, g, h\}$

$A^c \cap B^c = \{e, f, g, h\} \cap \{b, c, f, g, h\}$
$= \{f, g, h\}$
Since, LHS = RHS
Hence proved.

ii) $(A \cap B)^c = A^c \cup B^c$

LHS = $(A \cap B)^c$

$A \cap B = \{a, b, c, d\} \cap \{a, d, e\}$
$= \{a, d\}$
$(A \cap B)^c = U - (A \cap B)$
$= \{a, b, c, d, e, f, g, h\} - \{a, d\}$

$= \{b, c, e, f, g, h\}$

RHS = $A^c \cup B^c$

$A^c = U - A$
$= \{a, b, c, d, e, f, g, h\} - \{a, b, c, d\}$
$= \{e, f, g, h\}$
$B^c = U - B$
$= \{a, b, c, d, e, f, g, h\} - \{a, d, e\}$

= {b, c, f, g, h}

$A^c U B^c$ = {e, f, g, h} U {b, c, f, g, h}

= {b, c, e, f, g, h}

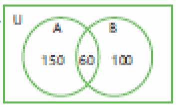
Since, LHS = RHS

Hence proved

## Exercise 4.4

1. **In a school, all the students play cricket or badminton o both. 150 students play cricket, 100 students play badminton and 60 students play both games. Show his information using the Venn diagram.**

   **Solution:**

   Let's denote each set first.

   A = people who play cricket

   B = People who play badminton

   Also, A∩B = 60

   Note that the size of set AUB can be found by adding the sizes of sets A and B, and then subtracting the size of set A∩B.

   n(AUB) = n(A) + n(B) – n(A∩B)

   = 150 + 100 – 60

   = 190

   So, there are 190 students in total who play at least one of the games.

2. **There are 200 students who appear in the admission test, of whom 96 students apply for section-A and 64 students apply for section-B. If 50 students applied to neither of the two groups, how many students applied to both groups.**

   **Solution:**

   Number of students in section A = 96

   Number of students in section B = 64

   Number of students in neither section = 50

   Total number of students = 200

   Number of students in both sections = x = ?

   We know that

   Number of students in section A + Number of students in section B + Number of students in both sections + Number of students in neither section = Total number of students

   (96 – x) + (64 – x) + x + 50 = 200

   210 – x = 200

   210 – 200 = x

   x = 10

   Therefore, 10 students applied to both section A and B.

3. **The following Venn diagram shows the data of kids and they choose pie, cake and cookies. Look at it and answer the questions.**
   a) How many kids choose only cake?
   b) How many kids choose only pie?
   c) How many kids choose only cookies?
   d) How many choose both cakes but not cookies?
   e) How many kids choose all the three dessert?
   f) How many choose at least two desserts?

   **Solution:**

   a) There are 12 kids who choose only cake.
   b) There are 10 kids who choose only pie.
   c) There are 14 kids who choose only cookies.
   d) There are 3 kids who choose both cakes but not cookies.
   e) There are 4 kids who choose all the three dessert.
   f) There are 22 kids who choose at least two desserts.

4. **Create a Venn diagram to illustrate the given information and find the values of the variables.**

| Total people | jogging | Cycling | Both jogging and cycling | Neither jogging nor cycling |
|---|---|---|---|---|
| 82 | 22 | 18 | $x$ | $3x$ |

**Solution:**

Let $x$ be the number of people who like both jogging and cycling.

U= Total people

A= people who like jogging

B= people who like cycling

So, $A \cap B = x$.

Also, people who like neither jogging nor cycling $= 3x$.

As a universal set has all the elements of the sets under consideration, we can write by looking at the Venn diagram that:

Number of elements in U = Number of people only in A + Number of people only in B + Number of people in $A \cap B$ + Number of people in neither A nor in B.

Since the number of people in U= 82

$82 = (22 - x) + (18 - x) + x + 3x$

$82 = 40 - 2x + 4x$

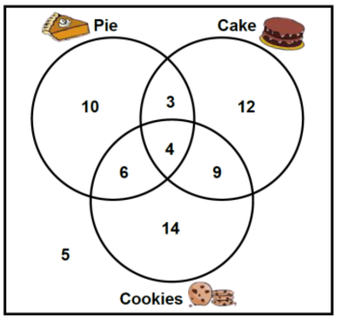
$82 - 40 = 2x$

$42 = 2x$

Dividing 2 on both sides

$\frac{42}{2} = \frac{2x}{2}$

$x = 21$

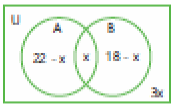


So, 21 people like both jogging and cycling.

Putting $x$=21 in 3$x$

3$x$=3(21) =63

Sixty-three people like neither jogging nor cycling.




## Unit Evaluation 4

**1. Encircle the correct answer.**

a) The number of subsets in a set {0, 1, 4} is:

i) 2    ii) 4    **iii) 8**    iv) 16

b) The set $\{x/x \in Z \wedge -1000 < x < 0\}$ is:

**i) finite**    ii) infinite    iii) subset    iv) empty set

c) $A \cup (B \cap C)$ =

i) $A \cap (B \cap C)$    ii) $(A \cup B) \cap C$    iii) $(A \cap B) \cup C$    **iv) $(A \cup B) \cap (A \cup C)$**

d) If X = {w, x, y, z} and U = {a, b, c, d, ⋯, z}, then the complement of set X is:

i) {a, b, c, d, ⋯, w}    ii) {a, b, c, d, ⋯, v}

**iii) {b, c, d, e, f, ⋯, v}**    iv) {a, b, c, d, ⋯, u}

e) The associative property of sets under intersection is:

i) $A \cup (B \cup C) = (A \cup B) \cup C$    ii) $A \cap (B \cup C) = (A \cup B) \cap C$

iii) $A \cup (B \cap C) = (A \cap B) \cup C$    **iv) $A \cap (B \cap C) = (A \cap B) \cap C$**

f) $(A \cup B)^c$ =

i) $A \cup B^c$    ii) $A^c \cup B$    iii) $(A \cap B)^c$    **iv) $A^c \cap B^c$**

**2. Fill in the blanks.**

a) In the **descriptive** form, the set is represented a statement using well-defined words.

b) For any set B, the **power** set of B is written as P (B).

c) The general formula for the number of elements in a power set having n elements is **$2^n$**.

d) The power set of the set A = {+, -, ×, ÷} has **16** elements.

e) The number of all subsets of the set {a, b, c, d} is **16**.

**3. Write the given sets in set builder notation.**

a) A = {5, 10, 15, 20, …, 50}

b) b) B = {- 160, -159, -158, …, 0, 1, 2, …, 160}

c) C = The set of even numbers between 31 and 45

d) D = The set of prime numbers from 51 upto 70.

**Solution:**

a) A = {5, 10, 15, 20, …, 50}
Set builder form: A = {x / x is the set of multiples of 5 upto 50}

b) B = {- 160, -159, -158, …, 0, 1, 2, …, 160}
Set builder form: B = {x / x ϵ Z ∧ -160 ≤ x ≤ 160}

c) C = The set of even numbers between 31 and 45
Set builder form: C = {x / x ϵ E ∧ 31 < x < 45}

d) D = The set of prime numbers from 51 upto 70.
Set builder form: D = {x / x ϵ P ∧ 51 ≤ x ≤ 70}

**4. Write the given sets in descriptive form and tabular form.**

a) A = {x/x∈ $N \wedge x \leq 8$}
b) B = {x/x∈ $P \wedge 90 \leq x \leq 105$}
c) C = {x/x∈ $Z \wedge -6 \leq x \leq 3$}
d) D = {x/x∈ $O \wedge -1 < x < 1$}

**Solution:**

a) A = {x/x∈ $N \wedge x \leq 8$}
Descriptive form: A = The set of natural numbers upto 8
Tabular form: A = {1, 2, 3, 4, 5, 6,7, 8}

b) B = {x/x∈ $P \wedge 90 \leq x \leq 105$}
Descriptive form: B = The set of prime numbers from 90 upto 105
Tabular form: B = {91, 93, 97, 101, 103}

c) C = {x/x∈ $Z \wedge -6 \leq x \leq 3$}
Descriptive form: C = The set of integers from -6 upto 3
Tabular form: C = {-6, -5, -4, -3, -2, -1, 0, 1, 2, 3}

d) D = {x/x∈ $O \wedge -1 < x < 1$}
Descriptive form: D = The set of odd numbers between -1 and 1
Tabular form: D = { }

**5. Write the power sets of the given sets.**

a) {1, 2}
b) {∅}
c) {+, -, ×}
d) {e, f, g, h}

**Solution:**

a) {1, 2}
Power sets are: {ϕ, {1}, {2}, {1, 2}}

b) {∅}
Power sets are: {ϕ, {ϕ}}

c) {+, -, ×}
Power sets are: {ϕ, {+}, {-}, {×}, {+, -}, {+, ×}, {-, ×}, {+, -, ×}}

d) {e, f, g, h}

Power sets are: {ϕ, {e}, {f}, {g}, {h}, {e, f}, {e, g}, {e, h}, {f, g}, {f, h}, {g, h}, {e, f, g}, {e, f, h}, {e, g, h}, {f, g, h}, {e, f, g, h}}

6. **State De-Morgan's Laws. If A = {2, 4, 6, 8}, B = {1, 3, 5, 7, 9}, and U = {1, 2, 3, …, 10}, then verify De-Morgan's law and show it using a Venn diagram.**
**Solution:**

i) $(A \cup B)^c = A^c \cap B^c$
LHS = $(A \cup B)^c$
$A \cup B$ = {2, 4, 6, 8} U {1, 3, 5, 7, 9}
= {1, 2, 3, 4, 5, 6, 7, 8, 9}
$(A \cup B)^c$ = U – (AUB)
= {1, 2, 3, …, 10} – {1, 2, 3, 4, 5, 6, 7, 8, 9}
= {10}
RHS = $A^c \cap B^c$
$A^c$ = U – A
= {1, 2, 3, …, 10} – {2, 4, 6, 8}
= {1, 3, 5, 7, 9, 10}
$B^c$ = U – B
= {1, 2, 3, …, 10} – {1, 3, 5, 7, 9}

= {2, 4, 6, 8, 10}

$A^c \cap B^c$ = {1, 3, 5, 7, 9, 10} ∩ {2, 4, 6, 8, 10}
= {10}
Since, LHS = RHS
Hence proved.

ii) $(A \cap B)^c = A^c \cup B^c$

LHS = $(A \cap B)^c$

$A \cap B$ = {2, 4, 6, 8} ∩ {1, 3, 5, 7, 9}
= { }
$(A \cap B)^c$ = U – (A∩B)
= {1, 2, 3, …, 10} – { }

= {1, 2, 3, …, 10}

RHS = $A^c \cup B^c$

$A^c$ = U – A
= {1, 2, 3, …, 10} – {2, 4, 6, 8}
= {1, 3, 5, 7, 9, 10}
$B^c$ = U – B
= {1, 2, 3, …, 10} – {1, 3, 5, 7, 9}

= {2, 4, 6, 8, 10}

$A^c \cup B^c$ = {1, 3, 5, 7, 9, 10} U {2, 4, 6, 8, 10}
= {1, 2, 3, …, 10}
Since, LHS = RHS
Hence proved

7. **Verify the associative property of sets with respect to union and intersection if sets are:**
   A = {3, 6, 9, 12, 15}, B = {5, 10, 15, 20, 25, 30}, and C = {6, 12, 18, 24, 30}
   **Solution:**

   **Associative property of sets with respect to union**

   (AUB)UC = AU(BUC)
   LHS = (AUB)UC
   AUB = {3, 6, 9, 12, 15} U {5, 10, 15, 20, 25, 30}
   = {3, 5, 6, 9, 10, 12, 15, 20, 25, 30}
   (AUB)UC = {3, 5, 6, 9, 10, 12, 15, 20, 25, 30} U {6, 12, 18, 24, 30}
   = {3, 5, 6, 9, 10, 12, 15, 18, 20, 24, 25, 30}
   RHS = AU(BUC)
   BUC = {5, 10, 15, 20, 25, 30} U {6, 12, 18, 24, 30}
   = {5, 6, 10, 12, 15, 18, 20, 24, 25, 30}
   AU(BUC) = {3, 6, 9, 12, 15} U {5, 6, 10, 12, 15, 18, 20, 24, 25, 30}
   = {3, 5, 6, 9, 10, 12, 15, 18, 20, 24, 25, 30}
   Since, LHS = RHS
   Hence proved.

   **Associative property of sets with respect to intersection**

   (A∩B)∩C = A∩(B∩C)
   LHS = (A∩B)∩C
   A∩B = {3, 6, 9, 12, 15} ∩ {5, 10, 15, 20, 25, 30}
   = {15}
   (A∩B) ∩C = {15} ∩ {6, 12, 18, 24, 30}
   = { }
   RHS = A∩(B∩C)
   B∩C = {5, 10, 15, 20, 25, 30} ∩ {6, 12, 18, 24, 30}
   = {30}
   A∩ (B∩C) = {3, 6, 9, 12, 15} ∩ {30}
   = { }
   Since, LHS = RHS

8. **In a class 50 students, 20 students like mathematics and 18 students like English. Eight students like both, but four students do not like either. Show this information using a Venn diagram.**
   **Solution:**

   Let $x$ be the number of students who like both mathematics and English.
   U= Total students
   A= Students who like mathematics
   B= Students who like English
   So, $A \cap B = x$.

   Also, students who like neither jogging nor cycling = $4x$.

   As a universal set has all the elements of the sets under consideration, we can write by looking at the Venn diagram that:

Number of elements in U = Number of students only in A + Number of students only in B + Number of students in A∩B + Number of students in neither A nor in B.
Since the number of students in U= 50

$50 = (20 - x) + (18 - x) + x + 4x$

50 = 38 – 2x + 5x
50 – 38 = 3x
12 = 3x
Dividing 2 on both sides

$\frac{12}{3} = \frac{3x}{3}$

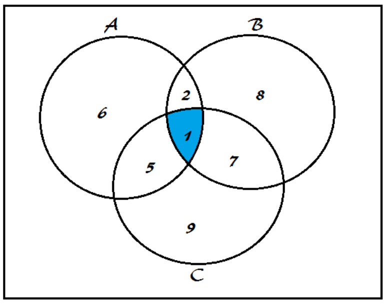
x = 4
So, 4 students like both mathematics and English.

Putting $x$=4 in $4x$

$4x = 4(4) = 16$

Sixteen students like neither mathematics nor English.

9. **The following Venn diagram shows the number of elements in three sets. Look at the diagram and answer the following questions.**
a) How many elements in set A?
b) How many elements in set B?
c) How many elements in set C?
d) How many elements in sets A and B?
e) How many elements in sets B and C?
f) How many elements in sets A and C?
g) How many common elements in sets in sets A, B and C?




**Solution:**
a) There are 6 elements in set A.
b) There are 8 elements in set B.
c) There are 9 elements in set C.
d) There are 6 common elements in set A and B.
e) There are 8 common elements in set B and C.
f) There are 3 common elements in set A and C.
g) There is 1 element common in set A, B, and C.

## Learning Check 5.1

1. **Identify whether a given sequence is arithmetic or geometric.**

a) 3, 7, 11, 15, … b) 729, 243, 81, … c) 240, 246, 252, 258, …
d) 6, 36, 216, … e) 512, 494, 476, 458, … f) 117649, 16807, 2401, …

**Solution:**

a) 3, 7, 11, 15, …
In this case, we check the common difference:
7 – 3 = 4, 11 – 7 = 4, 15 – 11 = 4
Therefore, this sequence is arithmetic with a common difference of 4.

b) 729, 243, 81, …
In this case, we check the common ratio:
243/729 = 1/3, 81/243 = 1/3
Therefore, this sequence is geometric with a common ratio of 1/3.

c) 240, 246, 252, 258, …
In this case, we check the common difference:
246 – 240 = 6, 252 – 246 = 6, 258 – 252 = 6
Therefore, this sequence is arithmetic with a common difference of 6.

d) 6, 36, 216, …
In this case, we check the common ratio:
36/6 = 6, 216/36 = 6
Therefore, this sequence is geometric with a common ratio of 6.

e) 512, 494, 476, 458, …
In this case, we check the common difference:
494 – 512 = – 18, 476 – 494 = – 18 458 – 476 = – 18
Therefore, this sequence is arithmetic with a common difference of -18.

f) 117649, 16807, 2401, …
In this case, we check the common difference:
16807/117649 = 1/7, 2401/16807 = 1/7
Therefore, this sequence is geometric with a common ratio of 1/7.

2. **Find the next three terms of each sequence by using the term-to-term rule.**

a) 32, 35, 38, 41, … b) 11, 17, 23, 29, … c) -13, -8, -3, 2, …
d) –9, –5, –1, 3, … e) –42, –22, –2, 18, … f) 64, 128, 192, 256, …

**Solution:**

a) 32, 35, 38, 41, …
Observe the sequence. The term-to-term rule helps us find the next terms in the sequence:
32, 35, 38, 41, …
First, find the common difference between the consecutive terms.
35 − 32 = 3, 38 − 35 = 3, 41 − 38 = 3
The difference between the two terms is 3, so the next terms of the sequence can be obtained by adding 3 to each previous term.
The next three terms of the sequence are:
41 + 3 = 44, 44 + 3 = 47, 47 +3 = 50
The sequence is:
32, 35, 38, 41, 44, 47, 50.

b) 11, 17, 23, 29, …
Observe the sequence. The term-to-term rule helps us find the next terms in the sequence:

11, 17, 23, 29, …
First, find the common difference between the consecutive terms.
$17 - 11 = 6$, $23 - 17 = 6$, $29 - 23 = 6$
The difference between the two terms is 6, so the next terms of the sequence can be obtained by adding 6 to each previous term.
The next three terms of the sequence are:
$29 + 6 = 35$, $35 + 6 = 41$, $41 + 6 = 47$
The sequence is:
11, 17, 23, 29, 35, 41, 47
c) –13, –8, –3, 2, …
Observe the sequence. The term-to-term rule helps us find the next terms in the sequence:
–13, –8, –3, 2, …
First, find the common difference between the consecutive terms.
$-8 - (-13) = -8 + 13 = 5$, $-3 - (-8) = -3 + 8 = 5$, $2 - (-3) = 2 + 3 = 5$
The difference between the two terms is 5, so the next terms of the sequence can be obtained by adding 5 to each previous term.
The next three terms of the sequence are:
$2 + 5 = 7$, $7 + 5 = 12$, $12 + 5 = 17$
The sequence is:
–13, –8, –3, 2, 7, 12, 17
d) –9, –5, –1, 3, …
Observe the sequence. The term-to-term rule helps us find the next terms in the sequence:
–9, –5, –1, 3, …
First, find the common difference between the consecutive terms.
$-5 - (-9) = -5 + 9 = 4$, $-1 - (-5) = -1 + 5 = 4$, $3 - (-1) = 3 + 1 = 4$
The difference between the two terms is 4, so the next terms of the sequence can be obtained by adding 4 to each previous term.
The next three terms of the sequence are:
$3 + 4 = 7$, $7 + 4 = 11$, $11 + 4 = 15$
The sequence is:
–9, –5, –1, 3, 7, 11, 15.
e) –42, –22, –2, 18, …
Observe the sequence. The term-to-term rule helps us find the next terms in the sequence:
–42, –22, –2, 18, …
First, find the common difference between the consecutive terms.
$-22 - (-42) = -22 + 42 = 20$, $-2 - (-22) = -2 + 22 = 20$, $18 - (-2) = 18 + 2 = 20$
The difference between the two terms is 20, so the next terms of the sequence can be obtained by adding 20 to each previous term.
The next three terms of the sequence are:
$18 + 20 = 38$, $38 + 20 = 58$, $58 + 20 = 78$
The sequence is:
–42, –22, –2, 18, 38, 58, 78.
f) 64, 128, 192, 256, …
Observe the sequence. The term-to-term rule helps us find the next terms in the sequence:
64, 128, 192, 256, …
First, find the common difference between the consecutive terms.

$128 - 64 = 64$, $192 - 128 = 64$, $256 - 192 = 64$

The difference between the two terms is 64, so the next terms of the sequence can be obtained by adding 64 to each previous term.

The next three terms of the sequence are:

$256 + 64 = 320$, $320 + 64 = 384$, $384 + 64 = 448$

The sequence is:

64, 128, 192, 256, 320, 384, 448.

**3. Find the 5th, 8th and 15th term of each sequence by using the position-to-term rule.**

a) 1, 2, 3, 4, … b) –5, –10, –15, –20, … c) 8, 16, 24, 32, …

d) 17, 18, 19, 20, … e) 10, 20, 30, 40, … f) 21, 22, 23, 24, ...

**Solution:**

a) 1, 2, 3, 4, …

In the sequence,

1, 2, 3, 4, …

$a_1 = 1$

$d = 1$ $[a_2 - a_1 = 2 - 1 = 1]$

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= 1 + (n - 1)(1)$

$= 1 + n - 1$

$= n$

$a_n = n$

Hence,

$a_5 = 5$

$a_8 = 8$

$a_{15} = 15$

b) –5, –10, –15, –20, …

In the sequence,

–5, –10, –15, –20, …

$a_1 = -5$

$d = -5$ $[a_2 - a_1 = -10 - (-5) = -10 + 5 = -5]$

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= -5 + (n - 1)(-5)$

$= -5 - 5n + 5$

$a_n = -5n$

Hence,

$a_5 = -5\,(5) = -25$

$a_8 = -5\,(8) = -40$

$a_{15} = -5\,(15) = -75$

c) 8, 16, 24, 32, …

In the sequence,

8, 16, 24, 32, …

$a_1 = 8$

$d = 8$ $[a_2 - a_1 = 16 - 8 = 8]$

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= 8 + (n - 1)(8)$

$= 8 + 8n - 8$

$a_n = 8n$
Hence,
$a_5 = 8(5) = 40$
$a_8 = 8(8) = 64$
$a_{15} = 8\ (15) = 120$

d) 17, 18, 19, 20, …

In the sequence,
17, 18, 19, 20, …
$a_1 = 17$
$d = 1$ [$a_2 - a_1 = 18 - 17 = 1$]
Therefore,
$a_n = a_1 + (n - 1)\ d$
$= 17 + (n - 1)\ (1)$
$= 17 + n - 1$
$a_n = n + 16$
Hence,
$a_5 = 5 + 16 = 21$
$a_8 = 8 + 16 = 24$
$a_{15} = 15 + 16 = 31$

e) 10, 20, 30, 40, …

In the sequence,
10, 20, 30, 40, …
$a_1 = 10$
$d = 10$ [$a_2 - a_1 = 20 - 10 = 10$]
Therefore,
$a_n = a_1 + (n - 1)\ d$
$= 10 + (n - 1)\ (10)$
$= 10 + 10n - 10$
$a_n = 10n$
Hence,
$a_5 = 10(5) = 50$
$a_8 = 10\ (8) = 80$
$a_{15} = 10\ (15) = 150$

f) 21, 22, 23, 24, ...

In the sequence,
21, 22, 23, 24, …
$a_1 = 21$
$d = 1$ [$a_2 - a_1 = 22 - 21 = 1$]
Therefore,
$a_n = a_1 + (n - 1)\ d$
$= 21 + (n - 1)\ (1)$
$= 21 + n - 1$
$a_n = n + 20$
Hence,
$a_5 = 20 + 5 = 25$
$a_8 = 20 + 8 = 28$
$a_{15} = 20 + 15 = 35$

**4. Find the general term for each arithmetic sequence.**

a) 99, 110, 121, 132, … b) –31, –26, –21, –16, … c) 64, 82, 100, 118, …

d) $\frac{1}{2}$, $\frac{3}{4}$, 1, $\frac{5}{4}$, … e) 78,72,66, ... f) 1.25, 4.55, 7.85, 11.15, …

**Solution:**

a) 99, 110, 121, 132, …

In the sequence,

99, 110, 121, 132, …

$a_1 = 99$

$d = 11$ [$a_2 - a_1 = 110 - 99 = 11$]

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= 99 + (n - 1)(11)$

$= 99 + 11n - 11$

$a_n = 11n + 88$

Hence, general term of the given sequence is $11n + 88$.

b) –31, –26, –21, –16, …

In the sequence,

–31, –26, –21, –16, …

$a_1 = -31$

$d = 5$ [$a_2 - a_1 = -26 - (-31) = -26 + 31 = 5$]

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= -31 + (n - 1)(5)$

$= 99 + 5n - 5$

$a_n = 5n + 94$

Hence, general term of the given sequence is $5n + 94$.

c) 64, 82, 100, 118, …

In the sequence,

64, 82, 100, 118, …

$a_1 = 64$

$d = 18$ [$a_2 - a_1 = 82 - 64 = 18$]

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= 64 + (n - 1)(18)$

$= 99 + 18n - 18$

$a_n = 18n + 81$

Hence, general term of the given sequence is $18n + 81$.

d) $\frac{1}{2}$, $\frac{3}{4}$, 1, $\frac{5}{4}$, …

In the sequence,

$\frac{1}{2}$, $\frac{3}{4}$, 1, $\frac{5}{4}$, …

$a_1 = \frac{1}{2}$

$d = \frac{1}{4}$ [$a_2 - a_1 = \frac{3}{4} - \frac{1}{2} = \frac{1}{4}$]

Therefore,

$a_n = a_1 + (n - 1)\, d$

$= \frac{1}{2} + (n - 1)(\frac{1}{4})$

$= \frac{1}{2} + \frac{1}{4}n - \frac{1}{4}$

$a_n = \frac{1}{4}n + \frac{1}{4}$
$a_n = \frac{1}{4}(n + 1)$

Hence, general term of the given sequence is $\frac{1}{4}(n + 1)$.
e) 78,72,66, ...
In the sequence,
78, 72, 66, …
$a_1 = 78$
$d = -6$ [$a_2 - a_1 = 72 - 78 = -6$]
Therefore,
$a_n = a_1 + (n - 1)d$
$= 78 + (n - 1)(-6)$
$= 99 - 6n + 6$
$a_n = 105 - 6n$
Hence, general term of the given sequence is $105 - 6n$.
f) 1.25, 4.55, 7.85, 11.15, …
In the sequence,
1.25, 4.55, 7.85, 11.15, …
$a_1 = 1.25$
$d = 3.3$ [$a_2 - a_1 = 110 - 99 = 3.3$]
Therefore,
$a_n = a_1 + (n - 1)d$
$= 1.25 + (n - 1)(3.3)$
$= 1.25 + 3.3n - 3.3$
$a_n = 3.3n - 2.05$
Hence, general term of the given sequence is $3.3n - 2.05$.

5. **The first term of the sequence is 8 and the common difference is 14. Find the $18^{th}$ term of the arithmetic sequence.**
**Solution:**
Here $a_1 = 8$ and $d = 14$
Therefore,
$a_n = a_1 + (n - 1)d$
$= 8 + (n - 1)(14)$
$= 8 + 14n - 14$
$a_n = 14n - 6$
$a_{18} = 14(18) - 6$
$= 7 - 64$
$= 252 - 6$
$a_{18} = 246$
Therefore, the 18th term of the arithmetic sequence is 246.

6. **The first term of the arithmetic sequence is 12 and the 9th term of the sequence is 84. What is the common difference?**
**Solution:**
Here, $a_1 = 12$ and $a_9 = 84$
Therefore,
$a_n = a_1 + (n - 1)d$
$a_9 = 12 + (9 - 1)d$
$84 = 12 + 8d$
$84 - 12 = 8d$

72 = 8d
Dividing 9 on both sides
$\frac{72}{8} = \frac{8d}{8}$
$\Longrightarrow$ d = 9
Therefore, the common difference is 9.

7. **Amna decided to recite the Holy Quran. She recites 35 Ayats on the first day. She increases 6 Ayats daily. How many Ayats will she recite on the 12th day?**
**Solution:**
Here, $a_1 = 35$ and $d = 6$
Therefore,
$a_n = a_1 + (n - 1)\, d$
$a_{12} = 35 + (12 - 1)\,(6)$
$= 35 + (11)(6)$
$= 35 + 66$
$a_{12} = 101$
Therefore, Amna will recite 101 Ayats on the 12th day.

8. **Ibrahim decided to improve his English vocabulary. Each day, the number of his learnt words increases by 2. If he learnt 20 words on the 9th day, how many words did he learn on the first day?**
**Solution:**
Here, $a_9 = 20$ and $d = 2$
Therefore,
$a_n = a_1 + (n - 1)\, d$
$a_9 = a_1 + (9 - 1)\,(2)$
$20 = a_1 + (8)(2)$
$20 = a_1 + 16$
$20 - 16 = a_1$
$\Longrightarrow a_1 = 4$
Therefore, Ibrahim learned 4 words on the first day.

9. **Iffat decided to rent her house. The current rent for the house is Rs 12,000. If the rent of the house is increasing every year by Rs 1000, what will the rent of the house be in the 7th year?**
**Solution:**
Here, $a_1 = 12{,}000$ and $d = 1000$
Therefore,
$a_n = a_1 + (n - 1)\, d$
$a_7 = 12{,}000 + (7 - 1)\,(1000)$
$= 12{,}000 + (6)\,(1000)$
$= 12{,}000 + 6000$
$a_7 = 18{,}000$
Therefore, the rent of the house in the 7th year will be Rs 18,000.

## Learning Check 5.2

1. **Separate the open and closed sentences.**

a) $7 + 2x = 1$ b) $3 - 19 = -4 \times 4$ c) $x - 1 = 1 - 2x$ d) $3x + y = 20$
e) $-8x + 4 = 20$ f) $35 \div 5 = 12 - 5$ g) $41 - 21 = 21 - 41$ h) $\frac{x+5}{3} = 8$

**Solution:**

a) $7 + 2x = 1$

This is an open sentence because it contains the variable 'x'.

b) $3 – 19 = - 4 \times 4$

This is a closed sentence because it does not contain any variables.

c) $x – 1 = 1 – 2x$

This is an open sentence because it contains the variable 'x'.

d) $3x + y = 20$

This is an open sentence because it contains the variables 'x'.

e) $– 8x +4 = 20$

This is an open sentence because it contains the variable 'x'.

f) $35 \div 5 = 12 – 5$

This is a closed sentence because it does not contain any variables.

g) $41 – 21 = 21 – 41$

This is a open sentence because it does not contain any variables.

h) $\frac{x+5}{3} = 8$

This is an open sentence because it contains the variable 'x'.

**2. Identify and separate the equations and inequalities.**

a) $2 + 4z = -7$ b) $t – 5 > -4$ c) $5d = 25$ d) $-2r +7 > -1$
e) $7t + 4 \leq 10$ f) $6y -2y = 8$ g) $m^2 -16 = 11$ h) $2n^2 – 1 \geq 20$
i) $3k + 14 < 21$ j) $a^2 + 2b^2 = 8$

**Solution:**

a) $2 + 4z = -7$

This is an equation because it contains an equal sign.

b) $t – 5 > -4$

This is an inequality because it contains an inequality symbol.

c) $5d = 25$

This is an equation because it contains an equal sign.

d) $-2r +7 > -1$

This is an inequality because it contains an inequality symbol.

e) $7t + 4 \leq 10$

This is an inequality because it contains an inequality symbol.

f) $6y - 2y = 8$

This is an equation because it contains an equal sign.

g) $m^2 - 16 = 11$

This is an equation because it contains an equal sign.

h) $2n^2 – 1 \geq 20$

This is an inequality because it contains an inequality symbol.

i) $3k + 14 < 21$

This is an inequality because it contains an inequality symbol.

j) $a^2 + 2b^2 = 8$

This is an equation because it contains an equal sign.

3. **Add the following polynomials.**
   a) $2x^2 + 3x + 7, – x^2 + 2x – 5, 5x^2 – 5x + 2$
   b) $2y^3 + 5y^2z + 7z^3, -5y^3 + 9yz^2 – 4y^2z, 8y^3 – 10z^3 + 3y^2z$
   c) $a^2 – 7a + 1, 2a^2 – 7a – 3, 3a^2 + 3a – 10$
   d) $4z^3 + 2z^2 + 2z – 6, -5z^3 – 5z^2 - 8z + 17, 2z^3 + 10z – 12$
   e) $u^4 – 3u^3v + 4u^2v^2 – 8, v^4 + uv^3 – 4u^2v^2, 3u^3v – 6uv^3 + 18$

**Solution:**

a) $2x^2 + 3x + 7, – x^2 + 2x – 5, 5x^2 – 5x + 2$
$= (2x^2 + 3x + 7) + (– x^2 + 2x – 5) + (5x^2 – 5x + 2)$
$= 2x^2 + 3x + 7 – x^2 + 2x – 5 + 5x^2 – 5x + 2$
$= 2x^2 – x^2 + 5x^2 + 3x + 2x – 5x + 7 – 5 + 2$
$= 6x^2 + 0 + 4$
$= 6x^2 + 4$

b) $2y^3 + 5y^2z + 7z^3, – 5y^3 + 9yz^2 – 4y^2z, 8y^3 – 10z^3 + 3y^2z$
$= (2y^3 + 5y^2z + 7z^3) + (– 5y^3 + 9yz^2 – 4y^2z) + (8y^3 – 10z^3 + 3y^2z)$
$= 2y^3 + 5y^2z + 7z^3 – 5y^3 + 9yz^2 – 4y^2z + 8y^3 – 10z^3 + 3y^2z$
$= 2y^3 – 5y^3 + 8y^3 + 5y^2z – 4y^2z + 3y^2z + 9yz^2 + 7z^3 – 10z^3$
$= 5y^3 + 4y^2z + 9yz^2 – 3z^3$

c) $a^2 – 7a + 1, 2a^2 – 7a – 3, 3a^2 + 3a – 10$
$= (a^2 – 7a + 1) + (2a^2 – 7a – 3) + (3a^2 + 3a – 10)$
$= a^2 – 7a + 1 + 2a^2 – 7a – 3 + 3a^2 + 3a – 10$
$= a^2 + 2a^2 + 3a^2 – 7a – 7a + 3a + 1 – 3 – 10$
$= 6a^2 – 11a – 12$

d) $4z^3 + 2z^2 + 2z – 6, – 5z^3 – 5z^2 – 8z + 17, 2z^3 + 10z – 12$
$= (4z^3 + 2z^2 + 2z – 6) + (– 5z^3 – 5z^2 – 8z + 17) + (2z^3 + 10z – 12)$
$= 4z^3 + 2z^2 + 2z – 6 – 5z^3 – 5z^2 – 8z + 17 + 2z^3 + 10z – 12$
$= 4z^3 – 5z^3 + 2z^3 + 2z^2 – 5z^2 + 2z – 8z + 10z – 6 + 17 – 12$

$= z^3 - 3z^2 + 4z - 1$

e) $u^4 - 3u^3v + 4u^2v^2 - 8$, $v^4 + uv^3 - 4u^2v^2$, $3u^3v - 6uv^3 + 18$

$= (u^4 - 3u^3v + 4u^2v^2 - 8) + (v^4 + uv^3 - 4u^2v^2) + (3u^3v - 6uv^3 + 18)$

$= u^4 - 3u^3v + 4u^2v^2 - 8 + v^4 + uv^3 - 4u^2v^2 + 3u^3v - 6uv^3 + 18$

$= u^4 + v^4 - 3u^3v + 3u^3v + uv^3 - 6uv^3 + 4u^2v^2 - 4u^2v^2 - 8 + 18$

$= u^4 + v^4 + 0 - 5uv^3 + 0 + 10$

$= u^4 - 5uv^3 + v^4 + 10$

**4. Find the product of the following polynomials.**

a) $(a - 2b)(3a + 5b)$
b) $(p - q)(5p + 5q)$
c) $(1 - 2m)(7 + m)$
d) $(x - 9y)(12x + 108y)$
e) $(x + 10)(7 - 3x)$
f) $(3x + y)(x - 4y)$

**Solution:**

a) $(a - 2b)(3a + 5b)$

$(a - 2b)(3a + 5b) = a(3a + 5b) - 2b(3a + 5b)$
$= 3a^2 + 5ab - 6ab - 10b^2$
$= 3a^2 - ab - 10b^2$

b) $(p - q)(5p + 5q)$

$(p - q)(5p + 5q) = p(5p + 5q) - q(5p + 5q)$
$= 5p^2 + 5pq - 5pq - 5q^2$
$= 5p^2 + 0 - 5q^2$
$= 5p^2 - 5q^2$
$= 5(p^2 - q^2)$

c) $(1 - 2m)(7 + m)$

$(1 - 2m)(7 + m) = 1(7 + m) - 2m(7 + m)$
$= 7 + m - 14m - 2m^2$
$= -2m^2 - 13m + 7$

d) $(x - 9y)(12x + 108y)$

$(x - 9y)(12x + 108y) = x(12x + 108y) - 9y(12x + 108y)$
$= 12x^2 + 108xy - 108xy - 972y^2$
$= 12x^2 + 0 - 972y^2$
$= 12x^2 - 972y^2$

e) $(x + 10)(7 - 3x)$

$(x + 10)(7 - 3x) = x(7 - 3x) + 10(7 - 3x)$
$= 7x - 3x^2 + 70 - 30x$
$= -3x^2 - 23x + 70$

f) $(3x + y)(x - 4y)$

$(3x + y)(x - 4y) = 3x(x - 4y) + y(x - 4y)$
$= 3x^2 - 12xy + xy - 4y^2$
$= 3x^2 - 11xy - 4y^2$

**5. Divide the first polynomial by the second polynomial when:**

a) $(x^2 - 6x - 27) \div (x + 3)$
b) $(2a^4 + 7a^3 + 12a^2 - 9a - 27) \div (2a + 3)$
c) $(y^4 - 6y^3 - 15y^2 - 12y + 32) \div (y - 8)$
d) $(2p^3 - 7p^2 - 10p + 35) \div (p^2 - 5)$

e) $(2z^2 + 4z + 3) \div (z + 5)$
f) $(6q^3 + 30q^2 + 24q) \div 12q$
g) $(3x^3 + 4x^2 + 8x - 15) \div (x - 1)$
h) $(5x^2 + 11xy + 6y2) \div (5x + 6y)$

**Solution:**

a) $(x^2 - 6x - 27) \div (x + 3)$

$$
\begin{array}{r|rrr}
 & x & -9 & \\
\hline
x+3 & x^2 & -6x & -27 \\
 & {}_{-}x^2 & +{}_{-}3x & \\
\hline
 & & -9x & -27 \\
 & & -{}_{+}9x & -{}_{+}27 \\
\hline
 & & & 0
\end{array}
$$

So, $(x^2 - 6x - 27) \div (x + 3) = x - 9$
Here, the dividend = $(x^2 - 6x - 27)$
Divisor = $x + 3$, Quotient = $x - 9$ and Remainder = 0

b) $(2a^4 + 7a^3 + 12a^2 - 9a - 27) \div (2a + 3)$

$$
\begin{array}{r|rrrrr}
 & a^3 & +2a^2 & +3a & -9 & \\
\hline
2a+3 & 2a^4 & +7a^3 & +12a^2 & -9a & -27 \\
 & {}_{-}2a^4 & +{}_{-}3a^3 & & & \\
\hline
 & & 4a^3 & +12a^2 & -9a & -27 \\
 & & {}_{-}4a^3 & +{}_{-}6a^2 & & \\
\hline
 & & & 6a^2 & -9a & -27 \\
 & & & {}_{-}6a^2 & +{}_{-}9a & \\
\hline
 & & & & -18a & -27 \\
 & & & & -{}_{+}18a & -{}_{+}27 \\
\hline
 & & & & & 0
\end{array}
$$

So, $(2a^4 + 7a^3 + 12a^2 - 9a - 27) \div (2a + 3) = a^3 + 2a^2 + 3a - 9$
Here, the dividend = $(2a^4 + 7a^3 + 12a^2 - 9a - 27)$
Divisor = $2a + 3$, Quotient = $a^3 + 2a^2 + 3a - 9$ and Remainder = 0

c) $(y^4 - 6y^3 - 15y^2 - 12y + 32) \div (y - 8)$

$$
\begin{array}{r|rrrrr}
 & y^3 & +\,2y^2 & +\,y & -\,4 & \\
\hline
y-8 & y^4 & -\,6y^3 & -\,15y^2 & -\,12y & +\,32 \\
 & {}_{-}y^4 & -\,{}_{+}8y^3 & & & \\
\hline
 & & 2y^3 & -\,15y^2 & -\,12y & +\,32 \\
 & & {}_{-}2y^3 & -\,{}_{+}16y^2 & & \\
\hline
 & & & y^2 & -\,12y & +\,32 \\
 & & & {}_{-}y^2 & -\,{}_{+}8y & \\
\hline
 & & & & -\,4y & +\,32 \\
 & & & & -\,{}_{+}4y & +\,{}_{-}32 \\
\hline
 & & & & & 0
\end{array}
$$

So, ($y^4 – 6y^3 – 15y^2 – 12y + 32$) ÷ ($y – 8$) = *$y^3 + 2y^2 + y – 4$*
Here, the dividend = ($y^4 – 6y^3 – 15y^2 – 12y + 32$)
Divisor = $y – 8$, Quotient = *$y^3 + 2y^2 + y – 4$* and Remainder = 0

d) ($2p^3 – 7p^2 – 10p + 35$) ÷ ($p^2 – 5$)

$$
\begin{array}{r|rrrr}
 & 2p & -\,7 & & \\
\hline
p^2-5 & 2p^3 & -\,7p^2 & -\,10p & +\,35 \\
 & {}_{-}2p^3 & & -\,{}_{+}10p & \\
\hline
 & & -\,7p^2 & & +\,35 \\
 & & -\,{}_{+}7p^2 & & +\,{}_{-}35 \\
\hline
 & & & & 0
\end{array}
$$

So, ($2p^3 – 7p^2 – 10p + 35$) ÷ ($p^2 – 5$) = *$2p – 7$*
Here, the dividend = ($2p^3 – 7p^2 – 10p + 35$)
Divisor = $p^2 – 5$, Quotient = *$2p – 7$* and Remainder = 0

e) ($2z^2 + 4z + 3$) ÷ ($z + 5$)

$$
\begin{array}{r|rrr}
 & 2z & -\,6 & \\
\hline
z+5 & 2z^2 & +\,4z & +\,3 \\
 & {}_{-}2z^2 & +\,{}_{-}10z & \\
\hline
 & & -\,6z & +\,3 \\
 & & -\,{}_{+}6z & -\,{}_{+}30 \\
\hline
 & & & 33
\end{array}
$$

So, ($2z^2 + 4z + 3$) ÷ ($z + 5$) = *$2z – 6$*
Here, the dividend = ($2z^2 + 4z + 3$)
Divisor = $z + 5$, Quotient = *$2z – 6$* and Remainder = 33

f) $(6q^3 + 30q^2 + 24q) \div 12q$

$$
\begin{array}{r|l}
 & 0.5q^2 + 2.5q + 2 \\
\hline
12q & 6q^3 + 30q^2 + 24q + 0 \\
 & \underline{-6q^3} \\
 & \quad 30q^2 + 24q + 0 \\
 & \quad \underline{-30q^2} \\
 & \qquad 24q + 0 \\
 & \qquad \underline{-24q} \\
 & \qquad\qquad 0
\end{array}
$$

So, $(6q^3 + 30q^2 + 24q) \div 12q = \frac{1}{2}q^2 + \frac{5}{2}q + 2$

Here, the dividend = $(6q^3 + 30q^2 + 24q)$

Divisor = $12q$, Quotient = $\frac{1}{2}q^2 + \frac{5}{2}q + 2$ and Remainder = 0

g) $(3x^3 + 4x^2 + 8x – 15) \div (x – 1)$

$$
\begin{array}{r|l}
 & 3x^2 + 7x + 15 \\
\hline
x - 1 & 3x^3 + 4x^2 + 8x - 15 \\
 & \underline{_-3x^3 - {_+3x^2}} \\
 & \quad 7x^2 + 8x - 15 \\
 & \quad \underline{_-7x^2 - {_+7x}} \\
 & \qquad 15x - 15 \\
 & \qquad \underline{_-15x - {_+15}} \\
 & \qquad\qquad 0
\end{array}
$$

So, $(3x^3 + 4x^2 + 8x – 15) \div (x – 1) = 3x^2 + 7x + 15$

Here, the dividend = $(3x^3 + 4x^2 + 8x – 15)$

Divisor = $x – 1$, Quotient = $3x^2 + 7x + 15$ and Remainder = 0

h) $(5x^2 + 11xy + 6y^2) \div (5x + 6y)$

$$
\begin{array}{r|l}
 & x + y \\
\hline
5x + 6y & 5x^2 + 11xy + 6y^2 \\
 & \underline{5x^2 + 6xy} \\
 & 5xy + 6y^2 \\
 & \underline{5xy + 6y^2} \\
 & 0
\end{array}
$$

So, $(5x^2 + 11xy + 6y^2) \div (5x + 6y) = x + y$

Here, the dividend = $(5x^2 + 11xy + 6y^2)$

Divisor = $5x + 6y$, Quotient = $x + y$ and Remainder = 0

6. **The difference between the two polynomials is $7a^4 + 4a^3 + 3a^2b - b^4$. If one polynomial is $5a^4 - 3a^3 - 5a^2b + 4b^4$, find the other.**
   **Solution:**
   First polynomial = $5a^4 - 3a^3 - 5a^2b + 4b^4$
   Difference between two polynomials = $7a^4 + 4a^3 + 3a^2b - b^4$
   Let x be another polynomial.
   According to given condition.
   $5a^4 - 3a^3 - 5a^2b + 4b^4 - x = 7a^4 + 4a^3 + 3a^2b - b^4$
   $5a^4 - 3a^3 - 5a^2b + 4b^4 - (7a^4 + 4a^3 + 3a^2b - b^4) = x$
   $5a^4 - 3a^3 - 5a^2b + 4b^4 - 7a^4 - 4a^3 - 3a^2b + b^4 = x$
   $5a^4 - 7a^4 - 3a^3 - 4a^3 - 5a^2b - 3a^2b + 4b^4 + b^4 = x$
   $\Rightarrow x = -2a^4 - 7a^3 - 8a^2b + 5b^4$
   Therefore, the other polynomial is $-2a^4 - 7a^3 - 8a^2b + 5b^4$
7. **What should be subtracted from $16x^2 - 24xy + 9y^2$ to get $2x^2 + y^2 - xy + 12$?**
   **Solution:**
   Let A be the subtrahend from first polynomial to get 2nd polynomial as:
   $16x^2 - 24xy + 9y^2 - A = 2x^2 + y^2 - xy + 12$
   $16x^2 - 24xy + 9y^2 - (2x^2 + y^2 - xy + 12) = A$
   $16x^2 - 24xy + 9y^2 - 2x^2 - y^2 + xy - 12 = A$
   $16x^2 - 2x^2 + 9y^2 - y^2 - 24xy + xy - 12 = A$
   $14x^2 + 8y^2 - 23xy - 12 = A$
   $\Rightarrow A = 14x^2 + 8y^2 - 23xy - 12$
   Therefore, we need to subtract $14x^2 - 23xy + 8y^2 - 12$.
8. **The product of two polynomials is $x^3 + 2x^2 + x + 12$. If one polynomial is $x + 3$ then find the other polynomial.**
   **Solution:**
   First polynomial = $x + 3$
   Product = $x^3 + 2x^2 + x + 12$
   Let A be the 2nd polynomial. Then
   Product = $x^3 + 2x^2 + x + 12$
   $A(x + 3) = x^3 + 2x^2 + x + 12$
   Dividing $x + 3$ on both sides
   $A = \frac{x^3 + 2x^2 + x + 12}{x + 3}$

$$
\begin{array}{r|l}
 & x^2 - x + 4 \\
\hline
x + 3 & x^3 + 2x^2 + x + 12 \\
 & \underline{\pm x^3 \pm 3x^2} \\
 & \quad -x^2 + x + 12 \\
 & \quad \underline{\mp x^2 \mp 3x} \\
 & \qquad 4x + 12 \\
 & \qquad \underline{\pm 4x \pm 12} \\
 & \qquad\qquad 0
\end{array}
$$

   $A = x^2 - x + 4$
   Therefore, the other polynomial is $x^2 - x + 4$.

**9. For what value of x is $x^3 + 5x^2 + 15x – 325$ exactly divisible by $(x – 5)$?**
**Solution:**

$$
\begin{array}{r|rrrrrrr}
 & x^2 & + & 10x & & + 65 & & \\
\hline
x - 5 & x^3 & + & 5x^2 & + & 15x & - & 325 \\
 & {}_{-}x^3 & - & {}_{+}5x^2 & & & & \\
\hline
 & & & 10x^2 & + & 15x & - & 325 \\
 & & & {}_{-}10x^2 & - & {}_{+}50x & & \\
\hline
 & & & & & 65x & - & 325 \\
 & & & & & {}_{-}65x & - & {}_{+}325 \\
\hline
 & & & & & & & 0
\end{array}
$$

So, $(x^3 + 5x^2 + 15x – 325) \div (x – 5) = x^2 + 10x + 65$
Here, the dividend = $(x^3 + 5x^2 + 15x – 325)$
Divisor = x – 5, Quotient = $x^2 + 10x + 65$ and Remainder = 0

**Learning Check 5.3**

**1. Simply the following by using the BODMAS rule.**

a) – 4a – [5a – 2{2b – (4a – 3a)}]
b) 12m – [12n – {3m – (12n – m) + 8n} + m]
c) 2x – [2y + {3z – 3x – (x -2y + 3z)}]    d) 10[x – 2{x – 2(x –2x + 3x)}]
e) $5h – [4g^2h \div h \{9gh^2 – (6g + h^3)\}]$    f) $4s + 3s \{3 – 2s^2(8s^3 \div 4s^2 + 4s)\}$
g) – 16(p – s) + 5(q – r) – 2[q + r + s – 3{r + s – 4(s – p)}]

**Solution:**

a) – 4a – [5a – 2{2b – (4a – 3a)}]
= – 4a – [5a – 2{2b – a}]
= – 4a – [5a – 4b + 2a]
= – 4a – 7a + 4b
= – 11a + 4b

b) 12m – [12n – {3m – (12n – m) + 8n} + m]
= 12m – [12n – {3m – 12n + m + 8n} + m]
= 12m – [12n – {4m – 4n} + m]
= 12m – [12n – 4m + 4n + m]
= 12m – [16n – 3m]
= 12m – 16n + 3m
= 15m – 16n

c) 2x – [2y + {3z – 3x – (x -2y + 3z)}]
= 2x – [2y + {3z – 3x – x + 2y – 3z}]
= 2x – [2y – 4x + 2y + 0]
= 2x – [– 4x + 4y]
= 2x + 4x – 4y
= 6x – 4y

d) $10[x - 2\{x - 2(x - 2x + 3x)\}]$
$= 10[x - 2\{x - 2(2x)\}]$
$= 10[x - 2\{x - 4x\}]$
$= 10[x - 2\{-3x\}]$
$= 10[x + 6x]$
$= 10[7x]$
$= 70x$

e) $5h - [4g^2h \div h\{9gh^2 - (6g + h^3)\}]$
$= 5h - [4g^2h \div h\{9gh^2 - 6g - h^3\}]$
$= 5h - [\frac{4g^2h}{h(9gh^2 - 6g - h^3)}]$
$= 5h - \frac{4g^2}{(9gh^2 - 6g - h^3)}$
$= \frac{5h(9gh^2 - 6g - h^3) - 4g^2}{9gh^2 - 6g - h^3}$
$= \frac{45gh^3 - 30gh - 5h^4 - 4g^2}{9gh^2 - 6g - h^3}$

f) $4s + 3s\{3 - 2s^2(8s^3 \div 4s^2 + 4s)\}$
$= 4s + 3s\{3 - 2s^2(\frac{8s^3}{4s^2} + 4s)\}$
$= 4s + 3s\{3 - 2s^2(2s + 4s)\}$
$= 4s + 3s\{3 - 2s^2(6s)\}$
$= 4s + 3s\{3 - 12s^3\}$
$= 4s + 9s - 36s^4$
$= 13s - 36s^4$

g) $-16(p - s) + 5(q - r) - 2[q + r + s - 3\{r + s - 4(s - p)\}]$
$= -16p + 16s + 5q - 5r - 2[q + r + s - 3\{r + s - 4(s - p)\}]$
$= -16p + 16s + 5q - 5r - 2[q + r + s - 3\{r + s - 4s + 4p\}]$
$= -16p + 16s + 5q - 5r - 2[q + r + s - 3\{r - 3s + 4p\}]$
$= -16p + 16s + 5q - 5r - 2[q + r + s - 3r + 9s - 12p]$
$= -16p + 16s + 5q - 5r - 2[q - 2r + 10s - 12p]$
$= -16p + 16s + 5q - 5r - 2q + 4r - 20s + 24p$
$= 8p + 3q - r - 4s$

## Learning Check 5.4

**1. Evaluate the following algebraic expressions by using identities.**

a) $(a + 2b)(a + 2b)$ b) $(2x - y)(2x + y)$ c) $(8m + 3n)(8m - 3n)$
d) $(4y^2 - 1)^2$ e) $(x^2 - y^2)^2$ f) $(3m + n)^2$
g) $(x^2 + 3)(x^2 - 3)$ h) $(12a + 7b)^2$ i) $(4z - 7)(4z - 7)$

**Solution:**

a) $(a + 2b)(a + 2b)$
$(a + 2b)(a + 2b) = (a + 2b)^2$
$= (a)^2 + 2(a)(2b) + (2b)^2$
$= a^2 + 4ab + 4b^2$

b) $(2x - y)(2x + y)$
$(2x - y)(2x + y) = (2x)^2 - (y)^2$
$= 4x^2 - y^2$

c) $(8m + 3n)(8m - 3n)$
$(8m + 3n)(8m - 3n) = (8m)^2 - (3n)^2$
$= 64m^2 - 9n^2$

d) $(4y^2 - 1)^2$
$(4y^2 - 1)^2 = (4y^2)^2 - 2(4y^2)(1) + (1)^2$
$= 16y^4 - 8y^2 + 1$

e) $(x^2 - y^2)^2$
$(x^2 - y^2)^2 = (x^2)^2 - 2(x^2)(y^2) + (y^2)^2$
$= x^4 - 2x^2y^2 + y^4$

f) $(3m + n)^2$
$(3m + n)^2 = (3m)^2 + 2(3m)(n) + (n)^2$
$= 9m^2 + 6mn + n^2$

g) $(x^2 + 3)(x^2 - 3)$
$(x^2 + 3)(x^2 - 3) = (x^2)^2 - (3)^2$
$= x^4 - 9$

h) $(12a + 7b)^2$
$(12a + 7b)^2 = (12a)^2 + 2(12a)(7b) + (7b)^2$
$= 144a^2 + 168ab + 49b^2$

i) $(4z - 7)(4z - 7)$
$(4z - 7)(4z - 7) = (4z - 7)^2$
$= (4z)^2 - 2(4z)(7) + (7)^2$
$= 16z^2 - 56z + 49$

**2. Evaluate the following by using the appropriate algebraic identities.**

a) $(99)^2$ b) $(4.5)^2$ c) $125 \times 125$ d) $44 \times 36$
e) $(301)^2$ f) $11^2 \times 11^2$ g) $15 \times 25$ h) $27 \times 33$

**Solution:**

a) $(99)^2$
$(99)^2 = (100 - 1)^2$
$= 100^2 - 2(100)(1) + (1)^2$
$= 10000 - 200 + 1$
$= 9801$

b) $(4.5)^2$
$(4.5)^2 = (4 + 0.5)2$
$= 4^2 + 2(4)(0.5) + (0.5)^2$
$= 15 + 4 + 0.25$
$= 20.25$

c) $125 \times 125$
$125 \times 125 = (125)^2$
$= (100 + 25)^2$
$= 100^2 + 2(100)(25) + 25^2$
$= 10000 + 5000 + 625$
$= 15625$

d) 44 × 36
44 × 36 = (40 + 4) (40 - 4)
= $40^2$ - $4^2$
= 1600 - 16
= 1584

e) $(301)^2$
$(301)^2$ = $(300 + 1)^2$
= $300^2$ + 2(300) (1) + $1^2$
= 90000 + 600 + 1
= 90601

f) $11^2 \times 11^2$
$11^2 \times 11^2$ = $(100 + 21)^2$
= $100^2$ + 2(100) (21) + $21^2$
= 10000 + 4200 + 441
= 14641

g) 15 × 25
15 × 25 = (20 - 5) (20 + 5)
= $20^2$ - $5^2$
= 400 - 25
= 375

h) 27 × 33
27 × 33 = (30 - 3) (30 + 3)
= (30)2 – (3)2
= 900 – 9
= 891

**3. Find the value of $x^2 + \frac{1}{9x^2}$ if $x + \frac{1}{3x} = 4$.**

**Solution:**

Given: $x + \frac{1}{3x} = 4$

To find: $x^2 + \frac{1}{9x^2}$

We have

$x + \frac{1}{3x} = 4$

Squaring on both sides

$(x + \frac{1}{3x})^2 = (4)^2$

$x^2 + 2(x)(\frac{1}{3x}) + (\frac{1}{3x})^2 = 16$

$x^2 + \frac{2}{3} + \frac{1}{9x^2} = 16$

$x^2 + \frac{1}{9x^2} = 16 - \frac{2}{3}$

$x^2 + \frac{1}{9x^2} = \frac{48 - 2}{3}$

$x^2 + \frac{1}{9x^2} = \frac{46}{3}$

4. **If a + b = 5 and ab = 8, then find the value of $a^2 + b^2$.**
   **Solution:**
   Given that: a + b = 5 and ab = 8
   To find: $a^2 + b^2$
   We have
   a + b = 5
   squaring on both sides
   $a^2 + 2(a)(b) + b^2 = (5)^2$
   $a^2 + 2(8) + b^2 = 25$
   $a^2 + 16 + b^2 = 25$
   $a^2 + b^2 = 25 – 16$
   $a^2 + b^2 = 9$
5. **If a + b = 6 and a – b = 5, then find $a^2 – b^2$.**
   **Solution:**
   Given that: a + b = 6 and a – b = 5
   To find: $a^2 – b^2$
   We know that
   $a^2 – b^2 = (a+ b) (a – b)$
   $= (6) (5)$
   $= 30$
6. **Find the value of $(\boldsymbol{xyz} + \mathbf{3})\,(\boldsymbol{xyz} - \mathbf{3})$.**
   **Solution:**
   $(xyz + 3)\,(xyz - 3) = (xyz)^2 – (3)^2$
   $= x^2y^2z^2 – 9$
7. **Find the value of $\boldsymbol{a^2} + \frac{\mathbf{1}}{\boldsymbol{b^2}}$ if $\mathbf{a} - \frac{\mathbf{1}}{\boldsymbol{b}} = \mathbf{3}$.**
   **Solution:**
   Given: $a – \frac{1}{b} = 3$
   To find: $a^2 + \frac{1}{b^2}$
   We have
   $a – \frac{1}{b} = 3$
   Squaring on both sides
   $(a – \frac{1}{b})^2 = (3)^2$
   $a^2 – 2(a)\,(\frac{1}{b}) + (\frac{1}{b})^2 = 9$
   $a^2 – 2 + \frac{1}{b^2} = 9$
   $a^2 + \frac{1}{b^2} = 9 + 2$
   $a^2 + \frac{1}{b^2} = 11$

## Learning Check 5.5

1. **Factorize the following algebraic expressions.**

| | | |
|---|---|---|
| a) 2a + 6 | b) 5x – 10 | c) $x^2 – x$ |
| b) $3x^3 + 12x^2$ | e) 7a – 7b + 7c | f) $3b^2 + 6b – 15$ |
| c) 5ax + 10ay – 25az | h) $7p^3 – 14p^2 – 21p$ | i) $10p^2q + 25pq^2 – 35pqr$ |
| j) 11ab – 22bc + 33cd | k) $a^2 + ab + 3a + 3b$ | l) $4a^2 + 4b^2 + a^2c^2 + b^2c^2$ |

**Solution:**

a) $2a + 6$
$2a + 6 = 2(a + 3)$
b) $5x - 10$
$5x - 10 = 5(x - 2)$
c) $x^2 - x$
$x^2 - x = x(x - 1)$
d) $3x^3 + 12x^2$
$3x^3 + 12x^2 = 3x^2(x + 4)$
e) $7a - 7b + 7c$
$7a - 7b + 7c = 7(a - b + c)$
f) $3b^2 + 6b - 15$
$3b^2 + 6b - 15 = 3(b^2 + 2b - 5)$
g) $5ax + 10ay - 25az$
$5ax + 10ay - 25az = 5a(x + 2y - 5z)$
h) $7p^3 - 14p^2 - 21p$
$7p^3 - 14p^2 - 21p = 7p(p^2 - 2p - 3)$
i) $10p^2q + 25pq^2 - 35pqr$
$10p^2q + 25pq^2 - 35pqr = 5pq(2p + 5q - 7r)$
j) $11ab - 22bc + 33cd$
$11ab - 22bc + 33cd = 11(ab - 2bc + 3cd)$
k) $a^2 + ab + 3a + 3b$
$a^2 + ab + 3a + 3b = a(a + b) + 3(a + b)$
$= (a + b)(a + 3)$
l) $4a^2 + 4b^2 + a^2c^2 + b^2c^2$
Re- arranging the terms
$4a^2 + 4b^2 + a^2c^2 + b^2c^2 = 4a^2 + a^2c^2 + 4b^2 + b^2c^2$
$= a^2(4 + c^2) + b^2(4 + c^2)$
$= (a^2 + b^2)(c^2 + 4)$

**2. Factorize the following algebraic expressions.**

a) $a^2 + 6a + 9$ b) $4a^2 + 20ab + 25b^2$ c) $9x^2 + 12xy + 4y^2$
d) $4a^2 - 12a + 9$ e) $9a^2 - 30ab + 25b^2$ f) $25p^2 + 70pq + 49q$
g) $49p^2 - 126pq + 81q^2$ h) $49t^2 + 154tv + 121v^2$ i) $t^2 - \frac{4}{5}tv + \frac{4}{25}v^2$
j) $16x^2 - 56xy + 49y^2$

**Solution:**

a) $a^2 + 6a + 9$
$a^2 + 6a + 9 = (a)^2 + 2(a)(3) + (3)^2$
$= (a + 3)^2$
$= (a + 3)(a + 3)$
b) $4a^2 + 20ab + 25b^2$
$4a^2 + 20ab + 25b^2 = (2a)^2 + 2(2a)(5b) + (5b)^2$
$= (2a + 5b)^2$
$= (2a + 5b)(2a + 5b)$
c) $9x^2 + 12xy + 4y^2$
$9x^2 + 12xy + 4y^2 = (3x)^2 + 2(3x)(2y) + (2y)^2$
$= (3x + 2y)^2$
$= (3X + 2y)(3x + 2y)$
d) $4a^2 - 12a + 9$
$4a^2 - 12a + 9 = (2a)^2 - 2(2a)(3) + (3)^2$

$= (2a - 3)^2$
$= (2a - 3)\ (2a - 3)$

e) $9a^2 - 30\ ab + 25b^2$
$9a^2 - 30\ ab + 25b^2 = (3a)^2 - 2\ (3a)\ (5b) + (5b)^2$
$= (3a - 5b)^2$
$= (3a - 5b)\ (3a - 5b)$

f) $25p^2 + 70pq + 49q$
$25p^2 + 70pq + 49q = (5p)^2 + 2\ (5p)\ (7q) + (7q)^2$
$= (5p + 7q)^2$
$= (5p + 7q)\ (5p + 7q)$

g) $49p^2 - 126pq + 81q^2$
$49p^2 - 126pq + 81q^2 = (7p)^2 - 2\ (7p)\ (9q) + (9q)^2$
$= (7p - 9q)^2$
$= (7p - 9q)\ (7p - 9q)$

h) $49t^2 + 154tv + 121v^2$
$49t^2 + 154tv + 121v^2 = (7t)^2 + 2\ (7t)\ (11v) + (11v)^2$
$= (7t + 11v)^2$
$= (7t + 11v)\ (7t + 11v)$

i) $t^2 - \frac{4}{5}tv + \frac{4}{25}v^2$
$t^2 - \frac{4}{5}tv + \frac{4}{25}v^2 = t^2 - 2(t)\ (\frac{2}{5}\ v) + (\frac{2}{5}\ v)^2$
$= (t - \frac{2}{5}\ v)^2$
$= (t - \frac{2}{5}\ v)\ (t - \frac{2}{5}\ v)$

j) $16x^2 - 56xy + 49y^2$
$16x^2 - 56xy + 49y^2 = (4x)^2 - 2\ (4x)\ (7y) + (7y)^2$
$= (4x - 7y)^2$
$= (4x - 7y)\ (4x - 7y)$

## Learning Check 5.6

**1. Evaluate the following expressions by using algebraic identities.**

a) $(2a + 1)^3$ b) $(p + \frac{1}{p})^3$ c) $(x + \frac{y}{3})^3$ d) $(x - 3y)^3$
e) $(6x + 11y)^3$ f) $(3a - 5b)^3$ g) $(2q - \frac{1}{2q})^3$ h) $(2 - t)^3$
i) $(3 + 4m)^3$ j) $(\frac{1}{z} + 4)^3$ k) $(m - 12n)^3$ l) $(10y + 7z)^3$

**Solution:**

a) $(2a + 1)^3$
$(2a + 1)^3 = (2a)^3 + 3(2a)^2\ (1) + 3(2a)\ (1)^2 + (1)^3$
$= 8a^3 + 3(4a^2)\ (1) + (6a)\ (1) + 1$
$= 8a^3 + 12a^2 + 6a + 1$

b) $(p + \frac{1}{p})^3$
$(p + \frac{1}{p})^3 = (p)^3 + 3(p)^2\ (\frac{1}{p}) + 3(p)\ (\frac{1}{p})^2 + (\frac{1}{p})^3$
$= p^3 + 3(p^2)\ (\frac{1}{p}) + 3(p)\ (\frac{1}{p})^2 + \frac{1}{p^3}$
$= p^3 + 3p + \frac{3}{p} + \frac{1}{p^3}$

c) $(x + \frac{y}{3})^3$

$(x + \frac{y}{3})^3 = (x)^3 + 3(x)^2\,(\frac{y}{3}) + 3(x)\,(\frac{y}{3})^2 + (\frac{y}{3})^3$

$= x^3 + 3(x^2)\,(\frac{y}{3}) + 3(x)\,(\frac{y}{3})^2 + \frac{y^3}{27}$

$= x^3 + x^2y + \frac{1}{3}\,xy^2 + \frac{1}{27}\,y^3$

d) $(x - 3y)^3$

$(x - 3y)^3 = (x)^3 - 3(x)^2\,(3y) + 3(x)\,(3y)^2 - (3y)^3$

$= x^3 - 3(x^2)\,(3y) + 3(x)\,(9y^2) - 27y^3$

$= x^3 - 9x^2y + 27xy^2 - 27y^3$

e) $(6x + 11y)^3$

$(6x + 11y)^3 = (6x)^3 + 3(6x)^2\,(11y) + 3(6x)\,(11y)^2 + (11y)^3$

$= 216x^3 + 3(36x^2)\,(11y) + 3(6x)\,(121y^2) + 1331y^3$

$= 216x^3 + 1188x^2y + 2178xy^2 + 1331y^3$

f) $(3a - 5b)^3$

$(3a - 5b)^3 = (3a)^3 - 3(3a)^2\,(5b) + 3(3a)\,(5b)^2 - (5b)^3$

$= 27a^3 - 3(9a^2)\,(5b) + 3(3a)\,(25b^2) - 125b^3$

$= 27a^3 - 135a^2b + 225ab^2 - 125b^3$

g) $(2q - \frac{1}{2}q)^3$

$(2q - \frac{1}{2q})^3 = (2q)^3 - 3(2q)^2\,(\frac{1}{2q}) + 3(2q)\,(\frac{1}{2q})^2 - (\frac{1}{2q})^3$

$= 8q^3 - 3(4q^2)\,(\frac{1}{2q}) + 3(2q)\,(\frac{1}{4q^2}) - \frac{1}{8q^3}$

$= 8q^3 - 6q + \frac{3}{2q} - \frac{1}{8q^3}$

h) $(2 - t)^3$

$(2 - t)^3 = (2)^3 - 3(2)^2\,(t) + 3(2)\,(t)^2 - (t)^3$

$= 8 - 3(4)\,(t) + 3(2)\,(t^2) - t^3$

$= 8 - 12t + 6t^2 - t^3$

i) $(3 + 4m)^3$

$(3 + 4m)^3 = (3)^3 + 3(3)^2\,(4m) + 3(3)\,(4m)^2 + (4m)^3$

$= 27 + 3(9)\,(4m) + 3(3)\,(16m^2) + 64m^3$

$= 27 + 108m + 144m^2 + 64m^3$

j) $(\frac{1}{z} + 4)^3$

$(\frac{1}{z} + 4)^3 = (\frac{1}{z})^3 + 3(\frac{1}{z})^2\,(4) + 3(\frac{1}{z})\,(4)^2 + (4)^3$

$= \frac{1}{z^3} + 3(\frac{1}{z})^2\,(4) + 3(\frac{1}{z})\,(16) + 64$

$= \frac{1}{z^3} + 12\,(\frac{1}{z^2}) + 48(\frac{1}{z}) + 64$

k) $(m - 12n)^3$

$(m - 12n)^3 = (m)^3 - 3(m)^2\,(12n) + 3(m)\,(12n)^2 - (12n)^3$

$= m^3 - 3(m^2)\,(12n) + 3(m)\,(144n^2) - 1778n^3$

$= m^3 - 36m^2n + 432mn^2 - 1778n^3$

l) $(10y + 7z)^3$

$(10y + 7z)^3 = (10y)^3 + 3(10y)^2\,(7z) + 3(10y)\,(7z)^2 + (7z)^3$

$= 1000y^3 + 3(10oy^2)\,(7z) + 3(10y)\,(49z^2) + 343z^3$

$= 1000y^3 + 210y^2z + 1470yz^2 + 343z^3$

**2. Find the value of $x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x} = -5$.**

**Solution:**

Given: $x - \frac{1}{x} = -5$

To find: $x^3 - \frac{1}{x^3}$

We have

$x - \frac{1}{x} = -5$

Cubing on both sides

$(x)^3 - 3(x)\left(\frac{1}{x}\right)\left(x - \frac{1}{x}\right) - \left(\frac{1}{x}\right)^3 = (-5)^3$

$(x)^3 - 3\left(x - \frac{1}{x}\right) - \left(\frac{1}{x}\right)^3 = (-5)^3$

Put $x - \frac{1}{x} = -5$

$x^3 - 3(-5) - \frac{1}{x^3} = -125$

$x^3 + 15 - \frac{1}{x^3} = -125$

$x^3 - \frac{1}{x^3} = -125 - 15$

$x^3 - \frac{1}{x^3} = -140$

**3. Find the value of $x^3 + \frac{1}{x^3}$ when**

**a) $x + \frac{1}{x} = 4$ b) $x + \frac{1}{x} = -7$**

**Solution:**

a) $x + \frac{1}{x} = 4$

cubing on both sides

$(x)^3 + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) + \left(\frac{1}{x}\right)^3 = (4)^3$

$(x)^3 + 3\left(x + \frac{1}{x}\right) + \left(\frac{1}{x}\right)^3 = (4)^3$

Put $x + \frac{1}{x} = 4$

$x^3 + 3(4) + \frac{1}{x^3} = 64$

$x^3 + 12 + \frac{1}{x^3} = 64$

$x^3 + \frac{1}{x^3} = 64 - 12$

$x^3 + \frac{1}{x^3} = 52$

b) $x + \frac{1}{x} = -7$

cubing on both sides

$(x)^3 + 3(x)\left(\frac{1}{x}\right)\left(x + \frac{1}{x}\right) + \left(\frac{1}{x}\right)^3 = (-7)^3$

$(x)^3 + 3\left(x + \frac{1}{x}\right) + \left(\frac{1}{x}\right)^3 = (-7)^3$

Put $x + \frac{1}{x} = -7$

$x^3 + 3(-7) + \frac{1}{x^3} = -343$

$x^3 - 21 + \frac{1}{x^3} = -343$

$x^3 + \frac{1}{x^3} = -343 + 21$
$x^3 + \frac{1}{x^3} = -322$

**4. Find the value of $x^3 + y^3$ when $x + y = 8$ and $xy = -5$.**
**Solution:**
Given that: $x + y = 8$ and $xy = -5$
To find: $x^3 + y^3$
We have
$x + y = 8$
cubing on both sides
$(x)^3 + 3(x)(y)(x + y) + (y)^3 = (8)^3$
$x^3 + 3xy(x + y) + y^3 = 512$
put $x + y = 8$ and $xy = -5$
$x^3 + 3(-5)(8) + y^3 = 512$
$x^3 - 120 + y^3 = 512$
$x^3 + y^3 = 512 + 120$
$x^3 + y^3 = 632$

**5. Find the value of $x^3 - y^3$ when $x - y = 10$ and $xy = 6$.**
**Solution:**
Given that: $x - y = 10$ and $xy = 6$
To find: $x^3 - y^3$
We have
$x - y = 10$
cubing on both sides
$(x)^3 - 3(x)(y)(x - y) - (y)^3 = (10)^3$
$x^3 - 3xy(x - y) - y^3 = 1000$
put $x - y = 10$ and $xy = 6$
$x^3 - 3(6)(10) - y^3 = 1000$
$x^3 - 180 - y^3 = 1000$
$x^3 - y^3 = 1000 + 180$
$x^3 - y^3 = 1180$

**6. Find the value of $xy$ when $x^3 + y^3 = 10$ and $x + y = 5$.**
**Solution:**
We have
$x + y = 5$
cubing on both sides
$(x)^3 + 3(x)(y)(x + y) + (y)^3 = (5)^3$
$x^3 + 3(xy)(x + y) + y^3 = 125$
$x^3 + y^3 + 3(xy)(x + y) = 125$
put $x^3 + y^3 = 10$ and $x + y = 5$
$10 + 3(xy)(5) = 125$
$15xy = 125 - 10$
$15xy = 115$
Dividing 15 on both sides
$\frac{15xy}{15} = \frac{115}{15}$
$xy = 7\frac{2}{3}$

## Unit Check 5

**1. Encircle the correct option.**

a) The 8th term of the sequence −9, −4, 1, 6, … is:
i) 5 ii) 11 iii) 21 **iv) 26**

b) The formula to find the general term of an arithmetic sequence is:
i) a + (n + 1) d **ii) a + (n – 1) d** iii) a – (n – 1) d iv) a + (2n – 1) d

c) The result of division of $5x^2 – 15x$ by $5x$ is:
i) $x + 3$ ii) $2x + 3$ iii) $x2 + 3$ **iv) $x – 3$**

d) The result of multiplication of $(3x + 7)$ and $(3x – 7)$ is:

i) $9x^2 + 14x – 49$ **ii) $9x^2 – 49$** iii) $9x^2 – 21x + 81$ iv) $3x^2 – 49$

e) $(a + b)^2 – (a – b)^2 =$
i) $2(a^2 + b^2)$ ii) $a^2 + b^2$ **iii) 4ab** iv) ab

**2. Fill in the blanks.**

a) The common difference in the sequence −43, −14, 15, 44, …. Is **29**.
b) $a^3 – 3a^2b + 3ab^2 – b^3 =$ **$(a – b)^3$**.
c) The process of finding factors of an algebraic expression is called **factorization**.
d) The sum of $x^2 + 3x –19$ and $4x^3 + 5x + 16$ is **$4x^3 + x^2 + 8x – 3$**.
e) The factors of $x^2 + 2x – 3$ are **$(x – 1)(x + 3)$**.

**3. Find the 9th, 14th and 25th terms of the following arithmetic sequences by using the general term of the arithmetic sequence.**

a) 14, 20, 26, 32, … b) 67, 58, 49, 40, … c) 105, 242, 379, 516, …

**Solution:**

a) 14, 20, 26, 32, …
In the sequence,
14, 20, 26, 32, …
$a_1 = 14$
$d = 6$ $[a_2 – a_1 = 20 – 14 = 6]$
Therefore,
$a_n = a_1 + (n - 1) d$
$a_9 = 14 + (9 - 1)(6)$
$= 14 + (8)(6)$
$= 14 + 48$
$a_9 = 62$
*Now*
$a_{14} = 14 + (14 - 1)(6)$
$= 14 + (13)(6)$
$= 14 + 78$
$a_{14} = 92$
*and*
$a_{25} = 14 + (25 - 1)(6)$
$= 14 + (24)(6)$
$= 14 + 144$
$a_{25} = 158$

b) 67, 58, 49, 40, …
In the sequence,
67, 58, 49, 40, …
$a_1 = 67$

$d = -9$ [$a_2 - a_1 = 58 - 67 = -9$]
Therefore,
$a_n = a_1 + (n - 1)\, d$
$a_9 = 67 + (9 - 1)(-9)$
$= 67 + (8)(-9)$
$= 67 - 72$
$a_9 = -5$
*Now*
$a_{14} = 67 + (14 - 1)(-9)$
$= 67 + (13)(-9)$
$= 67 - 117$
$a_{14} = -50$
*and*
$a_{25} = 67 + (25 - 1)(-9)$
$= 67 + (24)(-9)$
$= 67 - 216$
$a_{25} = -149$

c) 105, 242, 379, 516, …
In the sequence,
105, 242, 379, 516, …
$a_1 = 105$
$d = 137$ [$a_2 - a_1 = 242 - 105 = 137$]
Therefore,
$a_n = a_1 + (n - 1)\, d$
$a_9 = 105 + (9 - 1)(137)$
$= 105 + (8)(137)$
$= 105 + 1096$
$a_9 = 1201$
*Now*
$a_{14} = 105 + (14 - 1)(137)$
$= 105 + (13)(137)$
$= 105 + 1781$
$a_{14} = 1886$
*and*
$a_{25} = 105 + (25 - 1)(137)$
$= 105 + (24)(137)$
$= 105 + 3288$
$a_{25} = 3393$

**4. The auditorium of a well-known school has 30 seats in the first row, 36 seats in the second row, 42 seats in the third row, and so on. Find the number of seats in the 8th and 12th rows if the seats are in the same pattern.**
**Solution:**

Here, $a_1 = 30$, $a_2 = 36$
$d = a_2 - a_1 = 36 - 30 = 6$
we know that
$a_n = a_1 + (n - 1)\, d$
$a_8 = 30 + (8 - 1)(6)$
$= 30 + (7)(6)$
$= 30 + 42$

$a_8 = 72$
*And*
$a_{12} = 30 + (12 - 1)(6)$
$= 30 + (11)(6)$
$= 30 + 66$
$a_{12} = 96$

5. **If the first term of the sequence is –8 and the difference between the terms is 12, what is the 18th term of the sequence?**
**Solution:**

Here, $a_1 = -8$, $d = 12$
we know that
$a_n = a_1 + (n - 1)d$
$a_{18} = -8 + (18 - 1)(12)$
$= -8 + (17)(12)$
$= -8 + 204$
$a_{18} = 196$

6. **If the first term of the sequence is 40 and the 20th term of the sequence is 648, what is the common difference between the terms?**
**Solution:**
Here, $a_1 = 40$ and $a_{20} = 648$
Therefore,
$a_n = a_1 + (n - 1)d$
$a_{20} = 40 + (20 - 1)d$
$648 = 40 + 19d$
$648 - 40 = 19d$
$608 = 19d$
Dividing 19 on both sides
$\frac{608}{19} = \frac{19d}{19}$
$\Longrightarrow d = 32$
Therefore, the common difference is 32.

7. **What is the sum of $6x^2 - 13xy + 23y^2$ and $-8x^2 - 2x^2y + 12xy$?**
**Solution:**
$(6x^2 - 13xy + 23y^2) + (-8x^2 - 2x^2y + 12xy)$
$= 6x^2 - 13xy + 23y^2 - 8x^2 - 2x^2y + 12xy$
$= 6x^2 - 8x^2 - 2x^2y - 13xy + 12xy + 23y^2$
$= -2x^2 - 2x^2y - xy + 23y^2$

8. **Subtract $4x^3 - 3x^2y + 6xy^2 - y^3 - 31$ from $3x^3 + 5x^2y + 6xy^2 + 3y^3 - 31$.**
**Solution:**
$(3x^3 + 5x^2y + 6xy^2 + 3y^3 - 31) - (4x^3 - 3x^2y + 6xy^2 - y^3 - 31)$
$= 3x^3 + 5x^2y + 6xy^2 + 3y^3 - 31 - 4x^3 + 3x^2y - 6xy^2 + y^3 + 31$
$= 3x^3 - 4x^3 + 5x^2y + 3x^2y + 6xy^2 - 6xy^2 + 3y^3 + y^3 - 31 + 31$
$= -x^3 + 8x^2y + 0 + 4y^3 + 0$
$= -x^3 + 8x^2y + 4y^3$

9. **Find the product of the following polynomials.**

a) $(x + 7y)(4x - 3y)$

b) $(4a^2 - 4a + 1)(a^2 + 32a + 56)$

**Solution:**

a) $(x + 7y)(4x - 3y)$

$= x(4x - 3y) + 7y(4x - 3y)$

$= 4x^2 - 3xy + 28xy - 21y^2$

$= 4x^2 + 25xy - 21y^2$

b) $(4a^2 - 4a + 1)(a^2 + 32a + 56)$

$= 4a^2(a^2 + 32a + 56) - 4a(a^2 + 32a + 56) + 1(a^2 + 32a + 56)$

$= 4a^4 + 128a^3 + 224a^2 - 4a^3 - 128a^2 - 224a + a^2 + 32a + 56$

$= 4a^4 + 128a^3 - 4a^3 + 224a^2 - 128a^2 + a^2 - 224a + 32a + 56$

$= 4a^4 + 124a^3 + 97a^2 - 192a + 56$

**10. Divide the first polynomial by the second polynomial when:**

a) $8x^3 - 14xy + 4x^2y - 6x,\ 2x$

b) $2x^3 + 7x^2 + 13x - 292,\ x - 4$

**Solution:**

a) $8x^3 - 14xy + 4x^2y - 6x,\ 2x$

$$
\begin{array}{r|l}
 & 4x^2 + 2xy - 7y \quad -3 \\
\hline
2x & 8x^3 + 4x^2y - 14xy - 6x \\
 & 8x^3 \\
\hline
 & 4x^2y - 14xy - 6x \\
 & 4x^2y \\
\hline
 & -14xy - 6x \\
 & -14xy \\
\hline
 & -6x \\
 & \underline{-6x}
\end{array}
$$

So, $(8x^3 - 14xy + 4x^2y - 6x) \div (2x) = 4x^2 + 2xy - 7y - 3$

Here, the dividend $= (8x^3 - 14xy + 4x^2y - 6x)$

Divisor $= 2x$, Quotient $= 4x^2 + 2xy - 7y - 3$ and Remainder $= 0$

b) $2x^3 + 7x^2 + 13x - 292,\ x - 4$

$$
\begin{array}{r|l}
 & 2x^2 + 15x + 73 \\
\hline
x - 4 & 2x^3 + 7x^2 + 13x - 292 \\
 & \underset{-}{}2x^3 \ - \underset{+}{}8x^2 \\
\hline
 & 15x^2 + 13x - 292 \\
 & \underset{-}{}15x^2 - \underset{+}{}60x \\
\hline
 & 73x - 292 \\
 & \underset{-}{}73x - \underset{+}{}292 \\
\hline
 & 0
\end{array}
$$

So, $(2x^3 + 7x^2 + 13x - 292) \div (x - 4) = 2x^2 + 15x + 73$

Here, the dividend $= (2x^3 + 7x^2 + 13x - 292)$

Divisor $= x - 4$, Quotient $= 2x^2 + 15x + 73$ and Remainder $= 0$

**11. Simplify the following by using the BODMAS rule.**

a) $4 - 8x\,\{3y - 6(4xy \div x - 3y) - 6\}$    b) $[3p + 8q\,\{4pq \div 2\,(6p - q + 8)\}]$

**Solution:**

a) $4 - 8x\,\{3y - 6(4xy \div x - 3y) - 6\}$

$= 4 - 8x\,\{3y - 6(\frac{4xy}{x} - 3y) - 6\}$

$= 4 - 8x\,\{3y - 6(4y - 3y) - 6\}$

$= 4 - 8x\,\{3y - 6(y) - 6\}$

$= 4 - 8x\,\{3y - 6y - 6\}$

$= 4 - 8x\,\{- 3y - 6\}$

$= 4 + 24xy + 48x$

$= 24xy + 48x + 4$

b) $[3p + 8q\,\{4pq \div 2\,(6p - q + 8)\}]$

$= [3p + 8q\,\{\frac{4pq}{2\,(6p - q + 8)}\}]$

$= [3p + 8q\,\{\frac{2pq}{6p - q + 8}\}]$

$= [3p + \frac{16pq^2}{6p - q + 8}]$

$= 3p + \frac{16pq^2}{6p - q + 8}$

$= \frac{3p\,(6p - q + 8) + 16pq^2}{6p - q + 8}$

$= \frac{18p^2 - 3pq + 24p + 16pq^2}{6p - q + 8}$

**12. What is the value of $a^2 - b^2$ when $a - b = 12$ and $a + b = 7$?**

**Solution:**

Given that: $a - b = 12$ and $a + b = 7$

To find: $a^2 - b^2$

We know that

$a^2 - b^2 = (a + b)(a - b)$

$= (12)(7)$

$= 84$

**13. Determine the value of $a^2 + b^2$ when $a - b = -5$ and $ab = 8$.**

**Solution:**

Given that: $a - b = -5$ and $ab = 8$

To find: $a^2 + b^2$

we have

$a - b = -5$

squaring on both sides

$(a - b)^2 = (-5)^2$

$a^2 - 2ab + b^2 = 25$

$a^2 - 2(8) + b^2 = 25$

$a^2 - 16 + b^2 = 25$

$a^2 + b^2 = 25 + 16$

$a^2 + b^2 = 41$

## Learning Check 6.1

1. **The cost of a bag is three times the cost of a geometry box. The cost of 2 bags and 3 geometry boxes is Rs 3960. Construct simultaneous linear equations in two variables for these conditions.**
   **Solution:**
   Let
   The price of a bag = Rs. x
   The price of geometry box = Rs. y
   According to the given condition:
   $x = 3y$ -----------------------------------------(i)
   $2x + 3y = 3960$ ------------------------------(ii)
   So, (i) and (ii) are the required simultaneous linear equations for the given conditions.
2. **The volume of apple juice is 3 liters less than the volume of orange juice and the total volume of apple and orange juice is 11 liters. Construct simultaneous linear equations in two variables for these conditions.**
   **Solution:**
   Let
   The volume of apple juice = x litres
   The volume of orange juice = y litres
   According to the given conditions:
   $x = y – 3$
   $x – y = – 3$ -------------------------------------(i)
   $x + y = 11$ --------------------------------------(ii)
   So, (i) and (ii) are the required simultaneous linear equations for the given conditions.
3. **The sum of two numbers is 82 and their difference is 23. Construct simultaneous linear equations in two variables for these conditions.**
   **Solution:**
   Let
   The first number = x
   The second number = y
   According to the given conditions:
   $x + y = 82$ ------------------------ (i)
   $x – y = 23$ -------------------------(ii)
   So, (i) and (ii) are the required simultaneous linear equations for the given conditions.

## Learning Check 6.2

1. **Solve the following simultaneous equations by using the elimination method.**

   a) $x + y = 7$; $x – y = 3$
   b) $x + y = 10$; $x - y = 4$
   c) $x + y = 14$; $x - y = 4$
   d) $x + y = 25$; $x – y = 5$
   e) $x + y = 11$; $x + 3y = 25$
   f) $2x + 3y = 65$; $5x + 3y = 95$
   g) $2x – 3y= 13$; $7x + 2y= 8$
   h) $5x + 2y = 23$; $2x + 7y = 3$

**Solution:**

a) $x + y = 7$; $x - y = 3$

$x + y = 7$ ——— (1)

$x - y = 3$ ——— (2)

Add equations (1) and (2),

$(x + y) + (x - y) = 7 + 3$

$x + y + x - y = 10$

$2x = 10$

Dividing 2 on both sides

$\frac{2x}{2} = \frac{10}{2}$

$x = 5$

Substitute this value of $x$ in equation (1),

$5 + y = 7$

Subtracting 5 on both sides

$5 + y - 5 = 7 - 5$

$y = 2$

Thus, $x = 5$ and $y = 2$ is the solution of the given simultaneous linear equations.

b) $x + y = 10$; $x - y = 4$

$x + y = 10$ ——— (1)

$x - y = 4$ ——— (2)

Add equations (1) and (2),

$(x + y) + (x - y) = 10 + 4$

$x + y + x - y = 14$

$2x = 14$

Dividing 2 on both sides

$\frac{2x}{2} = \frac{14}{2}$

$x = 7$

Substitute this value of $x$ in equation (1),

$7 + y = 10$

Subtracting 7 on both sides

$7 + y - 7 = 10 - 7$

$y = 3$

Thus, $x = 7$ and $y = 3$ is the solution of the given simultaneous linear equations.

c) $x + y = 14$; $x - y = 4$

$x + y = 14$ ——— (1)

$x - y = 4$ ——— (2)

Add equations (1) and (2),

$(x + y) + (x - y) = 14 + 4$

$x + y + x - y = 18$

$2x = 18$

Dividing 2 on both sides

$\frac{2x}{2} = \frac{18}{2}$

$x = 9$

Substitute this value of $x$ in equation (1),

$9 + y = 14$

Subtracting 9 on both sides

$9 + y - 9 = 14 - 9$

y = 5

Thus, $x$ = 9 and y = 5 is the solution of the given simultaneous linear equations.

d) $x$ + y = 25; $x$ – y = 5

$x$ + y = 25 ——— (1)

$x$ – y = 5 ——— (2)

Add equations (1) and (2),

(x + y) + (x – y) = 25 + 5

x + y + x – y = 30

2$x$ = 30

Dividing 2 on both sides

$\frac{2x}{2} = \frac{30}{2}$

$x$ = 15

Substitute this value of $x$ in equation (1),

15 + y = 25

Subtracting 15 on both sides

15 + y – 15 = 25 – 15

y = 10

Thus, $x$ = 15 and y = 10 is the solution of the given simultaneous linear equations.

e) $x$ + y = 11; $x$ + 3y = 25

$x$ + y = 11 ——— (1)

$x$ + 3y = 25 ——— (2)

Subtracting equations (1) from (2),

(x + 3y) – (x + y) = 25 – 11

x + 3y – x – y = 14

2$y$ = 14

Dividing 2 on both sides

$\frac{2y}{2} = \frac{14}{2}$

$y$ = 7

Substitute this value of $y$ in equation (1),

x + 7 = 11

Subtracting 7 on both sides

x + 7 – 7 = 11 – 7

x = 4

Thus, $x$ = 4 and y = 7 is the solution of the given simultaneous linear equations.

f) 2$x$ + 3y = 65; 5$x$ + 3y = 95

2$x$ + 3y = 65 ——— (1)

5$x$ + 3y = 95 ——— (2)

Subtracting equations (1) from (2),

(5$x$ + 3y) – (2$x$ + 3y) = 95 – 65

5$x$ + 3y – 2$x$ – 3y = 30

3$x$ = 30

Dividing 3 on both sides

$\frac{3x}{3} = \frac{30}{3}$

$x$ = 10

Substitute this value of $x$ in equation (1),

2*(10)* + 3y = 65

20 + 3y = 65
Subtracting 20 on both sides
20 + 3y – 20 = 65 – 20
3y = 45
Dividing 3 on both ides
$\frac{3y}{3} = \frac{45}{3}$
*y* = 15

Thus, *x* = 10 and y = 15 is the solution of the given simultaneous linear equations.

g) 2*x* – 3y= 13; 7*x* + 2y= 8
2*x* – 3y= 13 ——— (1)
7*x* + 2y= 8 ——— (2)
Multiplying equation (1) by 2 and equation (2) by 3
4x – 6y = 26 ——— (3)
21x + 6y = 24 ——— (4)
Add equations (3) and (4),
(4x – 6y) + (21x + 6y) = 26 + 24
4x – 6y + 21x + 6y = 50
25*x* = 50
Dividing 25 on both sides
$\frac{25x}{25} = \frac{50}{25}$
*x* = 2
Substitute this value of *x* in equation (1),
2*(2)* – 3y= 13
4 – 3y = 13
Subtracting 4 on both sides
4 – 3y – 4 = 13 – 4
– 3y = 9
Dividing – 3 on both sides
$\frac{-3y}{-3} = \frac{9}{-3}$
*y* = – 3

Thus, *x* = 2 and y = – 3 is the solution of the given simultaneous linear equations.

h) 5*x* + 2y = 23; 2*x* + 7y = 3
5*x* + 2y = 23 ——— (1)
2*x* + 7y = 3 ——— (2)
Multiplying equation (1) by 7 and equation (2) by 2
35x + 14y = 161 ——— (3)
4x + 14y = 6 ——— (4)
Subtracting equations (4) from (3),
(35x + 14y) – (4x + 14y) = 161 – 6
35x + 14y – 4x – 14y = 155
31*x* = 155
Dividing 31 on both sides
$\frac{31x}{31} = \frac{155}{31}$
*x* = 5
Substitute this value of *x* in equation (1),
5*(5)* + 2y = 23

$25 + 2y = 23$
Subtracting 25 on both sides
$25 + 2y - 25 = 23 - 25$
$2y = -2$
Dividing 2 on both sides
$\frac{2y}{2} = \frac{-2}{2}$
$y = -1$

Thus, $x = 5$ and $y = -1$ is the solution of the given simultaneous linear equations.

**2. Solve the following simultaneous equations by using the substitution method.**

a) $x + y = 9$; $x - y = 5$
b) $2x + y = 6$; $x - 3y = 17$
c) $x + y = 7$; $x - y = 3$
d) $x + y = 16$; $x - y = 4$
e) $x + y = 4$; $x - y = -10$
f) $x + y = -12$; $x - y = 2$
g) $2x + 3y = 52$; $5x - 2y = 16$
h) $15x + 6y = 9$; $10x - 18y = 28$

**Solution:**

a) $x + y = 9$; $x - y = 5$

$x + y = 9$ ——— (1)
$x - y = 5$ ——— (2)
From equation (2), we get
$x = 5 + y$ ——— (3)
Put this value of $x$ in equation (1),
$(5 + y) + y = 9$
$5 + y + y = 9$
Subtracting 5 on both sides
$5 + 2y - 5 = 9 - 5$
$2y = 4$
Dividing 2 on both sides
$\frac{2y}{2} = \frac{4}{2}$
$y = 2$
Substitute $y = 2$ in equation (3),
$x = 5 + 2 = 7$
Thus, $x = 7$ and $y = 2$ is the solution of the given equations.

b) $2x + y = 6$; $x - 3y = 17$

$2x + y = 6$ ——— (1)
$x - 3y = 17$ ——— (2)
From equation (2), we get
$x = 17 + 3y$ ——— (3)
Put this value of $x$ in equation (1),
$2(17 + 3y) + y = 6$
$34 + 6y + y = 6$
$7y + 34 = 6$
Subtracting 34 on both sides
$7y + 34 - 34 = 6 - 34$
$7y = -28$
Dividing 7 on both sides
$\frac{7y}{7} = \frac{-28}{7}$
$y = -4$
Substitute $y = -4$ in equation (3),

$x = 17 + 3(-4)$
$x = 17 - 12$
$x = 5$
Thus, $x = 5$ and $y = -4$ is the solution of the given equations.

c) $x + y = 7$; $x - y = 3$

$x + y = 7$ —— (1)
$x - y = 3$ —— (2)
From equation (2), we get
$x = 3 + y$ —— (3)
Put this value of $x$ in equation (1),
$(3 + y) + y = 7$
$2y + 3 = 7$
Subtracting 3 on both sides
$2y + 3 - 3 = 7 - 3$
$2y = 4$
Dividing 2 on both sides
$\frac{2y}{2} = \frac{4}{2}$
$y = 2$
Substitute $y = 2$ in equation (3),
$x = 3 + 2 = 5$
Thus, $x = 5$ and $y = 2$ is the solution of the given equations.

d) $x + y = 16$; $x - y = 4$

$x + y = 16$ —— (1)
$x - y = 4$ —— (2)
From equation (2), we get
$x = 4 + y$ —— (3)
Put this value of $x$ in equation (1),
$(4 + y) + y = 16$
$4 + y + y = 16$
$2y + 4 = 16$
Subtracting 4 on both sides
$2y + 4 - 4 = 16 - 4$
$2y = 12$
Dividing 2 on both sides
$\frac{2y}{2} = \frac{12}{2}$
$y = 6$
Substitute $y = 6$ in equation (3),
$x = 4 + 6 = 10$
Thus, $x = 10$ and $y = 6$ is the solution of the given equations.

e) $x + y = 4$; $x - y = -10$

$x + y = 4$ —— (1)
$x - y = -10$ —— (2)
From equation (2), we get
$x = y - 10$ —— (3)
Put this value of $x$ in equation (1),
$(y - 10) + y = 4$

y – 10 + y = 4
2y – 10 = 4
Adding 10 on both sides
2y – 10 + 10 = 4 + 10
2y = 14
Dividing 2 on both sides
$\frac{2y}{2} = \frac{14}{2}$
y = 7
Substitute y = 7 in equation (3),
$x$ = 7 – 10 = – 3
Thus, $x$ = – 3 and y = 7 is the solution of the given equations.

f) $x$ + y = –12; $x$ - y = 2

$x$ + y = –12 ——— (1)
$x$ – y = 2 ——— (2)
From equation (2), we get
$x$ = 2 + y ——— (3)
Put this value of $x$ in equation (1),
(2 + y) + y = –12
2 + y + y = – 12
2y + 2 = – 12
Subtracting 2 on both sides
2y + 2 – 2 = – 12 – 2
2y = – 14
Dividing 2 on both sides
$\frac{2y}{2} = \frac{-14}{2}$
y = – 7
Substitute y = – 7 in equation (3),
$x$ = 2 – 7 = – 5
Thus, $x$ = – 5 and y = – 7 is the solution of the given equations.

g) 2$x$ + 3y = 52; 5$x$ – 2y = 16

2$x$ + 3y = 52 ——— (1)
5$x$ – 2y = 16 ——— (2)
From equation (2), we get
*5x* = 2y + 16
*Dividing 5 on both sides*
$\frac{5x}{5} = \frac{2y+16}{5}$
x = $\frac{2y+16}{5}$ ——— (3)
Put this value of $x$ in equation (1),
$2(\frac{2y+16}{5})$ + 3y = 52
$\frac{4y+32}{5}$ + 3y = 52
Multiplying 5 on both sides, we get
4y + 32 + 15y = 260
19y + 32 = 260
Subtracting 32 on both sides
19y + 32 – 32 = 260 – 32

19y = 228
Dividing 19 on both sides
$\frac{19y}{19} = \frac{228}{19}$
y = 12
Substitute y = 12 in equation (3),
$x = \frac{2(12)+16}{5}$
$x = \frac{24+16}{5}$
$x = \frac{40}{5}$
x = 8
Thus, $x$ = 8 and y = 12 is the solution of the given equations.

h) 15$x$ + 6y = 9; 10$x$ – 18y = 28

15$x$ + 6y = 9 ——— (1)
10$x$ – 18y = 28 ——— (2)
From equation (2), we get
10$x$ = 18y + 28
Dividing 10 on both sides
$\frac{10x}{10} = \frac{18y+28}{10}$
$x = \frac{18y+28}{10}$ ——— (3)
Put this value of $x$ in equation (1),
$15\left(\frac{18y+28}{10}\right) + 6y = 9$
$\frac{3}{2}$ [2 (9y + 14)] + 6y = 9
3 (9y + 14) + 6y = 9
27y + 42 + 6y = 9
33y + 42 = 9
Subtracting 42 on both sides
33y + 42 – 42 = 9 – 42
33y = – 33
Dividing 33 on both sides
$\frac{33y}{33} = \frac{-33}{33}$
y = – 1
Substitute y = – 1 in equation (3),
$x = \frac{18(-1)+28}{10}$
$= \frac{-18+28}{10}$
$= \frac{10}{10} = 1$
Thus, $x$ = 1 and y = – 1 is the solution of the given equations.

**3. Solve the following system of linear equations graphically.**

a) $x$ + y = 7; $x$ - y = 5
b) 2$x$ - y = 4; $x$ - 2y = 7
c) 3$x$ - y = 0; 2$x$ + y = 5
d) 3$x$ - 5y = 6; 2$x$ - 3y = 5
e) $\frac{x}{3} + \frac{y}{5}$ = 2; 2x – y = 1
f) $\frac{x}{3}$ + 5y = 16; x + y = 6
g) 2$x$ + 5y = 12; 4$x$ + 3y = -4
h) 5$x$ - y = 4; 10$x$ - 4y = 6

**Solution:**

a) $x + y = 7$; $x - y = 5$

$x + y = 7$ -----------(i)
$x - y = 5$ -----------(ii)
First find the ordered pairs satisfying these equations.

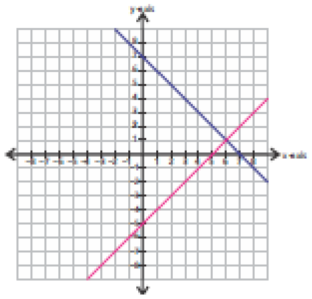
Put $x = 0$ in (i),
$x + y = 7$
$0 + y = 7$
$y = 7$
\ ($x$, y) = (0, 7)
Put y = 0 in (i),
$x + y = 7$
$x + (0) = 7$
$x = 7$
\ ($x$, y) = (7,0)
Put $x = 1$ in (i)
$x + y = 7$
$1 + y = 7$
$y = 7 – 1 = 6$
\ ($x$, y) = (1,6)
So, we get the coordinates of $x + y = 7$ as follows.

| | | | |
|---|---|---|---|
| x | 0 | 7 | 1 |
| y | 7 | 0 | 6 |

Now calculate the values of $x$ and y for the equation $x – y = 5$.
Put $x = 0$ in (ii),
$x - y = 5$
$(0) - y = 5$
$y = -5$
\ ($x$, y) = (0, -5)
Put y = 0 in (ii),
$x – (0) = 5$
$x = 5$
($x$, y) = (5,0)
Put $x = 1$ in (i),
$x - y = 5$
$(1) - y = 5$
$-y = 5 – 1$
$-y = 4$
$y = -4$
\ ($x$, y) = (1,-4)
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | 5 | 1 |
| y | -5 | 0 | - 4 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (6, 1), so this is the solution of the given simultaneous linear equations.

b) $2x$ - y = 4; $x$ - 2y = 7

$2x$ – y = 4 -----------(i)
$x$ – 2y = 7 -----------(ii)
First find the ordered pairs satisfying these equations.

Put $x$ = 0 in (i),
$2x$ – y = 4
2(0) – y = 4
– y = 4
y = – 4
($x$, y) = (0, – 4)
Put y = 0 in (i),
$2x$ – y = 4
$2x$ – 0 = 4
$2x$ = 4
*Dividing 2 on both sides*
$\frac{2x}{2} = \frac{4}{2}$
x = 2
($x$, y) = (2,0)
Put $x$ = 1 in (i)
2*(1)* – y = 4
2 – y = 4
– y = 4 – 2
– y = 2
y = – 2
($x$, y) = (1, – 2)
So, we get the coordinates of $x$ + y = 7 as follows.

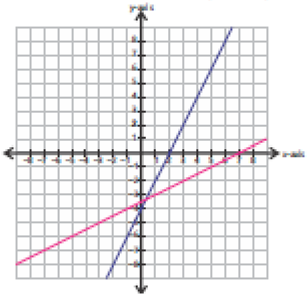
| | | | |
|---|---|---|---|
| x | 0 | 2 | 1 |
| y | – 4 | 0 | – 2 |

Now calculate the values of $x$ and y for the equation $x$ – 2y = 7.
Put $x$ = 0 in (ii),
$x$ – 2y = 7
(0) – 2y = 7
– 2y = 7
Dividing – 2 on both sides
$\frac{-2y}{-2} = \frac{7}{-2}$
($x$, y) = (0, $-\frac{7}{2}$)

Put y = 0 in (ii),
*x* – 2(0) = 7
*x* = 7
(*x*, y) = (7,0)
Put *x* = 1 in (i),
*x* – 2y = 7
(1) - 2y = 7
– 2y = 7 – 1
– 2y = 6
Dividing – 2 on both sides
$\frac{-2y}{-2} = \frac{6}{-2}$
y = – 3
\ (*x*, y) = (1, – 3)
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | 7 | 1 |
| y | $-\frac{7}{2}$ | 0 | – 3 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point $(\frac{1}{3}, -3\frac{1}{3})$, so this is the solution of the given simultaneous linear equations.

c) 3*x* – y = 0; 2*x* + y = 5
3*x* – y = 0 -----------(i)
2*x* + y = 5 -----------(ii)
First find the ordered pairs satisfying these equations.

Put *x* = 0 in (i),
3*x* – y = 0
3(0) – y = 0
y = 0
(*x*, y) = (0, 0)
Put y = 0 in (i),
3*x* – y = 0
*3x* – (0) = 0
*3x* = 0
*x = 0*
(*x*, y) = (0,0)
Put *x* = 1 in (i)
3*x* – y = 0
3(1) – y = 0
– y = – 3
y = 3

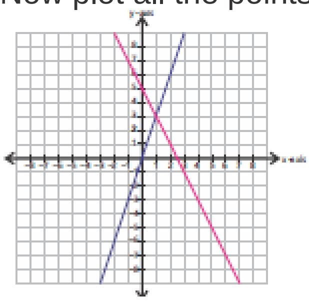
$(x, y) = (1,3)$
So, we get the coordinates of $x + y = 7$ as follows.

| | | | |
|---|---|---|---|
| x | 0 | 0 | 1 |
| y | 0 | 0 | 3 |

Now calculate the values of $x$ and y for the equation $2x + y = 5$.
Put $x = 0$ in (ii),
$2x + y = 5$
$2(0) + y = 5$
$y = 5$
$(x, y) = (0, 5)$
Put $y = 0$ in (ii)
$2x + y = 5$
$2x + (0) = 5$
$2x = 5$
*Dividing 2 on both sides*
$\frac{2x}{2} = \frac{5}{2}$
$(x, y) = (\frac{5}{2},0)$
Put $x = 1$ in (i),
$2x + y = 5$
$2(1) + y = 5$
$y = 5 – 2$
$y = 3$
$(x, y) = (1,3)$
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{5}{2}$ | 1 |
| y | 5 | 0 | 3 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (1, 3), so this is the solution of the given simultaneous linear equations.

d) $3x – 5y = 6$; $2x – 3y = 5$
$3x – 5y = 6$ -----------(i)
$2x – 3y = 5$ -----------(ii)
First find the ordered pairs satisfying these equations.

Put $x = 0$ in (i),
$3x – 5y = 6$
$3(0) – 5y = 6$
$– 5y = 7$
Dividing – 5 on both sides

$\frac{-5y}{-5} = \frac{7}{-5}$
$y = \frac{7}{-5}$
($x$, y) = (0, $\frac{7}{-5}$)
Put y = 0 in (i),
3$x$ – 5y = 6
3$x$ – 5(0) = 6
3$x$ = 6
Dividing 3 on both sides
$\frac{3x}{3} = \frac{6}{3}$
$x$ = 2
($x$, y) = (2,0)
Put $x$ = 1 in (i)
3$x$ – 5y = 6
3(1) – 5y = 6
– 5y = 6 – 3
– 5y = 3
Dividing – 5 on both sides
$\frac{-5y}{-5} = \frac{3}{-5}$
$y = \frac{3}{-5}$
($x$, y) = (1, $\frac{3}{-5}$)
So, we get the coordinates of $x$ + y = 7 as follows.

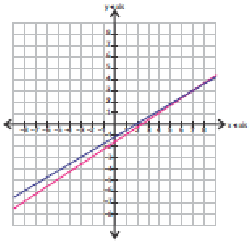
| | | | |
|---|---|---|---|
| x | 0 | 2 | 1 |
| y | $\frac{7}{-5}$ | 0 | $\frac{3}{-5}$ |

Now calculate the values of $x$ and y for the equation 2$x$ – 3y = 5.
Put $x$ = 0 in (ii),
2$x$ – 3y = 5
2(0) – 3y = 5
–3y = 5
$\frac{-3y}{-3} = \frac{5}{-3}$
$y = \frac{5}{-3}$
($x$, y) = (0, $\frac{5}{-3}$)
Put y = 0 in (ii)
2$x$ – 3y = 5
2$x$ – 3(0) = 5
2$x$ = 5
$\frac{2y}{2} = \frac{5}{2}$
$y = \frac{5}{2}$
($x$, y) = ($\frac{5}{2}$, 0)
Put $x$ = 1 in (i),
2$x$ – 3y = 5
2(1) – 3y = 5
– 3y = 5 – 2
– 3y = 3

$\frac{-3y}{-3} = \frac{3}{-3}$
$y = -1$
$(x, y) = (1, -1)$
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{5}{2}$ | 1 |
| y | $\frac{5}{-3}$ | 0 | – 1 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (7, 3), so this is the solution of the given simultaneous linear equations.

e) $\frac{x}{3} + \frac{y}{5} = 2$; $2x - y = 1$
$\frac{x}{3} + \frac{y}{5} = 2$ -----------(i)
$2x - y = 1$ -----------(ii)
First find the ordered pairs satisfying these equations.

Put $x = 0$ in (i),
$\frac{x}{3} + \frac{y}{5} = 2$
$\frac{0}{3} + \frac{y}{5} = 2$
$0 + \frac{y}{5} = 2$
$\frac{y}{5} = 2$
Multiplying 5 on both sides
y = 10
$(x, y) = (0, 10)$
Put y = 0 in (i),
$\frac{x}{3} + \frac{y}{5} = 2$
$\frac{x}{3} + \frac{0}{5} = 2$
$\frac{x}{3} + 0 = 2$
$\frac{x}{3} = 2$
Multiplying 3 on both sides
x = 6
$(x, y) = (6,0)$
Put $x = 1$ in (i)
$\frac{x}{3} + \frac{y}{5} = 2$
$\frac{1}{3} + \frac{y}{5} = 2$
Multiplying by LCM 15 on both sides
$15 \times \frac{1}{3} + 15 \times \frac{y}{5} = 15 \times 2$
$5 + 3y = 30$

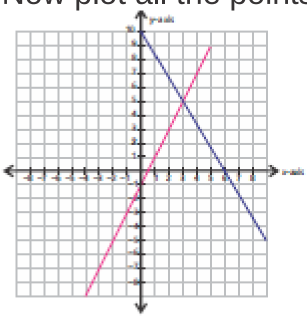
3y = 30 – 5
3y = 25
Dividing 3 on both sides
$\frac{3y}{3} = \frac{25}{3}$
y = $8\frac{1}{3}$
($x$, y) = (1, $8\frac{1}{3}$)
So, we get the coordinates of $\frac{x}{3} + \frac{y}{5} = 2$ as follows.

| | | | |
|---|---|---|---|
| x | 0 | 6 | 1 |
| y | 10 | 0 | $8\frac{1}{3}$ |

Now calculate the values of $x$ and y for the equation 2x – y = 1.
Put $x$ = 0 in (ii),
2x – y = 1
(0) – y = 1
y = – 1
($x$, y) = (0, – 1)
Put y = 0 in (ii)
2x – y = 1
*2x* – (0) = 1
*2x* = 1
*Dividing 2 on both sides*
$\frac{2x}{2} = \frac{1}{2}$
x = $\frac{1}{2}$
($x$, y) = ($\frac{1}{2}$, 0)
Put $x$ = 1 in (i),
2x – y = 1
2(1) – y = 1
– y = 1 – 2
– y = – 1
y = 1
($x$, y) = (1, 1)
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{1}{2}$ | 1 |
| y | – 1 | 0 | 1 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (3, 5), so this is the solution of the given simultaneous linear equations.

f) $\frac{x}{3}$ + 5y = 16; x + y = 6
$\frac{x}{3}$ + 5y = 16 -----------(i)
x + y = 6 -----------(ii)
First find the ordered pairs satisfying these equations.

Put $x$ = 0 in (i),
$\frac{x}{3}$ + 5y = 16
$\frac{0}{3}$ + 5y = 16
0 + 5y = 16
5y = 16
Dividing 5 on both sides
$\frac{5y}{5} = \frac{16}{5}$
y = $3\frac{1}{5}$
($x$, y) = (0, $3\frac{1}{5}$)
Put y = 0 in (i),
$\frac{x}{3}$ + 5y = 16
$\frac{x}{3}$ + 5(0) = 16
$\frac{x}{3}$ + 0 = 16
$\frac{x}{3}$ = 16
Multiplying 3 on both sides
x = 48
($x$, y) = (48, 0)
Put $x$ = 1 in (i)
$\frac{x}{3}$ + 5y = 16
$\frac{1}{3}$ + 5y = 16
Multiplying 3 on both sides, we get
1 + 15y = 48
15y = 48 – 1
15y = 47
Dividing 15 on both sides
$\frac{15y}{15} = \frac{47}{15}$
y = $3\frac{2}{15}$
($x$, y) = (1, $3\frac{2}{15}$)
So, we get the coordinates of $\frac{x}{3}$ + 5y = 16 as follows.

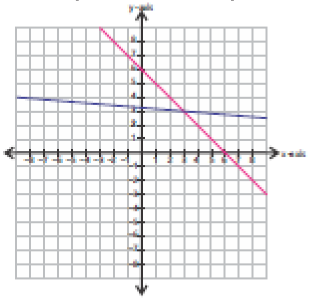
| | | | |
|---|---|---|---|
| x | 0 | 48 | 1 |
| y | $3\frac{1}{5}$ | 0 | $3\frac{2}{15}$ |

Now calculate the values of $x$ and y for the equation x + y = 6.
Put $x$ = 0 in (ii),
x + y = 6
(0) + y = 6
y = 6
($x$, y) = (0, 6)
Put y = 0 in (ii)

x + y = 6
$x$ + (0) = 6
$x$ = 6
($x$, y) = (6,0)
Put $x$ = 1 in (i),
x + y = 6
(1) + y = 6
y = 6 – 1
y = 5
($x$, y) = (1, 5)
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | 6 | 1 |
| y | 6 | 0 | 5 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (3, 3), so this is the solution of the given simultaneous linear equations.

g) 2$x$ + 5y = 12; 4$x$ + 3y = – 4
2$x$ + 5y = 12 -----------(i)
4$x$ + 3y = – 4 -----------(ii)
First find the ordered pairs satisfying these equations.

Put $x$ = 0 in (i),
2$x$ + 5y = 12
2(0) + 5y = 12
5y = 12
Dividing 5 on both sides
$\frac{5y}{5} = \frac{12}{5}$
y = $2\frac{2}{5}$
($x$, y) = (0, $2\frac{2}{5}$)
Put y = 0 in (i),
2$x$ + 5y = 12
2$x$ + 5(0) = 12
*2x = 12*
Dividing 2 on both sides
$\frac{2y}{2} = \frac{12}{2}$
$x$ = 6
($x$, y) = (6,0)
Put $x$ = 1 in (i)
2$x$ + 5y = 12

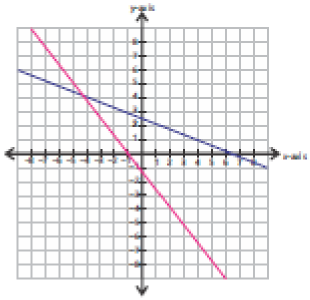
2(1) + 5y = 12
5y = 12 – 2
5y = 10
$\frac{5y}{5} = \frac{10}{5}$
$x$ = 2
($x$, y) = (1, 2)
So, we get the coordinates of 2$x$ + 5y = 12 as follows.

| | | | |
|---|---|---|---|
| x | 0 | 6 | 1 |
| y | $2\frac{2}{5}$ | 0 | 2 |

Now calculate the values of $x$ and y for the equation 4$x$ + 3y = – 4.
Put $x$ = 0 in (ii),
4$x$ + 3y = – 4
4(0) + 3y = – 4
3y = – 4
Dividing 3 on both sides
$\frac{3y}{3} = \frac{-4}{3}$
$x = \frac{-4}{3}$
($x$, y) = (0, $\frac{-4}{3}$)
Put y = 0 in (ii)
4$x$ + 3y = – 4
4$x$ + 3(0) = – 4
4$x$ = – 4
Dividing 4 on both sides
$\frac{4y}{4} = \frac{-4}{4}$
$x$ = – 1
($x$, y) = (– 1,0)
Put $x$ = 1 in (i),
4$x$ + 3y = – 4
4(1) + 3y = – 4
3y = – 4 – 4
3y = – 8
Dividing 3 on both sides
$\frac{3y}{3} = \frac{-8}{3}$
$y = -2\frac{2}{3}$
($x$, y) = (1, $-2\frac{2}{3}$)
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | 5 | 1 |
| y | $\frac{-4}{3}$ | – 1 | $-2\frac{2}{3}$ |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (- 4, 4), so this is the solution of the given simultaneous linear equations.

h) $5x - y = 4;\ 10x - 4y = 6$

$5x - y = 4$ -----------(i)

$10x - 4y = 6$ -----------(ii)

First find the ordered pairs satisfying these equations.

Put $x = 0$ in (i),

$5x - y = 4$

$5(0) - y = 4$

$0 - y = 4$

$y = -4$

$(x, y) = (0, -4)$

Put $y = 0$ in (i),

$5x - y = 4$

$5x - (0) = 4$

$5x = 4$

Dividing 5 on both sides

$\frac{5x}{5} = \frac{4}{5}$

$x = \frac{4}{5}$

$(x, y) = (\frac{4}{5}, 0)$

Put $x = 1$ in (i)

$5x - y = 4$

$5(1) - y = 4$

$-y = 4 - 5$

$-y = -1$

$y = 1$

$(x, y) = (1,1)$

So, we get the coordinates of $5x - y = 4$ as follows.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{4}{5}$ | 1 |
| y | – 4 | 0 | 1 |

Now calculate the values of $x$ and y for the equation $10x - 4y = 6$.

Put $x = 0$ in (ii),

$10x - 4y = 6$

$10(0) - 4y = 6$

$0 - 4y = 6$

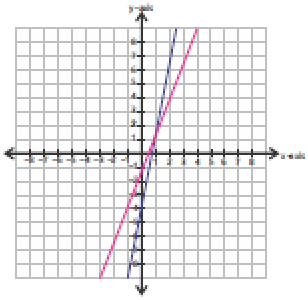
$\frac{-4y}{-4} = \frac{6}{-4}$

$y = \frac{3}{-2}$

$(x, y) = (0, \frac{3}{-2})$
Put y = 0 in (ii)
$10x - 4y = 6$
$10x - 4(0) = 6$
$10x = 6$
Dividing 10 on both sides
$\frac{10x}{10} = \frac{6}{10}$
$x = \frac{3}{5}$
$(x, y) = (\frac{3}{5}, 0)$
Put $x$ = 1 in (i),
$10x - 4y = 6$
$10(1) - 4y = 6$
$-4y = 6 - 10$
$-4y = -4$
Dividing – 4 on both sides
$\frac{-4x}{-4} = \frac{-4}{-4}$
y = 1
$(x, y) = (1, 1)$
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{3}{5}$ | 1 |
| y | $\frac{3}{-2}$ | 0 | 1 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point (1, 1), so this is the solution of the given simultaneous linear equations.

**4. Solve the following simultaneous linear equations using the factorization method.**

a) $5x + 10y = 15$; $3x + 12y = 9$
b) $4x + 8y = 12$; $7x + 14y = 14$
c) $9x + 9y = 18$; $2x + 8y = 4$
d) $3x + 12y = 3$; $6x + 12y = 24$
e) $11x + 33y = 44$; $5x + 15y = 30$
f) $x + y = 2$; $4x + 2y = 4$

**Solution:**

a) $5x + 10y = 15$; $3x + 12y = 9$
$5x + 10y = 15$ ------------ (i)
On LHS, 5 is the common factor, so take 5 as common.
$5(x + 2y) = 15$
Dividing both sides by 5, we get:
$x + 2y = 3$ ------------ (ii)
Now take the second equation.

$3x + 12y = 9$ ------------ (iii)
On LHS, 3 is a common factor:
$3(x + 4y) = 9$
Dividing both sides by 3, we get:
$x + 4y = 3$ ------------ (iv)
Now subtract equations (ii) from (iv),
$x + 4y – (x + 2y) = 3 - 3$
$x + 4y – x – 2y = 0$
$2y = 0$ or $y = 0$
Put the value of $x$ in the equation (ii) we have:
$x + 2 (0) = 3$
$x + 0 = 3$
$x = 3$
So, the solution set is: $(x, y) = (3, 0)$.

b) $4x + 8y = 12; 7x + 14y = 14$
$4x + 8y = 12$ ------------ (i)
On LHS, 4 is the common factor, so take 4 as common.
$4(x + 2y) = 12$
Dividing both sides by 4, we get:
$x + 2y = 3$ ------------ (ii)
Now take the second equation.
$7x + 14y = 14$ ------------ (iii)
On LHS, 7 is a common factor:
$7(x + 2y) = 2$
Dividing both sides by 7, we get:
$x + 2y = 2$ ------------ (iv)
Now subtract equations (iv) from (ii),
$x + 2y – (x + 2y) = 3 - 2$
$0 \neq -1$
So, solution does not exist.

c) $9x + 9y = 18; 2x + 8y = 4$
$9x + 9y = 18$ ------------ (i)
On LHS, 9 is the common factor, so take 9 as common.
$9(x + y) = 18$
Dividing both sides by 9, we get:
$x + y = 2$ ------------ (ii)
Now take the second equation.
$2x + 8y = 4$ ------------ (iii)
On LHS, 2 is a common factor:
$2(x + 4y) = 4$
Dividing both sides by 2, we get:
$x + 4y = 2$ ------------ (iv)
Now subtract equations (ii) from (iv)
$x + 4y – (x + y) = 2 - 2$
$x + 4y – x – y = 0$
$3y = 0$ or $y = 0$
Put the value of $x$ in the equation (ii) we have:
$x + 0 = 2$
$x = 2$
So, the solution set is: $(x, y) = (2, 0)$.

d) $3x + 12y = 3; 6x + 12y = 24$
$3x + 12y = 3$ ------------ (i)
On LHS, 3 is the common factor, so take 3 as common.
$3(x + 4y) = 3$
Dividing both sides by 3, we get:
$x + 4y = 1$ ------------ (ii)
Now take the second equation.
$6x + 12y = 24$ ------------ (iii)
On LHS, 6 is a common factor:
$6(x + 2y) = 24$
Dividing both sides by 6, we get:
$x + 2y = 4$ ------------ (iv)
Now subtract equations (iv) and (ii),
$x + 4y – (x + 2y) = 4 - 1$
$x + 4y – x – 2y = 3$
$2y = 3$ or $y = \frac{3}{2}$
Put the value of $x$ in the equation (ii) we have:
$x + 4\left(\frac{3}{2}\right) = 1$
$x + 2(3) = 1$
$x + 6 = 1$
$x = 1 – 6$
$x = – 5$
So, the solution set is: $(x, y) = \left(– 5, \frac{3}{2}\right)$.

e) $11x + 33y = 44; 5x + 15 y = 30$
$11x + 33y = 44$ ------------ (i)
On LHS, 11 is the common factor, so take 11 as common.
$11(x + 3y) = 44$
Dividing both sides by 11, we get:
$x + 3y = 4$ ------------ (ii)
Now take the second equation.
$5x + 15y = 30$ ------------ (iii)
On LHS, 5 is a common factor:
$5(x +3y) = 30$
Dividing both sides by 5, we get:
$x + 3y = 6$ ------------ (iv)
Now subtract equations (ii) from (iv),
$x + 3y – (x + 3y) = 6 - 4$
$x + 3y – x – 3y = 2$
$0 \neq 2$
So, solution does not exist.

f) $x + y = 2; 4x + 2y = 4$
$x + y = 2$ ------------ (i)
$4x + 2y = 4$ ------------ (ii)
On LHS, 2 is the common factor, so take 2 as common.
$2(2x + y) = 4$
Dividing both sides by 2, we get:
$2x + y = 2$ ------------ (iii)
Now subtract equations (i) from (iii),
$2x + y – (x + y) = 2 - 2$

$2x + y - x - y = 0$
$x = 0$
Put the value of $x$ in the equation (i) we have:
$0 + 2y = 2$
$2y = 2$
Dividing 2 on both sides, we get
$y = 1$
So, the solution set is: $(x, y) = (0, 1)$.

## Learning Check 6.3

1. **The sum of two numbers is 15 and their difference is 3. Find the numbers.**
**Solution:**
Let
The first number = x
The second number = y
According to first condition
$x + y = 15$ ---------------------- (i)
According to second condition
$x - y = 3$ ----------------------- (ii)
Add equations (i) and (ii),
$(x + y) + (x - y) = 15 + 3$
$x + y + x - y = 18$
$2x = 18$
Dividing 2 on both sides
$\frac{2x}{2} = \frac{18}{2}$
$x = 9$
Substitute this value of $x$ in equation (i),
$9 + y = 15$
Subtracting 9 on both sides
$9 + y - 9 = 15 - 9$
$y = 6$
Therefore,
The first number = x = 9
The second number = y = 6
2. **There is a certain pair of numbers such that the sum of twice the first number and three times the second number is 61. However, when we subtract five times the second number from four times the first number, the answer is 45. Find the numbers.**
**Solution:**
Let
The first number = x
The second number = y
According to first condition
$2x + 3y = 61$ ---------------------- (i)
According to second condition
$4x - 5y = 45$ ---------------------- (ii)

Multiplying equation (i) by 2
4x – 6y = 122 ---------------------- (iii)
Subtracting equations (iii) and (ii)
(4x – 5y) – (4x – 6y) = 45 – 122
4x – 5y – 4x + 6y = 45 – 122
*y* = – 77
Substitute this value of *y* in equation (i),
2x + 3y = 61
2x + 3 (– 77) = 61
2x – 231 = 61
2x = 61 + 231
2x = 292
Dividing 2 on both sides
$\frac{2x}{2} = \frac{292}{2}$
x = 146
Therefore,
The first number = x = 146
The second number = y = – 77

**3. Zara bought five pencils and two sharpeners for Rs 62, while Madeeha bought eight pencils and two sharpeners for Rs 92. Find the price of a pencil and a sharpener.**
**Solution:**
Let
The price of a pencil = Rs. x
The price of a sharpeners = Rs. y
According to first condition
5x + 2y = 62 ---------------------- (i)
According to second condition
8x + 2y = 92 ----------------------- (ii)
Subtracting equations (i) from (ii)
(8x + 2y) – (5x + 2y) = 92 – 62
8x + 2y – 5x – 2y = 30
*3x* = 30
*Dividing 3 on both sides*
$\frac{3x}{3} = \frac{30}{3}$
x = 10
Substitute this value of *x* in equation (i),
5x + 2y = 62
2 (10) + 2y = 62
20 + 2y = 62
2y = 62 – 20
2y = 42
Dividing 2 on both sides
$\frac{2y}{2} = \frac{42}{2}$
y = 21
Therefore,

The price of a pencil = x = Rs. 10
The price of a sharpener = y = Rs. 21

4. **Ahmad bought 6 chairs and 2 tables for Rs 6300, while Ali bought 2 chairs and one table of the same quality from the same shop for Rs 2600. Find the price of a chair and a table.**
**Solution:**
Let
The price of a chair = Rs. x
The price of a table = Rs. y
According to first condition
$6x + 2y = 6300$ ---------------------- (i)
On LHS, 2 is the common factor, so take 2 as common.
$2(3x + y) = 6300$
Dividing both sides by 2, we get:
$3x + y = 3150$ ------------------------ (ii)
According to second condition
$2x + y = 2600$ ----------------------- (iii)
Subtracting equations (iii) from (ii)
$(3x + y) – (2x + y) = 3150 – 2600$
$3x + y – 2x – y = 550$
$x = 550$
Substitute this value of $x$ in equation (iii),
$2x + y = 2600$
$2 (550) + y = 2600$
$1100 + y = 2600$
$y = 2600 – 1100$
$y = 1500$
Therefore,
The price of a chair = x = Rs. 550
The price of a table = y = Rs. 1500

5. **A shopkeeper has prepared bags of potatoes and carrots. Bag 'A' contains 5 kg of potatoes and 2 kg of carrots, while bag 'B' contains 7 kg of potatoes and 3 kg of carrots. If the cost of bag A is Rs 850 and that of bag B is Rs 1200, find the price of 1 kg of potatoes and 1 kg of carrots.**
**Solution:**
Let
The price of 1 kg potato = Rs. x
The price of 1 kg carrot = Rs. y
According to first condition
$5x + 2y = 850$ ---------------------- (i)
According to second condition
$7x + 3y = 1200$ --------------------- (ii)
Multiplying equation (i) by 3 and equation (ii) by 2, we get
$15x + 6y = 2550$ -------------------- (iii)
$14x + 6y = 2400$ -------------------- (iv)
Subtracting equations (iv) from (iii)
$(15x + 6y) – (14x + 6y) = 2550 – 2400$
$15x + 6y – 14x – 6y = 150$

*x* = 150
Substitute this value of *x* in equation (i),
5x + 2y = 850
5 (150) + 2y = 850
750 + 2y = 850
2y = 850 – 750
2y = 100
Dividing 2 on both sides
$\frac{2y}{2} = \frac{100}{2}$
y = 50
Therefore,
The price of 1 kg potato = x = Rs. 150
The price of 1 kg carrot = y = Rs. 50

**6. The father's current age is four times that of his son. After five years, the father's age will be three times that of his son. Find the present ages of the son and the father.**

**Solution:**
Let
Son's age = x years
Father's age = y years
According to first condition
y = 4x ------------------ (i)
After 5 years
Son's age = (x + 5) years
Father's age = (y + 5) years
According to second condition
(y + 5) = 3 (x + 5) --------------- (ii)
Substitute y = 4x in equation (ii)
(4x + 5) = 3 (x + 5)
4x + 5 = 3x + 15
4x – 3x = 15 – 5
x = 10
substitute x = 10 in equation (i)
y = 4 (10)
y = 40
Therefore,
Son's age = x = 10 years
Father's age = y = 40 years

## Learning Check 6.4

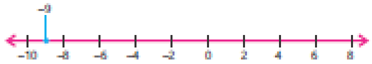
**7. Solve the following linear inequalities. Also represent the solution for each on a number line.**

| | | | |
|---|---|---|---|
| a) $-5x - 2 > 13$ | b) $11x \leq -99$ | c) $4x - 6 > 24$ | d) $3x + 4 \leq 22$ |
| e) $9x > 54$ | f) $4(x + 4) \geq 16$ | g) $13x - 2 \leq 28$ | h) $-6x > 48$ |

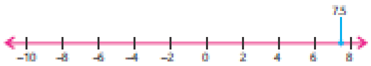
**Solution:**
a) $-5x - 2 > 13$
First, isolate the variable.
Add 2 to both sides.

– $5x$ – 2 > 13
– 5x – 2 + 2 > 13 + 2
– 5x > 15
Dividing both sides by – 5
$\frac{-5x}{-5} < \frac{15}{-5}$
$x$ < – 3
So, $x$ < – 3 is the solution to this inequality. We can say that the solution to this inequality is "all the numbers less than – 3."
Let's show this solution on a number line.

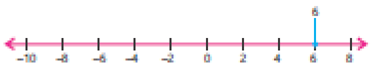

b) $11x \leq -99$
First, isolate the variable.
Divide both sides by 11.
$\frac{11x}{11} \leq \frac{-99}{11}$
x ≤ – 9
So, x ≤ – 9 is the solution to this inequality. We can say that
the solution to this inequality is "all the numbers less than or equal to – 9."
Let's show this solution on a number line.




c) $4x$ – 6 > 24
First, isolate the variable.
Add 6 to both sides.
$4x$ – 6 + 6 > 24 + 6
4x > 32
Dividing both sides by 4
$\frac{4x}{4} > \frac{30}{4}$
$x$ > 7.5
So, $x$ > 7.5 is the solution to this inequality. We can say that the solution to this inequality is "all the numbers greater than 7.5."
Let's show this solution on a number line.




d) $3x + 4 \leq 22$
First, isolate the variable.
Subtract 4 from both sides.
$3x + 4 - 4 \leq 22 - 4$
3x ≤ 18
Divide both sides by 3
$\frac{3x}{3} \leq \frac{18}{3}$
x ≤ 6
So, x ≤ 6 is the solution to this inequality. We can say that the solution to this inequality is "all the numbers less than or equal to 6."
Let's show this solution on a number line.



e) $9x > 54$

First, isolate the variable.

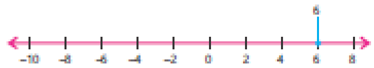
Dividing both sides by 9

$\frac{9x}{9} > \frac{54}{9}$

$x > 6$

So, $x > 6$ is the solution to this inequality. We can say that the solution to this inequality is “all the numbers greater than 6.”

Let's show this solution on a number line.




f) $4(x + 4) \geq 16$

First, isolate the variable.

Dividing both sides by 4

$\frac{4\,(x+4)}{4} \geq \frac{16}{4}$

$x + 4 \geq 4$

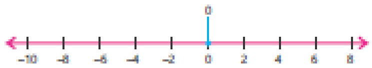
Subtract to both sides by 4.

$X + 4 - 4 \geq 4 - 4$

$x \geq 0$

So, $x \geq 0$ is the solution to this inequality. We can say that the solution to this inequality is “all the numbers greater than or equal to 0.”

Let's show this solution on a number line.




g) $13x - 2 \leq 28$

First, isolate the variable.

Add 2 from both sides.

$13x - 2 + 2 \leq 28 + 2$

$13x \leq 30$

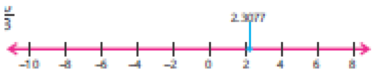
Divide both sides by 13

$\frac{13x}{13} \leq \frac{30}{13}$

$x \leq 2.3077$

So, $x \leq 2.3077$ is the solution to this inequality. We can say that the solution to this inequality is “all the numbers less than or equal to $x \leq$ 2.3077.”

Let's show this solution on a number line.




h) $-6x > 48$

First, isolate the variable.

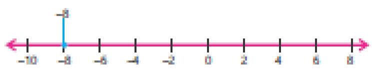
Dividing both sides by – 6

$\frac{-6x}{-6} > \frac{48}{-6}$

$x < -8$

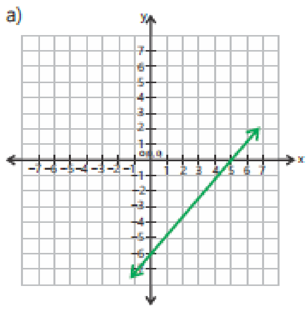
So, $x < -8$ is the solution to this inequality. We can say that the solution to this inequality is “all the numbers less than – 8.”

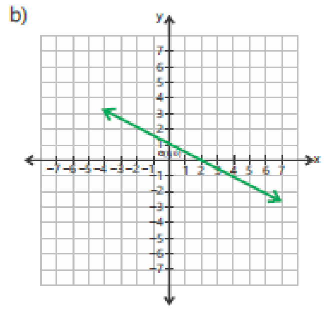
Let's show this solution on a number line.

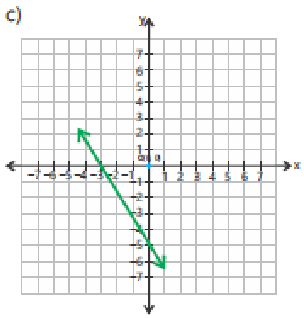


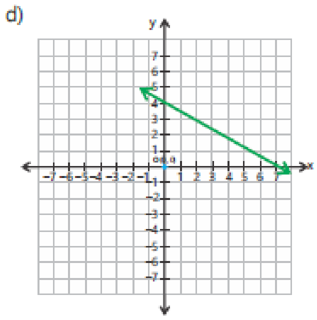
## Learning Check 6.5

**1. Find and interpret the gradient of each straight line. Also describe the y-intercept.**



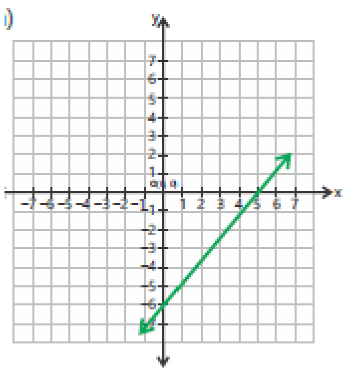







**Solution:**

a)



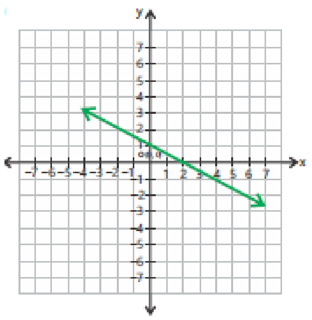
The gradient is positive as the line is going upward from left to right.

Gradient = $\frac{Vertical\ change}{Horizontal\ change} = \frac{6}{5}$

The line is crossing the y-axis at (0, – 6). So, the y-intercept is – 6.

The line is crossing the *x*-axis at (5, 0). So, the *x*-intercept is 5.

b)



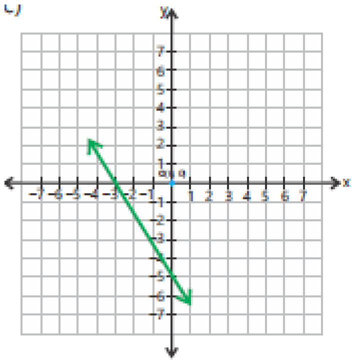
The gradient is negative as the line is going downward from left to right.

Gradient = $\frac{Vertical\ change}{Horizontal\ change} = -\frac{1}{2}$

The line is crossing the y-axis at (0,1). So, the y-intercept is 1.

The line is crossing the *x*-axis at (2, 0). So, the *x*-intercept is 2.

c)



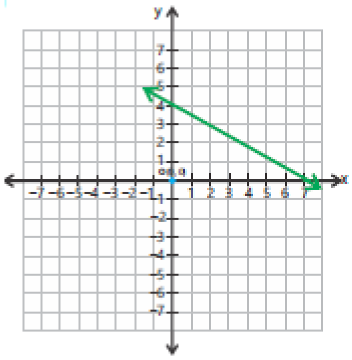
The gradient is negative as the line is going downward from left to right.

Gradient = $\frac{Vertical\ change}{Horizontal\ change} = -\frac{5}{3}$

The line is crossing the y-axis at (0, – 5). So, the y-intercept is – 5.

The line is crossing the *x*-axis at (– 3, 0). So, the *x*-intercept is – 3.

d)



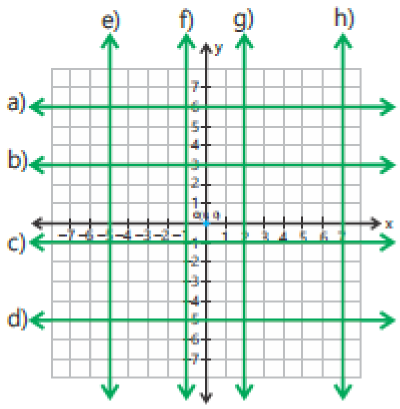
The gradient is negative as the line is going downward from left to right.

Gradient = $\frac{Vertical\ change}{Horizontal\ change} = -\frac{4}{7}$

The line is crossing the y-axis at (0, 4). So, the y-intercept is 4.

The line is crossing the *x*-axis at (7, 0). So, the *x*-intercept is 7.

**2. Write the equations for the following vertical and horizontal lines.**




**Solution:**

a) The equation of a horizontal line is y = *6*. In a horizontal line, the y-value is constant. The horizontal line goes through *6* on the y-axis.

b) The equation of a horizontal line is y = *3*. In a horizontal line, the y-value is constant. The horizontal line goes through *3* on the y-axis.

c) The equation of a horizontal line is y = – 1. In a horizontal line, the y-value is constant. The horizontal line goes through – 1 on the y-axis.

d) The equation of a horizontal line is y = – 5. In a horizontal line, the y-value is constant. The horizontal line goes through – 5 on the y-axis.

e) The equation of *a* vertical line is *x* = – 5. In a vertical line, the *x*-value is constant. This graph is showing an equation of a vertical line.

f) The equation of *a* vertical line is *x* = – 1. In a vertical line, the *x*-value is constant. This graph is showing an equation of a vertical line.

g) The equation of *a* vertical line is *x* = *2*. In a vertical line, the *x*-value is constant. This graph is showing an equation of a vertical line.

h) The equation of *a* vertical line is *x* = *7*. In a vertical line, the *x*-value is constant. This graph is showing an equation of a vertical line.

## Learning Check 6.6

1. **Find the value of *x* or y and complete the tables.**

a)

| $x$ | | | | |
|---|---|---|---|---|
| $y = 3x - 5$ | | | | |

b)

| $x$ | | | | |
|---|---|---|---|---|
| $y = 2x + 3$ | | | | |

c)

| $x$ | | | | |
|---|---|---|---|---|
| $y = -x + 2$ | | | | |

**Solution:**

a) y = 3x – 5

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.
Substitute x = – 2, – 1, 0, 1, 2
y = 3 (– 2) – 5 = – 6 – 5 = – 11
y = 3 (– 1) – 5 = – 3 – 5 = – 8
y = 3 (0) – 5 = 0 – 5 = – 5
y = 3 (1) – 5 = 3 – 5 = – 2
y = 3 (2) – 5 = 6 – 5 = 1

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = 3x – 5 | – 11 | – 8 | – 5 | – 2 | 1 |

b) y = 2x + 3
Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.
Substitute x = – 2, – 1, 0, 1, 2
y = 2 (– 2) + 3 = – 4 + 3 = – 1
y = 2 (– 1) + 3 = – 2 + 3 = 1
y = 2 (0) + 3 = 0 + 3 = 3
y = 2 (1) + 3 = 2 + 3 = 5
y = 2 (2) + 3 = 4 + 3 = 7

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = 2x + 3 | – 1 | 1 | 3 | 5 | 7 |

c) y = – x + 2
Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.
Substitute x = – 2, – 1, 0, 1, 2
y = – 2 + 2 = 0
y = – 1 + 2 = 1
y = 0 + 2 = 2
y = 1 + 2 = 3
y = 2 + 2 = 4

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = – x + 2 | 0 | 1 | 2 | 3 | 4 |

**2. Plot the graphs for the following equations of straight lines.**

a) y = x b) y = 3x c) y = $\frac{1}{2}x$ d) y = $\frac{5}{2}x$
e) y = $-x$ f) y = $-2x$ g) y = $-3x$

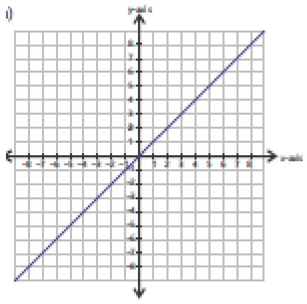
**Solution:**

a) y = x

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | – 2 | – 1 | 0 | 1 | 2 |

We can see that the graph is a straight line passing through the origin (0, 0).

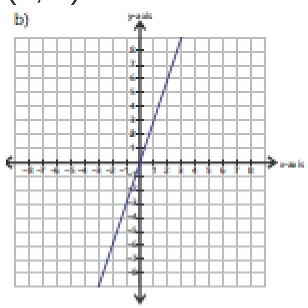


b) y = 3x

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | – 6 | – 3 | 0 | 3 | 6 |

We can see that the graph is a straight line passing through the origin (0, 0).

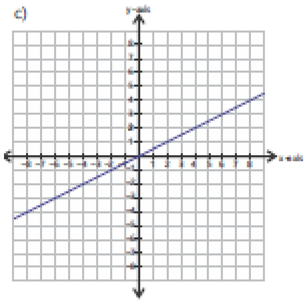


c) y = $\frac{1}{2}x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 4 | – 2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| y = x | – 2 | – 1 | 0 | 1 | 2 |

We can see that the graph is a straight line passing through the origin (0, 0).

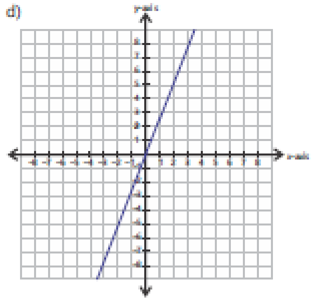



d) y = $\frac{5}{2}x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 4 | – 2 | 0 | 2 | 4 |
|---|---|---|---|---|---|
| y = x | – 10 | – 5 | 0 | 5 | 10 |

We can see that the graph is a straight line passing through the origin (0, 0).

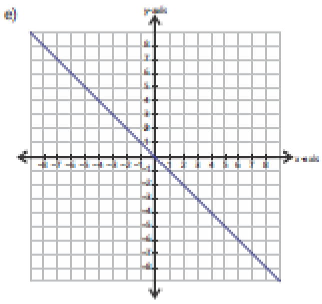



e) y = $-x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | 2 | 1 | 0 | –1 | –2 |

We can see that the graph is a straight line passing through the origin (0, 0).

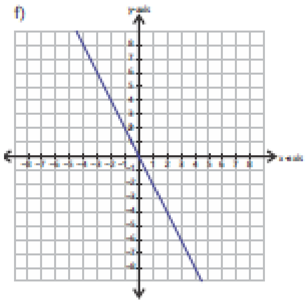



f) y = $-2x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | 4 | 2 | 0 | – 2 | – 4 |

We can see that the graph is a straight line passing through the origin (0, 0).

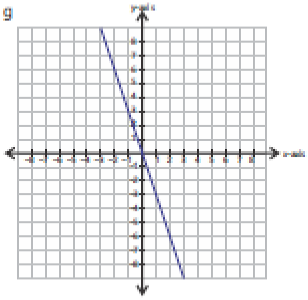


g) y = $-3x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | 6 | 3 | 0 | – 3 | – 6 |

We can see that the graph is a straight line passing through the origin (0, 0).



**3. Plot the graphs for the following equations of straight lines.**

a) y = 2*x* + 1 b) y = 4*x* – 2 c) y = 3*x* – 1 d) y = *x* + 2

e) $y = \frac{1}{2}x - \frac{5}{2}$ f) y = 4x – 3

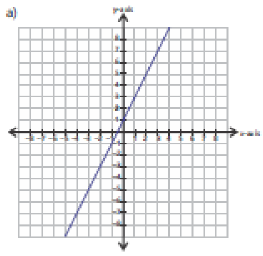
**Solution:**

a) y = 2*x* + 1

Let's find some ordered pairs that satisfy this equation by entering a few *x* or y values and getting the corresponding y or *x* values.

| | | | |
|---|---|---|---|
| x | 0 | $-\frac{1}{2}$ | 1 |
| y | 1 | 0 | 3 |

Plot these ordered pairs to get the required graph of the equation.



Choose any two points on the line, draw a right-angled triangle, and calculate the gradient.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{2}{1}$ = 2

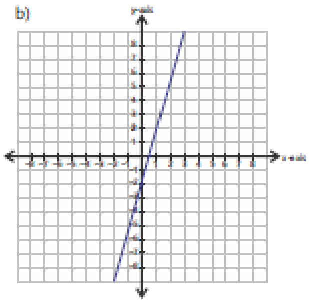
Now find the y-intercept. When $x = 0$, the value of y is 1. This line cuts the y-axis at y = 1. So, the y intercept is 1.
Observe the equation again.
$y = 2x + (1)$
We can see that in this form, 2 is representing the gradient (slope) and 1 is representing the y-intercept of the straight line.
In general, when the equation of a straight line is in the form $y = mx + c$, m represents the gradient (or slope), and c represents the y-intercept of the line.

b) $y = 4x - 2$

Let's find some ordered pairs that satisfy this equation by entering a few $x$ or y values and getting the corresponding y or $x$ values.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{1}{2}$ | 1 |
| y | $-2$ | 0 | 2 |

Plot these ordered pairs to get the required graph of the equation.




Choose any two points on the line, draw a right-angled triangle, and calculate the gradient.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{4}{1} = 4$

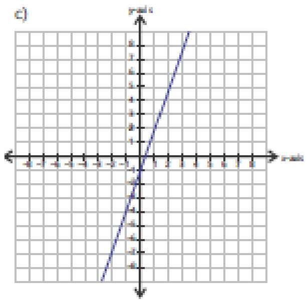
Now find the y-intercept. When $x = 0$, the value of y is $-2$. This line cuts the y-axis at $y = -2$. So, the y intercept is -2.
Observe the equation again.
$y = 4x + (-2)$
We can see that in this form, 4 is representing the gradient (slope) and $-2$ is representing the y-intercept of the straight line.
In general, when the equation of a straight line is in the form $y = mx + c$, m represents the gradient (or slope), and c represents the y-intercept of the line.

c) $y = 3x - 1$

Let's find some ordered pairs that satisfy this equation by entering a few $x$ or y values and getting the corresponding y or $x$ values.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{1}{3}$ | 1 |
| y | $-1$ | 0 | 2 |

Plot these ordered pairs to get the required graph of the equation.



Choose any two points on the line, draw a right-angled triangle, and calculate the gradient.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{3}{1}$ = 3

Now find the y-intercept. When *x* = 0, the value of y is $-1$. This line cuts the y-axis at y = - 1. So, the y intercept is -1.

Observe the equation again.

y = 3*x* +(- 1)

We can see that in this form, 3 is representing the gradient (slope) and - 1 is representing the y-intercept of the straight line.

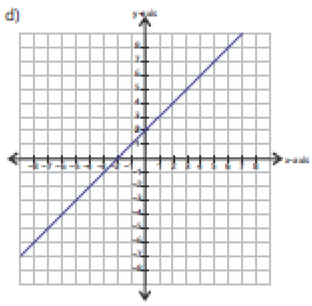
In general, when the equation of a straight line is in the form y = m*x* + c, m represents the gradient (or slope), and c represents the y-intercept of the line.

d) y = *x* + 2

Let's find some ordered pairs that satisfy this equation by entering a few *x* or y values and getting the corresponding y or *x* values.

| | | | |
|---|---|---|---|
| x | 0 | $-2$ | 1 |
| y | 2 | 0 | 3 |

Plot these ordered pairs to get the required graph of the equation.



Choose any two points on the line, draw a right-angled triangle, and calculate the gradient.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{1}{1}$ = 1

Now find the y-intercept. When *x* = 0, the value of y is 2. This line cuts the y-axis at y = 2. So, the y intercept is 2.

Observe the equation again.

y = *x* +(2)

We can see that in this form, 1 is representing the gradient (slope) and 2 is representing the y-intercept of the straight line.

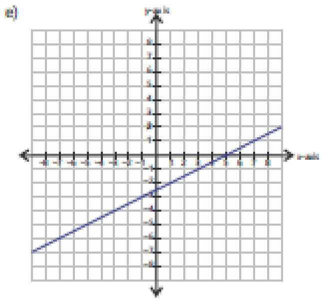
In general, when the equation of a straight line is in the form y = m$x$ + c, m represents the gradient (or slope), and c represents the y-intercept of the line.

e) $y = \frac{1}{2}x - \frac{5}{2}$

Let's find some ordered pairs that satisfy this equation by entering a few $x$ or y values and getting the corresponding y or $x$ values.

| | | | |
|---|---|---|---|
| x | 0 | 5 | 1 |
| y | $-\frac{5}{2}$ | 0 | – 2 |

Plot these ordered pairs to get the required graph of the equation.



Choose any two points on the line, draw a right-angled triangle, and calculate the gradient.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{1}{2}$

Now find the y-intercept. When $x$ = 0, the value of y is $-\frac{5}{2}$. This line cuts the y-axis at y = $-\frac{5}{2}$. So, the y intercept is $-\frac{5}{2}$.

Observe the equation again.

y = $\frac{1}{2}x + (-\frac{5}{2})$

We can see that in this form, $\frac{1}{2}$ is representing the gradient (slope) and $\frac{5}{2}$ is representing the y-intercept of the straight line.

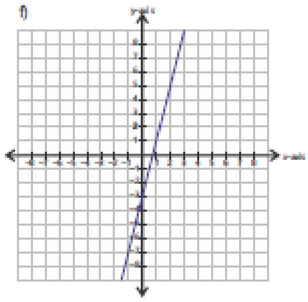
In general, when the equation of a straight line is in the form y = m$x$ + c, m represents the gradient (or slope), and c represents the y-intercept of the line.

f) y = 4x – 3

Let's find some ordered pairs that satisfy this equation by entering a few $x$ or y values and getting the corresponding y or $x$ values.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{3}{4}$ | 1 |
| y | – 3 | 0 | 1 |

Plot these ordered pairs to get the required graph of the equation.



Choose any two points on the line, draw a right-angled triangle, and calculate the gradient.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{4}{1} = 4$

Now find the y-intercept. When *x* = 0, the value of y is – 3. This line cuts the y-axis at y = – 3. So, the y intercept is -3.

Observe the equation again.

y = 4*x* +(– 3)

We can see that in this form, 4 is representing the gradient (slope) and – 3 is representing the y-intercept of the straight line.

In general, when the equation of a straight line is in the form y = m*x* + c, m represents the gradient (or slope), and c represents the y-intercept of the line.

4. **Identify the gradient and y-intercept for the following equations, and then plot the graphs and verify the value of the gradient and the y-intercept. (If the equation is not in slope intercept form, first write it in slope intercept form and then solve it.**

a) *x* – y = 5 b) 2*x* – y = – 4 c) – 4*x* – y = – 2

d) y = 3*x* + 3 e) y + *x* = 2 f) y = – 2*x* – 3

**Solution:**

a) *x* – y = 5

x – 5 = y

y = x – 5

To find the gradient and y-intercept, compare the given equation with the standard slope intercept form of the equation, i.e., y = m*x* + c.

y = x – 5

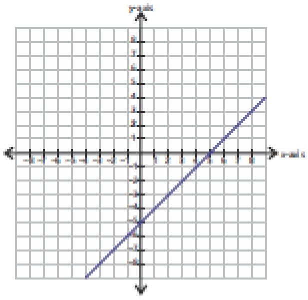
By comparison, we can see that m = 1 and the y-intercept = – 5.

Now let's draw a graph for this equation.

For this, let's find some ordered pairs satisfying this equation by putting a few values of *x* or y and getting the corresponding values of y or *x*.

| x | 0 | 5 | 1 |
|---|---|---|---|
| y | – 5 | 0 | – 4 |

Plot these ordered pairs to get the required graph of the equation.



To verify the gradient and y-intercepts, choose any two points on the line and draw a right-angled triangle, then calculate the gradient, e.g., (5,0) (0, – 5) etc.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{-5}{-5} = 1$

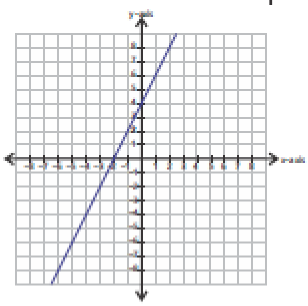
Now find the y-intercept. When $x$ = 0, the value of y is – 5. This line cuts the y-axis at y = –5. So, the y-intercept is –5. Hence verified.

b) $2x - y = -4$
$2x + 4 = y$
$y = 2x + 4$

To find the gradient and y-intercept, compare the given equation with the standard slope intercept form of the equation, i.e., y = m$x$ + c.
y = 2x + 4
By comparison, we can see that m = 2 and the y-intercept = 4.
Now let's draw a graph for this equation.
For this, let's find some ordered pairs satisfying this equation by putting a few values of $x$ or y and getting the corresponding values of y or $x$.

| | | | |
|---|---|---|---|
| x | 0 | – 2 | 1 |
| y | 4 | 0 | 6 |

Plot these ordered pairs to get the required graph of the equation.



To verify the gradient and y-intercepts, choose any two points on the line and draw a right-angled triangle, then calculate the gradient, e.g., (– 2,0), (0, 4) etc.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{4}{2} = 2$

Now find the y-intercept. When $x$ = 0, the value of y is 4. This line cuts the y-axis at y = 4. So, the y-intercept is 4. Hence verified.

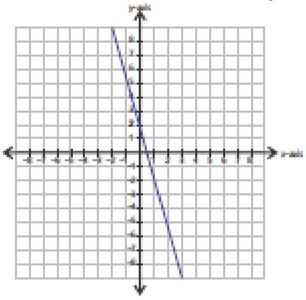
c) $-4x - y = -2$
$-y = 4x - 2$
$y = -4x + 2$

To find the gradient and y-intercept, compare the given equation with the standard slope intercept form of the equation, i.e., y = m$x$ + c.

y = – 4x + 2
By comparison, we can see that m = – 4 and the y-intercept = 2.
Now let's draw a graph for this equation.
For this, let's find some ordered pairs satisfying this equation by putting a few values of *x* or y and getting the corresponding values of y or *x*.

| | | | |
|---|---|---|---|
| x | 0 | $\frac{1}{2}$ | 1 |
| y | 2 | 0 | – 2 |

Plot these ordered pairs to get the required graph of the equation.



To verify the gradient and y-intercepts, choose any two points on the line and draw a right-angled triangle, then calculate the gradient, e.g., ($\frac{1}{2}$,0) (0, 2) etc.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{2}{-\frac{1}{2}} = -4$

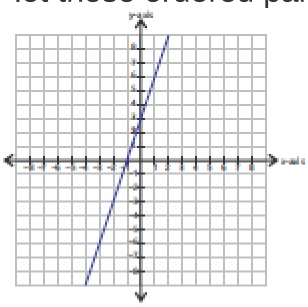
Now find the y-intercept. When *x* = 0, the value of y is 2. This line cuts the y-axis at y = 2. So, the y-intercept is 2. Hence verified.

d) y = 3*x* + 3

To find the gradient and y-intercept, compare the given equation with the standard slope intercept form of the equation, i.e., y = m*x* + c.
y = 3*x* + 3
By comparison, we can see that m = 3 and the y-intercept = 3.
Now let's draw a graph for this equation.
For this, let's find some ordered pairs satisfying this equation by putting a few values of *x* or y and getting the corresponding values of y or *x*.

| | | | |
|---|---|---|---|
| x | 0 | – 1 | 1 |
| y | 3 | 0 | 6 |

Plot these ordered pairs to get the required graph of the equation.



To verify the gradient and y-intercepts, choose any two points on the line and draw a right-angled triangle, then calculate the gradient, e.g., (– 1,0) (0, 3) etc.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{3}{1} = 3$

Now find the y-intercept. When $x = 0$, the value of y is 3. This line cuts the y-axis at y = 3. So, the y-intercept is 3. Hence verified.

e) $y + x = 2$

$y = 2 – x$

To find the gradient and y-intercept, compare the given equation with the standard slope intercept form of the equation, i.e., $y = mx + c$.

$y = – x + 2$

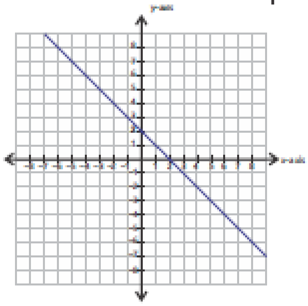
By comparison, we can see that m = – 1 and the y-intercept = 2.

Now let's draw a graph for this equation.

For this, let's find some ordered pairs satisfying this equation by putting a few values of $x$ or y and getting the corresponding values of y or $x$.

| | | | |
|---|---|---|---|
| x | 0 | 2 | 1 |
| y | 2 | 0 | 1 |

Plot these ordered pairs to get the required graph of the equation.



To verify the gradient and y-intercepts, choose any two points on the line and draw a right-angled triangle, then calculate the gradient, e.g., (2,0) (0, 2) etc.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{2}{-2} = – 1$

Now find the y-intercept. When $x = 0$, the value of y is 2. This line cuts the y-axis at y = 2. So, the y-intercept is 2. Hence verified.

f) $y = – 2x – 3$

To find the gradient and y-intercept, compare the given equation with the standard slope intercept form of the equation, i.e., $y = mx + c$.

$y = – 2x – 3$

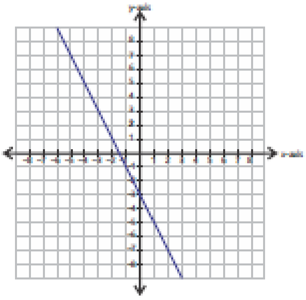
By comparison, we can see that m = – 2 and the y-intercept = – 3.

Now let's draw a graph for this equation.

For this, let's find some ordered pairs satisfying this equation by putting a few values of $x$ or y and getting the corresponding values of y or $x$.

| | | | |
|---|---|---|---|
| x | 0 | $-\frac{3}{2}$ | 1 |
| y | – 3 | 0 | – 5 |

Plot these ordered pairs to get the required graph of the equation.



To verify the gradient and y-intercepts, choose any two points on the line and draw a right-angled triangle, then calculate the gradient, e.g., $(-\frac{3}{2},0)$ (0, – 3) etc.

Gradient = $\frac{Vertical\ change}{\text{Horizontal change}} = \frac{-3}{\frac{3}{2}} = -2$

Now find the y-intercept. When *x* = 0, the value of y is – 3. This line cuts the y-axis at y = –3. So, the y-intercept is –3. Hence verified.

**5. Find the slope intercept form of each equation of the straight lines described below.**

a) gradient 2, y-intercept –1
b) gradient −2, y-intercept 2
c) gradient 2, y-intercept –1
d) gradient $-\frac{5}{2}$, y-intercept −2
e) gradient $\frac{1}{2}$, passing through the origin
f) gradient 3, passing through (0, 0)

**Solution:**

a) gradient 2, y-intercept –1
Slope-intercept form of equation is:
y = mx + c ----------- (i)
Where m represents the gradient and c represents the y-intercept.
So, m = 2 and c = – 1
Substituting m = 2 and c = – 1 in (i)
y = (2)x + (– 1)
y = 2x – 1
Therefore, slope intercept form is y = 2x – 1.

b) gradient −2, y-intercept 2
Slope-intercept form of equation is:
y = mx + c ----------- (i)
Where m represents the gradient (slope) and c represents the y-intercept.
So, m = − 2 and c = 2
Substituting m = − 2 and c = 2 in (i)
y = (− 2) x + (2)
y = − 2x + 2
Therefore, slope intercept form is y = − 2x + 2.

c) gradient 2, y-intercept –1
Slope-intercept form of equation is:
y = mx + c ----------- (i)
Where m represents the gradient (slope) and c represents the y-intercept.
So, m = 2 and c = – 1
Substituting m = 2 and c = – 1 in (i)

$y = (2)x + (-1)$
$y = 2x - 1$
Therefore, slope intercept form is $y = 2x - 1$.

d) gradient $-\frac{5}{2}$, y-intercept $-2$
Slope-intercept form of equation is:
$y = mx + c$ ----------- (i)
Where m represents the gradient (slope) and c represents the y-intercept.
So, $m = -\frac{5}{2}$ and $c = -2$
Substituting $m = -\frac{5}{2}$ and $c = -2$ in (i)
$y = (-\frac{5}{2})x + (-2)$
$y = -\frac{5}{2}x - 2$
Therefore, slope intercept form is $y = -\frac{5}{2}x - 2$.

e) gradient $\frac{1}{2}$, passing through the origin
Slope-intercept form of equation is:
$y = mx + c$ ----------- (i)
Where m represents the gradient (slope) and c represents the y-intercept.
So, $m = \frac{1}{2}$ and $c = 0$
Substituting $m = \frac{1}{2}$ and $c = 0$ in (i)
$y = (\frac{1}{2})x + (0)$
$y = \frac{1}{2}x$
Therefore, slope intercept form is $y = \frac{1}{2}x$ .

f) gradient 3, passing through (0, 0)
Slope-intercept form of equation is:
$y = mx + c$ ----------- (i)
Where m represents the gradient (slope) and c represents the y-intercept.
So, $m = 3$ and $c = 0$
Substituting $m = 3$ and $c = 0$ in (i)
$y = (3)x + (0)$
$y = 3x$
Therefore, slope intercept form is $y = 3x$.

## Unit 6 – Linear Equations and their Graphs

### Unit Check 6

**1. Encircle the correct option.**

a) The gradient is positive when:

i) The line moves downward from left to right.
ii) The line moves downward from right to left.
iii**) The line moves upward from left to right.**
iv) The line moves upward from right to left.

b) If a straight line is crossing the y-axis at (0, -2), the y-intercept is:

| | | | |
|---|---|---|---|
| i) 0 | ii) 2 | **iii)** – **2** | iv) –1 |

c) To find the x-intercept of a line, we put:

| | | | |
|---|---|---|---|
| i) x = 1 | **ii) y = 0** | iii) x = 0 | iv) y = 1 |

d) Which of these lines passes through the origin?

| | | | |
|---|---|---|---|
| i) y = 2*x* + 6 | **ii) y = 3*x*** | iii) y = 3*x* – 4 | iv) y = 4*x* – 1 |

e) The equation of a straight line is:

| | | | |
|---|---|---|---|
| i) $x = y^2$ | **ii) y = m*x* + c** | iii) $y^2 = \frac{x}{2} + c$ | iv) $x^2 = cy$ |

**2. Fill in the blanks.**

a) The measure of steepness is known as **gradient.**
b) If the horizontal change is 3 and the gradient is 2, then the vertical change is
**6.**
c) In a horizontal line, the **y** value is constant.
d) The **gradient** of a line is undefined when the line is vertical.
e) The equation of a line of type y=mx always passes through the **origin.**
f) The solution of the inequality –5x – 3 < 7 is **x > – 2.**

**3. Find the solution of each simultaneous linear equation using the elimination method.**

a) $3x + 5y = 2$, $x + 3y = 8$ b) $y = 5x + 3$, $3x = 5y + 10$

**Solution:**

a) $3x + 5y = 2$, $x + 3y = 8$

$3x + 5y = 2$ ——— (1)
$x + 3y = 8$ ——— (2)
*Multiplying equation (2) by 3*
$3x + 9y = 24$ ——— (3)
Subtract equation (1) from (3),
$(3x + 9y) – (3x + 5y) = 24 – 2$
$3x + 9y – 3x – 5y = 22$
$4y = 22$
Dividing 2 on both sides
$\frac{4x}{4} = \frac{22}{4}$
$x = 5\frac{1}{2}$
Substitute this value of *x* in equation (1),
$3\ (\frac{22}{4}) + 5y = 2$
$3\ (\frac{11}{2}) + 5y = 2$

$\frac{33}{2}$ + 5y = 2
Multiplying 2 with each term, we get
33 + 10y = 4
10y = 4 – 33
10y = – 29
Dividing 10 o both sides, we get
y = $-\frac{29}{10}$
y = $-2\frac{9}{10}$
Thus, $x$ = $5\frac{1}{2}$ and y = $-2\frac{9}{10}$ is the solution of the given simultaneous linear equations.

b) y = 5$x$ + 3, 3$x$ = 5y + 10

y = 5$x$ + 3 ——— (1)
3$x$ = 5y + 10 ——— (2)
*From equation (1)*
*y = 5x + 3*
*– 3 = 5x – y*
$5x - y = -3$ ——— (3)
*From equation (2)*
*3x = 5y + 10*
*3x – 5y = 10* ——— (4)
*Multiplying equation (3) by 5*
*25x – 5y = – 15* ——— (5)
Subtract equation (4) from (5),
(*25x – 5y*) – (*3x – 5y*) = –15 – 10
25x – 5y – 3x + 5y = –25
22x = *– 25*
Dividing 22 on both sides
$\frac{22x}{22} = -\frac{25}{22}$
$x$ = $-1\frac{3}{22}$
Substitute this value of $x$ in equation (1),
y = 5 ($-\frac{25}{22}$) + 3
y = $-\frac{125}{22}$ + 3
y = $\frac{-125+66}{22}$
y = $\frac{-59}{22}$
y = $-2\frac{15}{22}$
Thus, $x$ = $-1\frac{3}{22}$ and y = $-2\frac{15}{22}$ is the solution of the given simultaneous linear equations.

**4. Find the solution of each simultaneous linear equation using the substitution method.**

a) 2y – 3$x$ = – 9, 7 – 3$x$ = 8y

b) 5$x$ + y= -7, 3$x$ – $\frac{1}{2}$y = 5

**Solution:**

a) $2y - 3x = -9,\ 7 - 3x = 8y$

$2y - 3x = -9$ ——— (1)
$7 - 3x = 8y$ ——— (2)
From equation (1), we get
$y = \frac{3x-9}{2}$ ——— (3)
Put this value of *y* in equation (2),
$7 - 3x = 8\left(\frac{3x-9}{2}\right)$
$7 - 3x = 4(3x - 9)$
$7 - 3x = 12x - 36$
$7 + 36 = 12x + 3x$
$43 = 15x$
Dividing 15 on both sides
$\frac{43}{15} = \frac{15x}{15}$
$x = 2\frac{13}{15}$
Substitute $x = 2\frac{13}{15}$ in equation (3), we have
$y = \frac{3x-9}{2}$
$y = \frac{1}{2}\left[3\left(\frac{43}{15}\right) - 9\right]$
$y = \frac{1}{2}\left[\frac{43}{5} - 9\right]$
$y = \frac{1}{2}\left(\frac{43-45}{5}\right)$
$y = \frac{1}{2}\left(\frac{-2}{5}\right)$
$y = -\frac{1}{5}$
Thus, $x = 2\frac{13}{15}$ and $y = -\frac{1}{5}$ is the solution of the given equations.

b) $5x + y = -7,\ 3x - \frac{1}{2}y = 5$

$5x + y = -7$ ——— (1)
$3x - \frac{1}{2}y = 5$ ——— (2)
From equation (2)
$3x - \frac{1}{2}y = 5$
Multiplying 2 on both sides
$6x - y = 10$
$6x - 10 = y$
$y = 6x - 10$ ——— (3)
Put this value of *y* in equation (1),
$5x + y = -7$
$5x + 6x - 10 = -7$
$11x = -7 + 10$
$11x = 3$
Dividing 11 on both sides
$\frac{11x}{11} = \frac{3}{11}$
$x = \frac{3}{11}$

Substitute x = $\frac{3}{11}$ in equation (3)
y = 6 ($\frac{3}{11}$) – 10
y = $\frac{18}{11}$ – 10
y = $\frac{18-110}{11}$
$y = \frac{-92}{11}$
$y = -8\frac{4}{11}$
Thus, x = $\frac{3}{11}$ and $y = -8\frac{4}{11}$ is the solution of the given equations.

**5. Find the solution of each simultaneous linear equation using the graphical method.**

a) 3*x* – y = 5, *x* + 7y = 14 b) *x* + 4y = 9, 7*x* + 7y = 14

**Solution:**

a) 3*x* – y = 5, *x* + 7y = 14

3*x* – y = 5 -----------(i)
*x* + 7y = 14 -----------(ii)
First find the ordered pairs satisfying these equations.

Put *x* = 0 in (i),
3*x* – y = 5
3 (0) – y = 5
– y = 5
y = – 5
(*x*, y) = (0, – 5)
Put y = 0 in (i),
*3x* – y = 5
*3x* – (0) = 5
*3x* = 5
*Dividing 5 on both sides*
$\frac{3x}{3} = \frac{5}{3}$
x = $1\frac{2}{3}$
(*x*, y) = ($1\frac{2}{3}$,0)
Put *x* = 1 in (i)
*3x* – y = 5
3(1) – y = 5
– y = 5 – 3
– y = 2
y = – 2
(*x*, y) = (1, – 2)
So, we get the coordinates of *x* + y = 7 as follows.

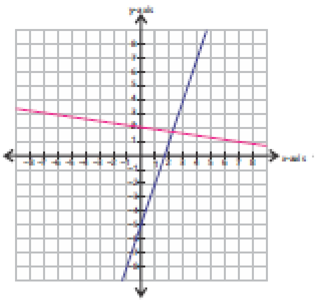
| | | | |
|---|---|---|---|
| x | 0 | $1\frac{2}{3}$ | 1 |
| y | – 5 | 0 | – 2 |

Now calculate the values of *x* and y for the equation *x* + 7y = 14.
Put *x* = 0 in (ii),
*x* + 7y = 14
(0) + 7y = 14

7y = 14
Dividing 7 on both sides, we get
y = $\frac{14}{7}$
y = 2
($x$, y) = (0, 2)
Put y = 0 in (ii)
$x$ + 7y = 14
$x$ + 7(0) = 14
$x$ = *14*
($x$, y) = (14,0)
Put $x$ = 1 in (i),
$x$ + 7y = 14
(1) + 7y = 14
7y = 14 – 1
7y = 13
Dividing 7 on both sides
$\frac{7y}{7} = \frac{13}{7}$
y = $1\frac{6}{7}$
($x$, y) = (1, $1\frac{6}{7}$)
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | 14 | 1 |
| y | 2 | 0 | $1\frac{6}{7}$ |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point ($2\frac{5}{22}$, $1\frac{15}{22}$), so this is the solution of the given simultaneous linear equations.

b) $x$ + 4y = 9, 7$x$ + 7y = 14

$x$ + 4y = 9 -----------(i)
7$x$ + 7y = 14 -----------(ii)
First find the ordered pairs satisfying these equations.

Put $x$ = 0 in (i),
$x$ + 4y = 9
0 + 4y = 9
4y = 9
Dividing 4 on both sides
$\frac{4y}{4} = \frac{9}{4}$
y = $2\frac{1}{4}$

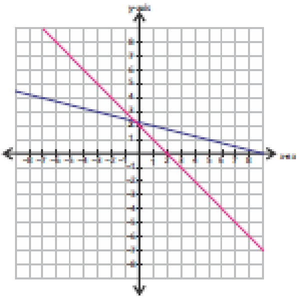
$(x, y) = (0, 2\frac{1}{4})$
Put y = 0 in (i),
$x + 4y = 9$
$x + 4(0) = 9$
$x = 9$
$(x, y) = (9, 0)$
Put $x = 1$ in (i)
$x + 4y = 9$
$1 + 4y = 9$
$4y = 9 – 1$
$4y = 8$
Dividing 4 on both sides
$\frac{4y}{4} = \frac{8}{4}$
$y = 2$
$(x, y) = (1,2)$
So, we get the coordinates of $x + y = 7$ as follows.

| | | | |
|---|---|---|---|
| x | 0 | 9 | 1 |
| y | $2\frac{1}{4}$ | 0 | 2 |

Now calculate the values of $x$ and y for the equation $7x + 7y = 14$.
Put $x = 0$ in (ii),
$7x + 7y = 14$
$7(0) + 7y = 14$
$7y = 14$
Dividing 7 on both sides
$\frac{7y}{7} = \frac{14}{7}$
$y = 2$
$(x, y) = (0, 2)$
Put y = 0 in (ii),
$7x + 7(0) = 14$
$7x = 14$
Dividing 7 on both sides
$\frac{7y}{7} = \frac{14}{7}$
$y = 2$
$(x, y) = (2, 0)$
Put $x = 1$ in (i),
$7(1) + 7y = 14$
$7y = 14 – 7$
$7y = 7$
Dividing 7 on both sides
$\frac{7y}{7} = \frac{7}{7}$
$y = 1$
$(x, y) = (1, 1)$
So, we get the coordinates as follows.

| | | | |
|---|---|---|---|
| x | 0 | 2 | 1 |
| y | 2 | 0 | 1 |

Now plot all the points from both equations on the graph.



Now plot the coordinates of the points we get from the two equations. The lines intersect at point $(-\frac{1}{3}, 2\frac{1}{3})$, so this is the solution of the given simultaneous linear equations.

6. **The sum of the weights of Hina and Ariba is 68 kg. The difference between the weights of Hina and Ariba is 22 kg. What is the weight of Hina and Ariba?**

**Solution:**

Let
Hina's weight = x kg
Ariba's weight = y kg
According to first condition
x + y = 68 ---------------------- (i)
According to second condition
x – y = 22 --------------------- (ii)
Adding equations (i) and (ii)
(x + y) + (x – y) = 68 + 22
x + y + x – y = 90
*2x* = 90
*Dividing 2 on both sides*
$\frac{2x}{2} = \frac{90}{2}$
x = 45
Substitute this value of *x* in equation (i),
x + y = 68
45 + y = 68
y = 68 – 45
y = 23
Thus,
Hina's weight = x = 45 kg
Ariba's weight = y = 23 kg

7. **Solve the given inequalities and show them on a number line.**

a) $5x \geq 2$ b) $-4x < 5$ c) $2x + 5 < 6$ d) $\frac{1}{2}x + 7 = 15$

**Solution:**

a) $5x \geq 2$

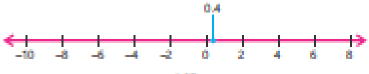
First, isolate the variable.
Dividing both sides by 5
$\frac{5x}{5} \geq \frac{2}{5}$

$x \geq \frac{2}{5}$

$x \geq 0.4$

So, $x \geq 0.4$ is the solution to this inequality. We can say that the solution to this inequality is "all the numbers greater than or equal to 0.4."

Let's show this solution on a number line.




b) $-4x < 5$

First, isolate the variable.

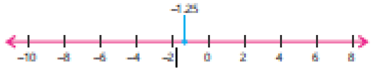
Dividing both sides by – 4

$\frac{-4x}{-4} < \frac{5}{-4}$

$x < -1.25$

So, $x < -1.25$ is the solution to this inequality. We can say that the solution to this inequality is "all the numbers less than – 1.25."

Let's show this solution on a number line.




c) $2x + 5 < 6$

First, isolate the variable.

Subtract 5 to both sides.

$2x + 5 - 5 < 6 - 5$

$2x < 1$

Dividing both sides by 2

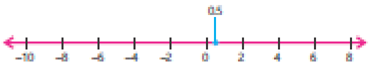
$\frac{2x}{2} < \frac{1}{2}$

$x < \frac{1}{2}$

$x < 0.5$

So, $x < 0.5$ is the solution to this inequality. We can say that the solution to this inequality is "all the numbers less than 0.5."

Let's show this solution on a number line.




d) $\frac{1}{2}x + 7 = 15$

First, isolate the variable.

Subtract 7 to both sides.

$\frac{1}{2}x + 7 - 7 = 15 - 7$

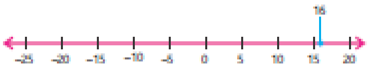
$\frac{1}{2}x = 8$

Multiplying both sides by 2

$x = 16$

So, $x = 16$ is the solution to this equation. We can say that the solution to this equation is the number 16.

Let's show this solution on a number line.



**8. Draw a graph for each linear equation in two variables.**

a) y = 4$x$ b) y = 4$x$ + 3 c) y = $\frac{1}{2}x$ d) y= 3$x$ + 4

e) y = 7$x$ f) y = 5$x$ + 4 g) y = 4$x$ - $\frac{2}{5}$ h) y = 2$x$

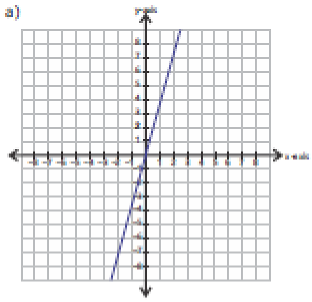
**Solution:**

a) y = 4$x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of $x$ and find the corresponding values of y, or vice versa.

| | | | | | |
|---|---|---|---|---|---|
| x | – 2 | – 1 | 0 | 1 | 2 |
| y = x | – 8 | – 4 | 0 | 4 | 8 |

We can see that the graph is a straight line passing through the origin (0, 0).

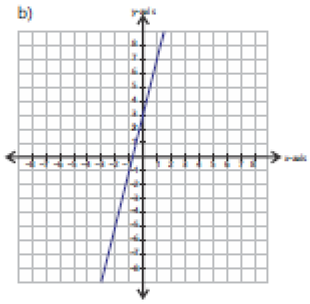



b) y = 4$x$ + 3

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of $x$ and find the corresponding values of y, or vice versa.

| | | | |
|---|---|---|---|
| x | 0 | $-\frac{3}{4}$ | 1 |
| y = x | 3 | 0 | 7 |

We can see that the graph is a straight line passing through the origin (0, 0).

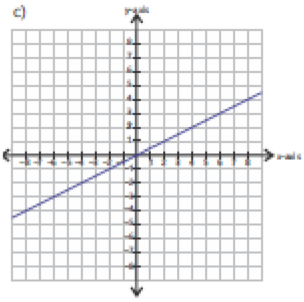



c) y = $\frac{1}{2}x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of $x$ and find the corresponding values of y, or vice versa.

| | | | | | |
|---|---|---|---|---|---|
| x | – 4 | –2 | 0 | 2 | 4 |
| y = x | – 2 | – 1 | 0 | 1 | 2 |

We can see that the graph is a straight line passing through the origin (0, 0).

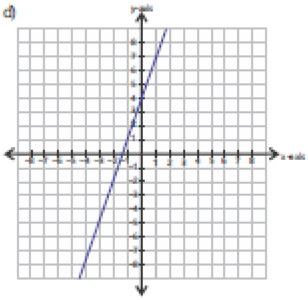


d) y= 3*x* + 4

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | 0 | $-\frac{4}{3}$ | 1 |
|---|---|---|---|
| y = x | 4 | 0 | 7 |

We can see that the graph is a straight line passing through the origin (0, 0).

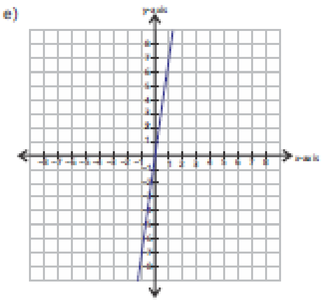


e) y = 7*x*

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | – 14 | – 7 | 0 | 7 | 14 |

We can see that the graph is a straight line passing through the origin (0, 0).

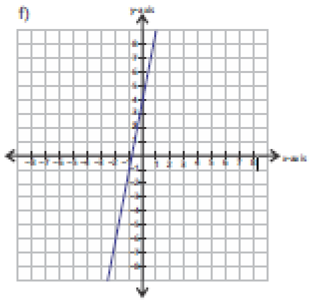


f) y = 5*x* + 4

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | 0 | $-\frac{4}{5}$ | 1 |
|---|---|---|---|
| y = x | 4 | 0 | 9 |

We can see that the graph is a straight line passing through the origin (0, 0).

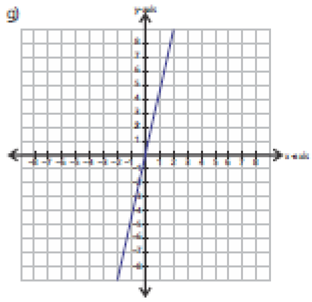


g) $y = 4x - \frac{2}{5}$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

| x | 0 | $\frac{1}{10}$ | 1 |
|---|---|---|---|
| y = x | $-\frac{2}{5}$ | 0 | $3\frac{3}{5}$ |

We can see that the graph is a straight line passing through the origin (0, 0).

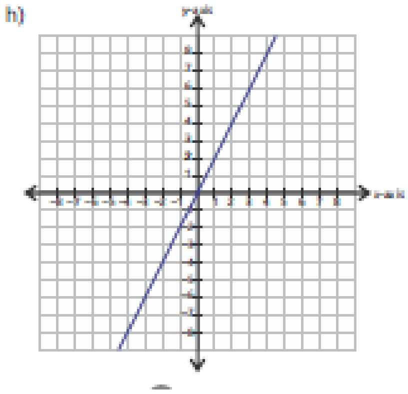


h) $y = 2x$

Let's find a few ordered pairs for the equation that satisfy it. Use Trial and Error method. Put the values of *x* and find the corresponding values of y, or vice versa.

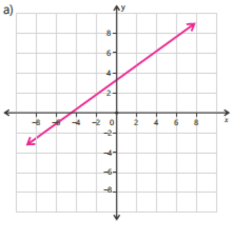
| x | – 2 | – 1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| y = x | – 4 | – 2 | 0 | 2 | 4 |

We can see that the graph is a straight line passing through the origin (0, 0).

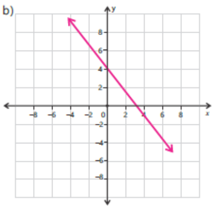


**9. Find and interpret the gradient of each line. Also find the y-intercept.**

a)



b)



**Solution:**

a) The gradient is positive as the line is going upward from left to right.

Gradient = $\frac{Vertical\ change}{Horizontal\ change} = \frac{3}{5}$

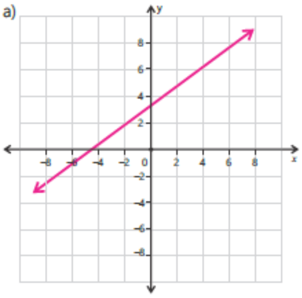
The line is crossing the y-axis at (0, 3).
So, the y-intercept is 3.
The line is crossing the *x*-axis at (– 5, 0).
So, the *x*-intercept is – 5.

b) The gradient is negative as the line is going downward from left to right.

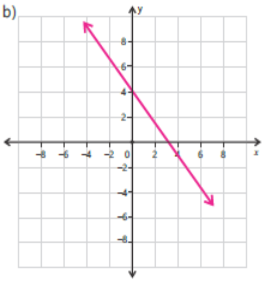
Gradient = $\frac{Vertical\ change}{Horizontal\ change} = -\frac{4}{3}$

The line is crossing the y-axis at (0,4).
So, the y-intercept is 4.
The line is crossing the *x*-axis at (3, 0).
So, the *x*-intercept is 3.

a)

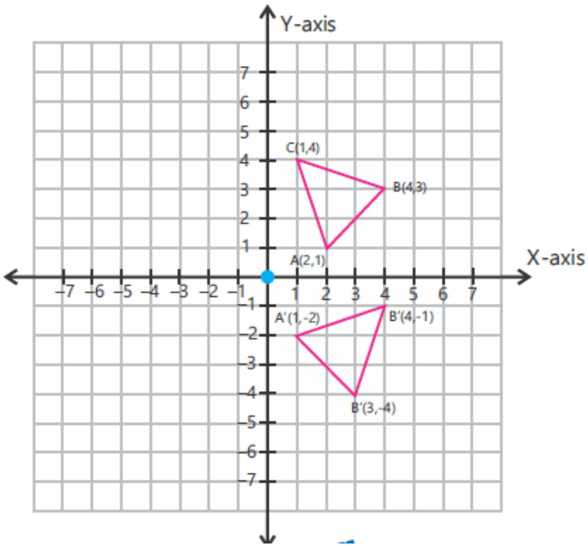


b)



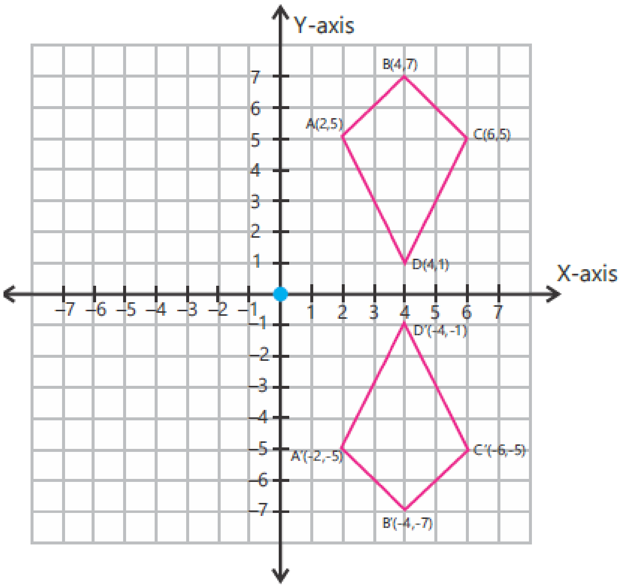
## Learning Check 7.1

1. **Draw a triangle on the coordinate plane. Then rotate it through $90^{o}$ clockwise about the origin.**
   **Solution:**
   Let’s say the triangular coordinate points are A (2, 1), B (4, 3) and C (1, 4).
   To rotate the triangle ABC about the origin 90° clockwise, we would follow the rule (*x*, y) → (y, - *x*), where the y-value of the original point becomes the new *x*-value with the same sign and the *x*-value of the original point becomes the new y-value with the opposite sign.
   Let's apply the rule to the vertices to create the new triangle A'B'C':
   A (2, 1) becomes A' (1, - 2)
   B (4, 3) becomes B' (3, - 4)
   C (1, 4) becomes C' (4, - 1)
   Mark the coordinates that will be the vertices of the reflected object/image.
   Join the vertices.
   The triangle A'B'C' is the required rotated image of triangle ABC.




   **Note:** (Students can choose any points to make a square shape and rotate it $90^0$ clockwise.)

2. **Draw a kite on the coordinate plane. Then rotate it through $180^{o}$ clockwise about the origin.**
   **Solution:**
   Let’s say the kite coordinate points are A (2, 1), B (4, 3) and C (1, 4).
   To rotate the kite ABCD about the origin 180° clockwise, we would follow the rule (*x*, y) → (- x, - *y*), where the x-value of the original point becomes the new *x*-value with the opposite sign and the *y*-value of the original point becomes the new y-value with the opposite sign.
   Let's apply the rule to the vertices to create the new kite A'B'C'D':
   A (2, 5) becomes A' (- 2, - 5)
   B (4, 7) becomes B' (- 4, - 7)
   C (6, 5) becomes C' (- 6, - 5)
   D (4, 1) becomes D' (- 4, - 1)
   Mark the coordinates that will be the vertices of the reflected object/image. Join the vertices. The kite A'B'C'D' is the required rotated image of the kite ABCD.




**Note:** (Students can choose any points to make a square shape and rotate it $180^0$ clockwise.)

3. **Find the values of *x* and y for P' if an image having point P (2, -9) is rotated:** a) $90^O$ clockwise  b) $180^O$ anticlockwise  c) $270^O$ clockwise

**Solution:**

a) $90^O$ clockwise

To rotate the point P (2, - 9) about the origin 90° clockwise, we would follow the rule (*x*, y) → (y, - *x*), where the x-value of the original point becomes the new *y*-value with the same sign and the *y*-value of the original point becomes the new x-value with the opposite sign.
Let's apply the rule to the vertex P (2, - 9). So
P (2, - 9) becomes P' (- 9, - 2).

b) $180^O$ anticlockwise

To rotate the point P (2, - 9) about the origin 180° anticlockwise, we would follow the rule (*x*, y) → (- x, - y), where the x-value of the original point becomes the new *x*-value with the opposite sign and the *y*-value of the original point becomes the new y-value with the opposite sign.
Let's apply the rule to the vertex P (2, - 9). So
P (2, - 9) becomes P' (- 2, 9).

c) $270^O$ clockwise

To rotate the point P (2, - 9) about the origin 270° clockwise, we would follow the rule (*x*, y) → (- y, *x*), where the x-value of the original point becomes the new *y*-value with the opposite sign and the *y*-value of the original point becomes the new x-value with the same sign.
Let's apply the rule to the vertex P (2, - 9). So
P (2, - 9) becomes P' (9, 2).

4. **Find the values of *x* and y for P' if an image having point P (-5, -3) is rotated:**

a) $90^O$ anticlockwise  b) $180^O$ clockwise  c) $270^O$ anticlockwise

**Solution:**

a) $90^O$ anticlockwise

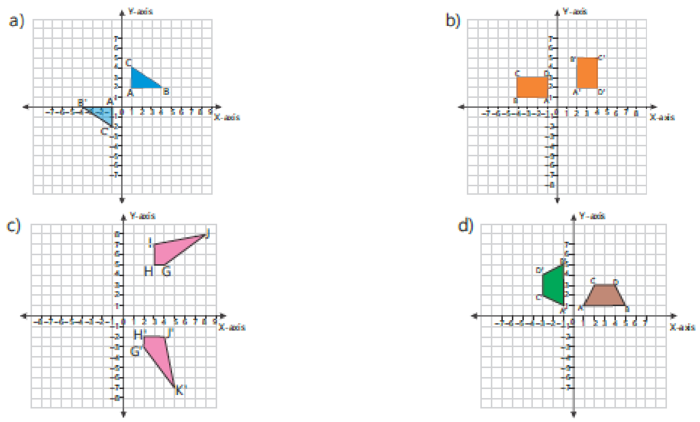
To rotate the point P (- 5, - 3) about the origin 90° anticlockwise, we would follow the rule (*x*, y) → (- y, *x*), where the x-value of the original point

becomes the new *y*-value with the opposite sign and the *y*-value of the original point becomes the new x-value with the same sign.
Let's apply the rule to the vertex P (- 5, - 3). So
P (- 5, - 3) becomes P' (3, - 5).

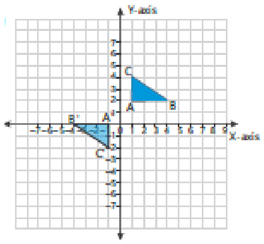
b) $180^{O}$ clockwise
To rotate the point P (- 5, - 3) about the origin 180° clockwise, we would follow the rule (*x*, y) → (- x, - y), where the x-value of the original point becomes the new *x*-value with the opposite sign and the *y*-value of the original point becomes the new y-value with the opposite sign.
Let's apply the rule to the vertex P (- 5, - 3). So
P (- 5, - 3) becomes P' (5, 3).

c) $270^{O}$ anticlockwise
To rotate the point P (- 5, - 3) about the origin 270° anticlockwise, we would follow the rule (*x*, y) → (y, - *x*), where the x-value of the original point becomes the new *y*-value with the same sign and the *y*-value of the original point becomes the new x-value with the opposite sign.
Let's apply the rule to the vertex P (- 5, - 3). So
P (- 5, - 3) becomes P' (- 3, 5).

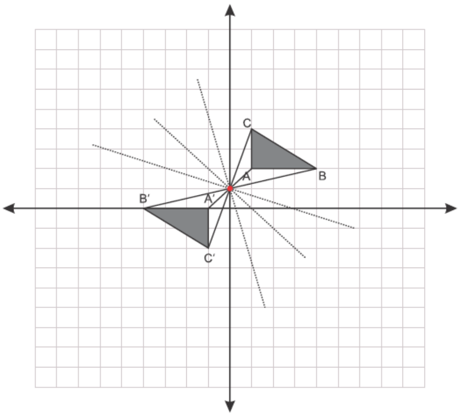
**5. Copy the given shapes onto cartesian graph paper. Find their centre of rotation.**



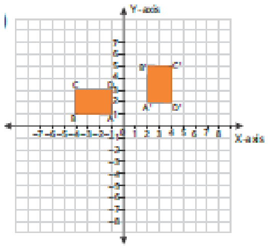
**Solution:**
a)



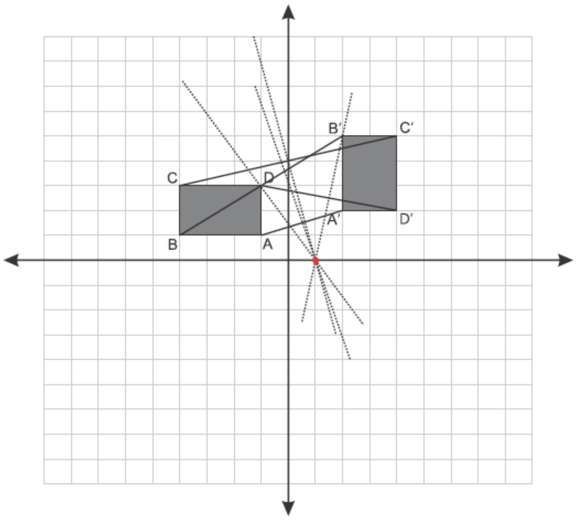
To find the centre of rotation, follow the given steps:
**Step 1:** Draw a line joining the corresponding vertices of both images, i.e.,
AA', BB' and CC', respectively.
**Step 2:** Construct the perpendicular bisector of these segments,
AA', BB' and CC'.
The point X (0,1) where the three perpendicular bisectors meet is the centre of rotation.



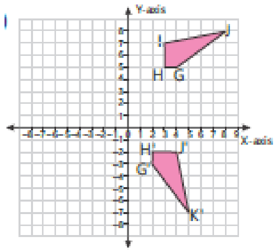

b)




To find the centre of rotation, follow the given steps:
**Step 1:** Draw a line joining the corresponding vertices of both images, i.e.,
AA', BB', CC' and DD', respectively.
**Step 2:** Construct the perpendicular bisector of these segments,
AA', BB', CC' and DD'.
The point X (1,0) where the three perpendicular bisectors meet is the centre of rotation.




c)




To find the centre of rotation, follow the given steps:

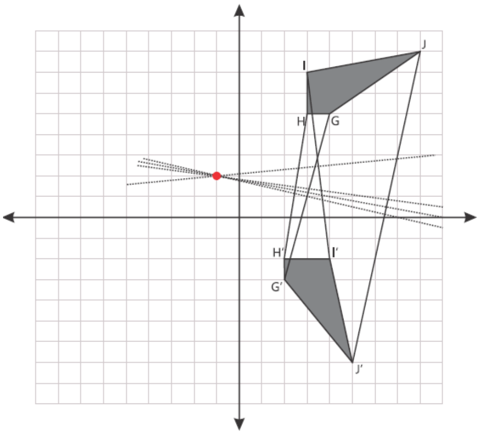
**Step 1:** Draw a line joining the corresponding vertices of both images, i.e.,

GG', HH', II' and JJ', respectively.

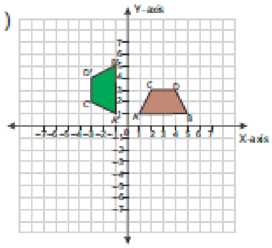
**Step 2:** Construct the perpendicular bisector of these segments,

GG', HH', II' and JJ'.

The point X (- 1,2) where the three perpendicular bisectors meet is the centre of rotation.




d)




To find the centre of rotation, follow the given steps:

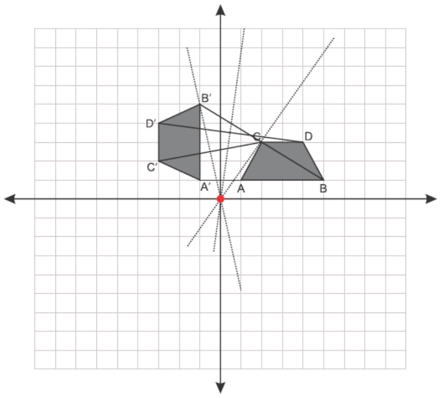
**Step 1:** Draw a line joining the corresponding vertices of both images, i.e.,

AA', BB', CC' and DD', respectively.

**Step 2:** Construct the perpendicular bisector of these segments,

AA', BB', CC' and DD'.

The point X (0,1) where the three perpendicular bisectors meet is the centre of rotation.

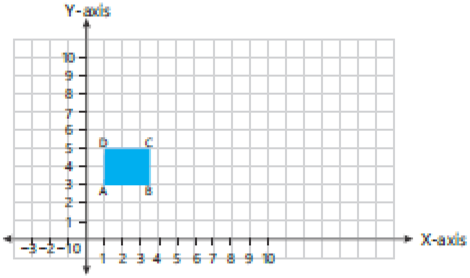


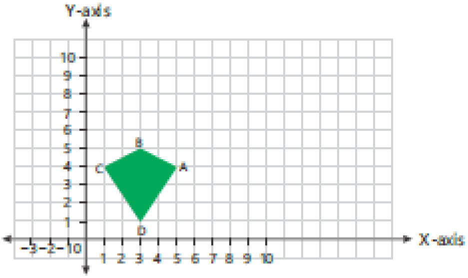
## Learning Check 7.2

1. **Copy the given shapes onto cartesian graph paper. Enlarge them by the given scale factor and centre of enlargement.**

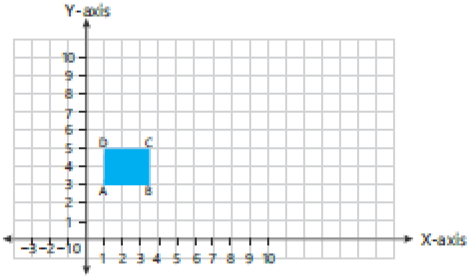
   a) Scale factor is = 2 and centre of enlargement = (1, -1)

   b) Scale factor is = 3 and centre of enlargement = (1, 1)




a)




**Step 1:** Identify and note the coordinates of the shape to be enlarged. Here,

A (1, 3), B (3.5, 3), C (3.5, 5) and D (1, 5) are the coordinates of the vertices of the given triangle.

The centre of enlargement is given, i.e., P (1, – 1).

**Step 2:** Count and note the distance of each vertex from the centre of enlargement.

First consider vertex A. Let's find the coordinates for A' of the enlarged shape.

The distance of P from A is (1 – 1, 3 + 1) or (0, 4).

Here the scale factor is 2, so multiply the coordinates of this distance by 2.

The distance of P from A' is (0 × 2, 4 × 2) or (0, 8).

So, A' will be at a distance of (0, 8) from P. Count and mark point A'. Similarly, find the coordinates for B' of the enlarged shape.

The distance of P from B is (3.5 – 1, 3 + 1) or (2.5, 4). Here the scale factor is 2, so multiply the coordinates of this distance by 2.

The distance of P from B' is (2.5 × 2, 4 × 2) or (5, 8). So, B' will be at a distance of (5, 8) from P.

Count and mark point B'.

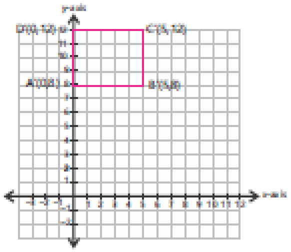
Similarly, find the coordinates for C' of the enlarged shape.

The distance of P from C is (3.5 – 1, 5 + 1) or (2.5, 6). Here the scale factor is 2, so multiply the coordinates of this distance by 2.

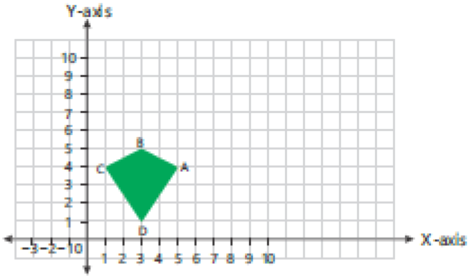
The distance of P from C' is (2.5 × 2, 6 × 2) or (5, 12). So, C' will be at a distance of (5, 12) from P. Count and mark point C'.

Similarly, D' will be at a distance of (0, 12) from P. Mark it and join the vertices.

A'B'C'D' is the required enlarged figure.



b)




**Step 1:** Identify and note the coordinates of the shape to be enlarged.
Here,

A (5, 4), B (3, 5), C (1, 4) and D (3, 1) are the coordinates of the vertices of the given triangle.

The centre of enlargement is given, i.e., P (1, 1).

**Step 2:** Count and note the distance of each vertex from the centre of enlargement.

First consider vertex A. Let's find the coordinates for A' of the enlarged shape.

The distance of P from A is (5 – 1, 4 – 1) or (4, 3).

Here the scale factor is 3, so multiply the coordinates of this distance by 3.

The distance of P from A' is (4 × 3, 3 × 3) or (12, 9).

So, A' will be at a distance of (12, 9) from P. Count and mark point A'. Similarly, find the coordinates for B' of the enlarged shape.

The distance of P from B is (3 – 1, 5 – 1) or (2, 4). Here the scale factor is 3, so multiply the coordinates of this distance by 3.

The distance of P from B' is (2 × 3, 4 × 3) or (6, 12). So, B' will be at a distance of (6, 12) from P.

Count and mark point B'.

Similarly, find the coordinates for C' of the enlarged shape.

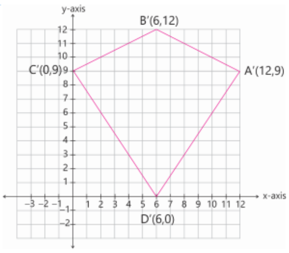
The distance of P from C is (1 – 1, 4 – 1) or (0, 3). Here the scale factor is 3, so multiply the coordinates of this distance by 3.

The distance of P from C' is (0 × 3, 3 × 3) or (0, 9). So, C' will be at a distance of (0, 9) from P.

Count and mark point C'

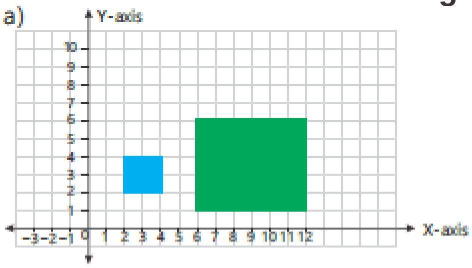
Similarly, D' will be at a distance of (6, 0) from P. Mark it and join the vertices.

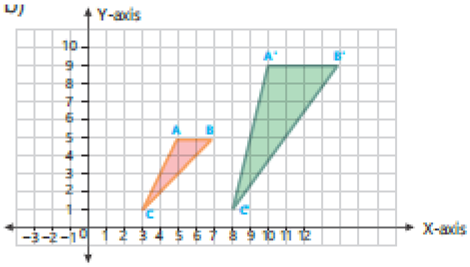
A'B'C'D' is the required enlarged figure.




2. **Copy the given shapes onto cartesian graph paper. Find their scale factor and the centre of enlargement.**

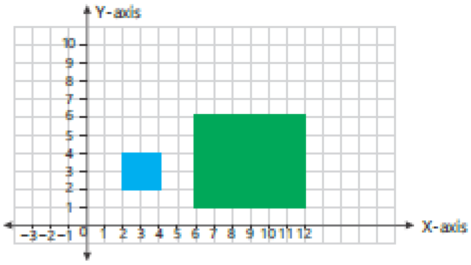
a)



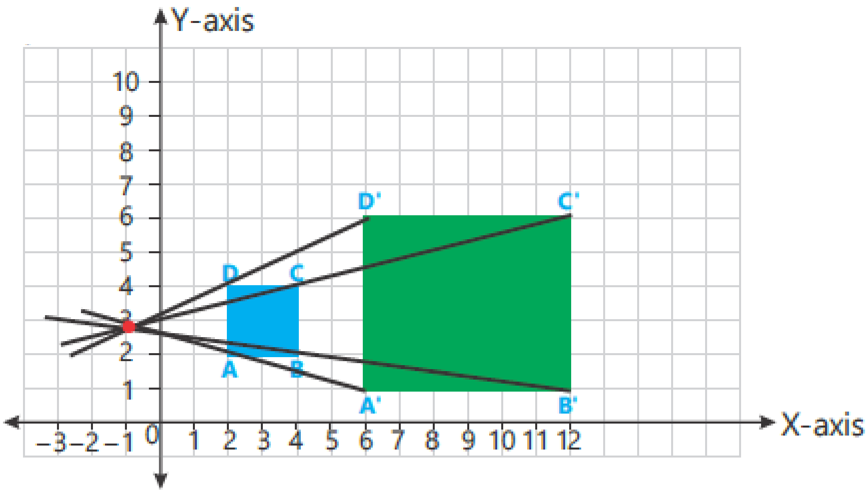

**Solution:**

a)

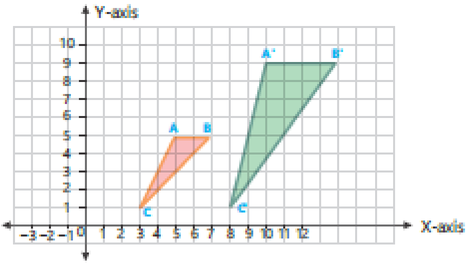



To find its centre of enlargement, join its vertices to the corresponding vertices of the original shape. The point where three vertices meet is the centre of enlargement. So, point P (-1,3) is the centre of enlargement. The scale factor is the ratio of the length of any side of the new image to the length of the corresponding side of the pre-image. Here, we can see that A'B' is 6 units long and the corresponding side AB is 2 units long.

So, here the scale factor is = $\frac{A'C'}{AC} = \frac{6}{2} = 3$

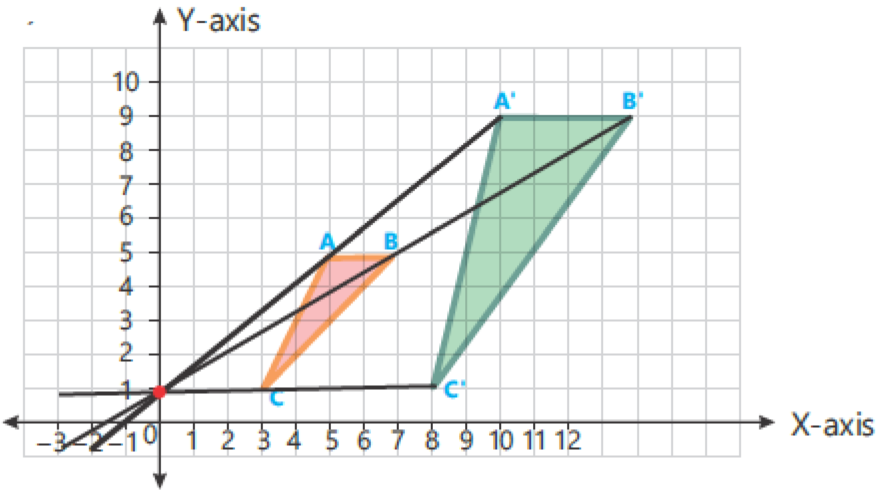


b)



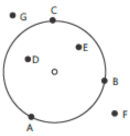

To find its centre of enlargement, join its vertices to the corresponding vertices of the original shape. The point where three vertices meet is the centre of enlargement. So, point P (-1,3) is the centre of enlargement. The scale factor is the ratio of the length of any side of the new image to the length of the corresponding side of the pre-image. Here, we can see that A'B' is 4 units long and the corresponding side AB is 2 units long.

So, here the scale factor is = $\frac{A'C'}{AC} = \frac{4}{2} = 2$




**Learning Check 7.3**

1. **Sort out the points from the given figure.**
   a) interior points b) exterior points c) concyclic points




**Solution:**

a)

Interior points are O, D and E.

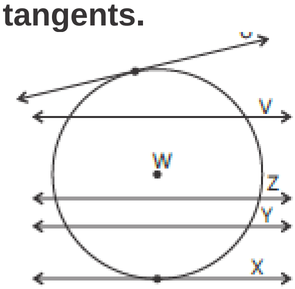
b)

Exterior points are g and F.

c)

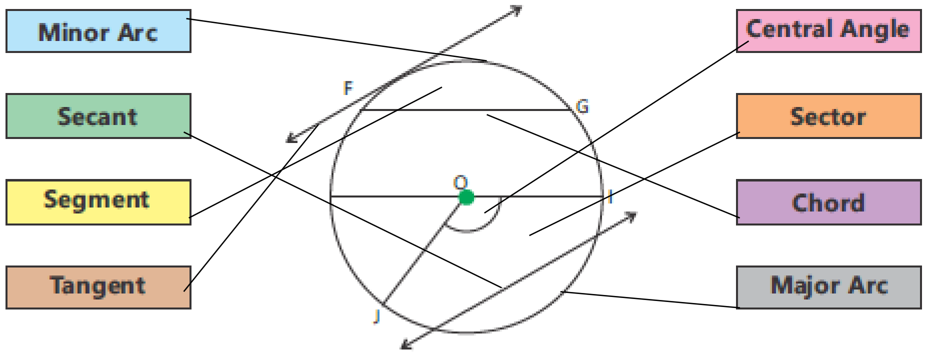
Concyclic points are A, B and C.

2. **Observe the given figure and write the names of the secants and tangents.**




**Solution:**
Tangent lines: U and X.
Secant lines: V, Y and Z.

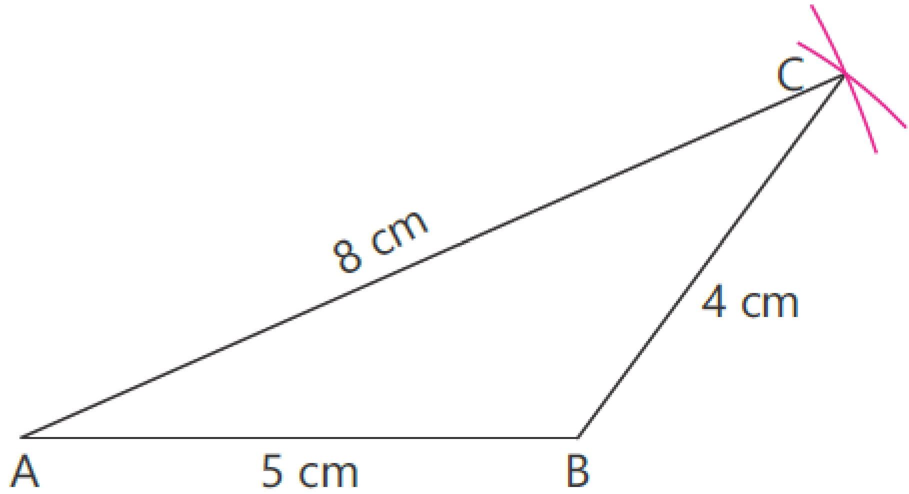
3. **Match the labels with the correct parts of the circle.**



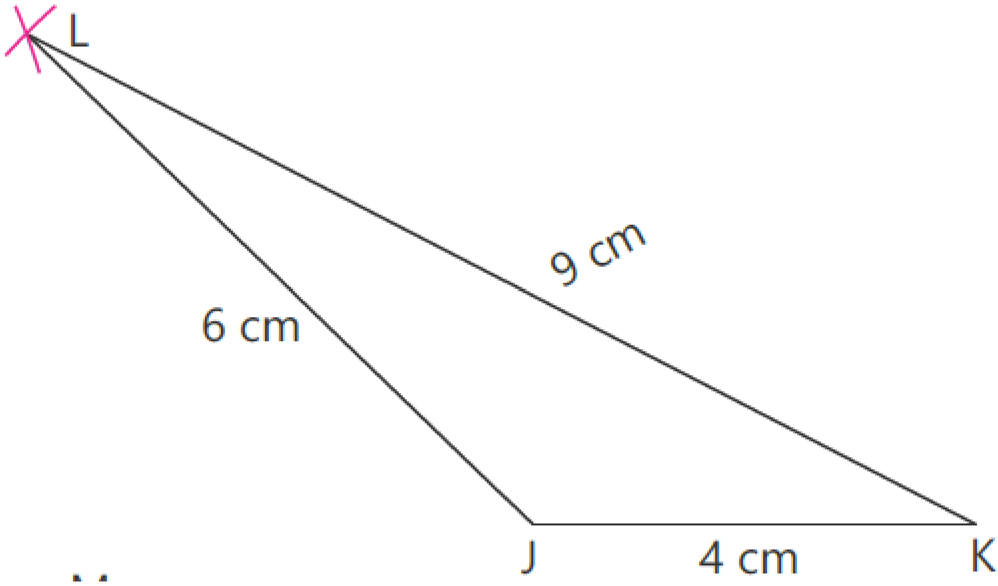

**Learning Check 7.4**

1. **Draw the following triangles using a compass and a ruler. (if possible)**
   a) $\overline{AB}$ = 5 cm, $\overline{AC}$ = 8 cm, $\overline{BC}$ = 4 cm   b) $\overline{JK}$ = 4 cm, $\overline{KL}$ = 9 cm, $\overline{JL}$ = 6 cm
   c) $\overline{XY}$ = 3 cm, $\overline{YZ}$ = 6 cm, $\overline{XZ}$ = 7 cm   d) $\overline{LN}$ = 5 cm, $\overline{LM}$ = 8 cm, $\overline{MN}$ = 9 cm

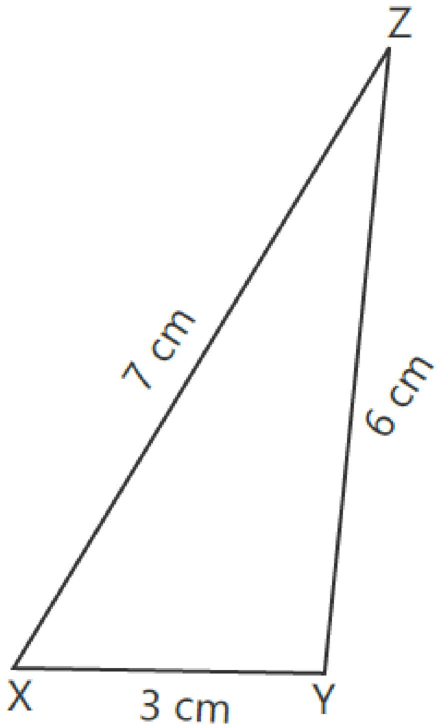
   **Solution:**
   a) $\overline{AB}$ = 5 cm, $\overline{AC}$ = 8 cm, $\overline{BC}$ = 4 cm
   **Steps of Construction**
   **Step I:** Draw a line segment $\overline{AB}$ = 5 cm with the help of a ruler.
   **Step II:** Place the pointer of the compass at point A and draw an arc of length 8 cm.
   **Step III:** Place the pointer of the compass at point B and draw an arc of radius 4 cm that cuts the previous arc at point C.
   **Step IV:** Join C to A and B.
   So, ΔABC is the required triangle.



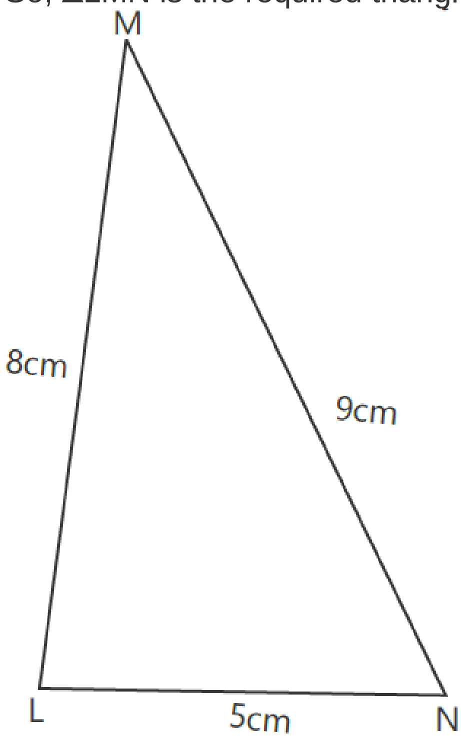

b) $\overline{JK}$ = 4 cm, $\overline{KL}$ = 9 cm, $\overline{JL}$ = 6 cm
**Step I:** Draw a line segment $\overline{JK}$ = 4 cm with the help of a ruler.
**Step II:** Place the pointer of the compass at point K and draw an arc of length 9 cm.
**Step III:** Place the pointer of the compass at point J and draw an arc of radius 6 cm that cuts the previous arc at point L.
**Step IV:** Join L to J and K.
So, ΔJKL is the required triangle.




c) $\overline{XY}$ = 3 cm, $\overline{YZ}$ = 6 cm, $\overline{XZ}$ = 7 cm
**Step I:** Draw a line segment $\overline{XY}$ = 3 cm with the help of a ruler.
**Step II:** Place the pointer of the compass at point Y and draw an arc of length 6 cm.
**Step III:** Place the pointer of the compass at point X and draw an arc of radius 7 cm that cuts the previous arc at point Z.
**Step IV:** Join Z to X and Y.
So, ΔXYZ is the required triangle.




d) $\overline{LN}$ = 5 cm, $\overline{LM}$ = 8 cm, $\overline{MN}$ = 9 cm
**Step I:** Draw a line segment $\overline{\mathrm{LN}}$ = 5 cm with the help of a ruler.
**Step II:** Place the pointer of the compass at point L and draw an arc of length 8 cm.
**Step III:** Place the pointer of the compass at point N and draw an arc of radius 9 cm that cuts the previous arc at point M.
**Step IV:** Join M to L and N.
So, ΔLMN is the required triangle.



**2. Draw the following triangles using a compass and a ruler. (if possible)**

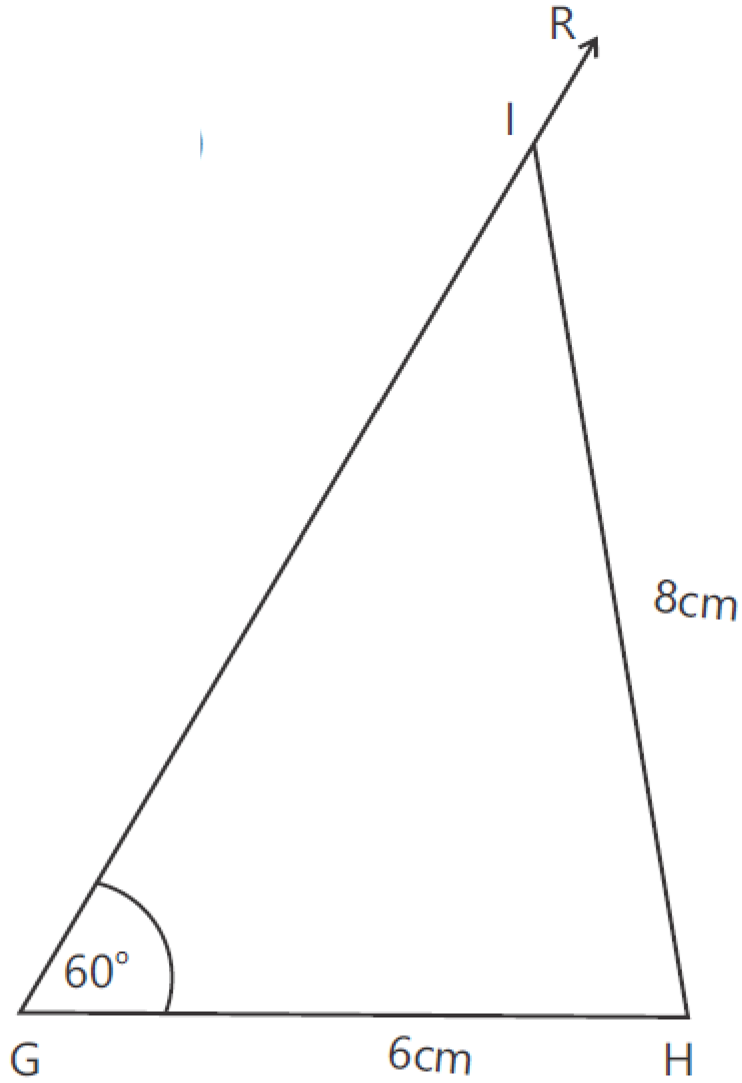
a) $\overline{GH}$ = 6 cm, $\overline{HI}$ = 8 cm, ∠G = 60°
b) $\overline{XY}$ = 7 cm, $\overline{YZ}$ = 8 cm, ∠Y = 45°
c) ∠M = 75°, $\overline{LM}$ = 4 cm, $\overline{MN}$ = 4 cm
d) $\overline{DE}$ = 9 cm, ∠E = 120°, $\overline{EF}$ = 8 cm

**Solution:**

a) $\overline{GH}$ = 6 cm, $\overline{HI}$ = 8 cm, ∠G = 60°

**Steps of Construction**

**Step I:** Draw a line segment $\overline{GH}$ = 6 cm with the help of a ruler.
**Step II:** Draw an arc of 60° at point G.
**Step III:** Place the pointer of the compass at point H and draw an arc of length 8 cm that cuts the arm of the angle $\overrightarrow{GR}$ at point I.
**Step IV:** Using a ruler join I to H.
So, ΔGHI is the required triangle.

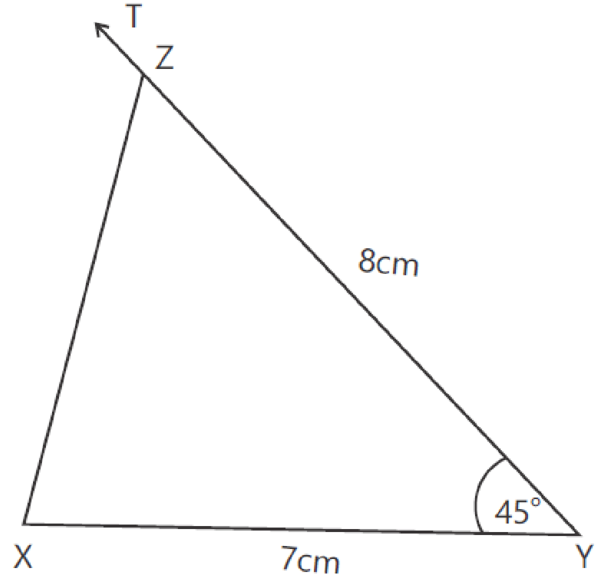



b) $\overline{XY}$ = 7 cm, $\overline{YZ}$ = 8 cm, ∠Y = 45°

**Steps of Construction**

**Step I:** Draw a line segment $\overline{XY}$ = 7 cm with the help of a ruler.
**Step II:** Draw an arc of 45° at point Y.
**Step III:** Place the pointer of the compass at point Y and draw an arc of length 8 cm that cuts the arm of the angle $\overrightarrow{YT}$ at point Z.
**Step IV:** Using a ruler join Z to X.

So, ΔGHI is the required triangle.




c) $\angle M = 75°$, $\overline{LM} = 4$ cm, $\overline{MN} = 4$ cm

**Steps of Construction**

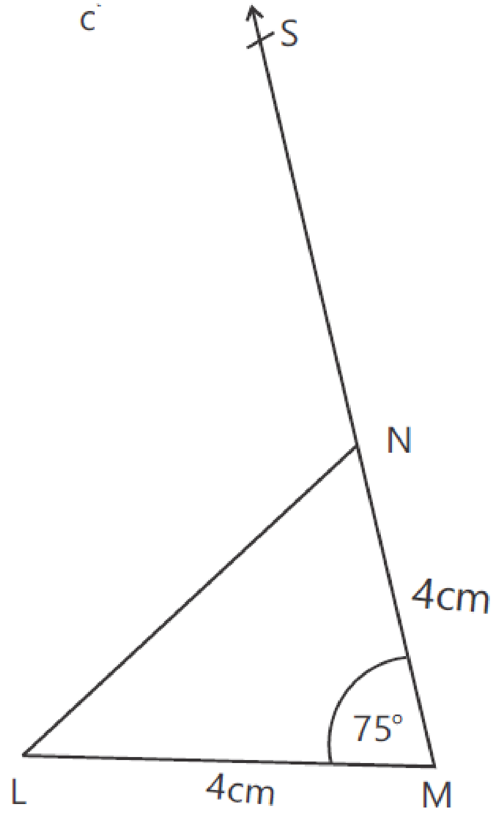
**Step I:** Draw a line segment $\overline{LM} = 4$ cm with the help of a ruler.

**Step II:** Draw an arc of 75° at point M.

**Step III:** Place the pointer of the compass at point M and draw an arc of length 4 cm that cuts the arm of the angle $\overrightarrow{MS}$ at point N.

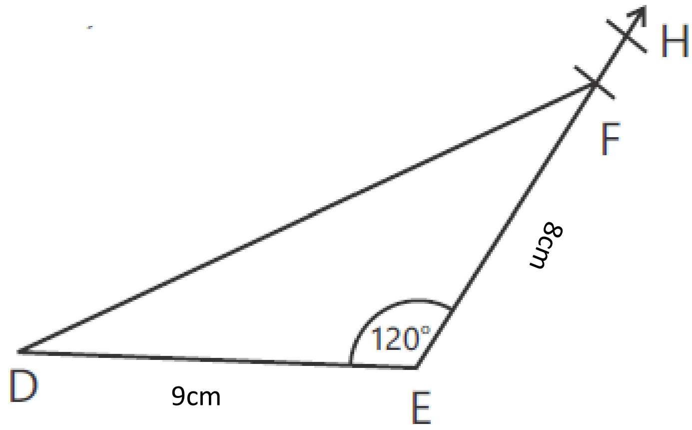
**Step IV:** Using a ruler join N to L.

So, ΔLMN is the required triangle.




d) $\overline{DE} = 9$ cm, $\angle E = 120°$, $\overline{EF} = 8$ cm

**Steps of Construction**
**Step I:** Draw a line segment $\overline{DE}$ = 9 cm with the help of a ruler.
**Step II:** Draw an arc of 120° at point E.
**Step III:** Place the pointer of the compass at point E and draw an arc of length 8 cm that cuts the arm of the angle $\overrightarrow{EH}$ at point F.
**Step IV:** Using a ruler join F to D.
So, ΔDEF is the required triangle.



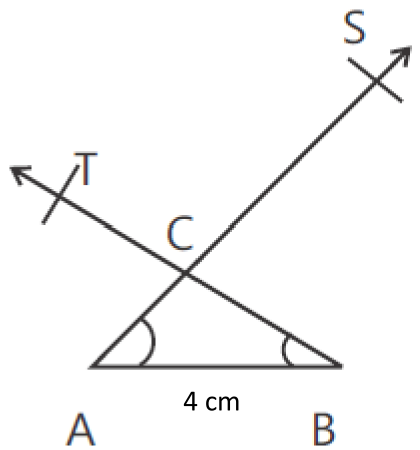

**3. Draw the following triangles using a compass and a ruler. (if possible)**

a) $\overline{AB}$ = 4 cm, ∠A = 45°, ∠B = 30°
b) ∠X = 45°, ∠Y = 75°, $\overline{XY}$ = 7 cm
c) $\overline{LN}$ = 7 cm, ∠L = 60°, ∠N = 105°
d) ∠E = 15°, ∠F = 105°, $\overline{EF}$ = 5 cm

**Solution:**

a) $\overline{AB}$ = 4 cm, ∠A = 45°, ∠B = 30°

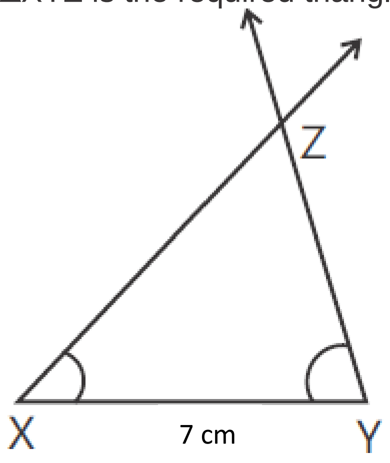
**Steps of Construction**
**Step I:** Draw a line segment $\overline{AB}$ of length 4 cm with the help of a ruler.
**Step II:** Draw an angle of 45° at point A.
**Step III:** Similarly, at point B draw ∠ABC = 30°.
$\overrightarrow{AS}$ and $\overrightarrow{BT}$ cut each other at point C.
ΔABC is the required triangle.




b) ∠X = 45°, ∠Y = 75°, $\overline{XY}$ = 7 cm

**Steps of Construction**
**Step I:** Draw a line segment $\overline{XY}$ of length 7 cm with the help of a ruler.
**Step II:** Draw an angle of 45° at point X.
**Step III:** Similarly, at point Y draw ∠XYZ = 75°.

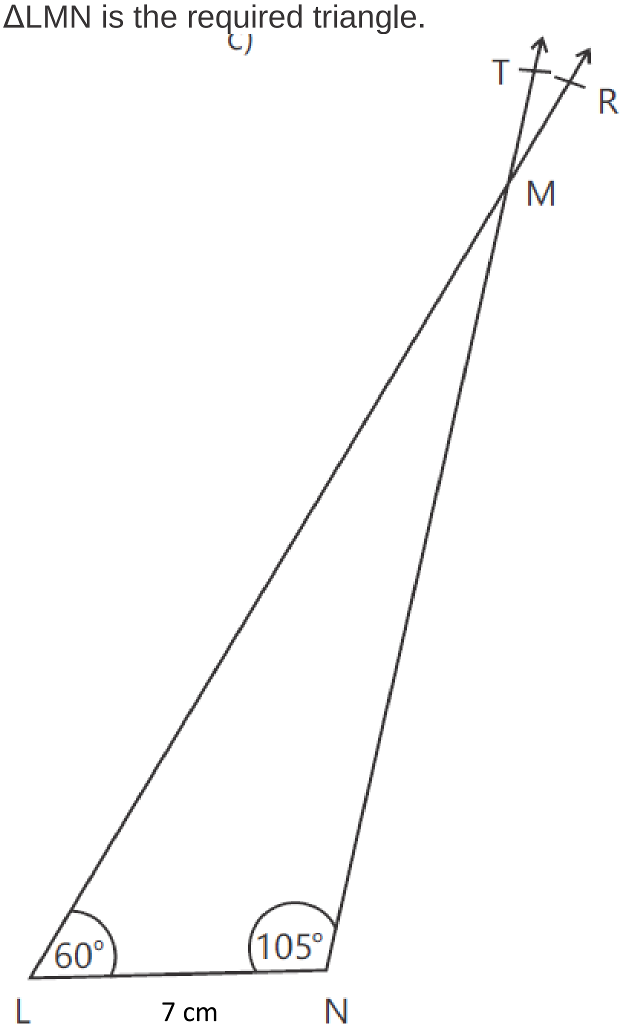
$\overrightarrow{XS}$ and $\overrightarrow{YT}$ cut each other at point Z.
ΔXYZ is the required triangle.




c) $\overline{LN}$ = 7 cm, ∠L = 60°, ∠N = 105°

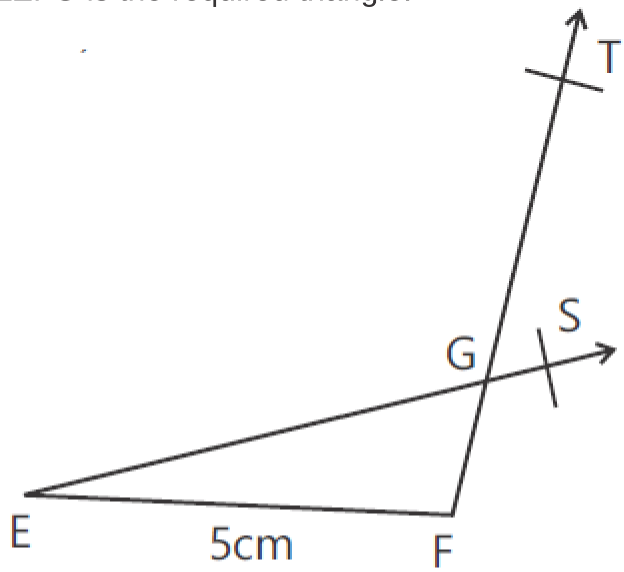
**Steps of Construction**

**Step I:** Draw a line segment $\overline{LN}$ of length 7 cm with the help of a ruler.
**Step II:** Draw an angle of 60° at point L.
**Step III:** Similarly, at point N draw ∠LNM = 30°.
$\overrightarrow{LR}$ and $\overrightarrow{NT}$ cut each other at point M.
ΔLMN is the required triangle.



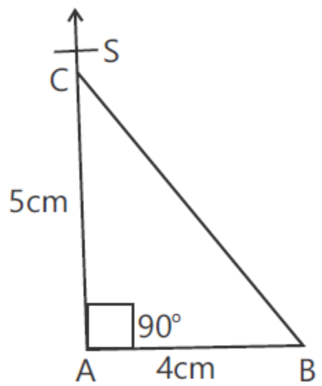

d) ∠E = 15°, ∠F = 105°, $\overline{EF}$ = 5 cm

**Steps of Construction**
**Step I:** Draw a line segment $\overline{EF}$ of length 5 cm with the help of a ruler.
**Step II:** Draw an angle of 15° at point E.
**Step III:** Similarly, at point F draw ∠EFG = 105°.
$\overrightarrow{ES}$ and $\overrightarrow{FT}$ cut each other at point G.
ΔEFG is the required triangle.




4. **Draw the following right-triangles using a compass and a ruler. (if possible)**
   a) $\overline{AB}$ = 4 cm, $\overline{AC}$ (Hypotenuse) = 5 cm   b) $\overline{XY}$ = 2 cm, $\overline{YZ}$ (Hypotenuse) = 4 cm
   c) $\overline{LN}$ = 7 cm, $\overline{LM}$ (Hypotenuse) = 9 cm   d) $\overline{DE}$ = 4 cm, $\overline{DF}$ (Hypotenuse) = 6 cm

   **Solution:**
   a) $\overline{AB}$ = 4 cm, $\overline{AC}$ (Hypotenuse) = 5 cm
   **Steps of Construction**
   **Step I:** Draw a line segment $\overline{AB}$ of length 4 cm with the help of a ruler.
   **Step II:** Draw an angle of 90° at point A.
   **Step III:** Place the pointer of the compass at point B and draw an arc of length 5 cm that cuts arm AS of ∠90° at point C.
   **Step IV:** Join the point B to C. So, ΔABC is the required triangle.



b) $\overline{XY}$ = 2 cm, $\overline{YZ}$ (Hypotenuse) = 4 cm

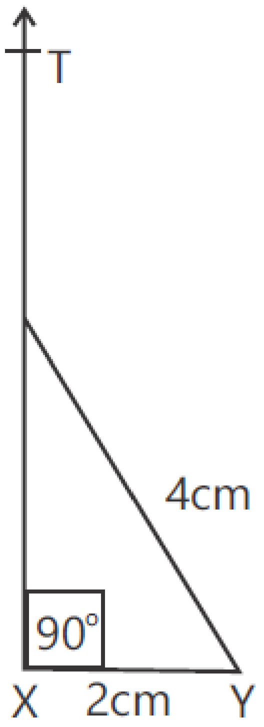
**Steps of Construction**

**Step I:** Draw a line segment $\overline{XY}$ of length 2 cm with the help of a ruler.

**Step II:** Draw an angle of 90° at point X.

**Step III:** Place the pointer of the compass at point Y and draw an arc of length 4 cm that cuts arm XT of ∠90° at point Z.

**Step IV:** Join the point Y to Z. So, XYZ is the required triangle.




c) $\overline{LN}$ = 7 cm, $\overline{LM}$ (Hypotenuse) = 9 cm

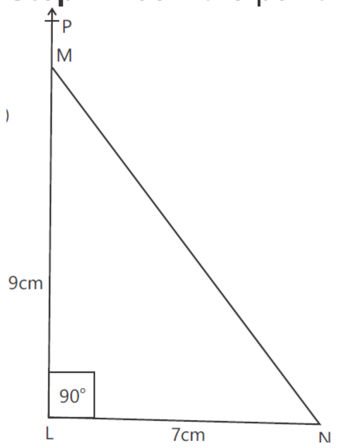
**Steps of Construction**

**Step I:** Draw a line segment $\overline{LN}$ of length 7 cm with the help of a ruler.

**Step II:** Draw an angle of 90° at point L.

**Step III:** Place the pointer of the compass at point N and draw an arc of length 9 cm that cuts arm LP of ∠90° at point M.

**Step IV:** Join the point N to M. So, ΔLMN is the required triangle.



d) $\overline{DE}$ = 4 cm, $\overline{DF}$ (Hypotenuse) = 6 cm

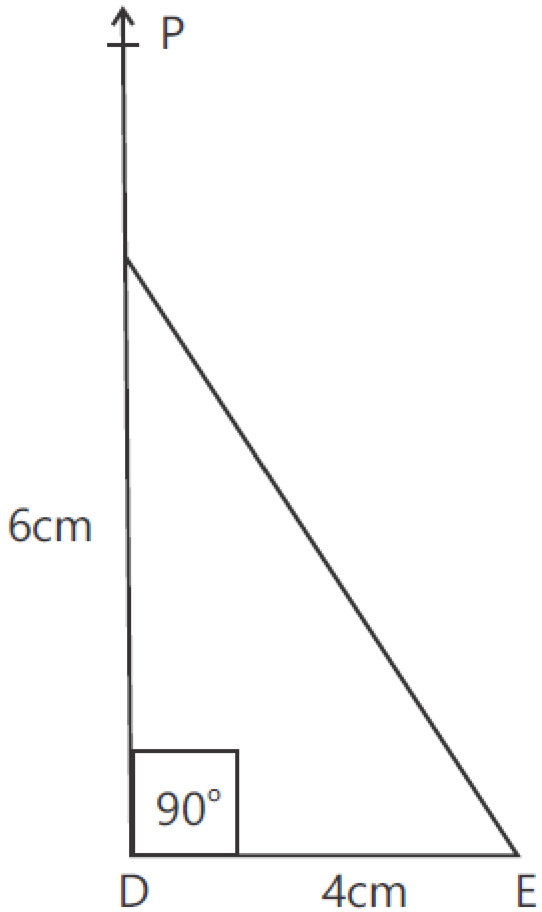
**Steps of Construction**

**Step I:** Draw a line segment $\overline{DE}$ of length 4 cm with the help of a ruler.

**Step II:** Draw an angle of 90° at point D.

**Step III:** Place the pointer of the compass at point E and draw an arc of length 6 cm that cuts arm DP of ∠90° at point F.

**Step IV:** Join the point E to F. So, ΔDEF is the required triangle.




**Learning Check 7.5**

1. **Construct a square for each of the given lengths.**

a) 4.8 cm b) 5.2 cm c) 5.6 cm

**Solution:**

a) 4.8 cm

**Steps of Construction**

**Step I:** Draw a line segment m$\overline{AB}$ of 4.8 cm with the help of a ruler.

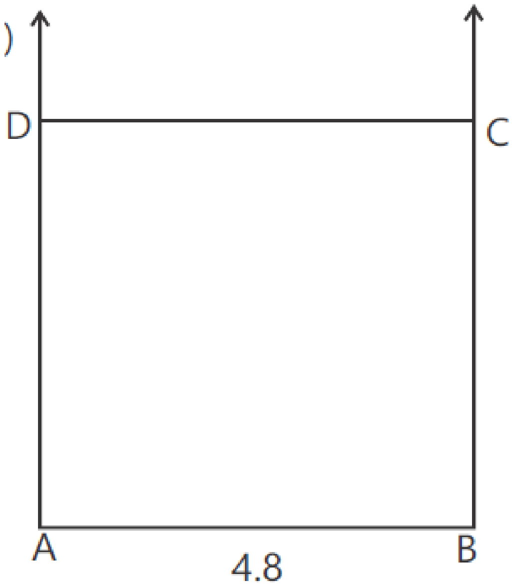
**Step II:** By using a protractor or set squares, draw a right angle at point A, i.e., ∠A = 90º.

**Step III:** Similarly, by using a protractor, draw a right angle at point B, i.e., ∠B = 90º.

**Step IV:** Mark points C and D on each of the newly drawn arms of the two right angles, respectively, such that $\overline{AD}$ = $\overline{BC}$ = 4.8 cm.

**Step V:** Join C to D.

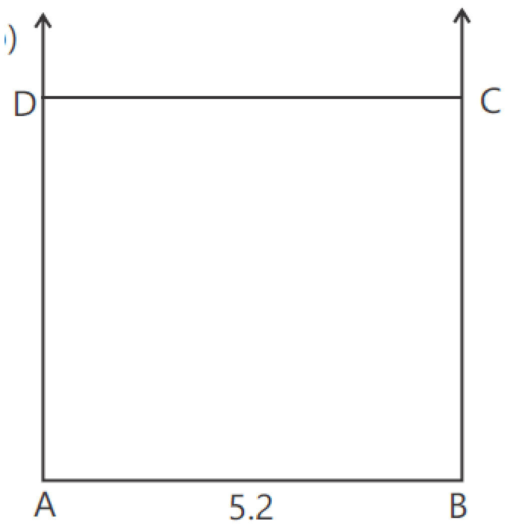
ABCD is the required square.




b) 5.2 cm

**Steps of Construction**

**Step I:** Draw a line segment m$\overline{AB}$ of 5.2 cm with the help of a ruler.
**Step II:** By using a protractor or set squares, draw a right angle at point A, i.e., ∠A = 90º.
**Step III:** Similarly, by using a protractor, draw a right angle at point B, i.e., ∠B = 90º.
**Step IV:** Mark points C and D on each of the newly drawn arms of the two right angles, respectively, such that $\overline{AD}$ = $\overline{BC}$ = 5.2 cm.
**Step V:** Join C to D.
ABCD is the required square.

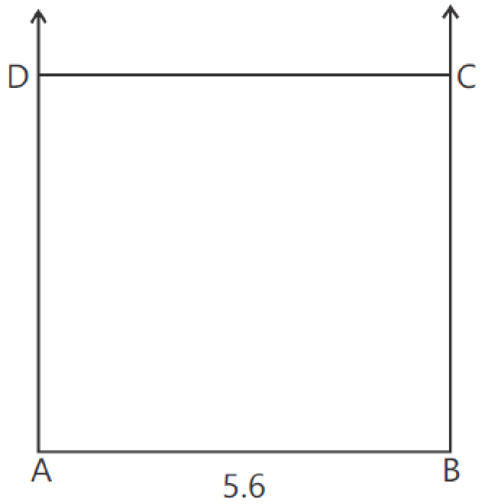



c) 5.6 cm

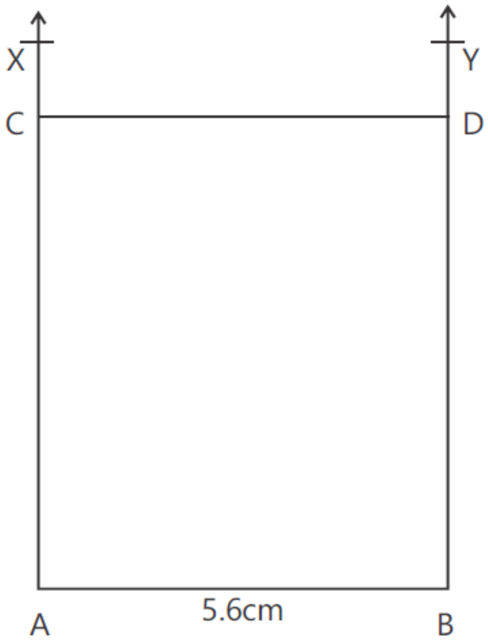
**Steps of Construction**

**Step I:** Draw a line segment m$\overline{AB}$ of 5.6 cm with the help of a ruler.
**Step II:** By using a protractor or set squares, draw a right angle at point A, i.e., ∠A = 90º.
**Step III:** Similarly, by using a protractor, draw a right angle at point B, i.e., ∠B = 90º.

**Step IV:** Mark points C and D on each of the newly drawn arms of the two right angles, respectively, such that $\overline{AD} = \overline{BC}$ = 5.6 cm.
**Step V:** Join C to D.
ABCD is the required square.

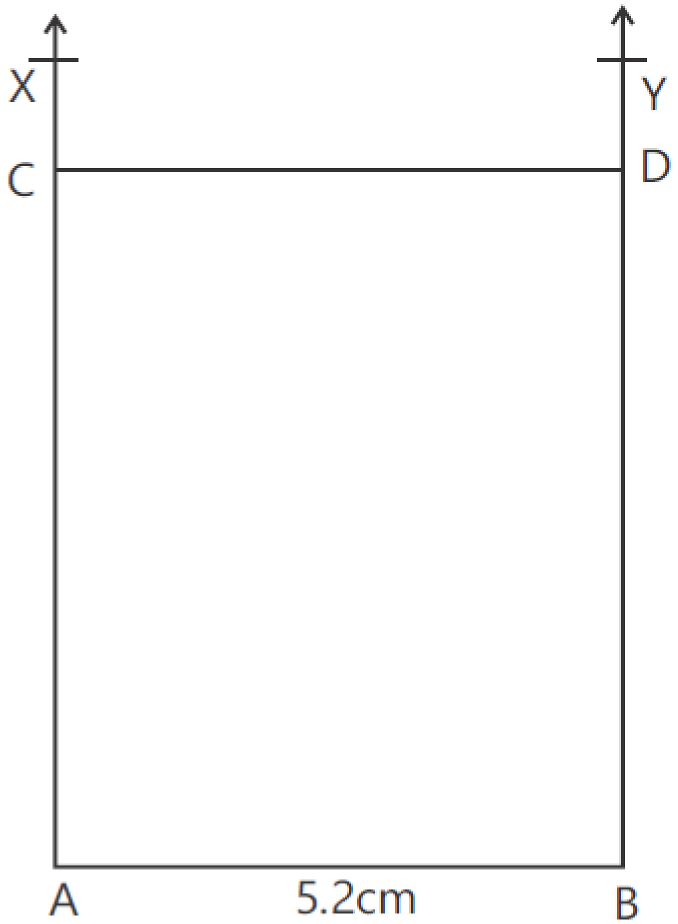



2. **Construct a rectangle ABCD when:**
a) $\overline{AB}$ = 5.6 cm, $\overline{AC}$ = 6.4 cm b) $\overline{AB}$ = 5.2 cm, $\overline{AC}$ = 7.2 cm
**Solution:**
a) $\overline{AB}$ = 5.6 cm, $\overline{AC}$ = 6.4 cm
**Steps of Construction**
**Step I:** Draw $\overline{AB}$ = 5.6 cm with the help of a ruler.
**Step II:** Draw an angle of 90° at A and B.
**Step III:** With A as the centre, draw an arc of radius 6.4 cm that cuts AX at point C.
**Step IV:** Using the same opening of the compasses, draw an arc with B as the centre, cutting BY at point D.
**Step V:** Join C with D.
ABCD is the required rectangle.

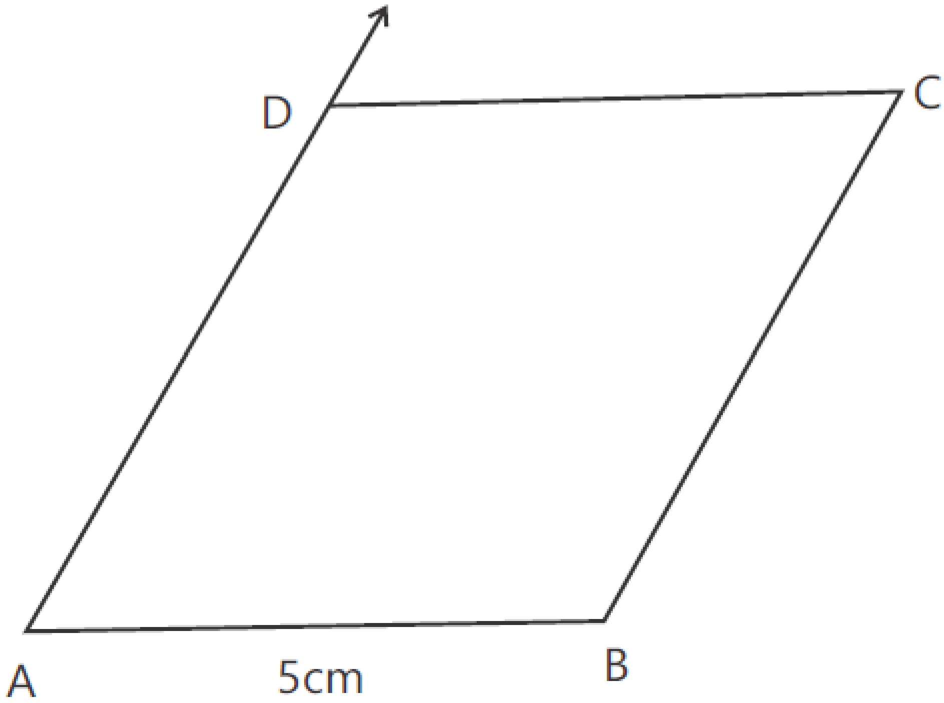



b) $\overline{AB}$ = 5.2 cm, $\overline{AC}$ = 7.2 cm

**Steps of Construction**
**Step I:** Draw $\overline{AB}$ = 5.2 cm with the help of a ruler.
**Step II:** Draw an angle of 90$^o$ at A and B.
**Step III:** With A as the centre, draw an arc of radius 7.2 cm that cuts AX at point C.
**Step IV:** Using the same opening of the compasses, draw an arc with B as the centre, cutting BY at point D.
**Step V:** Join C with D.
ABCD is the required rectangle.




3. **Construct a rhombus ABCD when the measurement of its side and included angle are given as:**
   a) $\overline{AB}$ = 5 cm, ∠A = 60°　　　　b) $\overline{AB}$ = 5.8 cm, ∠B = 135°
   **Solution:**
   a) $\overline{AB}$ = 5 cm, ∠A = 60°
   **Steps of Construction**
   **Step I:** Draw a line segment $\overline{AB}$ equal to 5 cm with the help of a ruler.
   **Step II:** At point A, construct a 60$^o$ angle.
   **Step III:** On the arm of this angle, mark a point D such that $\overline{AD}$ = 5 cm.
   **Step IV:** Draw an arc of 5 cm radius above B with a compass.
   **Step V:** Place the end of the compass at D, stretch it to 5 cm, and draw an arc that intersects the previous arc, naming the point where they meet as C.
   **Step VI:** Join C to D and B to C.
   ABCD is the required rhombus.




b) $\overline{AB}$ = 5.8 cm, ∠B = 135°

**Steps of Construction**

**Step I:** Draw a line segment $\overline{\mathrm{AB}}$ equal to 5.8 cm with the help of a ruler.

**Step II:** At point B, construct a 135° angle.

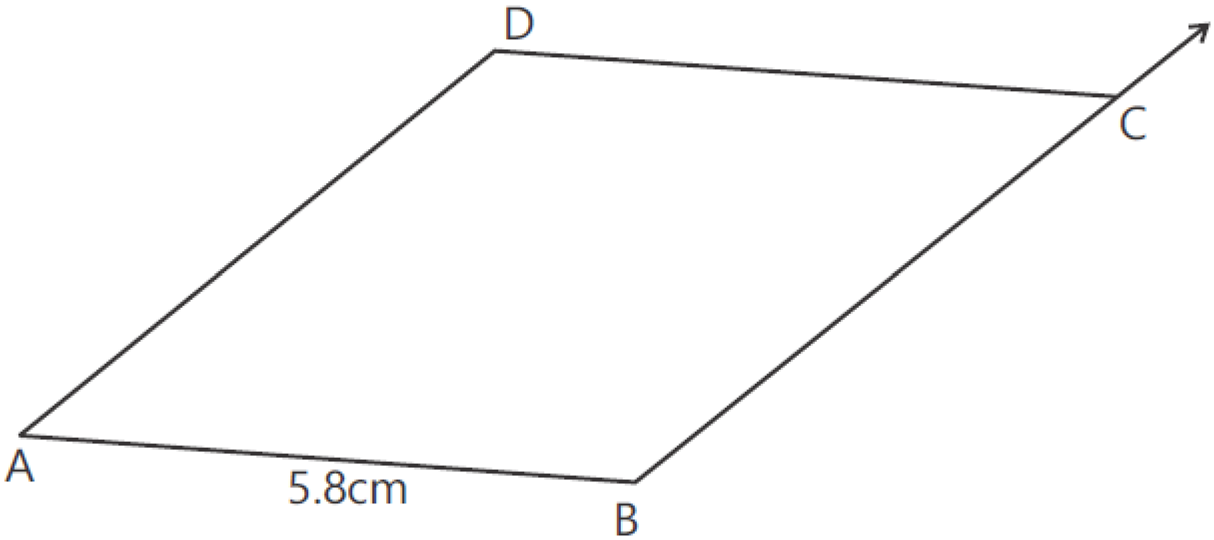
**Step III:** On the arm of this angle, mark a point C such that $\overline{\mathrm{BC}}$ = 5.8 cm.

**Step IV:** Draw an arc of 5.8 cm radius above A with a compass.

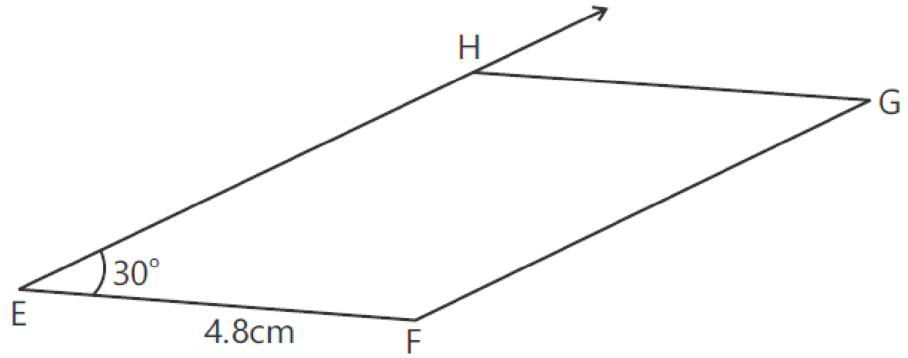
**Step V:** Place the end of the compass at C, stretch it to 5.8 cm, and draw an arc that intersects the previous arc, naming the point where they meet as D.

**Step VI:** Join C to D and B to C.

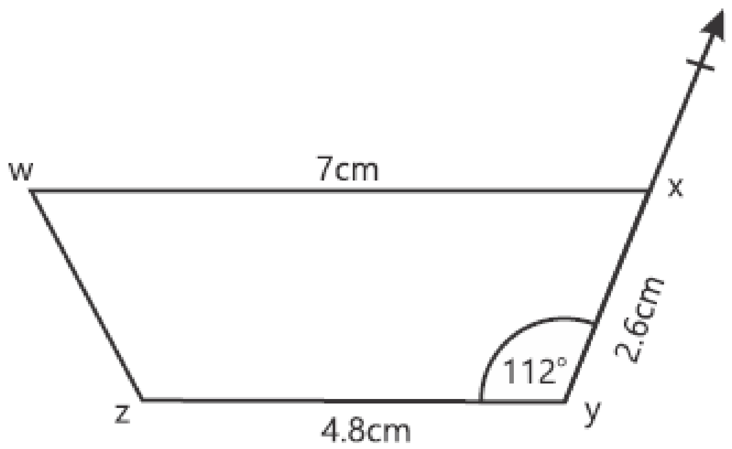
ABCD is the required rhombus.



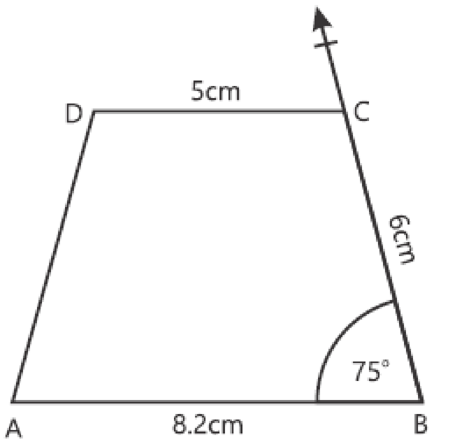
4. **Construct a parallelogram EFGH when EF = 4.8 cm, EH = 6cm and the angle between them is 30°.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw a line segment EF equal to 4.8 cm with the help of a ruler.
   **Step II:** At point E, construct a $30^o$ angle.
   **Step III:** On the arm of this angle, mark a point H such that EH = 6 cm.
   **Step IV:** Draw an arc of 6 cm radius above F with a compass.
   **Step V:** Place the end of the compass at H, stretch it to 6 cm, and draw an arc that intersects the previous arc, naming the point where they meet as G.
   **Step VI:** Join G to H and F to G.
   EFGH is the required parallelogram.



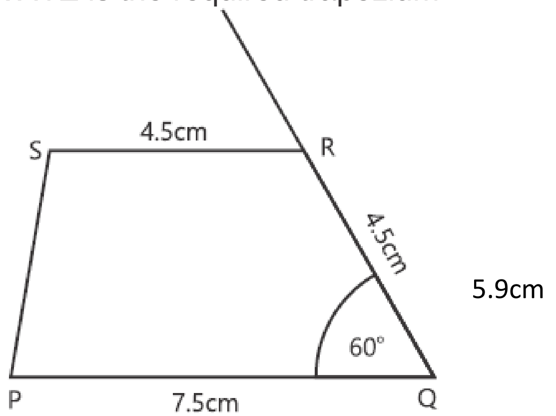

5. **Draw a trapezium WXYZ such that $\overline{WX}$ || $\overline{YZ}$, $\overline{ZY}$ = 4.8 cm, $\overline{WX}$ = 7 cm, $\overline{XY}$ = 2.6 cm and ∠XYZ = 112°.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw line $\overline{ZY}$ = 4.8 cm with the help of a ruler.
   **Step II:** Draw an angle of 112°, taking Y as a vertex, using a protractor.
   **Step III:** Mark point W with the help of a ruler such that $\overline{XY}$ = 2.6 cm.
   **Step IV:** Use a set square and a ruler to draw a line parallel to $\overline{ZY}$ through W of 7 cm and mark point X.
   **Step V:** Join Z to X.
   WXYZ is the required trapezium.



6. **Draw a trapezium ABCD in which $\overline{AB} \parallel \overline{DC}$, $\overline{AB}$ = 8.2 cm, $\overline{BC}$ = 6 cm, $\overline{DC}$ = 5 cm and ∠B = 75°.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw line $\overline{AB}$ = 8.2 cm with the help of a ruler.
   **Step II:** Draw an angle of 75°, taking B as a vertex, using a protractor.
   **Step III:** Mark point C with the help of a ruler such that $\overline{BC}$ = 6 cm.
   **Step IV:** Draw a line segment DC parallel to AB and make DC equal in length to DC = 5 cm.
   **Step V:** Draw another arc taking DC = 7.5 cm, taking A as a vertex, which cuts the previous arc at point D.
   **Step VI:** Join C to D.
   WXYZ is the required trapezium.




7. **Draw a trapezium PQRS in which $\overline{PQ} \parallel \overline{RS}$, $\overline{PQ}$ = 7.5 cm, $\overline{QR}$ = 5.9 cm, $\overline{SR}$ = 4.5 cm and ∠Q = 60°.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw line $\overline{PQ}$ = 7.5 cm with the help of a ruler.
   **Step II:** Draw an angle of 60°, taking Q as a vertex, using a protractor.
   **Step III:** Mark point S with the help of a ruler such that $\overline{QS}$ = 5.9 cm.
   **Step IV:** Draw a line segment SR parallel to PQ and make SR equal in length to $\overline{SR}$ = 7.5 cm.
   **Step V:** Draw another arc taking $\overline{SR}$ = 4.5 cm, taking P as a vertex, which cuts the previous arc at point R.
   **Step V:** Join R to P.
   WXYZ is the required trapezium.



**8. Construct a kite PQRS in which $\overline{PQ}$ = 3 cm, $\overline{QR}$ = 5 cm and $\overline{PR}$ (diagonal)=7cm.**

**Solution:**

**Steps of Construction**

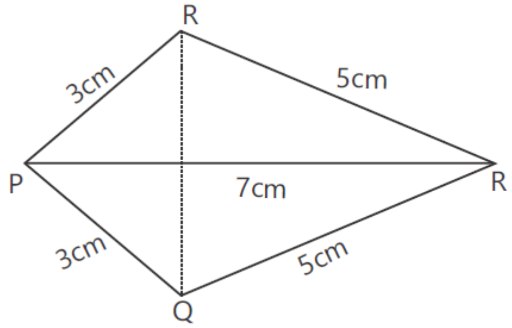
**Step I:** Draw a line segment $\overline{PR}$ = 7 cm with the help of a ruler.

**Step II:** With P as the centre, draw arcs of radius 3 cm above and below $\overline{PR}$.

**Step III:** With R as the centre, draw arcs of radius 5 cm above and below $\overline{PR}$ that intersect the previous arcs at points Q and S.

**Step IV:** Join P with Q, Q with R, R with S and S with P.

PQRS is the required kite.




**9. Construct a kite KLMN in which $\overline{KL}$ = 2.5 cm, $\overline{LM}$ = 4.5 cm and $\overline{KM}$ (diagonal)= 5.5cm.**

**Solution:**

**Steps of Construction**

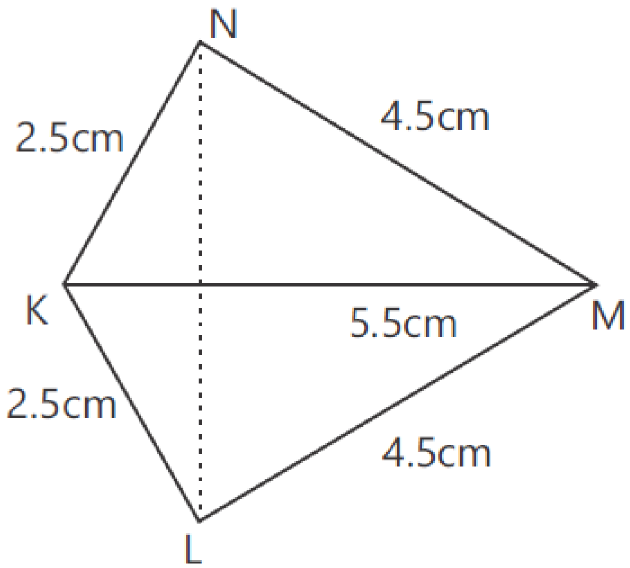
**Step I:** Draw a line segment $\overline{KM}$ = 5.5 cm with the help of a ruler.

**Step II:** With K as the centre, draw arcs of radius 2.5 cm above and below $\overline{KM}$.

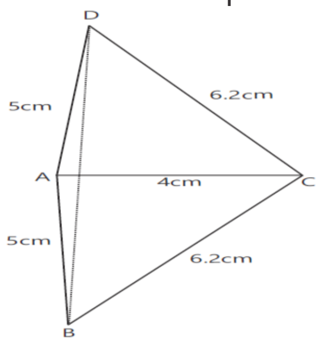
**Step III:** With M as the centre, draw arcs of radius 4.5 cm above and below $\overline{KM}$ that intersect the previous arcs at points L and N.

**Step IV:** Join K with L, L with M, M with N and n with K.

KLMN is the required kite.

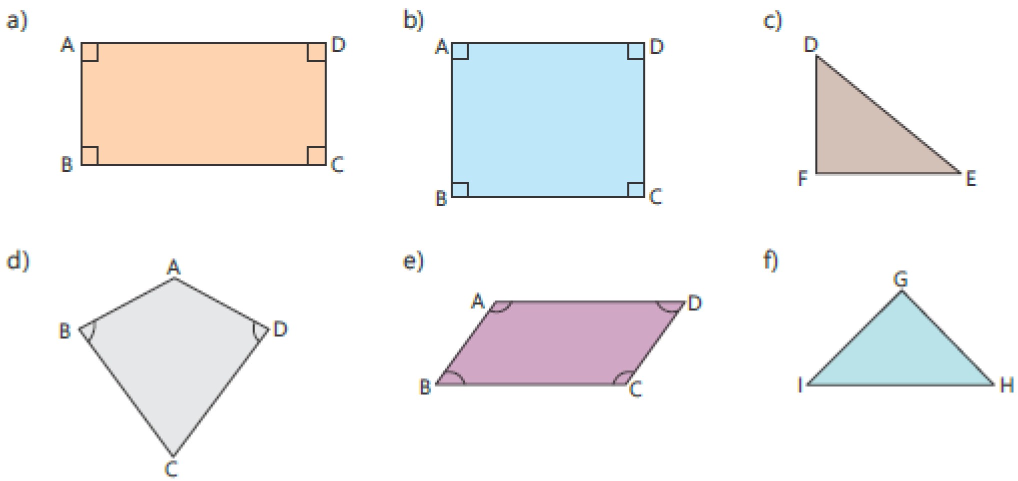


10. **Construct a kite with the length of its diagonal being 4 cm and the length of its sides being 5 cm and 6.2 cm, respectively.**
**Solution:**
**Steps of Construction**
**Step I:** Draw a line segment $\overline{AC}$ = 4 cm with the help of a ruler.
**Step II:** With A as the centre, draw arcs of radius 5 cm above and below $\overline{AC}$.
**Step III:** With C as the centre, draw arcs of radius 6.2 cm above and below $\overline{AC}$ that intersect the previous arcs at points B and D.
**Step IV:** Join A with B, B with C, C with D and D with A.
ABCD is the required kite.




**Learning Check 7.6**

1. **Draw angle bisectors for these shapes. Also draw perpendicular side bisectors.**




**Solution:**
a)

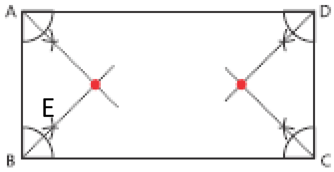

**Angles Bisector:**
**Steps of Construction**
**Step I:**
Take B as the centre and draw an arc of any appropriate radius using a compass.
**Step II:**

Without changing the radius, mark two arcs by taking center.

**Step III:**

Join B to E and extend it. This ray $\overrightarrow{BE}$ is the required angle bisector of angle B.

Similarly, bisect the other three angles.




**Sides Bisector:**

**Steps of Construction**

**Step I:**

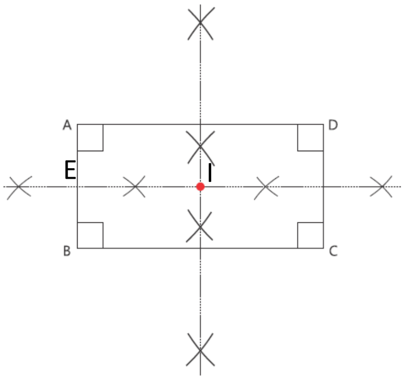
Using a compass, draw two arcs with A as the centre on both sides of AB with a radius slightly greater than half the length of segment AB.

**Step II:**

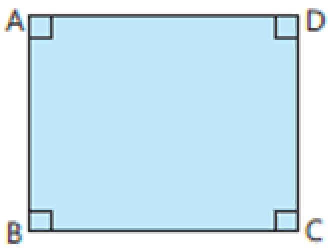
Similarly, using the same radius, take B as the centre and draw two arcs on both sides of AB.

**Step III:**

Join the point of intersection that cuts AB at E. This line is the perpendicular bisector of AB. Similarly, draw line bisectors of sides BC, CD and DA. The point I where the four bisectors of the sides of the rectangle meet.




b)




**Angles Bisector:**

**Steps of Construction**

**Step I:**

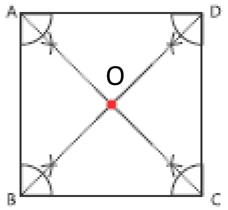
Take B as the centre and draw an arc of any appropriate radius using a compass.

**Step II:**

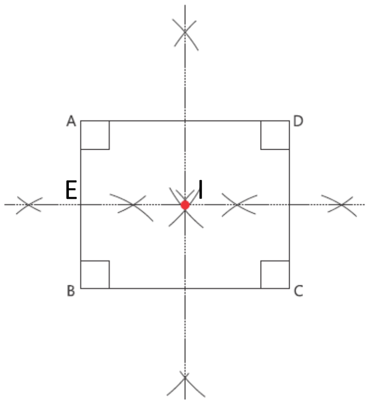
Without changing the radius, mark two arcs by taking center.

**Step III:**

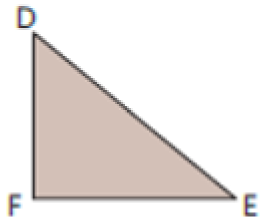
Join B to E and extend it. This ray $\overrightarrow{BE}$ is the required angle bisector of angle B.
Similarly, bisect the other three angles.
The bisectors meet at point O.




**Sides Bisector:**
**Steps of Construction**
**Step I:**
Using a compass, draw two arcs with A as the centre on both sides of AB with a radius slightly greater than half the length of segment AB.
**Step II:**
Similarly, using the same radius, take B as the centre and draw two arcs on both sides of AB.
**Step III:**
Join the point of intersection that cuts AB at E. This line is the perpendicular bisector of AB. Similarly, draw line bisectors of sides BC, CD and DA. The point I where the four bisectors of the sides of the square.

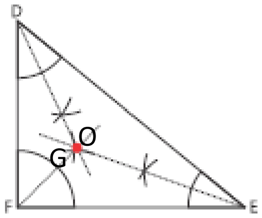



c)




**Angles Bisector:**
**Steps of Construction**
**Step I:**
Take F as the centre and draw an arc of any appropriate radius using a compass.
**Step II:**
Without changing the radius, mark two arcs by taking centre.
**Step III:**
Join F to G and extend it. This ray $\overrightarrow{FG}$ is the required angle bisector of angle F.

Similarly, bisect the other two angles. The bisectors meet at point O. This point O is called its **incenter**.




**Sides Bisector:**

**Steps of Construction**

**Step I:**

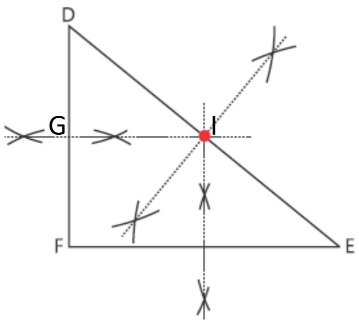
Using a compass, draw two arcs with F as the centre on both sides of FD with a radius slightly greater than half the length of segment FD.

**Step II:**

Similarly, using the same radius, take D as the centre and draw two arcs on both sides of FD.

**Step III:**

Join the point of intersection that cuts FD at G. This line is the perpendicular bisector of FD. Similarly, draw line bisectors of sides DE and EF. The point I where the three bisectors of the sides of the triangle meet is called the **circumcenter**.

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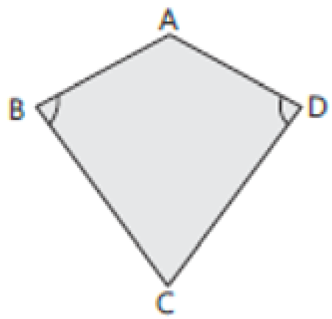



**Angles Bisector:**

**Steps of Construction**

**Step I:**

Take B as the centre and draw an arc of any appropriate radius using a compass.

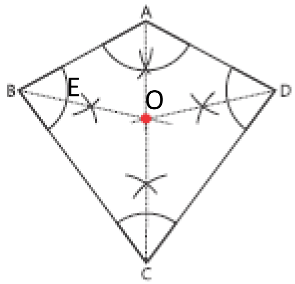
G

**Step II:**

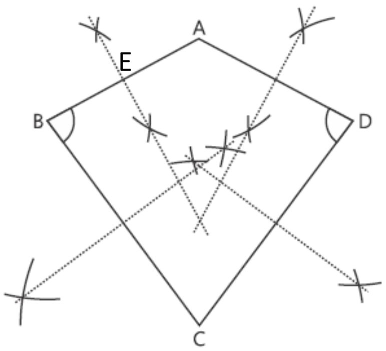
Without changing the radius, mark two arcs by taking two centres of the given arc that meet at a single point.

**Step III:**

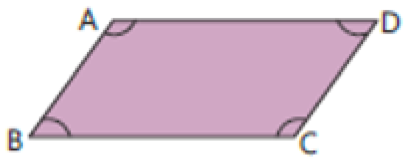
Join B to E and extend it. This ray $\overrightarrow{BE}$ is the required angle bisector of angle B. Similarly, bisect the other three angles.
The bisectors meet at point O.



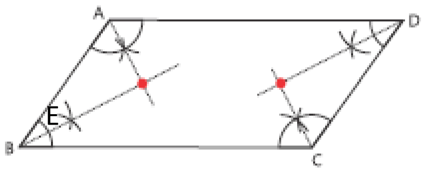

**Sides Bisector:**
**Steps of Construction**
**Step I:**
Using a compass, draw two arcs with A as the centre on both sides of AB with a radius slightly greater than half the length of segment AB.
**Step II:**
Similarly, using the same radius, take B as the centre and draw two arcs on both sides of AB.
**Step III:**
Join the point of intersection that cuts AB at E. This line is the perpendicular bisector of AB. Similarly, draw line bisectors of sides BC, CD and DA. So, there is no single point of intersection of four perpendicular lines.




e)




**Angles Bisector:**
**Steps of Construction**
**Step I:**
Take B as the centre and draw an arc of any appropriate radius using a compass.
**Step II:**
Without changing the radius, mark two arcs by taking as the centre.
**Step III:**
Join B to E and extend it. This ray $\overrightarrow{BE}$ is the required angle bisector of angle B. Similarly, bisect the other three angles.




**Sides Bisector:**

**Steps of Construction**

**Step I:**

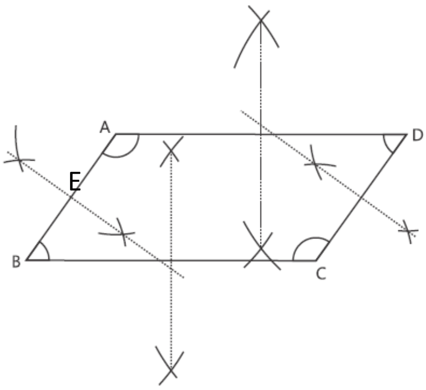
Using a compass, draw two arcs with A as the centre on both sides of AB with a radius slightly greater than half the length of segment AB.

**Step II:**

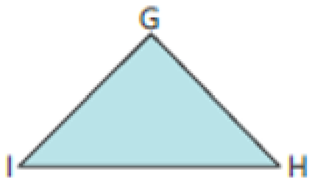
Similarly, using the same radius, take B as the centre and draw two arcs on both sides of AB.

**Step III:**

Join the point of intersection that cuts AB at E. This line is the perpendicular bisector of AB. Similarly, draw line bisectors of sides BC, CD and DA. So, there is no single point of intersection of four perpendicular lines.




f)




**Angles Bisector:**

**Steps of Construction**

**Step I:**

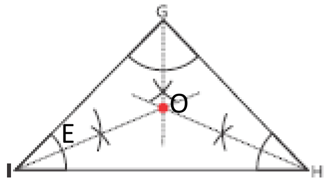
Take I as the centre and draw an arc of any appropriate radius using a compass.

**Step II:**

Without changing the radius, mark two arcs by taking as the centre.

**Step III:**

Join I to R and extend it. This ray IR is the required angle bisector of angle GHI. Similarly, bisect the other two angles. The bisectors meet at point O. This point O is called its **incenter**.




**Sides Bisector:**

**Steps of Construction**

**Step I:**

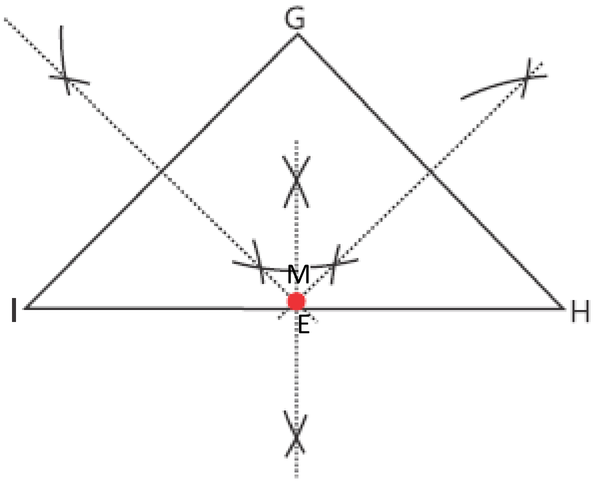
Using a compass, draw two arcs with I as the centre on both sides of HI with a radius slightly greater than half the length of segment HI.

**Step II:**

Similarly, using the same radius, take H as the centre and draw two arcs on both sides of HI.

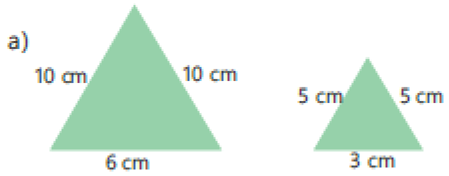
**Step III:**

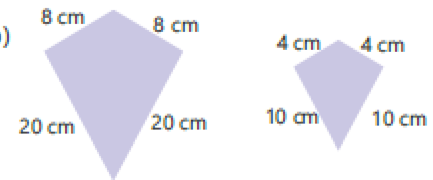
Join the point of intersection that cuts HI at J. This line is the perpendicular bisector of HI. Similarly, draw line bisectors of sides GH and IG. The point M where the three bisectors of the sides of the triangle meet is called the **circumcenter**.



## Learning Check 7.7

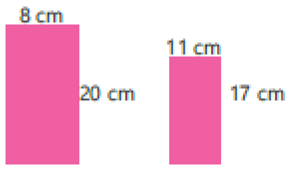
1. **Tick (✓) the pair of congruent figures and circle the pair of similar figures.**

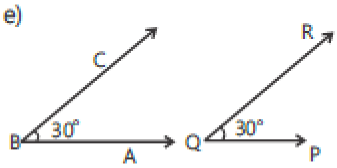
a)


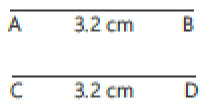

b)



c)

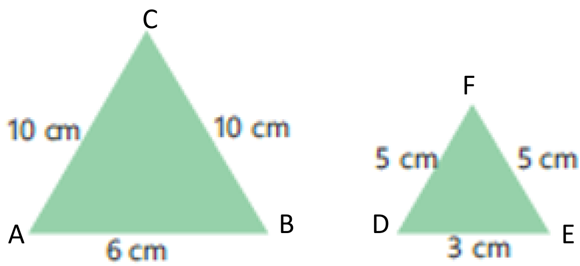


d)


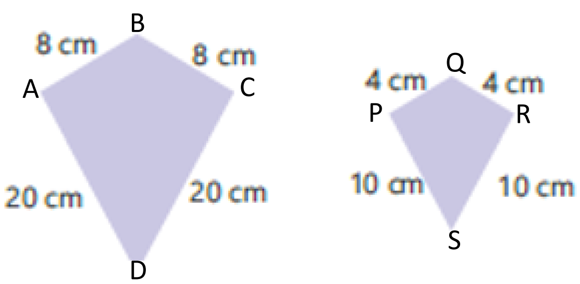

e)



f)



**Solution:**

a)



The corresponding angles are equal of the given figures:
∠A = ∠D, ∠B = ∠E and ∠C = ∠F
All sides of first figure are proportional to the corresponding sides of second figure.
$\frac{\overline{AB}}{\overline{DE}} = \frac{\overline{BC}}{\overline{EF}} = \frac{\overline{AC}}{\overline{DF}}$ = 2 cm
So, these figures are similar figures.
These figures are not congruent because their corresponding sides are different.

b)


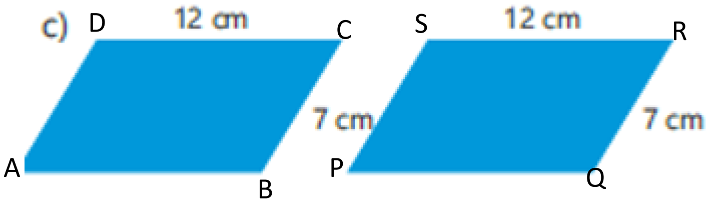

The corresponding angles are equal of the given figures:
∠A = ∠P, ∠B = ∠Q, ∠C = ∠R and ∠D = ∠S
All sides of first figure are proportional to the corresponding sides of second figure.

$\frac{\overline{AB}}{\overline{PQ}} = \frac{\overline{BC}}{\overline{QR}} = \frac{\overline{CD}}{\overline{RS}} = \frac{\overline{DA}}{\overline{SP}}$ = 2 cm

So, these figures are similar figures.

These figures are not congruent because their corresponding sides are different.

c)




In the above figures, ABCD ≅ PQRS. We read it as "ABCD is congruent to PQRS."

We can see that:

$\overline{AB} \cong \overline{PQ}$, $\overline{BC} \cong \overline{QR}$, $\overline{CD} \cong \overline{RS}$ and $\overline{DA} \cong \overline{SP}$

So, all the corresponding sides are equal. Hence, the above figures are congruent figures.

The corresponding angles are equal of the given figures:

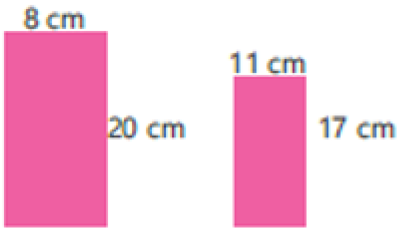
$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and $\angle D = \angle S$

All sides of first figure are proportional to the corresponding sides of second figure.

$\frac{\overline{AB}}{\overline{PQ}} = \frac{\overline{BC}}{\overline{QR}} = \frac{\overline{CD}}{\overline{RS}} = \frac{\overline{DA}}{\overline{SP}}$ = 1 cm

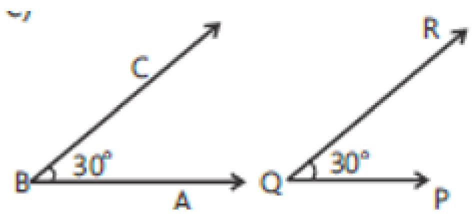
So, these figures are similar figures.

d)




Look at the rectangles. They look similar in shape, but their sizes are not the same. So, we can say that the rectangles are similar to each other, but they are not congruent as they have different sizes. The figures that have the same shape but not the same size are called similar figures.

e)



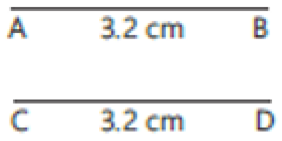

In the above figures, ABC ≅ PQR. We read it as "ABC is congruent to PQR."

We can see that:

$\overline{BA} \cong \overline{QP}$, $\angle B \cong \angle Q$ and $\overline{BC} \cong \overline{QR}$

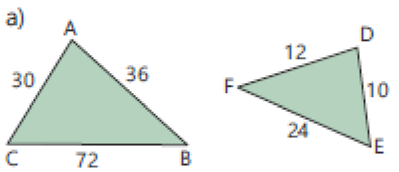
So, all the corresponding sides and included angle are equal. Hence, the above figures are congruent figures.

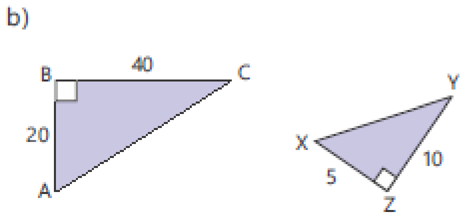
f)




In the above figures, AB ≅ CD. We read it as "AB is congruent to CD."
We can see that:
$\overline{AB} \cong \overline{CD}$
So, all the corresponding sides are equal. Hence, the above figures are congruent figures.

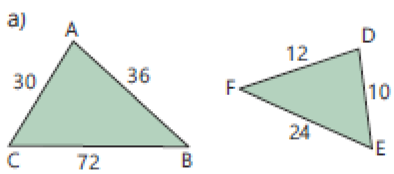
2. **Apply properties of similar figures and check if the triangles are similar or not.**



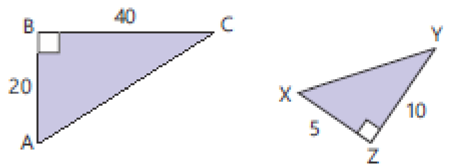

**Solution:**

a)




Look at the triangles. They look similar in shape, but their sizes are not the same. So, we can say that the triangles are similar to each other, but they are not congruent as they have different sizes. The figures that have the same shape but not the same size are called similar figures.

b)

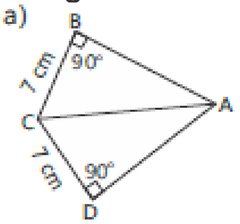



Look at the triangles. They look similar in shape, but their sizes are not the same. So, we can say that the triangles are similar to each other, but they are not congruent as they have different sizes. The figures that have the same shape but not the same size are called similar figures.

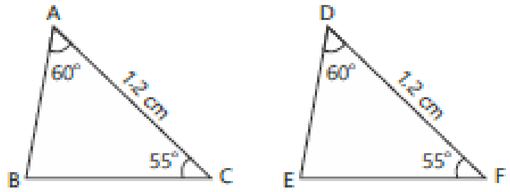
## Learning Check 7.8

1. **Apply properties of congruent triangles and find if the following triangles are congruent or not.**

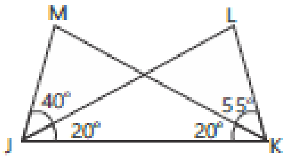
a)



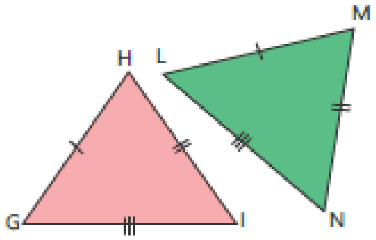

b)




c)

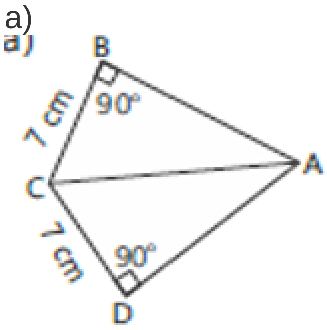



d)



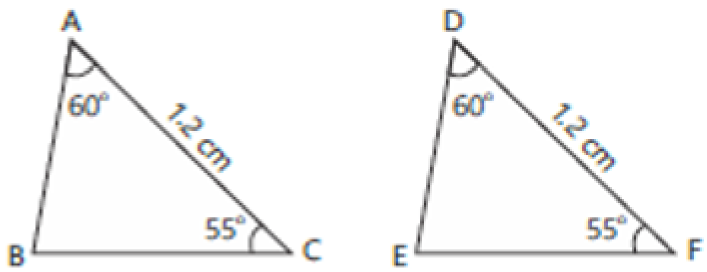

**Solution:**

a)



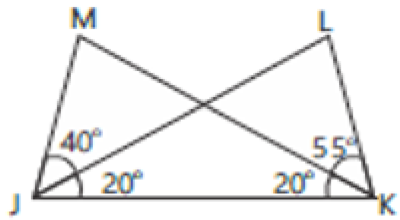

Look at the given triangles:
The measure of two sides of triangle ABC are equal to the corresponding sides of triangle ADC.
$\overline{AB} = \overline{AD}$, $\overline{BC} = \overline{CD}$
Also, the included ∠B of triangle ABC is equal to the corresponding included ∠D of triangle ADC.
∠B = ∠D
So, ΔDABC ≅ ΔADC.

b)



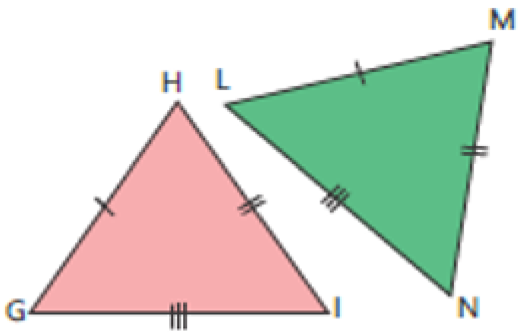

Look at triangle ABC and triangle DEF.
The two angles ∠A and ∠C and the non-included side AB is equal to the corresponding angles ∠D and ∠F and one non-included side DE of the other triangle DEF.
∠A = ∠D, ∠C =∠F and AB = DE.
∴ ΔABC ≅ ΔDEF

c)



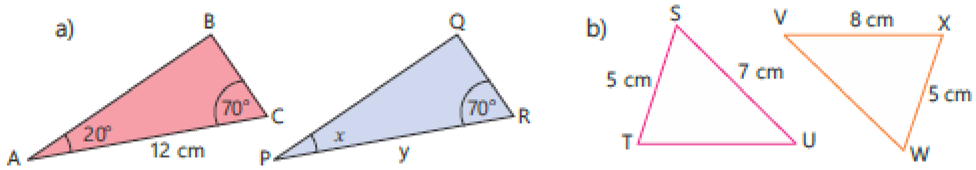

The hypotenuse JK is the common side of both triangles.
The length of the base of both triangles are also equal. So,
JK = JK and JM = KL
So, ΔJKL ≅ ΔKJM.

d)

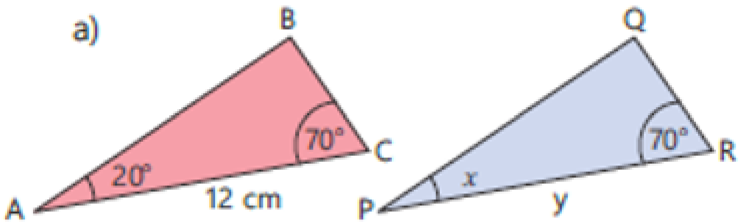



Look at the ΔGHI and ΔLMN.
The measures of all sides of the triangle GHI are equal to the corresponding sides of the triangle LMN, i.e.,
$\overline{GH} = \overline{LM}$, $\overline{HI} = \overline{MN}$ and $\overline{GI} = \overline{LN}$
So, ΔGHI ≅ ΔLMN.

**2. Find the unknown sides or angles of the given congruent triangles.**

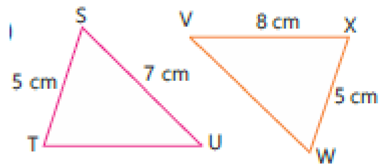



**Solution:**
a)



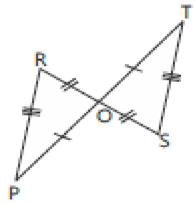

Look at triangle ABC and triangle PQR.
The two angles ∠A and ∠C and their included side $\overline{AC}$ is equal to the corresponding angle ∠P and ∠R and their included side $\overline{PR}$ of the other triangle PQR.
∠P = ∠A or x = $20^0$ and
$\overline{PR} = \overline{AC}$ or y = 12 cm
Hence, ΔGHI ≅ ΔLMN.

b)




In the triangles STU and VWX, the three sides of the triangle STU are equal to the three sides of triangle VWX.
$\overline{SU}$ = $\overline{VW}$ = 7 cm, $\overline{TU}$ = $\overline{VX}$ = 8 cm and $\overline{ST}$ = $\overline{WX}$ = 5 cm.
Hence, ΔGHI ≅ ΔLMN.

3. **Look at the figure. RS and TP bisect each other.**
   a) State the three pairs of equal sides in the two triangles POR and TO
   b) Which of the following statements is true?
   $\Delta POR \cong \Delta STO$ or $\Delta POR \cong \Delta TOS$




**Solution:**

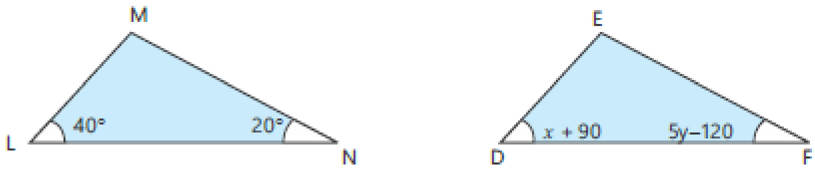
a)

In the triangles POR and TOS, the three sides of the triangle POR are equal to the three sides of triangle TOS.
$\overline{PO}$ = $\overline{OT}$ , $\overline{OR}$ = $\overline{OS}$ and $\overline{PR}$ = $\overline{ST}$.
Hence, ΔPOR ≅ ΔTOS.

b)

The correct statement is ΔPOR ≅ ΔTOS.
In this case, we have
$\overline{PR}$ = $\overline{TS}$ (corresponding sides), $\overline{OR}$ = $\overline{OS}$ (corresponding sides), and ∠POR = ∠TOS (since RS and TP bisect each other, the angles at O are congruent).
Therefore, by the SAS property, we can conclude that ΔPOR ≅ ΔTOS.

4. DLMN **and** DDEF **are congruent by Angle-Side-Angle congruence property.**
   **Find the value of *x* and y.**




**Solution:**

In ΔLMN and ΔDEF

∠D = ∠L

x + $90^0$ = $40^0$

x = $40^0$ – $90^0$

x = – $50^0$

As, LN = DF

Also,

∠F = ∠N

$5y$ – $120^0$ = $20^0$

$5y$ = $20^0$ + $120^0$

$5y$ = $140^0$

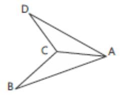
Dividing 5 on both sides

$\frac{5y}{5} = \frac{140^0}{5}$

y = $28^0$

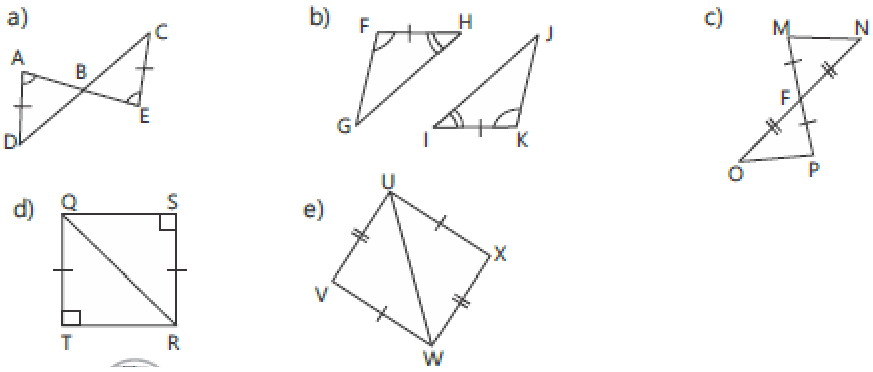
So, according to ASA property, ΔLMN is congruent to ΔDEF.

5. **In triangles ABC and ADC, $\overline{BC}$ = $\overline{DC}$ and $\overline{BA}$ = $\overline{DA}$ Prove that the triangles are congruent.**

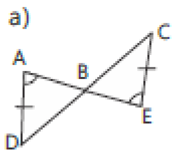



**Proof:**
In ΔABC and ΔADC
$\overline{BA}$ = $\overline{DA}$ (given)
$\overline{BC}$ = $\overline{DC}$ (given)
$\overline{AC}$ = $\overline{AC}$ (common)
By the side-side-side (SSS) property,
∴ ΔABC ≅ΔADC

6. **Prove the congruence of these triangles. Also state the property that is applied to prove the congruence.**

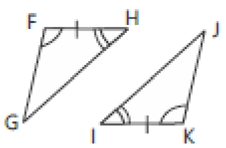



a)



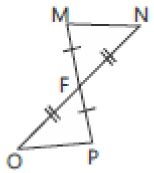

Look at the given triangles:
The measure of two sides of triangle ABD are equal to the corresponding sides of triangle BCE.
$\overline{AD} \cong \overline{CE}$, $\overline{AB} \cong \overline{BE}$
Also, the included ∠A of triangle ABC is equal to the corresponding included ∠E of triangle DEF.
∠A ≅ ∠E
∴ ΔABD ≅ ΔBCE.

b)



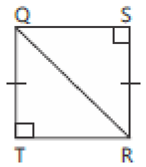

Look at triangle HFG and triangle IKJ.
The two angles ∠H and ∠F and their included side $\overline{HF}$ is equal to the corresponding angle ∠I and ∠K and their included side $\overline{IK}$ of the other triangle PQR, i.e.
∠H ≅ ∠I, ∠F ≅ ∠K and $\overline{HF} \cong \overline{IK}$.
∴ ΔHFG ≅ ΔIKJ.

c)



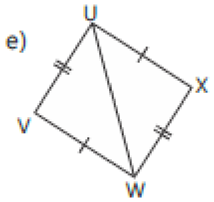

Look at the given triangles:
The measure of two sides of triangle MNF are equal to the corresponding sides of triangle POF.
$\overline{MF} \cong \overline{PF}$, $\overline{OF} \cong \overline{NF}$
Also, the included ∠F of triangle MNF is equal to the corresponding included ∠F of triangle POF.
∠F ≅ ∠F
So, ΔMNF ≅ ΔPOF.

d)




Look at the triangles RSQ and RTQ.
The length of one side and the hypotenuse of triangle RST are congruent to the corresponding side and the hypotenuse of triangle RTQ, i.e.,
$\overline{QR} \cong \overline{QR}$ and $\overline{RS} \cong \overline{QT}$.
So, ΔRSQ ≅ ΔRTQ.

e)



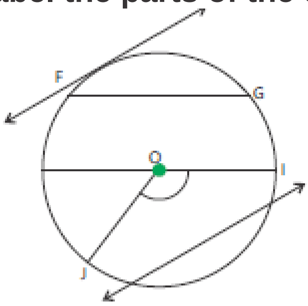

Look at the ΔUVW and ΔUXW.
The measures of all sides of the triangle UVW are equal to the corresponding sides of the triangle UXW, i.e.,
$\overline{VW} \cong \overline{UX}$, $\overline{UV} \cong \overline{WX}$ and $\overline{UW} \cong \overline{UW}$
So, ΔUVW ≅ ΔUXW.

## Unit Evaluation 7

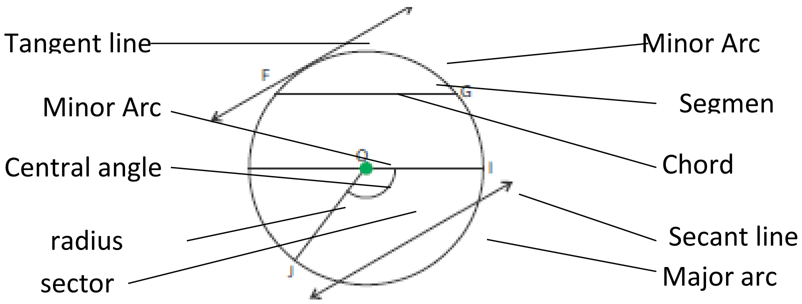
1. **Encircle the correct option.**
   a) Rotating 90° anticlockwise an image about the origin, the point P (*x*, y) switched to:
   i) (-*x*, -y) ii) (*x*, -y) iii) (-*x*, y) **iv) (-y, *x*)**
   b) If a line intersects a circle at two distinct points, this line is called:
   **i) secant** ii) tangent iii) sector iv) arc
   c) A line that passes through a circle and intersects it at two points is called:
   **i) secant** ii) tangent iii) sector iv) arc
   d) The objects or shapes that have the same shape and size are called:
   **i) congruent** ii) similar iii) sector iv) equal

e) If a line touches a circle at only one point, then this line is called:
i) secant ii) arc **iii) tangent** iv) sector

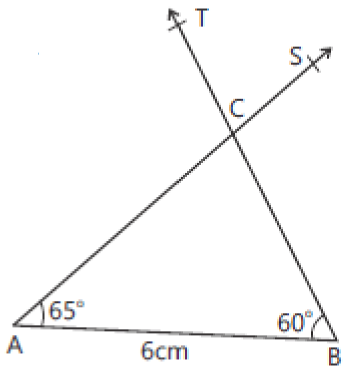
2. **Fill in the blanks.**
   a) The point around which a shape rotates is called **centre of rotation**.
   b) A part or portion of the circumference of a circle is called **bisectors**.
   c) The **similar figures** of an angle divide the angle into two equal angles.
   d) The figures that have the same shape but not the same size is called **congruent**.
   e) Angles and sides of **congruent** figures are the same.
3. **Label the parts of the circle.**



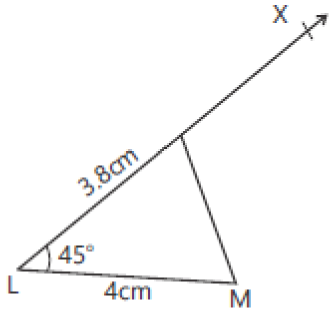

**Label:**



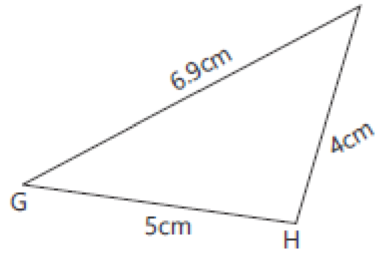

4. **Construct a triangle ABC when ∠B = 60°, ∠A = 45° and the length of one side is 6 cm.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw a line segment $\overline{AB}$ of length 6 cm with the help of a ruler.
   **Step II:** Draw an angle of 45° at point A.
   **Step III:** Similarly, at point B draw ∠ABC = 60°.
   $\overrightarrow{AS}$ and $\overrightarrow{BT}$ cut each other at point C.
   ΔABC is the required triangle.



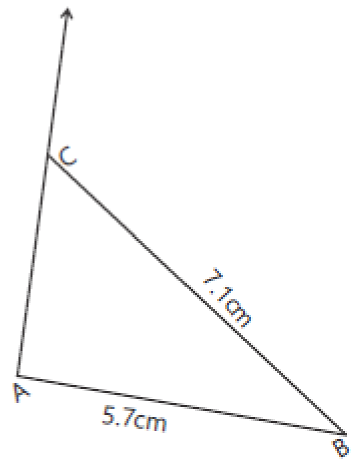
5. **Construct a triangle LMN when ∠L = 45° and the lengths of two sides are 4 cm and 3.8 cm.**
   **Solution:**
   **Step I:** Draw a line segment $\overline{LM}$ = 4 cm with the help of a ruler.
   Step **II:** Draw an arc of 45° at point L.
   **Step III:** Place the pointer of the compass at point L and draw an arc of length 3.8 cm that cuts the arm of the angle $\overrightarrow{LX}$ at point N.
   Step **IV:** Using a ruler join N to M.
   So, ΔLMN is the required triangle.



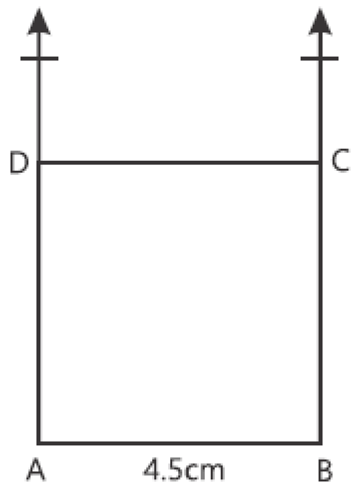

6. **Construct a triangle GHI when its three sides are GH = 5 cm, IG= 6.9 cm and HI = 4 cm.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw a line segment $\overline{\mathrm{GH}}$ = 5 cm with the help of a ruler.
   **Step II:** Place the pointer of the compass at point G and draw an arc of length 6.9 cm.
   **Step III:** Place the pointer of the compass at point H and draw an arc of radius 4 cm that cuts the previous arc at point I.
   **Step IV:** Join I to G and H.
   So, ΔGHI is the required triangle.




7. **Draw a right-angled triangle when the length of its hypotenuse is 7.1 cm and its base is 5.7 cm. Draw its angle and side bisectors as well.**
   **Solution:**
   **Steps of Construction**
   **Step I:** Draw a line segment AB of length 5.7 cm with the help of a ruler.
   **Step II:** Draw an angle of 90° at point A.
   **Step III:** Place the pointer of the compass at point B and draw an arc of length 7.1 cm that cuts arm AS of ∠90° at point C.
   **Step IV:** Join the point B to C. So, ΔABC is the required triangle.

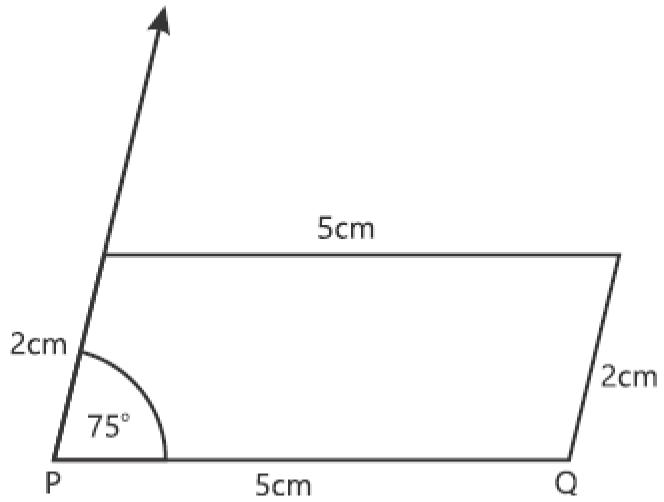



8. **Draw a square ABCD with side AB = 4.5 cm.**
**Solution:**
**Steps of Construction**
**Step I:** Draw a line segment m$\overline{AB}$ of 4.5 cm with the help of a ruler.
**Step II:** By using a protractor or set squares, draw a right angle at point A, i.e., ∠A = 90º.
**Step III:** Similarly, by using a protractor, draw a right angle at point B, i.e., ∠B = 90º.
**Step IV:** Mark points C and D on each of the newly drawn arms of the two right angles, respectively, such that $\overline{AD}$ = $\overline{BC}$ = 4.5 cm.
**Step V:** Join C to D.
ABCD is the required square.

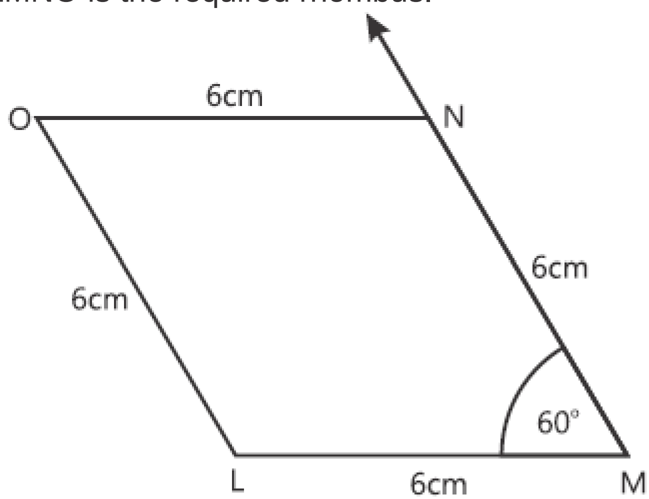



9. **Draw a parallelogram PQRS such that PQ = 5 cm, QR = 2 cm and ∠SPQ = 75°.**
**Solution:**
**Steps of Construction**
**Step I:** Draw a line segment $\overline{PQ}$ equal to 5 cm with the help of a ruler.
**Step II:** At point Q, construct a 75º angle.
**Step III:** On the arm of this angle, mark a point R such that $\overline{QR}$ = 2 cm.
**Step IV:** Draw an arc of 2 cm radius above P with a compass.
**Step V:** Place the end of the compass at R, stretch it to 5 cm, and draw an arc that intersects the previous arc, naming the point where they meet as S.

**Step VI:** Join R to S and P to S.
PQRS is the required parallelogram.

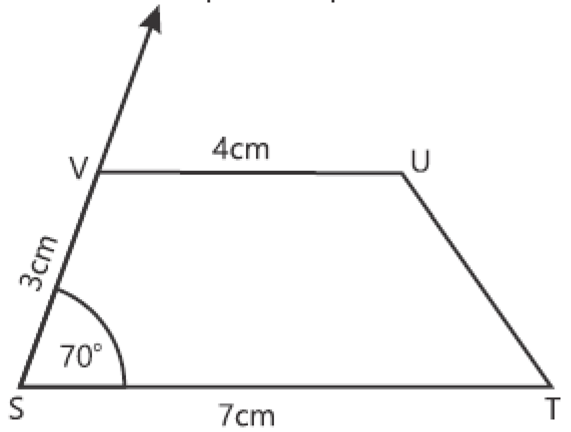



10. **Draw a rhombus LMNO such that LM = 6 cm and ∠LMN = 60°.**
**Solution:**
**Steps of Construction**
**Step I:** Draw a line segment $\overline{\mathrm{LM}}$ equal to 6 cm with the help of a ruler.
**Step II:** At point M, construct a 60° angle.
**Step III:** On the arm of this angle, mark a point M such that $\overline{\mathrm{MN}}$ = 6 cm.
**Step IV:** Draw an arc of 6 cm radius above L with a compass.
**Step V:** Place the end of the compass at N, stretch it to 5 cm, and draw an arc that intersects the previous arc, naming the point where they meet as O.
**Step VI:** Join N to O and L to O.
LMNO is the required rhombus.




11. **Draw a trapezium STUV such that ST = 7 cm, SV = 3 cm, VU = 4 cm, VST = 70° and VU II ST.**
**Solution:**
**Steps of Construction**
**Step I:** Draw line $\overline{ST}$ = 7 cm with the help of a ruler.
**Step II:** Draw an angle of 70°, taking S as a vertex, using a protractor.
**Step III:** Mark point V with the help of a ruler such that $\overline{SV}$ = 3 cm.
**Step IV:** Use a set square and a ruler to draw a line parallel to $\overline{ST}$ through V of 7 cm and mark point U.
**Step V:** Join V to U.

STUV is the required trapezium.




**12. Construct a kite with the length of its diagonal at 6 cm, the lengths of its sides at 7 cm, and 8 cm, respectively.**

**Solution:**

**Steps of Construction**

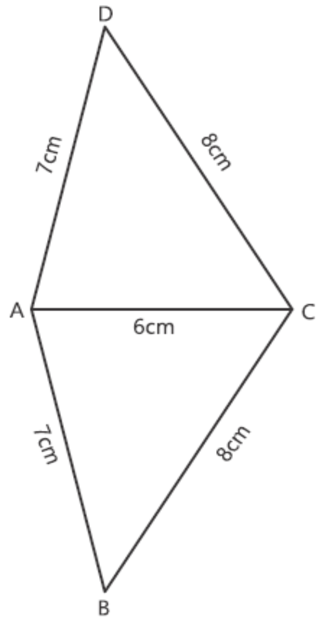
**Step I:** Draw a line segment $\overline{AC}$ = 6 cm with the help of a ruler.

**Step II:** With A as the centre, draw arcs of radius 7 cm above and below $\overline{AC}$.

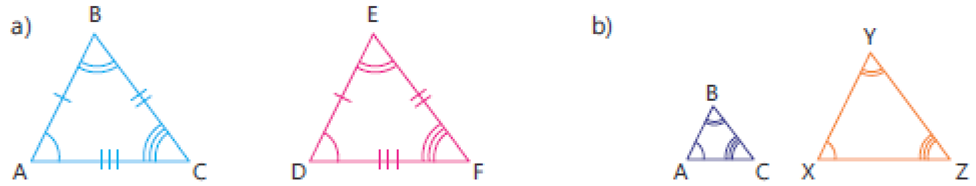
**Step III:** With C as the centre, draw arcs of radius 8 cm above and below $\overline{AC}$ that intersect the previous arcs at points B and D.

**Step IV:** Join A with B, B with C, C with D and D with A.

ABCD is the required kite.

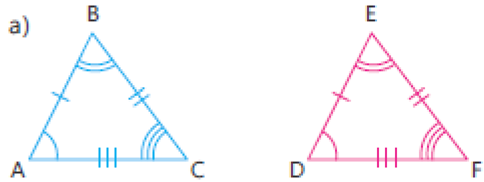



**13. Which pair of triangles is congruent?**



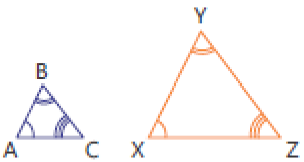
**Solution:**

a)



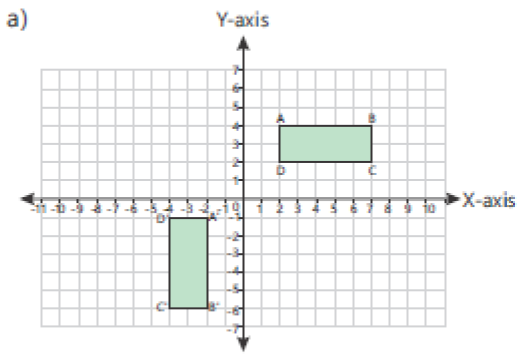

Look at the ΔABC and ΔDEF.
The measures of all sides of the triangle ABC are equal to the corresponding sides of the triangle DEF, i.e.,
$\overline{AB} = \overline{DE}$, $\overline{BC} = \overline{EF}$ and $\overline{AC} = \overline{DF}$
So, ΔABC ≅ ΔDEF.

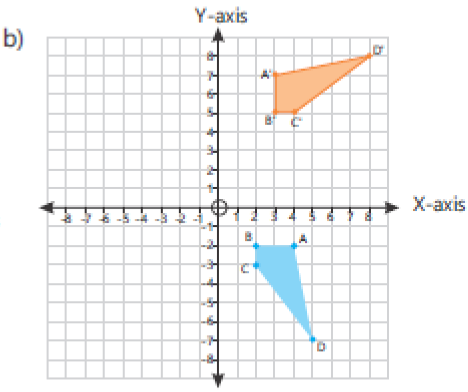
b)




Look at the triangles. They look similar in shape, but their sizes are not the same. So, we can say that the triangles are similar to each other, but they are not congruent as they have different sizes. The figures that have the same shape but not the same size are called similar figures.

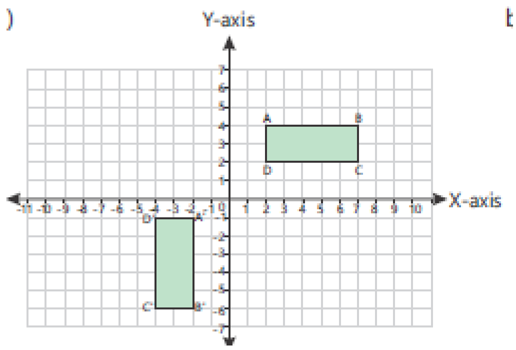
**14. Copy the given shapes onto cartesian graph paper and find the centre of rotation.**




**Solution:**

a)

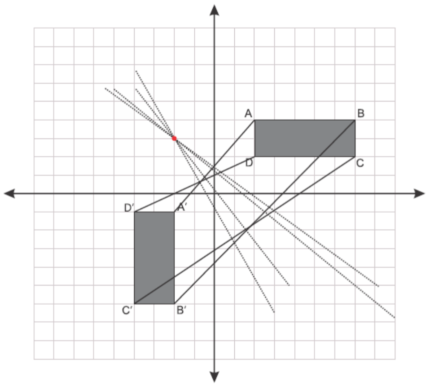



To find the centre of rotation, follow the given steps:

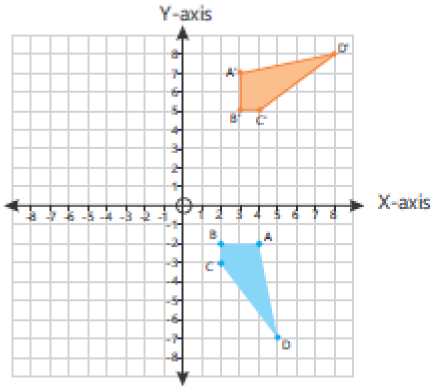
**Step 1:** Draw a line joining the corresponding vertices of both images, i.e.,
AA', BB', CC' and DD', respectively.

**Step 2:** Construct the perpendicular bisector of these segments, AA', BB', CC' and DD'.

The point X (- 2, 3) where the three perpendicular bisectors meet is the centre of rotation.




b)




To find the centre of rotation, follow the given steps:

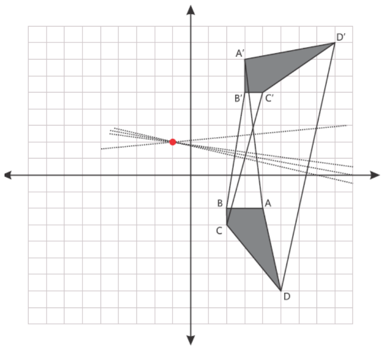
**Step 1:** Draw a line joining the corresponding vertices of both images, i.e.,

AA', BB', CC' and DD', respectively.

**Step 2:** Construct the perpendicular bisector of these segments,

AA', BB', CC' and DD'.

The point X (- 1, 2) where the three perpendicular bisectors meet is the centre of rotation.




15. **Draw a square on a coordinate plane. Then rotate it through 90° clockwise about the origin.**

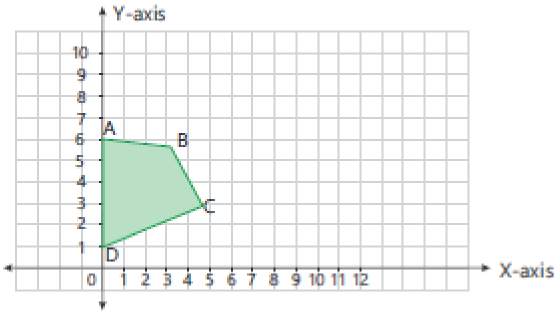
**Solution:**

To rotate the rectangle ABCD about the origin $90^0$ clockwise, we would follow the rule ($x$, y) → (y, - $x$), where the y-value of the original point becomes the new $x$-value with the same sign and the $x$-value of the original point becomes the new y-value with opposite sign.

Let's apply the rule to the vertices to create the new rectangle A'B'C'D':

A (1, 7) becomes A' (7, - 1)
B (6, 7) becomes B' (7, - 6)
C (6, 2) becomes C' (2, - 6)
D (1, 2) becomes D' (2, - 1)
Mark the coordinates that will be the vertices of the reflected object/image. Join the vertices.
The rectangle A'B'C'D' is the required rotated image of rectangle ABCD.
**Note:** (Students can choose any points to make a square shape and rotate it $90^0$ clockwise). See learning check 7.1 (question 1 and 2). Draw figures yourself.

16. **Copy the given shapes onto cartesian graph paper and draw an enlargement of the given shape using scale factor 3 and the centre of enlargement (-2, 1).**

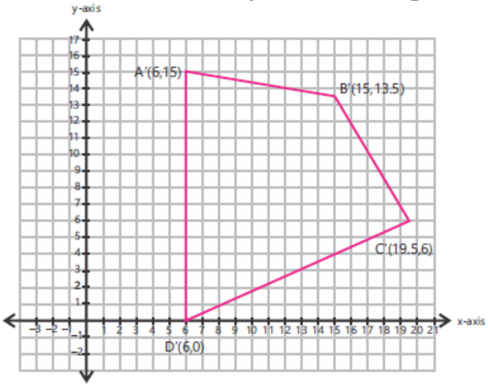



**Solution:**

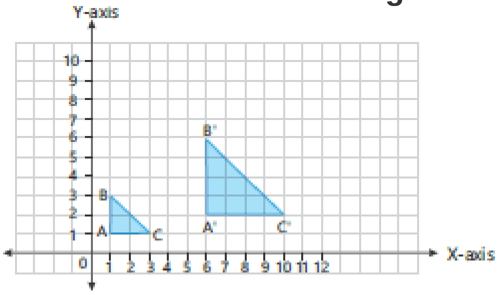
**Step 1:** Identify and note the coordinates of the shape to be enlarged. Here,
A (0, 6), B (3, 5.5), C (4.5, 3) and D (0, 1) are the coordinates of the vertices of the given triangle.
The centre of enlargement is given, i.e., P (-2, 1).

**Step 2:** Count and note the distance of each vertex from the centre of enlargement.
First consider vertex A. Let's find the coordinates for A' of the enlarged shape.
The distance of P from A is (0 + 2, 6 – 1) or (2, 5).
Here the scale factor is 3, so multiply the coordinates of this distance by 3.
The distance of P from A' is (2 × 3, 5 × 3) or (6, 15).
So, A' will be at a distance of (6, 15) from P. Count and mark point A'. Similarly, find the coordinates for B' of the enlarged shape.
The distance of P from B is (3 + 2, 5.5 – 1) or (5, 4.5). Here the scale factor is 3, so multiply the coordinates of this distance by 3.
The distance of P from B' is (5 × 3, 4.5 × 3) or (15, 13.5). So, B' will be at a distance of (15, 13.5) from P.
Count and mark point B'.
Similarly, find the coordinates for C' of the enlarged shape.
The distance of P from C is (4.5 + 2, 3 – 1) or (6.5, 2). Here the scale factor is 3, so multiply the coordinates of this distance by 3.
The distance of P from C' is (6.5 × 3, 2 × 3) or (19.5, 6). So, C' will be at a distance of (19.5, 6) from P.
Count and mark point C'

Similarly, D' will be at a distance of (6, 0) from P. Mark it and join the vertices.
A'B'C'D' is the required enlarged figure.

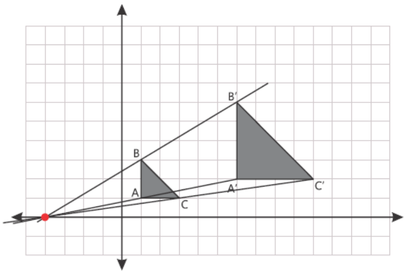



**17. Copy the given shapes onto cartesian graph paper and find the scale factor and centre of enlargement of these shapes.**




**Solution:**
To find its centre of enlargement, join its vertices to the corresponding vertices of the original shape. The point where three vertices meet is the centre of enlargement. So, point P (-4, 0) is the centre of enlargement.
The scale factor is the ratio of the length of any side of the new image to the length of the corresponding side of the pre-image. Here, we can see that A'B' is 4 units long and the corresponding side AB is 2 units long.

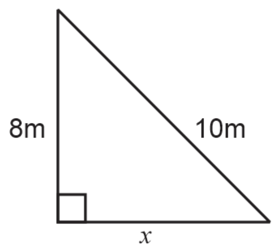
So, here the scale factor is = $\frac{A'B'}{AB} = \frac{4}{2}$ = 2 units



**Learning Check 8.1**

1. **Find the value of $x$ in each figure.**

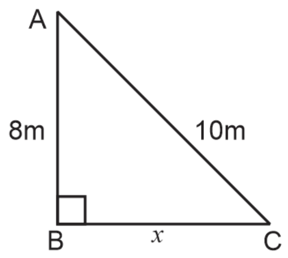
a)




**Solution:**

Name the three sides as A, B and C.

According to Pythagoras theorem:




$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$

Now put the values in the formula:

$(10m)^2 = (8m)^2 + x^2$

$100m^2 = 64m^2 + x^2$

$x^2 = 100m^2 - 64m^2$

$x^2 = 36m^2$

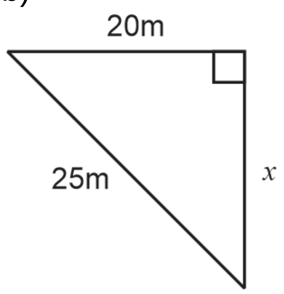
Taking square root on both sides:

$\sqrt{x^2} = \sqrt{36m^2}$

$x = \sqrt{(6m)^2}$

$x = 6m$

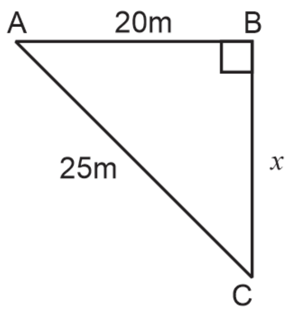
b)




**Solution:**

Name the three sides as A, B and C.

According to Pythagoras theorem:




$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$

$(25m)^2 = (20m)^2 + x^2$

$625m^2 = 400m^2 + x^2$

$x^2 = 625m^2 - 400m^2$

$x^2 = 225m^2$

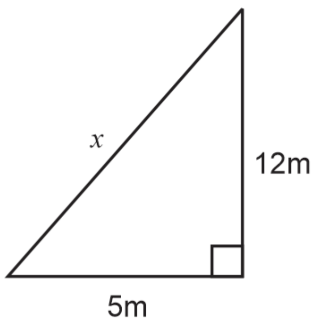
Taking square root on both sides:

$\sqrt{x^2} = \sqrt{225m^2}$

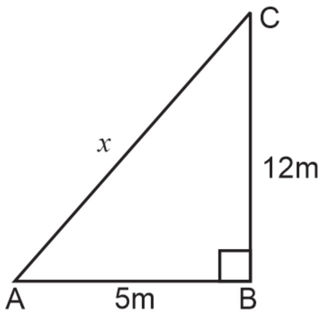
$x = \sqrt{(15m)^2}$

$x = 15m$

c)




**Solution:**




Name the three sides as A, B and C.
According to Pythagoras theorem:

$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$

$x^2 = (5m)^2 + (12m)^2$

$x^2 = 25m^2 + 144m^2$

$x^2 = 169m^2$

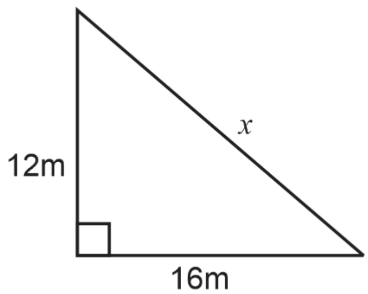
Taking square root on both sides:

$\sqrt{x^2} = \sqrt{169m^2}$

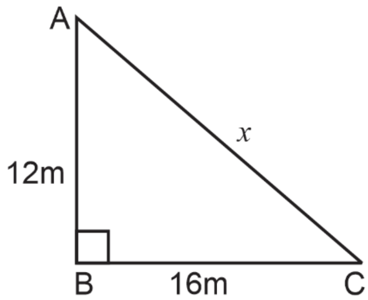
$x = \sqrt{(13m)^2}$

$x = 13m$

d)




**Solution:**




Name the three sides as A, B and C.
According to Pythagoras theorem:

$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$

$x^2 = (12m)^2 + (16m)^2$

$x^2 = 144m^2 + 256m^2$

$x^2 = 400m^2$

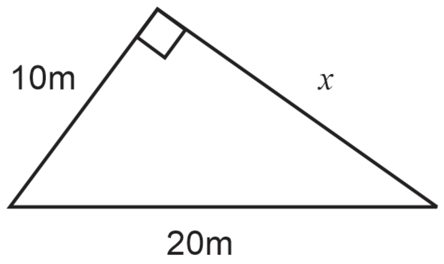
Taking square root on both sides:

$\sqrt{x^2} = \sqrt{400m^2}$

$x = \sqrt{(20m)^2}$

$x = 20m$

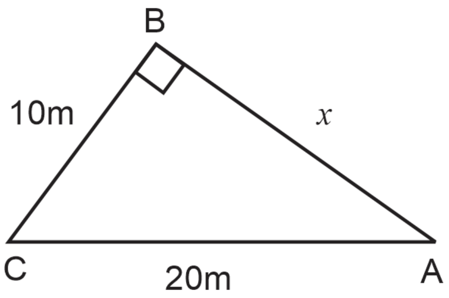
e)




**Solution:**

Name the three sides as A, B and C.

According to Pythagoras theorem:




$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(20m)^2 = x^2 + (10m)^2$

$400m^2 = x^2 + 100m^2$

$x^2 = 400m^2 - 100m^2$

$x^2 = 300m^2$

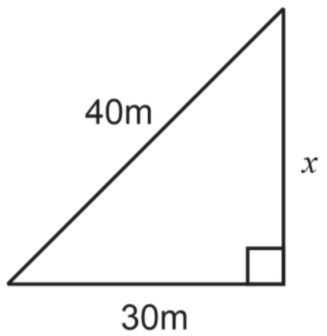
Taking square root on both sides:

$\sqrt{x^2} = \sqrt{300m^2}$

$x = \sqrt{(10)^2 \times 3}m$

$x = 10\sqrt{3}m$

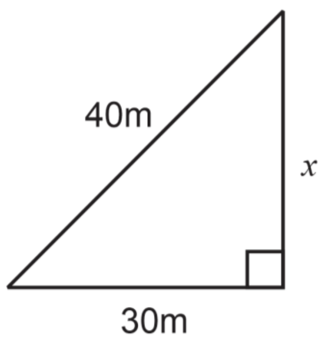
f)




**Solution:**

Name the three sides as A, B and C.

According to Pythagoras theorem:




$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(40m)^2 = (30m)^2 + x^2$

$1600m^2 = 900m^2 + x^2$

$x^2 = 1600m^2 - 900m^2$

$x^2 = 700m^2$

Taking square root on both sides:

$\sqrt{x^2} = \sqrt{700m^2}$

$x = \sqrt{(10)^2 \times 7}m$

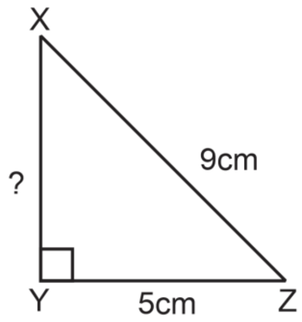
$x = 10\sqrt{7}m$

**2. If in $\triangle XYZ$ and $\angle Y = 90^o$, find the length of the missing side in each of the following.**

**a) $\overline{XZ} = 9cm, \overline{YZ} = 5cm$**

**Solution:**

According to Pythagoras theorem:




$$\left(\overline{XZ}\right)^2 = \left(\overline{XY}\right)^2 + \left(\overline{YZ}\right)^2$$
$$(9cm)^2 = \left(\overline{XY}\right)^2 + (5cm)^2$$
$$81cm^2 = \left(\overline{XY}\right)^2 + 25cm^2$$
$$\left(\overline{XY}\right)^2 = 81cm^2 - 25cm^2$$
$$\left(\overline{XY}\right)^2 = 56cm^2$$

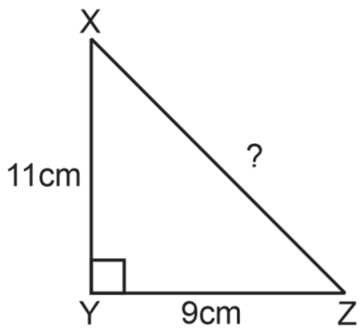
Taking square root on both sides:

$$\sqrt{\left(\overline{XY}\right)^2} = \sqrt{56cm^2}$$
$$\overline{XY} = \sqrt{(2)^2 \times 14}cm$$
$$\overline{XY} = 2\sqrt{14}cm$$

**b) $\overline{XY} = 11cm, \overline{YZ} = 9cm$**

**Solution:**

According to Pythagoras theorem:




$$\left(\overline{XZ}\right)^2 = \left(\overline{XY}\right)^2 + \left(\overline{YZ}\right)^2$$

Now put the values in the formulae:

$$\left(\overline{XZ}\right)^2 = (11cm)^2 + (9cm)^2$$
$$\left(\overline{XZ}\right)^2 = 121cm^2 + 81cm^2$$
$$\left(\overline{XZ}\right)^2 = 202cm^2$$

Taking square root on both sides:

$$\sqrt{\left(\overline{XZ}\right)^2} = \sqrt{202cm^2}$$
$$\overline{XZ} = \sqrt{202}cm$$

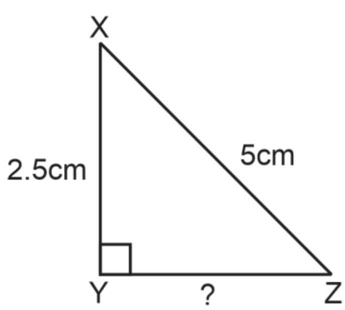
**c) $\overline{XY} = 2.5cm, \overline{XZ} = 5cm$**

**Solution:**

According to Pythagoras theorem:

$$\left(\overline{XZ}\right)^2 = \left(\overline{XY}\right)^2 + \left(\overline{YZ}\right)^2$$

Now put the values in the formulae:

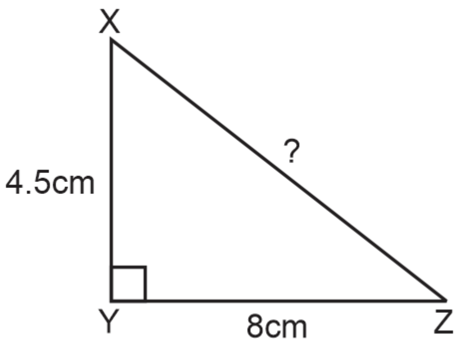



$$(5cm)^2 = (2.5cm)^2 + \left(\overline{YZ}\right)^2$$
$$25cm^2 = 6.25cm^2 + \left(\overline{YZ}\right)^2$$
$$\left(\overline{YZ}\right)^2 = 25cm^2 - 6.25cm^2$$
$$\left(\overline{YZ}\right)^2 = 18.75cm^2$$

Taking square root on both sides:

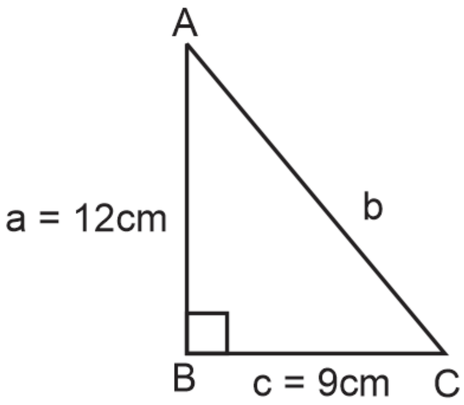
$$\sqrt{\left(\overline{YZ}\right)^2} = \sqrt{18.75cm^2}$$
$$\overline{YZ} = 4.3cm$$

**d)** $\boldsymbol{\overline{XY} = 4.5cm, \overline{YZ} = 8cm}$
**Solution:**
According to Pythagoras theorem:
$\left(\overline{XZ}\right)^2 = \left(\overline{XY}\right)^2 + \left(\overline{YZ}\right)^2$
Now put the values in the formulae:
$\left(\overline{XZ}\right)^2 = (4.5cm)^2 + (8cm)^2$
$\left(\overline{XZ}\right)^2 = 20.25cm^2 + 64cm^2$
$\left(\overline{XZ}\right)^2 = 84.25cm^2$
Taking square root on both sides:
$\sqrt{\left(\overline{XZ}\right)^2} = \sqrt{84.25cm^2}$
$\overline{XZ} = 9.18cm$

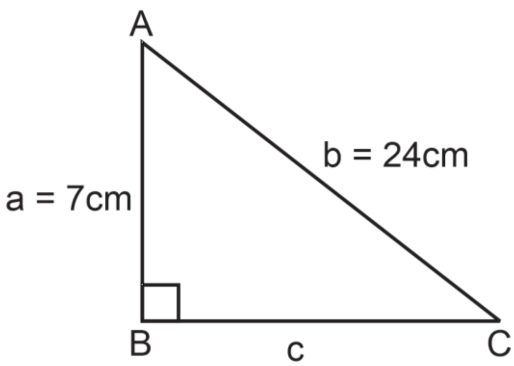



**3. In triangle ABC, $\boldsymbol{\angle B = 90^o}$ and b is the length of the hypotenuse, then find the length of the missing sides:**

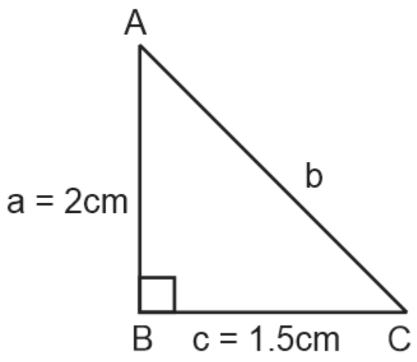
**a)** $\boldsymbol{a = 12cm, c = 9cm}$
**Solution:**
According to Pythagoras theorem:
$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$
Now put the values in the formulae:
$b^2 = (12cm)^2 + (9cm)^2$
$b^2 = 144cm^2 + 81cm^2$
$b^2 = 225cm^2$
Taking square root on both sides:
$\sqrt{b^2} = \sqrt{225cm^2}$
$b = \sqrt{(15cm)^2}$
$b = 15cm$




**b)** $\boldsymbol{b = 24cm, a = 7cm}$
**Solution:**
According to Pythagoras theorem:
$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$
Now put the values in the formulae:
$(24cm)^2 = (7cm)^2 + c^2$
$576cm^2 = 49cm^2 + c^2$
$c^2 = 576cm^2 - 49cm^2$
$c^2 = 527cm^2$
Taking square root on both sides:
$\sqrt{c^2} = \sqrt{527cm^2}$
$c = \sqrt{527}cm$



**c)** $\boldsymbol{a = 2cm, c = 1.5cm}$




**Solution:**

According to Pythagoras theorem:

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

Now put the values in the formulae:

$b^2 = (2cm)^2 + (1.5cm)^2$

$b^2 = 4cm^2 + 2.25cm^2$

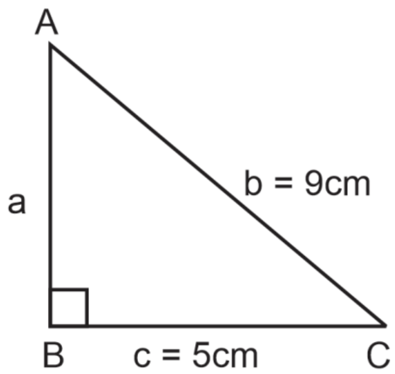
$b^2 = 6.25cm^2$

Taking square root on both sides:

$\sqrt{b^2} = \sqrt{6.25cm^2}$

$b = 2.5cm$

**d)** $\boldsymbol{b = 9cm, c = 5cm}$




**Solution:**

According to Pythagoras theorem:

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

Now put the values in the formulae:

$(9cm)^2 = a^2 + (5cm)^2$

$81cm^2 = a^2 + 25cm^2$

$a^2 = 81cm^2 - 25cm^2$

$a^2 = 56cm^2$

Taking square root on both sides:

$\sqrt{a^2} = \sqrt{56cm^2}$

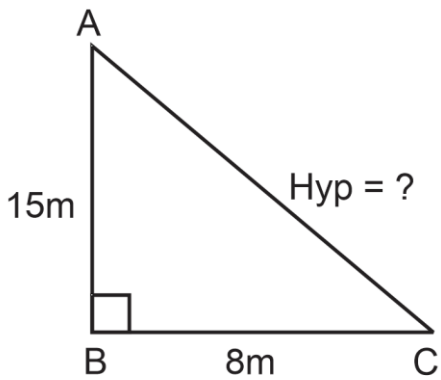
$a = \sqrt{2^2 \times 14}cm$

$a = 2\sqrt{14}cm$

**4. In a right-angled triangle, the length of the altitude is 15m and that of its base is 8m. Find the length of its hypotenuse.**

**Solution:**

According to Pythagoras theorem:




$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

Now put the values in the formulae:

$(Hyp)^2 = (15m)^2 + (8m)^2$

$(Hyp)^2 = 225m^2 + 64m^2$

$(Hyp)^2 = 289m^2$

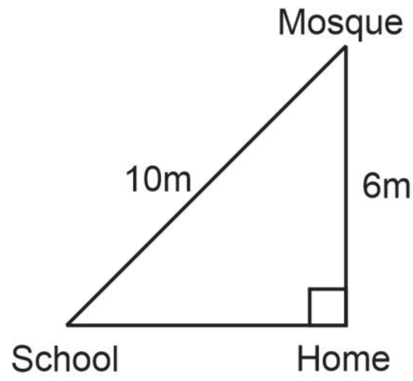
Taking square root on both sides:

$\sqrt{(Hyp)^2} = \sqrt{289m^2}$

$Hyp = \sqrt{17m^2}$

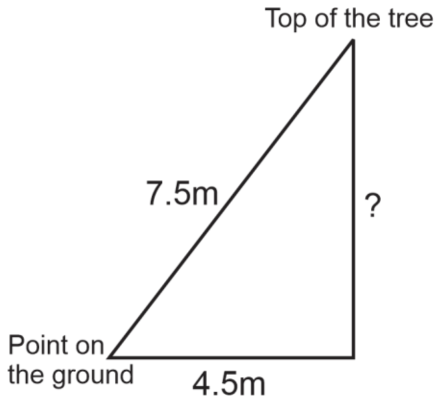
$Hyp = 17m$

5. **Observe the given figure and find the distance between the school and home.**
**Solution:**
According to Pythagoras theorem:
$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$
Let, School, home and mosque be A, B and C respectively.
Now put the values in the formulae:
$(10m)^2 = \left(\overline{AB}\right)^2 + (6m)^2$
$100m^2 = \left(\overline{AB}\right)^2 + 36m^2$
$\left(\overline{AB}\right)^2 = 100m^2 - 36m^2$
$\left(\overline{AB}\right)^2 = 64m^2$
Taking square root on both sides:
$\sqrt{\left(\overline{AB}\right)^2} = \sqrt{64m^2}$
$\overline{AB} = \sqrt{(8m)^2}$
$\overline{AB} = 8m$
The distance between home and school is 8m.

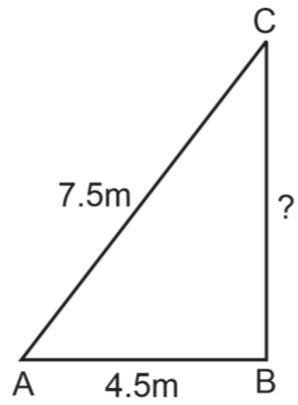



6. **A $3.4m$ long ladder touches the top of the wall. Find the height of the wall if the distance between the foot of the ladder and wall is $2.4m$.**
**Solution:**
Here, wall, ladder and ground makes a right angle triangle. Where wall is the perpendicular, distance between foot and ladder is base and length of ladder is hypotenuse.
Now, According to Pythagoras theorem:
$(Hpy)^2 = (Perp)^2 + (Base)^2$
Now, put the values in the formula:
$(3.4m)^2 = (Perp)^2 + (2.4m)^2$
$11.56m^2 = (Perp)^2 + 5.76m^2$
$(Perp)^2 = 11.56m^2 - 5.76m^2$
$= 5.8m^2$
Taking square root on both sides
$\sqrt{(Perp)^2} = \sqrt{5.8m^2}$
$Perp = 2.41m$
The height of wall is $2.41m$.

7. **The distance between the top of a tree and a point on the ground is $7.5m$. Find the height of the tree if the distance between the point on the ground and the tree is $4.5m$.**



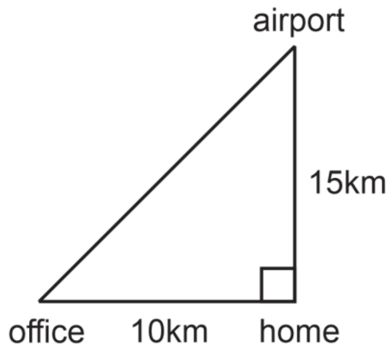
**Solution:**
Name the three sides as A, B and C.
According to Pythagoras theorem:




$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$
$(7.5m)^2 = (4.5m)^2 + \left(\overline{BC}\right)^2$
$56.25m^2 = 20.25m^2 + \left(\overline{BC}\right)^2$
$\left(\overline{BC}\right)^2 = 56.25m^2 - 20.25m^2$
$= 36m^2$
Taking square root on both sides:
$\sqrt{\left(\overline{BC}\right)^2} = \sqrt{36m^2}$
$\overline{BC} = \sqrt{(6m)^2}$
$\overline{BC} = 6m$
The height of the tree is $6m$.

8. **A ladder that is $2.8m$ long touches a wall when its foot is $1.2m$ away from the wall. Find the height of the wall.**
**Solution:**
Here, wall, ladder and ground makes the right triangle.
Let's consider the ladder as the hypotenuse, distance between ladder's foot and the ground as base and the wall as perpendicular.
According to Pythagoras theorem:
$(Hyp)^2 = (Perp)^2 + (Base)^2$
$(2.8m)^2 = (Perp)^2 + (1.2m)^2$
$7.84m^2 = (Perp)^2 + 1.44m^2$
$(Perp)^2 = 7.84m^2 - 1.44m^2$
$= 6.4m^2$
Taking square root on both sides:
$\sqrt{(Perp)^2} = \sqrt{6.4m^2}$
$Perp = 2.5m$
The height of wall is $2.5m$.

9. **See the figure shown here and find the distance between the office and the airport.**
**Solution:**
Let's consider office, home and airport as A, B and C respectively.
Now, according to Pythagoras theorem:




$\left(\overline{AC}\right)^2 = \left(\overline{AB}\right)^2 + \left(\overline{BC}\right)^2$
$\left(\overline{AC}\right)^2 = (10km)^2 + (15km)^2$
$\left(\overline{AC}\right)^2 = 100km^2 + 225km^2$
$\left(\overline{AC}\right)^2 = 325km^2$
Taking square root on both sides:
$\sqrt{\left(\overline{AC}\right)^2} = \sqrt{325km^2}$
$\overline{AC} = 18.03km$
The distance between the office and the airport is $18.03km$.

**10. The altitude of a right-angled triangle is $12cm$ and its hypotenuse is $18cm$ long. Find the length of the base of the triangle.**

**Solution:**

According to Pythagoras theorem:

$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$

$(18cm)^2 = (12cm)^2 + (\overline{BC})^2$

$324cm^2 = 144cm^2 + (\overline{BC})^2$

$(\overline{BC})^2 = 324cm^2 - 144cm^2$

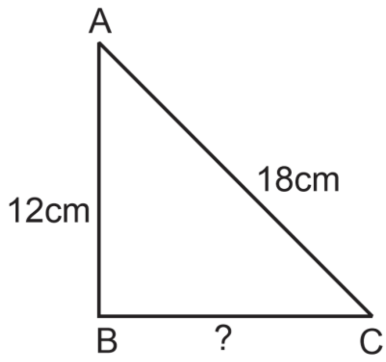
$(\overline{BC})^2 = 180cm^2$

Taking square root on both sides:

$\sqrt{(\overline{BC})^2} = \sqrt{180cm^2}$

$\overline{BC} = \sqrt{6^2 \times 5}cm$

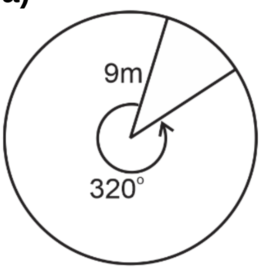
$\overline{BC} = 6\sqrt{5}cm$




## Learning Check 8.2

**1. Find the length of the arc and the area of the sector of each circle.**

**a)**




**Solution:**

Here, $\theta = 320^\circ, r = 9m, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{320^\circ}{360^\circ} \times 2(3.14)(9m)$

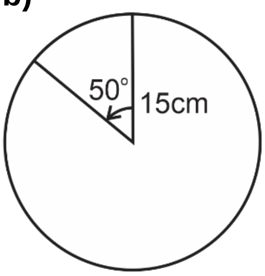
$= 50.24m$

And, Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{320^\circ}{360^\circ} \times (3.14)(9m)^2$

$= 226.08m^2$

**b)**



**Solution:**

Here, $\theta = 50^{\circ}, r = 15cm, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^{\circ}} \times 2\pi r$

$= \frac{50^{\circ}}{360^{\circ}} \times 2(3.14)(15cm)$

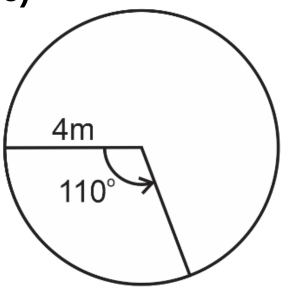
$= 13.08cm$

And, Area of a sector = $\frac{\theta}{360^{\circ}} \times \pi r^2$

$= \frac{50^{\circ}}{360^{\circ}} \times (3.14)(15cm)^2$

$= 98.13cm^2$

**c)**




**Solution:**

Here, $\theta = 110^{\circ}, r = 4m, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^{\circ}} \times 2\pi r$

$= \frac{110^{\circ}}{360^{\circ}} \times 2(3.14)(4m)$

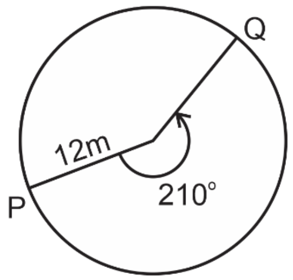
$= 7.68cm$

And, Area of a sector = $\frac{\theta}{360^{\circ}} \times \pi r^2$

$= \frac{110^{\circ}}{360^{\circ}} \times (3.14)(4m)^2$

$= 15.35m^2$

**d)**




**Solution:**

Here, $\theta = 210^{\circ}, r = 12m, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^{\circ}} \times 2\pi r$

$= \frac{210^{\circ}}{360^{\circ}} \times 2(3.14)(12m)$

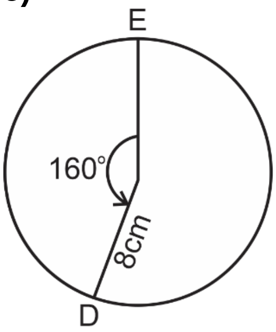
$= 43.96m$

And, Area of a sector = $\frac{\theta}{360^{\circ}} \times \pi r^2$

$= \frac{210^{\circ}}{360^{\circ}} \times (3.14)(12m)^2$

$= 263.76m^2$

**e)**




**Solution:**

Here, $\theta = 160^\circ, r = 8cm, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{160^\circ}{360^\circ} \times 2(3.14)(8cm)$

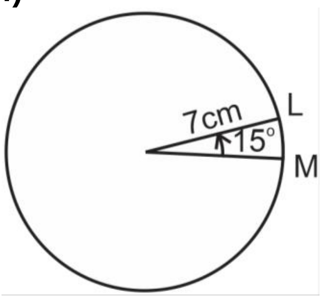
$= 22.33cm$

And, Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{160^\circ}{360^\circ} \times (3.14)(8cm)^2$

$= 89.32cm^2$

**f)**




**Solution:**

Here, $\theta = 15^\circ, r = 7cm, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{15^\circ}{360^\circ} \times 2(3.14)(7cm)$

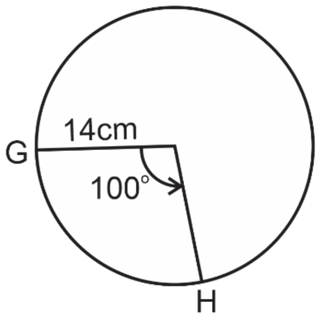
$= 1.83cm$

And, Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{15^\circ}{360^\circ} \times (3.14)(7cm)^2$

$= 6.41cm^2$

**g)**



**Solution:**

Here, $\theta = 100^\circ, r = 14cm, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{100^\circ}{360^\circ} \times 2(3.14)(14cm)$

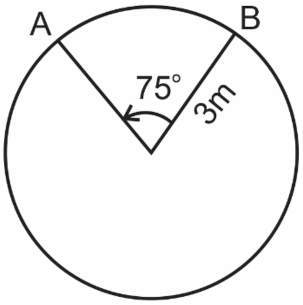
$= 24.42cm$

And, Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{100^\circ}{360^\circ} \times (3.14)(14cm)^2$

$= 170.96cm^2$

**h)**




**Solution:**

Here, $\theta = 75^\circ, r = 3m, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{75^\circ}{360^\circ} \times 2(3.14)(3m)$

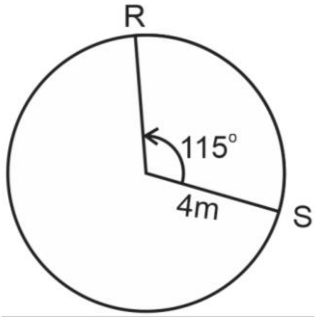
$= 3.93m$

And, Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{75^\circ}{360^\circ} \times (3.14)(3m)^2$

$= 5.89m^2$

**i)**




**Solution:**

Here, $\theta = 115^\circ, r = 4m, \pi = 3.14$

So, Arc length of circle = $\frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{115^\circ}{360^\circ} \times 2(3.14)(4m)$

$= 8.02m$

And, Area of a sector = $\frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{115^\circ}{360^\circ} \times (3.14)(4m)^2$

$= 16.05m^2$

**2. Find the arc length and area of the sector of a circle from the following measurements.**

**a)** $\boldsymbol{r = 6cm, \theta = 110^\circ}$

**Solution:**

Here, $\theta = 115^\circ, r = 6cm, \pi = 3.14$

So, length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{110^\circ}{360^\circ} \times 2(3.14)(6cm)$

$= 11.51cm$

and, Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{110^\circ}{360^\circ} \times (3.14)(6cm)^2$

$= 34.54cm^2$

**b)** $\boldsymbol{r = 15m, \theta = 120^\circ}$

**Solution:**

Here, $\theta = 120^\circ, r = 15m, \pi = 3.14$

So, length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{120^\circ}{360^\circ} \times 2(3.14)(15m)$

$= 31.4m$

and, Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{120^\circ}{360^\circ} \times (3.14)(15m)^2$

$= 235.5m^2$

**c)** $\boldsymbol{r = 12cm, \theta = 75^\circ}$

**Solution:**

Here, $\theta = 75^\circ, r = 12m, \pi = 3.14$

So, length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{75^\circ}{360^\circ} \times 2(3.14)(12cm)$

$= 15.7cm$

and, Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{75^\circ}{360^\circ} \times (3.14)(12cm)^2$

$= 94.2cm^2$

**d)** $\boldsymbol{r = 3cm, \theta = 80^\circ}$

**Solution:**

Here, $\theta = 80^\circ, r = 3cm, \ \pi = 3.14$

So, length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{80^\circ}{360^\circ} \times 2(3.14)(3cm)$

$= 4.19cm$

and, Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$
$= \frac{80^\circ}{360^\circ} \times (3.14)(3cm)^2$
$= 6.25cm^2$

**e)** $\boldsymbol{r = 12m, \theta = 120^\circ}$
**Solution:**
Here, $\theta = 120^\circ, r = 12m,\ \pi = 3.14$
So, length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$
$= \frac{120^\circ}{360^\circ} \times 2(3.14)(12m)$
$= 25.12m$
and, Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$
$= \frac{120^\circ}{360^\circ} \times (3.14)(12m)^2$
$= 150.72m^2$
**f)** $\boldsymbol{r = 10cm, \theta = 55^\circ}$
**Solution:**
Here, $\theta = 55^\circ, r = 10cm,\ \pi = 3.14$
So, length of arc $= \frac{\theta}{360^\circ} \times 2\pi r$
$= \frac{55^\circ}{360^\circ} \times 2(3.14)(10cm)$
$= 9.59cm$
and, Area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$
$= \frac{55^\circ}{360^\circ} \times (3.14)(10cm)^2$
$= 47.97cm^2$

3. **Find the area of a sector of a circular field having a diameter of 100m and a central angle of $180^\circ$.**
**Solution:**
Here, $\theta = 180^\circ, \pi = 3.14,$
and, $r = \frac{d}{2} = \frac{100m}{2} = 50m$
So, Area of a sector $= \frac{\theta}{360^\circ} \times \pi r^2$
$= \frac{160^\circ}{360^\circ} \times (3.14)(50m)^2$
$= 3925m^2$
So, Area of a sector of a circular field is $3925m^2$.

4. **Find the area of a piece of pizza having a radius of $8cm$ and a central angle of $45^\circ$. If the pizza has 8 such identical pieces, what is total area of the pizza?**
**Solution:**
Here, $\theta = 45^\circ, \pi = 3.14,\ r = 8cm$
So, Area of a piece of pizza $= \frac{\theta}{360^\circ} \times \pi r^2$
$= \frac{45^\circ}{360^\circ} \times (3.14)(8cm)^2$
$= 25.12cm^2$
Total area of the pizza $= Area\ of\ piece\ \times 8$
$= 25.12cm^2 \times 8$
$= 200.96cm^2$

**5. A pendulum like swing at an amusement park is moving in a circular arc at the end of a $50m$ arm. Find the distance travel by a rider at the end of the swing's arm if the angle made between the two extreme points $145^\circ$.**

**Solution:**

Here, $\theta = 145^\circ, \pi = 3.14,\ r = 50m$

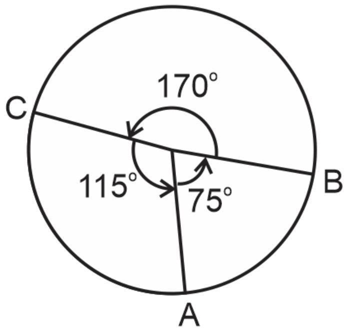
So, Distance travelled by the rider $= \frac{\theta}{360^\circ} \times 2\pi r$

$= \frac{145^\circ}{360^\circ} \times 2(3.14)(50m)$

$= 126.47m$

**6. A circular ground has been divided into three parts for different type of activities. If the diameter of this ground is $220m$. Find the area of each part and the length of the curved boundary of each circle.**

**Solution:**




Here, $r = \frac{d}{2} = \frac{220}{2}m = 110m$

So, Area of sector $\overline{AB} = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{75^\circ}{360^\circ} \times (3.14)(110m)^2$

$= 7915.42m^2$

So, Area of sector $\overline{BC} = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{170^\circ}{360^\circ} \times (3.14)(110m)^2$

$= 17941.61m^2$

So, Area of sector $\overline{AC} = \frac{\theta}{360^\circ} \times \pi r^2$

$= \frac{115^\circ}{360^\circ} \times (3.14)(110m)^2$

$= 12136.97m^2$

Now,

Length of curved boundary of a circle $= Area\ of\ \overline{AB} + Area\ of\ \overline{BC} + Area\ of\ \overline{AC}$

$= 7915.42m^2 + 17941.61m^2 + 12136.97m^2$

$= 37994m^2$

## Learning Check 8.3

1. **Find the surface area and volume of the sphere for the following radii $\left(\boldsymbol{\pi = \frac{22}{7}}\right)$**

**a) 4.2cm**

**Solution:**

Here, $r = 4.2cm, \pi = \frac{22}{7} = 3.14$

So, Surface area of sphere $= 4\pi r^2$

$= 4(3.14)(4.2cm)^2$

$= 221.56cm^2$

And, volume of the sphere $= \frac{4}{3}\pi r^3$

$= \left(\frac{4}{3}\right)(3.14)(4.2cm)^3$

$= 310.18cm^3$

**b) 63cm**

**Solution:**

Here, $r = 63cm, \pi = \frac{22}{7} = 3.14$

So, Surface area of sphere $= 4\pi r^2$

$= 4(3.14)(63cm)^2$

$= 49850.64cm^2$

And, volume of the sphere $= \frac{4}{3}\pi r^3$

$= \left(\frac{4}{3}\right)(3.14)(63cm)^3$

$= 1046863.44cm^3$

**c) 8.4m**

**Solution:**

Here, $r = 8.4m, \pi = \frac{22}{7} = 3.14$

So, Surface area of sphere $= 4\pi r^2$

$= 4(3.14)(8.4m)^2$

$= 886.23cm^2$

And, volume of the sphere $= \frac{4}{3}\pi r^3$

$= \left(\frac{4}{3}\right)(3.14)(8.4m)^3$

$= 2481.45m^3$

2. **The diameter of a sphere is 5m. Calculate the surface area and volume of the sphere $(\boldsymbol{\pi = 3.14})$**

**Solution:**

Here, $r = \frac{d}{2} = \frac{5}{2} = 2.5m, \pi = 3.14$

So, Surface area of sphere $= 4\pi r^2$

$= 4(3.14)(2.5m)^2$

$= 78.5m^2$

And, volume of the sphere $= \frac{4}{3}\pi r^3$

$= \left(\frac{4}{3}\right)(3.14)(2.5m)^3$

$= 65.42m^3$

3. **A spherical water tank has a radius of 2.1m.**
   **a) Calculate the cost of painting the water tank at the rate of Rs. 140 per $m^2$.**
   **Solution:**
   Here, $r = 2.1m, \pi = 3.14$
   First, we have to find the surface area of a water tank.
   So, Surface area of tank = $4\pi r^2$
   $= 4(3.14)(2.1m)^2$
   $= 55.3896m^2$
   Total cost of painting = Surface area × Cost per $m^2$
   $= 55.3896m^2 \times 140$
   $= Rs. 7754.54$
   **b) Find the capacity of the water tank $\left(\boldsymbol{\pi = \frac{22}{7}}\right)$**
   Here, $r = 2.1m, \pi = \frac{22}{7} = 3.14$
   So, volume (capacity) of tank $= \frac{4}{3}\pi r^3$
   $= \left(\frac{4}{3}\right)(3.14)(2.1m)^3$
   $= 38.77m^3$

4. **A hemispherical pool has a diameter of 20m. Find the capacity of the pool in litres.**
   **Solution:**
   Here, $r = \frac{d}{2} = \frac{20}{2} = 10m, \pi = 3.14$
   So, volume of the sphere $= \frac{4}{3}\pi r^3$
   $= \left(\frac{4}{3}\right)(3.14)(10m)^3$
   $= 2093.333m^3$
   As we know that $1m^3 = 1000$ litres
   So, Volume in litres = $2093.333 \times 1000$
   $= 2093333$ litres

5. **A Sphere of radius $\boldsymbol{1.5m}$ is made up of an iron sheet. Calculate its surface area.**
   **Solution:**
   Here, $r = 1.5m, \pi = 3.14$
   So, Surface area of sphere = $4\pi r^2$
   $= 4(3.14)(1.5m)^2$
   $= 28.26m^2$

## Learning Check 8.4

**1. Find the surface area of the following square pyramids.**

a)

**Solution:**

Here, $b = 23cm, l = 34cm$

So, surface area of square pyramid $= b^2 + 2bl$

$= (23cm)^2 + 2(23cm)(34cm)$

$= 2093cm^2$

b)

**Solution:**

Here, $b = 27cm, l = 35cm$

So, surface area of square pyramid $= b^2 + 2bl$

$= (27cm)^2 + 2(27cm)(35cm)$

$= 2619cm^2$

c)

**Solution:**

Here, $b = 32m, l = 45m$

So, surface area of square pyramid $= b^2 + 2bl$

$= (32m)^2 + 2(32m)(45m)$

$= 3904m^2$

d)

**Solution:**

Here, $b = 24m, l = 38m$

So, surface area of square pyramid $= b^2 + 2bl$

$= (24m)^2 + 2(24m)(38m)$

$= 2400m^2$

e)

**Solution:**

Here, $b = 18m, l = 37m$

So, surface area of square pyramid $= b^2 + 2bl$

$= (18m)^2 + 2(18m)(37m)$

$= 1656m^2$

f)

**Solution:**

Here, $b = 22cm, l = 50cm$

So, surface area of square pyramid $= b^2 + 2bl$

$= (22cm)^2 + 2(22cm)(50cm)$

$= 2684cm^2$

**2. Find the volume of each square pyramid having the given base length and altitude.**

**a)** $\boldsymbol{base\ length = 9cm, altitude = 5.5cm}$

**Solution:**

Here, $b = 9cm,\ \ h = 5.5cm$

So, Volume of square pyramid $= \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3} \times (9cm)^2(5.5cm)$

$= 148.5cm^3$

**b)** $\boldsymbol{base\ length = 7.1cm, altitude = 4.7cm}$

**Solution:**

Here, $b = 7.1cm, h = 4.7cm$

So, Volume of square pyramid $= \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3} \times (7.1cm)^2(4.7cm)$

$= 78.98cm^3$

**c)** $\boldsymbol{base\ length = 6.6cm, altitude = 4.5cm}$

**Solution:**

Here, $b = 6.6cm, h = 4.5cm$

So, Volume of square pyramid $= \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3} \times (6.6cm)^2(4.5cm)$

$= 65.34cm^3$

**d)** $\boldsymbol{base\ length = 12cm, altitude = 10cm}$

**Solution:**

Here, $b = 12cm, h = 10cm$

So, Volume of square pyramid $= \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3} \times (12cm)^2(10cm)$

$= 480cm^3$

**e)** $\boldsymbol{base\ length = 11cm, altitude = 8cm}$

**Solution:**

Here, $b = 11cm, h = 8cm$

So, Volume of square pyramid $= \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3} \times (11cm)^2(8cm)$

$= 322.67cm^3$

**f)** $\boldsymbol{base\ length = 17cm, altitude = 14cm}$

**Solution:**

Here, $b = 17cm, h = 14cm$

So, Volume of square pyramid $= \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3} \times (17cm)^2(14cm)$

$= 1348.67cm^3$

3. **An oil bottle is in the shape of a square pyramid. What is the capacity if the base length is $22cm$ and the perpendicular height (altitude) is $30cm$**
   **Solution:**
   Here, $b = 22cm,\ h = 30cm$
   So, capacity (volume) of oil bottle $= \frac{1}{3} \times b^2 \times h$
   $= \frac{1}{3} \times (22cm)^2(30cm)$
   $= 4840cm^3$

4. **A hall is in the shape of a square pramid. Its base length is $30$ metres. Slant height is $42$ metres and its altitude is 40 metres. Find**
   **a)** what is the cost of painting its outer triangle walls if the rate of painting is Rs. 975 per square metre?
   **Solution:**
   Here, $b = 30m, l = 42m$
   First we have to find surface area
   So, surface area of hall $= b^2 + 2bl$
   $= (30m)^2 + 2(30m)(42m)$
   $= 3420m^2$

   $Total\ cost\ of\ painting\ = surface\ area\ \times rate\ per\ m^2$
   $= 3420 \times 975$
   $= Rs. 3334500$

   **b)** What is the capacity of this hall?
   **Solution:**
   Here, $b = 30m,\quad h = 40m$
   So, volume (capacity) of hall $= \frac{1}{3} \times b^2 \times h$
   $= \frac{1}{3} \times (30m)^2(40m)$
   $= 12000m^3$

5. **For a picnic, a square pyramid shaped tent the prepared. How much fabric is used for making the four triangular surfaces if the base of this square pyramid is 4 metres and the slant height is 6 metres? Also find its capacity if the altitude is 5 metres.**
   **Solution:**
   Here, $b = 4m, l = 6m, h = 5m$
   So, surface area of tent $= b^2 + 2bl$
   $= (4m)^2 + 2(4m)(6m)$
   $= 64m^2$
   $64m^2$ fabric will be used for making four surfaces
   Now, capacity of tent $= \frac{1}{3} \times b^2 \times h$
   $= \frac{1}{3} \times (4m)^2(5m)$
   $= 26.67m^3$

**6. Find the surface area and volume of the pyramid shaped decoration piece if its base length is $20cm$, its slant height is $23cm$ and its altitude is $23cm$. Also find the cost of polishing its outer surface if the rate of polishing is Rs. 380 per square cm.**

**Solution:**

Here, $b = 20cm, l = 23cm,\ h = 23cm$

To find the cost of polishing, first we have to find surface area of decoration piece.

So, surface area of decoration piece $= b^2 + 2bl$

$= (20cm)^2 + 2(20cm)(23cm)$

$= 1320cm^2$

$Now, total\ cost\ of\ polishing = Surface\ area\ \times rate\ per\ cm^2$

$= 1320 \times 380$

$= Rs. 501600$

$Volume\ of\ a\ decoration\ piece = \frac{1}{3} \times b^2 \times h$

$= \frac{1}{3}(20cm)^2(23cm)$

$= 3066.67cm^3$

## Learning Check 8.5

**1. Find the total surface area of the cones of the following measurements: $(\boldsymbol{\pi = 3.14})$**

**a) $\boldsymbol{r = 5cm, l = 8cm}$**

**Solution:**

Surface area of cone $= \pi rl + \pi r^2$

$= (3.14)(5cm)(8cm) + (3.14)(5cm)^2$

$= 204.1cm^2$

**b) $\boldsymbol{r = 8.2cm, l = 20cm}$**

**Solution:**

Surface area of cone $= \pi rl + \pi r^2$

$= (3.14)(8.2cm)(20cm) + (3.14)(8.2cm)^2$

$= 726.09cm^2$

**c) $\boldsymbol{r = 3m, l = 4m}$**

**Solution:**

Surface area of cone $= \pi rl + \pi r^2$

$= (3.14)(3m)(4m) + (3.14)(3m)^2$

$= 65.94m^2$

**d) $\boldsymbol{r = 6cm, l = 8cm}$**

**Solution:**

Surface area of cone $= \pi rl + \pi r^2$

$= (3.14)(6cm)(8cm) + (3.14)(6cm)^2$

$= 263.76cm^2$

**2. Calculate the volume of the cones for the following measurement: $(\boldsymbol{\pi = 3.14})$**

**a) $\boldsymbol{r = 2.5cm, h = 12cm}$**

**Solution:**

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(2.5cm)^2(12cm)$

$= 78.5cm^3$

**b) $\boldsymbol{r = 3.2cm, h = 8.5cm}$**

**Solution:**

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(3.2cm)^2(8.5cm)$

$= 91.10cm^3$

**c) $\boldsymbol{r = 1.25m, h = 3m}$**

**Solution:**

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(1.25m)^2(3m)$

$= 4.91m^3$

**d) $\boldsymbol{r = 5cm, h = 13cm}$**

**Solution:**

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(5cm)^2(13cm)$

$= 340.17cm^3$

**3. The radius of the conical roof of a minar of a masjid is $1.25m$ and its slant height is $5m$. Find the cost of the metal sheet used to cover the roof at a rate of Rs. 250 per $\boldsymbol{m^2}$.**

**Solution:**

Here, $r = 1.25m, l = 5m$

To find the cost of metal sheet, first we have to find surface area.

So, Surface area of minar $= \pi rl + \pi r^2$

$= (3.14)(1.25m)(5m) + (3.14)(1.25m)^2$

$= 24.53125m^2$

Now,

$Total\ cost\ of\ metal\ sheet = Surface\ area\ \times rate\ per\ m^2$

$= 24.53125 \times 250$

$= Rs. 6132.81$

**4. The slant height of a conical funnel is $11cm$. How much metal plate is required to cover a conical funnel if the diameter of its base is $8.2cm$? $\left(\pi = \frac{22}{7}\right)$**

**Solution:**

Here, $l = 11cm, \ \ \pi = \frac{22}{7} = 3.14 \ and \ r = \frac{d}{2} = \frac{8.2}{2} = 4.1cm$

So, Surface area of funnel $= \pi rl + \pi r^2$

$= (3.14)(4.1cm)(11cm) + (3.14)(4.1cm)^2$

$= 194.3974cm^2$

$194.3974cm^2$ metal is required to cover funnel.

**5. The radius of a cone is $1.2cm$ and its slant height is $2.5cm$.**

**a) How much paper is required to cover the cone?**

**Solution:**

Here, $r = 1.2cm, l = 2.5cm$

So, Surface area of cone $= \pi rl + \pi r^2$

$= (3.14)(1.2cm)(2.5cm) + (3.14)(1.2cm)^2$

$= 13.9416cm^2$

**b) Find the cost of covering the cone at a rate of Rs. 90 per $m^2$.**

**Solution:**

Total cost = Surface area × rate per m$^2$

$= 13.9416 \times 90$

$= Rs. 1254.74$

**6. A conical vessel has a radius of $5cm$ and its slant height is $12cm$.**

**a) Calculate its total surface area.**

**Solution:**

Here, $r = 5cm, l = 12cm$

So, Surface area of conical vessel $= \pi rl + \pi r^2$

$= (3.14)(5cm)(12cm) + (3.14)(5cm)^2$

$= 266.9cm^2$

**b) Find the capacity of the conical vessel.**

**Solution:**

First we have to find height (h):

$l = \sqrt{r^2 + h^2}$

$12cm = \sqrt{(5cm)^2 + h^2}$

Take square of both sides:

$(12cm)^2 = (5cm)^2 + h^2$

$144cm^2 = (5cm)^2 + h^2$

$h^2 = 144cm^2 - 25cm^2$

$h^2 = 119cm^2$

Taking square root on both sides:

$\sqrt{h^2} = \sqrt{119cm^2}$

$h = 10.91cm$

Now, Capacity of conical vessel $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(5cm)^2(10.91cm)$

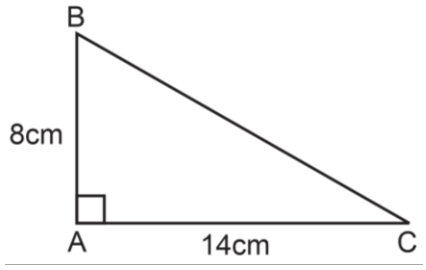
$= 285.48cm^3$

## Unit Evaluation 8

**3. Find the value of the unknown length (hypotenuse, perpendicular or base) by using Pythagoras's theorem.**

**a)**

**Solution:**




According to Pythagoras theorem

$(Hyp)^2 = (Perp)^2 + (Base)^2$

$(\overline{BC})^2 = (\overline{BA})^2 + (\overline{AC})^2$

$(\overline{BC})^2 = (8cm)^2 + (14cm)^2$

$(\overline{BC})^2 = 64cm^2 + 196cm^2$

$(\overline{BC})^2 = 260cm^2$

Taking square root on both sides:

$\sqrt{(\overline{BC})^2} = \sqrt{260cm^2}$

$\overline{BC} = \sqrt{2^2 \times 65cm^2}$

$\overline{BC} = 2\sqrt{65}$

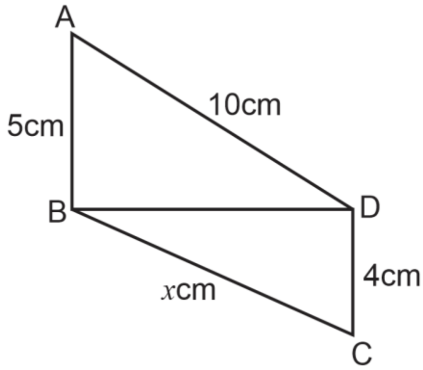
**b)**

**Solution:**

As we can see, there are to right triangles ABD and BDC. To find the value of $x$ first we have to find the base of triangle ABD.

So,

According to Pythagoras theorem:




$(Hyp)^2 = (Perp)^2 + (Base)^2$

$(\overline{AD})^2 = (\overline{AB})^2 + (\overline{BD})^2$

$(10cm)^2 = (5cm)^2 + (\overline{BD})^2$

$100cm^2 = 25cm^2 + (\overline{BD})^2$

$(\overline{BD})^2 = 100cm^2 - 25cm^2$

$(\overline{BD})^2 = 75cm^2$

Taking square root on both sides:

$\sqrt{(\overline{BD})^2} = \sqrt{75cm^2}$

$\overline{BD} = 8.66cm$

Now, for $\Delta ADC$

According to Pythagoras theorem:

$(Hyp)^2 = (Perp)^2 + (Base)^2$

$(\overline{BC})^2 = (\overline{BD})^2 + (\overline{DC})^2$

$(x)^2 = (8.66cm)^2 + (4cm)^2$

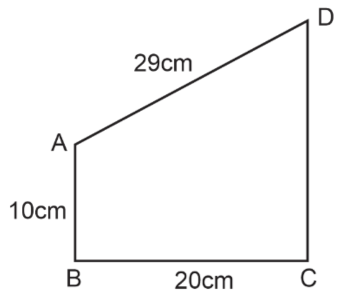
$x^2 = 90.9956cm$

Taking square root on both sides:

$\sqrt{x^2} = \sqrt{90.9956cm}$

$x = 9.54cm$

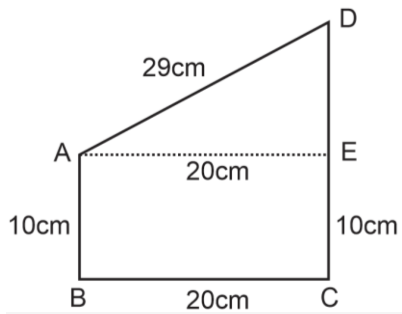
**c)**




**Solution:**

Let’s divide this shape into two shapes:

Here we’ve got the right triangle AED.




According to Pythagoras theorem:

$\left(\overline{AD}\right)^2 = \left(\overline{AE}\right)^2 + \left(\overline{ED}\right)^2$

$(29cm)^2 = (20cm)^2 + \left(\overline{ED}\right)^2$

$841cm^2 = 400cm^2 + \left(\overline{ED}\right)^2$

$\left(\overline{ED}\right)^2 = 841cm^2 - 400cm^2$

$\left(\overline{ED}\right)^2 = 441cm^2$

Taking square root on both sides:

$\sqrt{\left(\overline{ED}\right)^2} = \sqrt{441cm^2}$

$\overline{ED} = 21cm$

Now,

Let’s find the length of missing side CD:

$\overline{CD} = \overline{CE} + \overline{ED}$

$\overline{CD} = 21cm + 10cm$

$\overline{CD} = 31cm$

**4. The hypotenuse of a right-angled triangular ground is 45cm. If the length of one side of the triangular ground is 30m, what is the length of the other side?**

**Solution:**

According to Pythagoras theorem:

$(Hyp)^2 = (Perp)^2 + (Base)^2$

$c^2 = a^2 + b^2$

$(45m)^2 = (30m)^2 + b^2$

$2025m^2 = 900m^2 + b^2$

$b^2 = 2025m^2 - 900m^2$

$b^2 = 1125m^2$

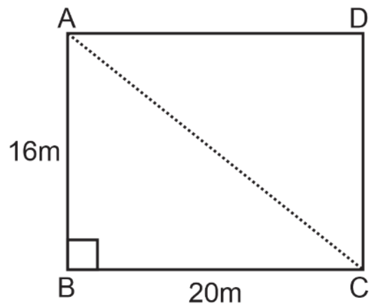
Taking square root on both sides:

$\sqrt{b^2} = \sqrt{1125m^2}$

$b = \sqrt{15^2 \times 5m^2}$

$b = 15\sqrt{5}m$

5. **Find the length of the diagonal of a rectangle-shaped computer lab. If the length of the computer lab is $20m$ and width is $16m$.**
   **Solution:**
   Let's find the length of diagonal ABC
   According to Pythagoras theorem:




   $(Hyp)^2 = (Perp)^2 + (Base)^2$
   $(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$
   $(\overline{AC})^2 = (16m)^2 + (20m)^2$
   $(\overline{AC})^2 = 256m^2 + 400m^2$
   $(\overline{AC})^2 = 656m^2$
   Taking square root on both sides:
   $\sqrt{(\overline{AC})^2} = \sqrt{656m^2}$
   $\overline{AC} = \sqrt{4^2 \times 41m^2}$
   $\overline{AC} = 4\sqrt{41}m$

6. **Find the area of the sector XYZ if the length of the radius is $8cm$ and angle is $315^\circ$.**
   **Solution:**
   Here, $r = 8cm,\ \theta = 315^\circ,\ \pi = 3.14$
   So, area of sector $= \frac{\theta}{360^\circ} \times \pi r^2$
   $= \frac{315^\circ}{360^\circ} \times (3.14)(8cm)^2$
   $= 175.84cm^2$

7. **Find the arc length AB if the angle is $50^\circ$ and the radius is $5cm$.**
   **Solution:**
   Here, $r = 5cm, \theta = 50^\circ, \pi = 3.14$
   So, Arc length of AB $= \frac{\theta}{360^\circ} \times 2\pi r$
   $= \frac{50^\circ}{360^\circ} \times 2(3.14)(5cm)$
   $= 4.36cm$

8. **Find the surface area and volume of the given pyramids.**
   **Solution:**
   **a)**
   Here, $b = 11m, l = 17m$
   So, Surface are of pyramid $= b^2 + 2bl$
   $= (11m)^2 + 2(11m)(17m)$
   $= 495m^2$
   Now, to find volume, first we have to find height $(h)$
   $l^2 = h^2 + \left(\frac{b}{2}\right)^2$
   $(17m)^2 - h^2 + \left(\frac{11}{2}\right)^2$
   $289m^2 = h^2 + 30.25m^2$
   $h^2 = 289m^2 - 30.25m^2$
   $h^2 = 258.75m^2$
   Taking square root on both sides:
   $\sqrt{h^2} = \sqrt{258.75m^2}$
   $h = 16.09m$

Now,
Volume of pyramid $= \frac{1}{3} \times b^2 \times h$
$= \frac{1}{3}(11m)^2(16.09m)$
$= 648.96m^3$

**b)**
Here, $b = 8cm, h = 7cm$
To find surface area, first we have to find slant height ($l$)
$l^2 = h^2 + \left(\frac{b}{2}\right)^2$
$l^2 = (7cm)^2 + \left(\frac{8cm}{2}\right)^2$
$l^2 = 49cm^2 + 16cm^2$
$l^2 = 65cm^2$
Taking square root on both sides:
$\sqrt{l^2} = \sqrt{65cm^2}$
$l = 8.06cm$
Now,
Surface area $= b^2 + 2bl$
$= (8cm)^2 + 2(8cm)(8.06cm)$
$= 192.96cm^2$
And,
volume of pyramid $= \frac{1}{3} \times b^2 \times h$
$= \frac{1}{3}(8cm)^2(7cm)$
$= 149.33cm^2$

9. **Find the surface area and volume of the given spheres.**
**a)**
**Solution:**
Here, $r = 5m, \pi = 3.14$
So, Surface area of sphere $= 4\pi r^2$
$= 4(3.14)(5m)^2$
$= 314m^2$
And, Volume of sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3}(3.14)(5m)^3$
$= 523.33m^3$

**b)**
**Solution:**
Here, $r = 18m, \pi = 3.14$
So, Surface area of sphere $= 4\pi r^2$
$= 4(3.14)(18m)^2$
$= 4049.44m^2$
And, Volume of sphere $= \frac{4}{3}\pi r^3$
$= \frac{4}{3}(3.14)(18m)^3$
$= 24416.64m^3$

**c)**
**Solution:**
Here, $r = \frac{d}{2} = \frac{20}{2} = 10m,\ \pi = 3.14$
So, Surface area of sphere = $4\pi r^2$
$= 4(3.14)(10m)^2$
$= 1256m^2$
And, Volume of sphere = $\frac{4}{3}\pi r^3$
$= 4(3.14)(10m)^3$
$= 4186.67m^3$

**10. Find the Surface area and volume of cones.**
**a)**
**Solution:**
Here, $r = 4cm, l = 5cm$
So, Surface area of cone = $\pi rl + \pi r^2$
$= (3.14)(4cm)(5cm) + (3.14)(4cm)^2$
$= 113.04cm^2$
Now, to find volume, first we have to find height $(h)$:
$l = \sqrt{r^2 + h^2}$
$5cm = \sqrt{(4cm)^2 + h^2}$
Taking square of both sides:
$(5cm)^2 = \left(\sqrt{(4cm)^2 + h^2}\right)^2$
$25cm^2 = 16cm^2 + h^2$
$h^2 = 25cm^2 - 16cm^2$
$h^2 = 9cm^2$
Taking square root on both sides:
$\sqrt{h^2} = \sqrt{9cm^2}$
$h = 3cm$
Now, Volume of cone = $\frac{1}{3}\pi r^2 h$
$= \frac{1}{3}(3.14)(4cm)^2(3cm)$
$= 50.24cm^2$
**b)**
**Solution:**
Here, $l = 5m, h = 3m$
First, we have to find radius $(r)$
$l = \sqrt{r^2 + h^2}$
Taking square on both sides
$l^2 = r^2 + h^2$
$r^2 = l^2 - h^2$
$r^2 = (5m)^2 - (3m)^2$
$r^2 = 25m^2 - 9m^2$
$r^2 = 16m^2$
Taking square root on both sides:
$\sqrt{r^2} = \sqrt{16m^2}$
$r = 4m$
Now, Surface area of cone = $\pi rl + \pi r^2$

$= (3.14)(4m)(5m) + (3.14)(4m)^2$
$= 113.04m^2$

And, Volume of cone $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}(3.14)(4m)^2(3m)$
$= 50.24m^3$

**c)**
**Solution:**
Here, $r = \frac{d}{2} = \frac{28}{2} = 14m,\ l = 50m, h = 48m$
So, Surface area $= \pi rl + \pi r^2$
$= (3.14)(14m)(50m) + (3.14)(14m)^2$
$= 2813.44m^2$
And, Volume of cone $= \frac{1}{3}\pi r^2 h$
$= \frac{1}{3}(3.14)(14m)^2(48m)$
$= 9847.04m^3$

**11. Find the surface area and volume of hemisphere having:**
**a) radius 6cm**
**Solution:**
Surface area of hemisphere $= 3\pi r^2$
$= 3(3.14)(6cm)^2$
$= 339.12cm^2$
And, Volume of hemisphere $= \frac{2}{3}\pi r^3$
$= \frac{2}{3}(3.14)(6cm)^3$
$= 452.16cm^3$

**b) radius 24m**
**Solution:**
Here, $r = \frac{d}{2} = \frac{24}{2} = 12m$
So, Surface area $= 3\pi r^2$
$= 3(3.14)(12m)^2$
$= 1356.48m^2$
And, Volume of hemisphere $= \frac{2}{3}\pi r^3$
$= \frac{2}{3}(3.14)(12m)^3$
$= 3617.28m^3$

**12. There is a spherical water tank whose radius is 3.5m.**
**a) Find the cost of painting water tank at the rate of Rs. 135 per $m^2$.**
**Solution:**
First, we have to find surface area
Here, $r = 3.5m$
So, Surface area of water tank $= 4\pi r^2$
$= 4(3.14)(3.5m)^2$
$= 153.86m^2$
Now, $Total\ cost\ of\ painting = Surface\ area\ \times rater\ per\ m^2$
$= 153.86 \times 135$
$= Rs. 20771.1$

**b) Find the capacity of the water tank.** $\left(\boldsymbol{\pi = \frac{22}{7}}\right)$

**Solution:**

Here, $r = 3.5m, \pi = \frac{22}{7} = 3.14$

So, capacity of tank $= \frac{4}{3}\pi r^3$

$= \frac{4}{3}(3.14)(3.5m)^3$

$= 179.50m^3$

**13. The height of a conical flask is $\boldsymbol{10cm}$, and the area of its circular base is $\boldsymbol{78.50cm^2}$.**

**Solution:**

First, we find radius $(r)$ from the following formula:

$A = \pi r^2$

$78.50cm^2 = (3.14)r^2$

$r^2 = \frac{78.50cm^2}{3.14}$

$r^2 = 25cm^2$

Taking square root on both sides.

$\sqrt{r^2} = \sqrt{25cm^2}$

$r = 5cm$

Now, we have to find slant height $(l)$:

$l^2 = h^2 + r^2$

$l^2 = (10cm)^2 + (5cm)^2$

$l^2 = 100cm^2 + 25cm^2$

$l^2 = 125cm^2$

$l = 11.18cm$

**a) Find the curved surface area of conical flask.**

**Solution:**

Curved surface area $= \pi rl$

$= (3.14)(5cm)(11.18cm)$

$= 175.53cm^2$

**b) Calculate its total surface area.**

**Solution:**

Total surface area $= \pi rl + \pi r^2$

$= (3.14)(5cm)(11.18cm) + (3.14)(5cm)^2$

$= 254.03cm^2$

**c) Find the capacity of the conical flask.**

**Solution:**

Capacity of conical flask $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}(3.14)(5cm)^2(10cm)$

$= 261.67cm^3$

## Learning Check 9.1

1. **For each of the following, state if the data would be discrete or continuous:**
   a) The number of animals in a zoo
   b) The length of a pencil
   c) The number of pages in a book
   d) The mass of a chair
   e) The time taken to cook a dish
   f) The number of books in a library
   g) The capacity of a pool
   h) The mass of a watermelon

   **Solution:**

   a) The number of animals in a zoo
   The number of animals in the zoo is in whole numbers. So, it is discrete data.
   b) The length of a pencil
   Length can be measured on a continuous scale, and it is possible to have fractional lengths. So, it is continuous data.
   c) The number of pages in a book
   The number of pages in a book is a whole number. So, it is discrete data.

   d) The mass of a chair
   Mass can be measured on a continuous scale, and it is possible to have fractional masses. So, it is continuous data.
   e) The time taken to cook a dish
   Time can be measured on a continuous scale, and it is possible to have fractional time intervals. So, it is continuous data.
   f) The number of books in a library
   The number of books is a whole number. So, it is discrete data.
   g) The capacity of a pool
   Capacity can be measured on a continuous scale, and it is possible to have fractional capacities. So, it is continuous data.
   h) The mass of a watermelon
   Mass can be measured on a continuous scale, and it is possible to have fractional masses. So, it is continuous data.

## Learning Check 9.2

1. **Find the range, variance, and standard deviation for each set of data.**
   a) 35, 79, 44 ,63, 92, 28, 20
   b) 400, 450, 520, 380, 495, 575, 444
   c) 6, 7, 10, 11, 14, 13, 16, 18, 25
   d) 83, 23, 24, 71, 52, 62, 63
   e) 63, 25, 43, 28, 72, 61, 45, 46, 13
   f) 8, 9, 8, 11, 13, 15, 10, 18, 20

   **Solution:**

   a) 35, 79, 44 ,63, 92, 28, 20
   **Step I:**
   In the give data, the greatest value is 92 and the smallest value is 20.
   $x_{\text{max}} = 92$, $x_{\text{min}} = 20$
   Range = $x_{\text{max}} - x_{\text{min}}$
   $= 92 - 20$
   $= 72$
   So, the range of this data is 72.
   **Step II:**
   Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 35 + 79 + 44 + 63 + 92 + 28 + 20 = 361

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{n} = \frac{361}{7}$ = 51.57

**Step III:**

Find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 51.57, take away 51.57 from each value.

Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 35 | 35 – 51.57 = – 16.57 | 274.56 |
| 79 | 79 – 51.57 = 27.43 | 752.40 |
| 44 | 44 – 51.57 = – 7.57 | 57.30 |
| 63 | 63 – 51.57 = 11.43 | 130.64 |
| 92 | 92 – 51.57 = 40.43 | 1634.58 |
| 28 | 28 – 51.57 = – 23.57 | 555.54 |
| 20 | 20 – 51.57 = – 31.57 | 996.66 |
| $\Sigma$ | | 4401.68 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma (x - \bar{x})^2}{n}$

$S^2 = \frac{4401.68}{7}$

$S^2$ = 628.82

This is the required variance for the given data values.

**Step IV:**

Putting the values in the formula, we get

Standard Deviation = S = $\sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$

S = $\sqrt{\frac{4401.68}{7}}$

S = $\sqrt{628.82}$

S = 25.08

So, the standard deviation of the given data is 25.08 (correct to 2 decimal figures).

b) 400, 450, 520, 380, 495, 575, 444

**Step I:**

In the give data, the greatest value is 575 and the smallest value is 380.

$x_{max}$ = 575, $x_{min}$ = 380

Range = $x_{max} - x_{min}$

= 575 – 380

= 195

So, the range of this data is 195.

**Step II:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 400 + 450 + 520 + 380 + 495 + 575 + 444 = 3264

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{3264}{7}$ = 466.29

**Step III:**
Find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 466.29, take away 466.29 from each value.
Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 400 | 400 – 466.29 = – 66.29 | 4394.36 |
| 450 | 450 – 466.29 = – 16.29 | 265.36 |
| 520 | 520 – 466.29 = 53.71 | 2884.76 |
| 380 | 380 – 466.29 = – 86.29 | 7445.96 |
| 495 | 495 – 466.29 = 28.71 | 824.26 |
| 575 | 575 – 466.29 = 108.71 | 11817.86 |
| 444 | 444 – 466.29 = – 22.29 | 496.84 |
| $\Sigma$ | | 28129.40 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x - \bar{x})^2}{n}$

$S^2 = \frac{28129.40}{7}$

$S^2 = 4018.49$

This is the required variance for the given data values.

**Step IV:**
Putting the values in the formula, we get

Standard Deviation = $S = \sqrt{\frac{\Sigma\,(x - \bar{x})^2}{n}}$

$S = \sqrt{\frac{28129.40}{7}}$

$S = \sqrt{4018.49}$

$S = 63.39$

So, the standard deviation of the given data is 63.39 (correct to 2 decimal points).

c) 6, 7, 10, 11, 14, 13, 16, 18, 25

**Step I:**
In the give data, the greatest value is 25 and the smallest value is 6.

$x_{\text{max}} = 25$, $x_{\text{min}} = 6$

Range = $x_{\text{max}} - x_{\text{min}}$
= 25 – 6
= 19

So, the range of this data is 19.

**Step II:**
Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 6 + 7 + 10 + 11 + 14 + 13 + 16 + 18 + 25 = 120

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{120}{9}$ = 13.33

**Step III:**
Find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 13.33, take away 13.33 from each value.

Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 6 | 6 – 13.33 = – 7.33 | 53.73 |
| 7 | 7 – 13.33 = – 6.33 | 40.07 |
| 10 | 10 – 13.33 = – 3.33 | 11.09 |
| 11 | 11 – 13.33 = – 2.33 | 5.43 |
| 14 | 14 – 13.33 = 0.67 | 0.45 |
| 13 | 13 – 13.33 = – 0.33 | 0.11 |
| 16 | 16 – 13.33 = 2.67 | 7.13 |
| 18 | 18 – 13.33 = 4.67 | 21.81 |
| 25 | 25 – 13.33 = 11.67 | 136.19 |
| $\Sigma$ | | 276.01 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma (x - \bar{x})^2}{n}$

$S^2 = \frac{276.01}{9}$

$S^2 = 30.67$

This is the required variance for the given data values.

**Step IV:**

Putting the values in the formula, we get

Standard Deviation = $S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$

$S = \sqrt{\frac{276.01}{9}}$

$S = \sqrt{30.67}$

$S = 5.54$

So, the standard deviation of the given data is 5.54 (correct to 2 decimal points).

d) 83, 23, 24, 71, 52, 62, 63

**Step I:**

In the give data, the greatest value is 83 and the smallest value is 23.

$x_{max} = 83$, $x_{min} = 23$

Range = $x_{max} - x_{min}$

= 83 – 23

= 60

So, the range of this data is 60.

**Step II:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 83 + 23 + 24 + 71 + 52 + 62 + 63 = 378

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{378}{7} = 54$

**Step III:**

Find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 54, take away 54 from each value.

Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 83 | 83 – 54 = 29 | 841 |
| 23 | 23 – 54 = – 31 | 961 |
| 24 | 24 – 54 = – 30 | 900 |
| 71 | 71 – 54 = 17 | 289 |
| 52 | 52 – 54 = – 2 | 4 |
| 62 | 62 – 54 = 8 | 64 |
| 63 | 63 – 54 = 9 | 81 |
| $\Sigma$ | | 3140 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x - \bar{x})^2}{n}$

$S^2 = \frac{3140}{7}$

$S^2 = 448.57$

This is the required variance for the given data values.

**Step IV:**

Putting the values in the formula, we get

Standard Deviation = S = $\sqrt{\frac{\Sigma\,(x - \bar{x})2}{n}}$

S = $\sqrt{\frac{3140}{7}}$

S = $\sqrt{448.57}$

S = 21.18

So, the standard deviation of the given data is 21.18 (correct to 2 decimal points).

e) 63, 25, 43, 28, 72, 61, 45, 46, 13

**Step I:**

In the give data, the greatest value is 72 and the smallest value is 13.

$x_{\text{max}} = 72$, $x_{\text{min}} = 13$

Range = $x_{\text{max}} - x_{\text{min}}$

= 72 – 13

= 59

So, the range of this data is 59.

**Step II:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 63 + 25 + 43 + 28 + 72 + 61 + 45 + 46 + 13 = 396

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{396}{9}$ = 44

**Step III:**

Find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 44, take away 44 from each value.

Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 63 | 63 – 44 = 19 | 361 |
| 25 | 25 – 44 = – 19 | 361 |
| 43 | 43 – 44 = – 1 | 1 |

| | | |
|---|---|---|
| 28 | 28 – 44 = – 16 | 256 |
| 72 | 72 – 44 = 28 | 784 |
| 61 | 61 – 44 = 17 | 289 |
| 45 | 45 – 44 = 1 | 1 |
| 46 | 46 – 44 = 2 | 4 |
| 13 | 13 – 44 = – 31 | 961 |
| $\Sigma$ | | 3018 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x-\bar{x})^2}{n}$

$S^2 = \frac{3018}{9}$

$S^2 = 335.33$

This is the required variance for the given data values.

**Step IV:**

Putting the values in the formula, we get

Standard Deviation = $S = \sqrt{\frac{\Sigma\,(x-\bar{x})^2}{n}}$

$S = \sqrt{\frac{3018}{9}}$

$S = \sqrt{335.33}$

$S = 18.31$

So, the standard deviation of the given data is 18.31 (correct to 2 decimal points).

f) 8, 9, 8, 11, 13, 15, 10, 18, 20

**Step I:**

In the give data, the greatest value is 20 and the smallest value is 8.

$x_{max} = 20$, $x_{min} = 8$

Range = $x_{max} - x_{min}$

= 20 – 8

= 12

So, the range of this data is 12.

**Step II:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 8 + 9 + 8 + 11 + 13 + 15 + 10 + 18 + 20 = 112

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{112}{9}$ = 12.44

**Step III:**

Find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 12.44, take away 12.44 from each value.

Take the square of each deviation from the mean, i.e., $(x-\bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 8 | 8 – 12.44 = – 4.44 | 19.71 |
| 9 | 9 – 12.44 = – 3.44 | 11.83 |
| 8 | 8 – 12.44 = – 4.44 | 19.71 |
| 11 | 11 – 12.44 = – 1.44 | 2.07 |
| 13 | 13 – 12.44 = 0.56 | 0.31 |
| 15 | 15 – 12.44 = 2.56 | 6.55 |
| 10 | 10 – 12.44 = – 2.44 | 5.95 |
| 18 | 18 – 12.44 = 5.56 | 30.91 |
| 20 | 20 – 12.44 = 7.56 | 57.15 |
| $\Sigma$ | | 154.19 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x - \bar{x})^2}{n}$

$S^2 = \frac{154.19}{9}$

$S^2 = 17.14$

This is the required variance for the given data values.

**Step IV:**

Putting the values in the formula, we get

Standard Deviation = $S = \sqrt{\frac{\Sigma\,(x - \bar{x})^2}{n}}$

$S = \sqrt{\frac{154.19}{9}}$

$S = \sqrt{17.14}$

$S = 4.14$

So, the standard deviation of the given data is 4.14 (correct to 2 decimal points).

2. **The following data shows the marks obtained by 15 students in the final science exam.**

   90, 82, 83, 59, 94, 70, 76, 32, 80, 87, 94, 62, 80, 72, 62

   Calculate:

   a) Range  b) Variance  c) Standard Deviation

   **Solution:**

   90, 82, 83, 59, 94, 70, 76, 32, 80, 87, 94, 62, 80, 72, 62

   **a) Range:**

   In the give data, the greatest value is 94 and the smallest value is 32.

   $x_{max} = 94$, $x_{min} = 32$

   Range = $x_{max} - x_{min}$

   = 94 – 32

   = 62

   So, the range of this data is 62.

   **b) Variance:**

   Find the mean. Add up all the data values, then divide them by the number of values.

   Sum = $\Sigma x$ = 90 + 82 + 83 + 59 + 94 + 70 + 76 + 32 + 80 + 87 + 94 + 62 + 80 + 72 + 62 = 1123

   Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{1123}{15} = 74.87$

Now, find the difference between the mean and each of the values.
Subtract the mean from each value to get the deviations from the mean.
Since $\bar{x}$ = 74.87, take away 74.87 from each value.
Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 90 | 90 – 74.87 = 15.13 | 228.92 |
| 82 | 82 – 74.87 =7.13 | 50.84 |
| 83 | 83 – 74.87 = 8.13 | 66.10 |
| 59 | 59 – 74.87 = – 15.87 | 251.86 |
| 94 | 94 – 74.87 = 19.13 | 365.96 |
| 70 | 70 – 74.87 = – 4.87 | 23.72 |
| 76 | 76 – 74.87 = 1.13 | 1.28 |
| 32 | 32 – 74.87 = – 42.87 | 1837.84 |
| 80 | 80 – 74.87 = 5.13 | 26.32 |
| 87 | 87 – 74.87 = 12.13 | 147.14 |
| 94 | 94 – 74.87 = 19.13 | 365.96 |
| 62 | 62 – 74.87 = – 12.87 | 165.64 |
| 80 | 80 – 74.87 = 5.13 | 26.32 |
| 72 | 72 – 74.87 = – 2.87 | 8.24 |
| 62 | 62 – 74.87 = – 12.87 | 165.64 |
| $\Sigma$ | | 3731.78 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x - \bar{x})^2}{n}$

$S^2 = \frac{3731.78}{15}$

$S^2$ = 248.78

This is the required variance for the given data values.

**c) Standard Deviation:**

Putting the values in the formula, we get

Standard Deviation = S = $\sqrt{\frac{\Sigma\,(x - \bar{x})^2}{n}}$

S = $\sqrt{\frac{3731.78}{15}}$

S = $\sqrt{248.79}$

S = 15.77

So, the standard deviation of the given data is 15.77 (correct to 2 decimal points).

3. **The following data shows the number of Hifz class students in a school during a 10-year period.**

28, 31, 45, 35, 30, 50, 54, 38, 40, 36

Calculate:

a) Range b) Variance c) Standard Deviation

**Solution:**

28, 31, 45, 35, 30, 50, 54, 38, 40, 36

**a) Range:**

In the give data, the greatest value is 54 and the smallest value is 28.

$x_{max}$ = 54, $x_{min}$ = 28

Range = $x_{max} - x_{min}$
= 54 – 28
= 26
So, the range of this data is 26.

**b) Variance:**
Find the mean. Add up all the data values, then divide them by the number of values.
Sum = $\Sigma x$ = 28 + 31 + 45 + 35 + 30 + 50 + 54 + 38 + 40 + 36 = 387
Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{387}{10}$ = 38.7
Now, find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 38.7, take away 38.7 from each value.
Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 28 | 28 – 38.7 = – 10.7 | 114.49 |
| 31 | 31 – 38.7 = – 7.7 | 59.29 |
| 45 | 45 – 38.7 = 6.3 | 39.69 |
| 35 | 35 – 38.7 = – 3.7 | 13.69 |
| 30 | 30 – 38.7 = – 8.7 | 75.69 |
| 50 | 50 – 38.7 = 11.3 | 127.69 |
| 54 | 54 – 38.7 = 15.3 | 234.09 |
| 38 | 38 – 38.7 = – 0.7 | 0.49 |
| 40 | 40 – 38.7 = 1.3 | 1.69 |
| 36 | 36 – 38.7 = – 2.7 | 7.29 |
| $\Sigma$ | | 674.1 |

Putting the values in the formula, we get
$S^2 = \frac{\Sigma (x - \bar{x})^2}{n}$
$S^2 = \frac{674.1}{10}$
$S^2$ = 67.41
This is the required variance for the given data values.

**c) Standard Deviation:**
Putting the values in the formula, we get
Standard Deviation = $S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$
$S = \sqrt{\frac{674.1}{10}}$
$S = \sqrt{67.41}$
S = 8.21

So, the standard deviation of the given data is 8.21 (correct to 2 decimal points).

4. **According to a library's weekly record, 42, 45, 58, 62, 70, 66 and 88 books were issued during the last 7 days. Find the range and standard deviation for this data.**
**Solution:**
42, 45, 58, 62, 70, 66, 88

**Range:**

In the give data, the greatest value is 88 and the smallest value is 42.

$x_{max} = 88$, $x_{min} = 42$

Range = $x_{max} - x_{min}$

= 88 – 42

= 46

So, the range of this data is 46.

**Standard Deviation:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 42 + 45 + 58 + 62 + 70 + 66 + 88 = 431

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{431}{7}$ = 61.57

Find the difference of the mean and each data value. Then take its square.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 42 | 42 – 61.57 = – 19.57 | 382.98 |
| 45 | 45 – 61.57 = – 16.57 | 274.56 |
| 58 | 58 – 61.57 = – 3.57 | 12.74 |
| 62 | 62 – 61.57 = 0.43 | 0.18 |
| 70 | 70 – 61.57 = 8.43 | 71.06 |
| 66 | 66 – 61.57 = 4.43 | 19.62 |
| 88 | 88 – 61.57 = 26.43 | 698.54 |
| $\Sigma$ | | 1459.68 |

Take the square of each deviation from the mean. This will result in positive numbers. Then find the sum of these squares.

Putting the values in the formula, we get

Standard Deviation = S = $\sqrt{\frac{\Sigma\ (x - \bar{x})^2}{n}}$

S = $\sqrt{\frac{1459.68}{7}}$

S = $\sqrt{208.53}$

S = 14.44

So, the standard deviation of the given data is 14.44 (correct to 2 decimal points).

5. **The following data shows the number of pages of 8 books.**

120, 225, 100, 154, 188, 204, 212, 170

Find the variance for this data. Also calculate the range.

**Solution:**

120, 225, 100, 154, 188, 204, 212, 170

**Variance:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 120 + 225 + 100 + 154 + 188 + 204 + 212 + 170 = 1373

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{1373}{8}$ = 171.63

Find the difference of the mean and each data value. Then take its square.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 120 | 120 – 171.63 = – 51.63 | 2665.66 |
| 225 | 225 – 171.63 = 53.37 | 2848.36 |
| 100 | 100 – 171.63 = – 71.63 | 5130.86 |
| 154 | 154 – 171.63 = – 17.63 | 310.82 |
| 188 | 188 – 171.63 = 16.37 | 267.98 |
| 204 | 204 – 171.63 = 32.37 | 1047.82 |
| 212 | 212 – 171.63 = 40.37 | 1629.74 |
| 170 | 170 – 171.63 = – 1.63 | 2.66 |
| $\Sigma$ | | 13903.90 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma (x - \bar{x})^2}{n}$

$S^2 = \frac{13903.90}{8}$

$S^2 = 1737.99$

This is the required variance for the given data values.

**Range:**

In the give data, the greatest value is 225 and the smallest value is 100.

$x_{max} = 225$, $x_{min} = 100$

Range = $x_{max} - x_{min}$

= 225 – 100

= 125

So, the range of this data is 125.

6. **The weekly income of 10 employees of a factory is:**

Rs 6000, Rs 65230, Rs 78600, Rs 8000, Rs 9350, Rs 9680, Rs 1030, Rs 1236, Rs 14340, Rs 15000.

Calculate the range, standard deviation, and variance for this data.

**Solution:**

6000, 65230, 78600, 8000, 9350, 9680, 1030, 1236, 14340, 15000

a) **Range:**

In the give data, the greatest value is 78600 and the smallest value is 1030.

$x_{max} = 78600$, $x_{min} = 1030$

Range = $x_{max} - x_{min}$

= 78600 – 1030

= 77570

So, the range of this data is 77570.

b) **Variance:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 6000 + 65230 + 78600 + 8000 + 9350 + 9680 + 1030 + 1236 + 14340 + 15000 = 208466

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{208466}{10}$ = 20846.6

Now, find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 20846.6, take away 20846.6 from each value.

Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 6000 | 6000 – 20846.6 = – 14846.6 | 220421531.6 |
| 65230 | 65230 – 20846.6 = 44383.4 | 1969886196 |
| 78600 | 78600 – 20846.6 = 57753.4 | 3335455212 |
| 8000 | 8000 – 20846.6 = – 12846.6 | 165035131.6 |
| 9350 | 9350 – 20846.6 = – 11496.6 | 132171811.6 |
| 9680 | 9680 – 20846.6 = – 11166.6 | 124692955.6 |
| 1030 | 1030 – 20846.6 = – 19816.6 | 392697635.6 |
| 1236 | 1236 – 20846.6 = – 19610.6 | 384575632.4 |
| 14340 | 14340 – 20846.6 = – 6506.6 | 42335843.56 |
| 15000 | 15000 – 20846.6 = – 5846.6 | 34182731.56 |
| $\Sigma$ | | 6801454682 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma (x - \bar{x})^2}{n}$

$S^2 = \frac{6801454682}{10}$

$S^2 = 680145468.2$

This is the required variance for the given data values.

**Standard Deviation:**

Putting the values in the formula, we get

Standard Deviation = $S = \sqrt{\frac{\Sigma (x - \bar{x})^2}{n}}$

$S = \sqrt{\frac{6801454682}{10}}$

$S = \sqrt{680145468.2}$

$S = 26079.60$

So, the standard deviation of the given data is 26079.60 (correct to 2 decimal points).

**Learning Check 9.3**

1. **The marks obtained by 25 students in a certain class test out of a total of 30 marks are given below.**

25, 19, 26, 15, 25, 28, 25, 27, 19, 28, 28, 19, 19, 15, 28, 19, 19, 25, 27, 25, 28, 25, 19, 26, 27

Construct a frequency distribution table having an equal class interval.

**Solution:**

First, arrange the given data in ascending order for convenience.

15, 15, 19, 19, 19, 19, 19, 19, 25, 25, 25, 25, 25, 26, 26, 27, 27, 27, 28, 28, 28, 28, 28, 28

Highest marks = 28

Lowest marks = 15

Let's make 5 classes with 3 as the size of the class interval.

| Class Intervals | Class Boundaries | Tally | Frequency (f) |
|---|---|---|---|
| 15 – 17 | 14.5 – 17.5 | II | 2 |
| 18 – 20 | 17.5 – 20.5 | ~~IIII~~ II | 7 |
| 21 – 23 | 20.5 – 23.5 | | 0 |
| 24 – 26 | 23.5 – 26.5 | ~~IIII~~ III | 8 |
| 27 – 29 | 26.5 – 29.5 | ~~IIII~~ III | 8 |
| Total | | | 25 |

This is the required frequency distribution table for the grouped data.

2. **Make a frequency distribution table of the weights in kilograms of 20 students in class VIII.**
   30, 39, 36.2, 42, 38, 42, 45, 39, 38.2, 41.5, 34, 44.8, 36.2, 38, 41.5, 44.8, 35, 41.5, 39, 42
   **Solution:**
   First, arrange the given data in ascending order for convenience.
   30, 34, 35, 36.2, 36.2, 38, 38, 38.2, 39, 39, 39, 41.5, 41.5, 41.5, 42, 42, 42, 44.8, 44.8, 45
   Highest marks = 28
   Lowest marks = 15
   Let's make 4 classes with 4 as the size of the class interval.

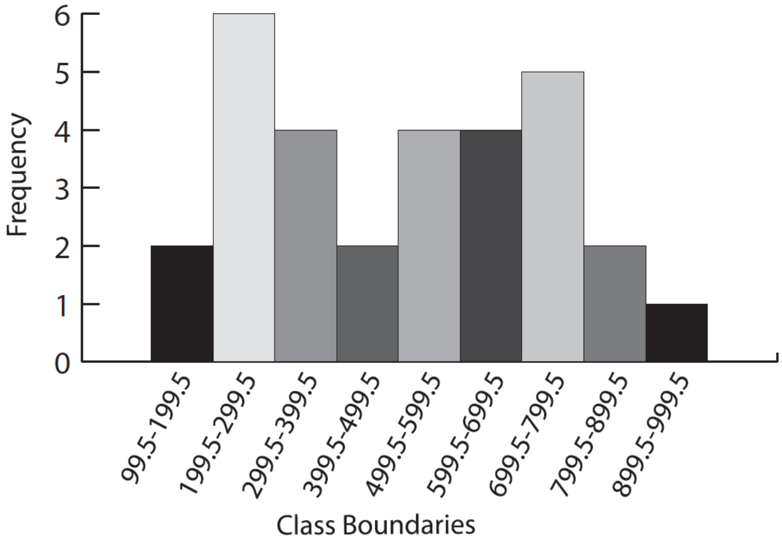
| Class Intervals | Class Boundaries | Tally | Frequency (f) |
|---|---|---|---|
| 30 – 33 | 29.5 – 33.5 | I | 1 |
| 34 – 37 | 33.5 – 37.5 | IIII | 4 |
| 38 – 41 | 37.5 – 41.5 | ~~IIII~~ I | 6 |
| 42 – 45 | 41.5 – 45.5 | ~~IIII~~ IIII | 9 |
| Total | | | 20 |

   This is the required frequency distribution table for the grouped data.

3. **Draw histogram for the following data values.**
   a) 228, 640, 772, 264, 937, 514, 763, 770, 908, 864, 950, 328, 306, 574, 233, 265, 163, 431, 807, 130, 532, 553, 576, 639, 675, 738, 205, 535, 368, 140
   b) 18, 25, 21, 20, 27, 18, 16, 17, 19, 19, 19, 23, 45, 12, 32, 30, 31, 36, 28, 25, 27, 30, 42, 40, 43, 44
   **Solution:**
   a) First, arrange the given data in ascending order for convenience.
      130, 140, 163, 205, 228, 233, 264, 265, 306, 328, 368, 431, 514, 532, 535, 553, 574, 576, 639, 640, 675, 738, 763, 770, 772, 807, 864, 908, 937, 950
      Highest number = 950
      Lowest number = 130
      Decide on the number of classes and the size of the class interval.
      Let's take 9 classes with size 100.

| Class Intervals | Class Boundaries | Tally | Frequency (f) |
|---|---|---|---|
| 100 – 199 | 99.5 – 199.5 | II | 2 |
| 200 – 299 | 199.5 – 299.5 | ~~IIII~~ I | 6 |
| 300 – 399 | 299.5 – 399.5 | IIII | 4 |
| 400 – 499 | 399.5 – 499.5 | II | 2 |
| 500 – 599 | 499.5 – 599.5 | IIII | 4 |
| 600 – 699 | 599.5 – 699.5 | IIII | 4 |
| 700 – 799 | 699.5 – 799.5 | ~~IIII~~ | 5 |
| 800 – 899 | 799.5 – 899.5 | II | 2 |
| 900 – 999 | 899.5 – 999.5 | I | 1 |
| | | | $\Sigma f = 30$ |

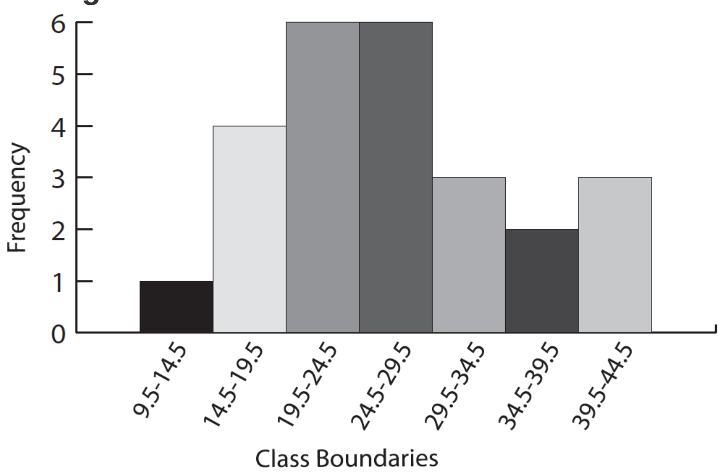
**Histogram:**




b) First, arrange the given data in ascending order for convenience.
12, 16, 17, 18, 18, 19, 19, 19, 20, 21, 23, 25, 25, 27, 27, 28, 30, 30, 31, 32, 36, 40, 42, 43, 44, 45
Highest number = 45
Lowest number = 12
Decide on the number of classes and the size of the class interval.
Let's take 7 classes with size 5.

| Class Intervals | Class Boundaries | Tally | Frequency (f) |
|---|---|---|---|
| 10 – 14 | 9.5 – 14.5 | II | 1 |
| 15 – 19 | 14.5 – 19.5 | IIII | 4 |
| 20 – 24 | 19.5 – 24.5 | ~~IIII~~ I | 6 |
| 25 – 29 | 24.5 – 29.5 | ~~IIII~~ I | 6 |
| 30 – 34 | 29.5 – 34.5 | III | 3 |
| 35 – 39 | 34.5 – 39.5 | II | 2 |
| 40 – 44 | 39.5 – 44.5 | III | 3 |
| | | | $\Sigma f = 25$ |

**Histogram:**

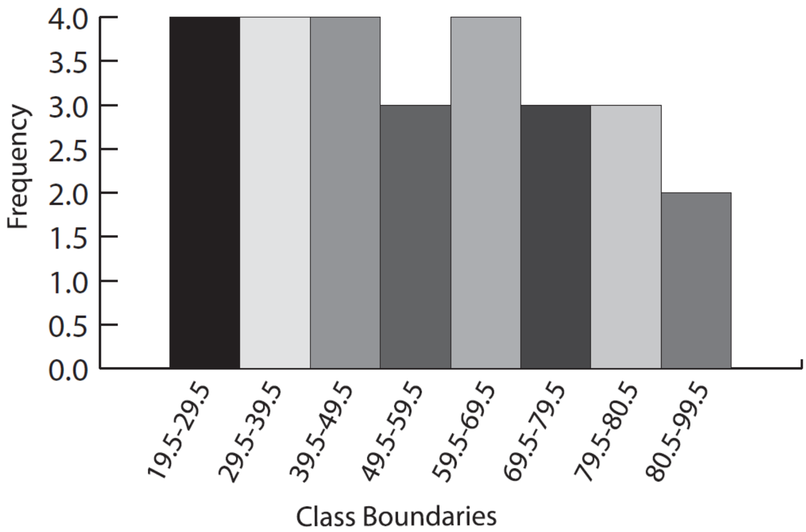



**4. Construct a histogram for the following data.**
42, 64, 36, 62, 41, 47, 87, 69, 65, 55, 83, 79, 34, 68, 25, 28, 46, 51, 74, 27, 29, 44, 59, 96, 40, 20, 50, 88, 86, 93

**Solution:**
First, arrange the given data in ascending order for convenience.
20, 25, 27, 28, 29, 34, 36, 40, 41, 42, 44, 46, 47, 50, 51, 55, 59, 62, 64, 65, 68, 69, 74, 79, 83, 86, 87, 88, 93, 96
Highest number = 96
Lowest number = 20
Decide on the number of classes and the size of the class interval.
Let's take 7 classes with size 5.

| Class Intervals | Class Boundaries | Tally | Frequency (f) |
|---|---|---|---|
| 20 – 29 | 19.5 – 29.5 | IIII | 4 |
| 30 – 39 | 29.5 – 39.5 | IIII | 4 |
| 40 – 49 | 39.5 – 49.5 | IIII | 4 |
| 50 – 59 | 49.5 – 59.5 | III | 3 |
| 60 – 69 | 59.5 – 69.5 | IIII | 4 |
| 70 – 79 | 69.5 – 79.5 | III | 3 |
| 80 – 80 | 79.5 – 80.5 | III | 3 |
| 90 – 99 | 80.5 – 99.5 | II | 2 |
| | | | $\Sigma f$ = 27 |

**Histogram**:

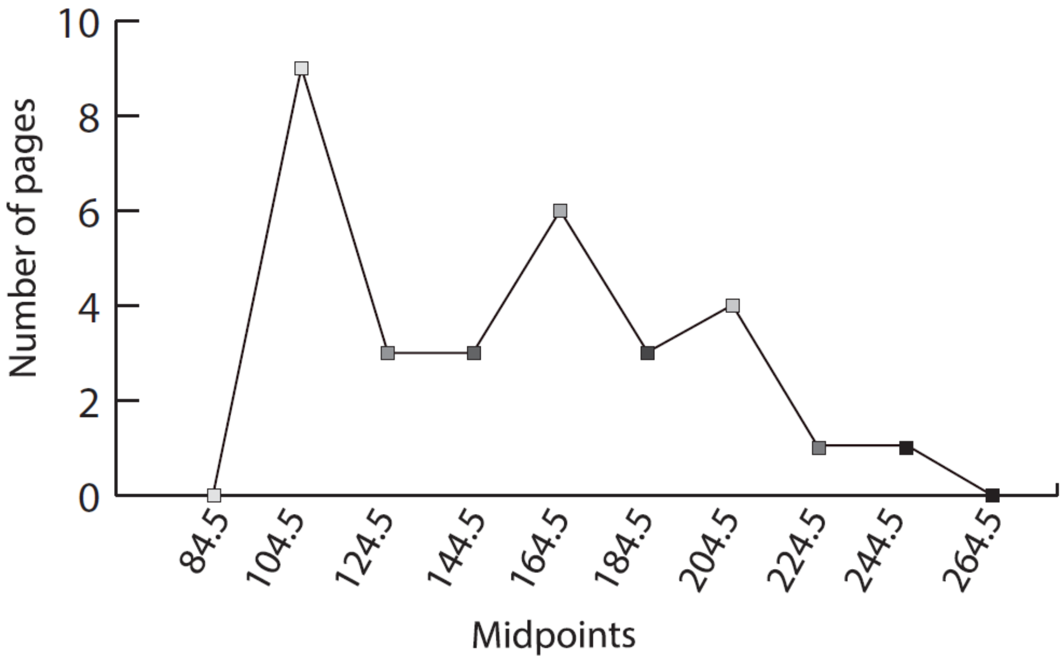



5. **The following data consists of the number of pages in 30 books of a home library. Construct a frequency polygon for this data.**
195, 206, 100, 98, 150, 210, 195, 106, 195, 168, 180, 212, 104, 195, 100, 216, 195, 209, 112, 99, 206, 116, 195, 100, 142, 100, 135, 98, 160, 155
**Solution:**
First, arrange the given data in ascending order for convenience.
98, 98, 99, 100, 100, 100, 100, 104, 106, 112, 116, 135, 142, 150, 155, 160, 168, 180, 195, 195, 195, 195, 195, 206, 206, 209, 210, 212, 216
Highest number = 216
Lowest number = 98
Decide on the number of classes and the size of the class interval.
Let's take 8 classes with size 20.
- First, add two extra classes, one before 98 and one after 216, of the same width but with 0 frequency.
- Next, find the class boundaries for each interval.
- Find the midpoint (or class mark) for each class.

| Class Intervals | Class Boundaries | Mid-Points | Tally | Frequency (f) |
|---|---|---|---|---|
| 75 – 94 | 7.5 – 94.5 | 84.5 | – | 0 |
| 95 – 114 | 94.5 – 114.5 | 104.5 | ~~IIII~~ IIII | 9 |
| 115 – 134 | 114.5 – 134.5 | 124.5 | III | 3 |
| 135 – 154 | 134.5 – 154.5 | 144.5 | III | 3 |
| 155 – 174 | 154.5 – 174.5 | 164.5 | ~~IIII~~ I | 6 |
| 175 – 194 | 174.5 – 194.5 | 184.5 | III | 3 |
| 195 – 214 | 194.5 – 214.5 | 204.5 | IIII | 4 |
| 215 – 234 | 214.5 – 234.5 | 224.5 | I | 1 |
| 235 – 254 | 234.5 – 254.5 | 244.5 | I | 1 |
| 255 – 274 | 254.5 – 274.5 | 264.5 | – | 0 |
| | | | | $\Sigma f = 30$ |

- Mark midpoints (class marks) along the x-axis and frequency along the y - axis using the appropriate scale.
- Plot all the midpoints (class marks) against the corresponding frequencies.

**Polygon:**




6. **Construct a frequency polygon for the given data.**

| Class Interval | Frequency |
|---|---|
| 91 – 100 | 6 |
| 101 – 110 | 3 |
| 111 – 120 | 3 |
| 121 – 130 | 5 |
| 131 – 140 | 8 |

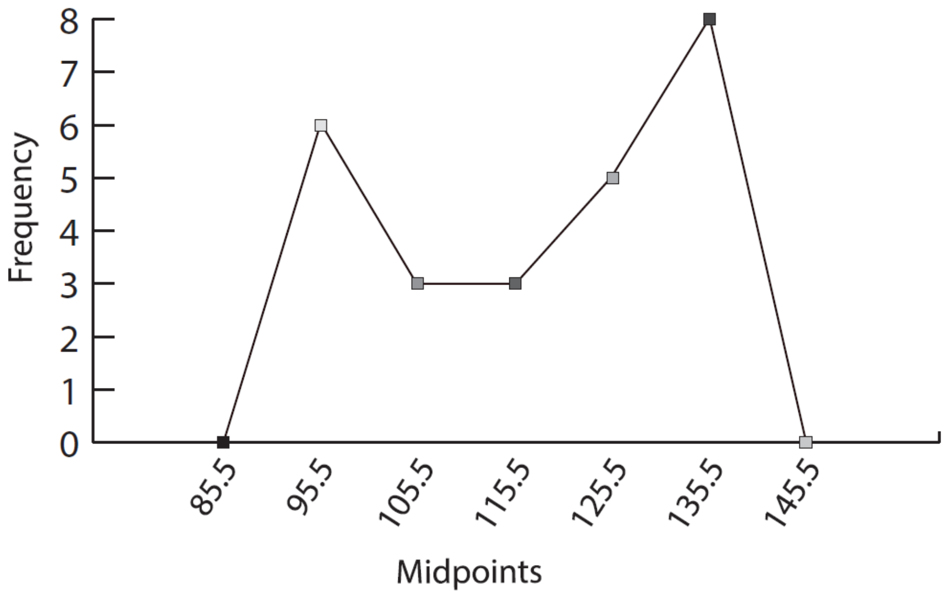
**Solution:**

- First, add two extra classes, one before 91 and one after 140, of the same width but with 0 frequency.
- Next, find the class boundaries for each interval.
- Find the midpoint (or class mark) for each class.

| Class Interval | Class Boundaries | Mid-Points | Frequency (f) |
|---|---|---|---|
| 81 – 90 | 80.5 – 90.5 | 85.5 | 0 |
| 91 – 100 | 90.5 – 100.5 | 95.5 | 6 |
| 101 – 110 | 100.5 – 110.5 | 105.5 | 3 |
| 111 – 120 | 110.5 – 120.5 | 115.5 | 3 |
| 121 – 130 | 120.5 – 130.5 | 125.5 | 5 |
| 131 – 140 | 130.5 – 140.5 | 135.5 | 8 |
| 141 – 150 | 140.5 – 150.5 | 145.5 | 0 |

- Mark midpoints (class marks) along the x-axis and frequency along the y - axis using the appropriate scale.
- Plot all the midpoints (class marks) against the corresponding frequencies.

**Polygon:**



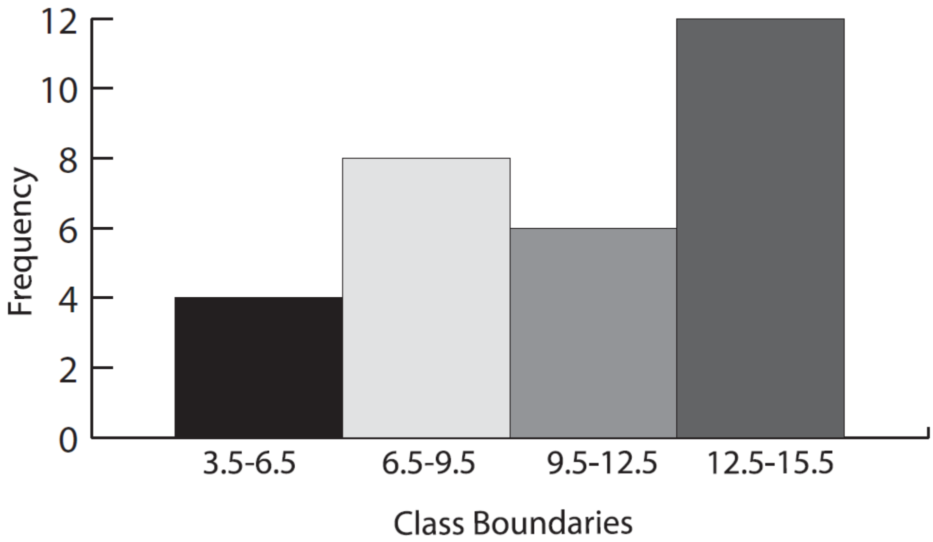

**7. Construct a histogram and frequency polygon for the following data.**

| Class Interval | Frequency |
|---|---|
| 4 – 6 | 4 |
| 7 – 9 | 8 |
| 10 – 12 | 6 |
| 13 – 15 | 12 |

**Solution:**

| Class Interval | Class Boundaries | Frequency |
|---|---|---|
| 4 – 6 | 3.5 – 6.5 | 4 |
| 7 – 9 | 6.5 – 9.5 | 8 |
| 10 – 12 | 9.5 – 12.5 | 6 |
| 13 – 15 | 12.5 – 15.5 | 12 |
| | | $\Sigma f = 30$ |

**Histogram:**




8. **Construct a frequency polygon for the given data.**

| Class Interval | Frequency |
|---|---|
| 150 – 153 | 7 |
| 154 – 157 | 7 |
| 158 – 161 | 15 |
| 162 – 165 | 10 |
| 166 – 169 | 5 |
| 170 – 173 | 6 |

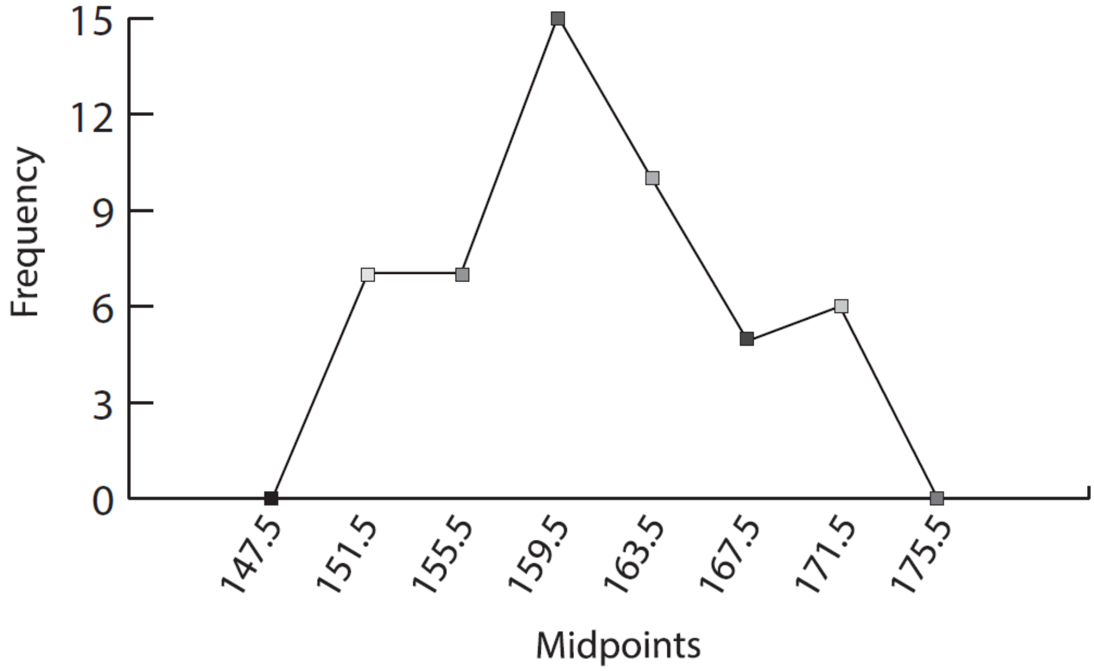
**Solution:**

- First, add two extra classes, one before 150 and one after 173, of the same width but with 0 frequency.
- Next, find the class boundaries for each interval.
- Find the midpoint (or class mark) for each class.

| Class Interval | Class Boundaries | Mid-Points | Frequency (f) |
|---|---|---|---|
| 146 – 149 | 145.5 – 149.5 | 147.5 | 0 |
| 150 – 153 | 149.5 – 153.5 | 151.5 | 7 |
| 154 – 157 | 153.5 – 157.5 | 155.5 | 7 |
| 158 – 161 | 157.5 – 161.5 | 159.5 | 15 |
| 162 – 165 | 161.5 – 165.5 | 163.5 | 10 |
| 166 – 169 | 165.5 – 169.5 | 167.5 | 5 |
| 170 – 173 | 169.5 – 173.5 | 171.5 | 6 |
| 174 – 177 | 173.5 – 177.5 | 175.5 | 0 |

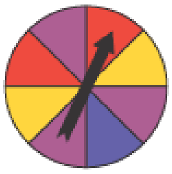
- Mark midpoints (class marks) along the x-axis and frequency along the y - axis using the appropriate scale.
- Plot all the midpoints (class marks) against the corresponding frequencies.

**Polygon:**




**Learning Check 9.4**

1. **A spinner is spun.**
   Event A: The arrow landed on purple
   Event B: The arrow landed on yellow.
   If the two events are mutually exclusive find:

   a) P(A∪B) b) P(A∩B)

   **Solution:**

   a) P(A∪B)
   The events A and B are mutually exclusive as these two events cannot happen at the same time. Either the arrow landed on purple or yellow. There is no colour which is both purple and yellow.
   The intersection of these two events A and B is the empty set. So, the probability of occurring at the same time is zero.
   P (A and B) = P(A∩B) = 0 (as the two events cannot have any common element).
   P (A or B) = P(A∪B)
   $= P(A) + P(B)$
   $= \frac{3}{8} + \frac{2}{8}$
   $= \frac{3+2}{8}$
   $= \frac{5}{8}$

   b) P (A∩B)
   The intersection of these two events A and B is the empty set. So, the probability of occurring at the same time is zero.
   P (A and B) = P(A∩B) = 0 (as the two events cannot have any common element).

2. **A single dice is rolled. What is the probability that the dots on the top are less than 3?**

**Solution:**
Total number of outcomes = 6
Number of favourable outcomes = 2
Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$
$= \frac{2}{6} = \frac{1}{3}$

3. **A single dice is rolled. What is the probability of getting a 3 or 4? Are the events mutually exclusive?**
**Solution:**
Let
A: Event that getting 3
B: Event that getting 4
The events A and B are mutually exclusive as these two events cannot happen at the same time. Either 3 or 4.
The intersection of these two events A and B is the empty set. So, the probability of occurring at the same time is zero.
P (A and B) = P(A∩B) = 0 (as the two events cannot have any common element).
There are total 6 outcomes.
P (A or B) = P(A∪B)
= P (A) + P (B)
$= \frac{1}{6} + \frac{1}{6}$
$= \frac{1+1}{6}$
$= \frac{2}{6}$
$= \frac{1}{3}$
Yes, these events are mutually exclusive.

4. **A bag contains 5 blue, 7 red, 3 green and 2 yellow marbles. Ali picked a marble randomly and recorded the result. Then he put the marble back in the bag. He again picked a marble randomly. Tell if the events are independent or not. Explain your answer. Also calculate the probability that the first marble is green and the second marble is red.**
**Solution:**
Since we put the first marble back in the bag before the second draw, the number of marbles and their colors remain the same for the second draw. Therefore, the probability of picking a red marble on the second draw does not depend on whether a green marble was picked on the first draw. Hence, the events are independent.
Now, we find the probability of independent events.
Total marbles = 5 (blue) + 7 (red) + 3 (green) + 2 (yellow) = 17
Probability of green ball = $\frac{Number\ of\ green\ balls}{Total\ balls} = \frac{3}{17}$
Probability of red ball = $\frac{Number\ of\ \text{red balls}}{Total\ balls} = \frac{7}{17}$
We know that
P (green and red) = P (G ∩ R)
= P (G) × P (R)
$= \frac{3}{17} \times \frac{7}{17}$
$= \frac{21}{289}$

5. **Two dice are rolled.**
Event A: The sum of the numbers shown on the two dice is 3.
Event B: Both of the numbers are odd.
Find:
a) The sample space b) If the two events are mutually exclusive
c) P(A∪B) d) P(A∩B)
**Solution:**
When two dice are thrown, the following results are obtained.

| . | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1,1) | (1,2) | (1,3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

a) The total sample space = 36
b) Event A: Sum of two dice is 3 = {(1, 2), (2, 1)} = 2
Event B: Both of numbers are odd = {(1,1), (1,3), (1, 5), (3,1), (3,3), (3, 5), (5,1), (5, 3), (5, 5)} = 9
There is no common element in both events. So, both events are mutually exclusive.
c) P(A∪B) = P (A) + P (B)
$$= \frac{2}{36} + \frac{9}{36}$$
$$= \frac{2+9}{36}$$
$$= \frac{11}{36}$$
$$= \frac{11}{36}$$
d) P(A∩B) = 0 (There is no common element in both events.

6. **A coin is tossed 4 times. What is the probability of getting tails every time?**
**Solution:**
When a fair coin is tossed, there are two equally likely outcomes: heads (H) or tails (T). Since each toss is independent of the others, the probability of getting tails on any single toss is $\frac{1}{2}$.
Probability of getting tails on all four tosses = P (T) × P (T) × P (T) × P (T)
$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$
$$= \frac{1}{16}$$
So, the probability of getting tails every time in four-coin tosses is $\frac{1}{16}$.

7. **Nida picks a card randomly.**
Event A: The randomly selected card will have a 7.
Event B: The randomly selected card will be even.
If the two events are mutually exclusive

| | | | | | |
|---|---|---|---|---|---|
| 4 | 7 | 7 | 0 | 3 | 8 |

find: a) P(A∪B)        b) P(A∩B)

**Solution:**

a) The total sample space = 36

Event A: Selected card will be 7 = 2

Event B: Selected card will be even = 4, 8 = 2

There is no common element in both events. So, both events are mutually exclusive.

P(A∪B) = P (A) + P (B)

$= \frac{2}{6} + \frac{2}{6}$

$= \frac{2+2}{6}$

$= \frac{4}{6}$

$= \frac{2}{3}$

b) P(A∩B) = 0 (There is no common element in both events.

## Learning Check 9.5

1. **Ahmad performed an experiment. He tossed a coin 50 times. The results are represented in the table below.**

| Outcome | Frequency |
|---|---|
| Heads | 27 |
| Tails | 23 |

a) Calculate the experimental probability of getting heads and compare it with the theoretical probability of getting heads.

b) Calculate the experimental probability of getting tails and compare it with the
the
theoretical probability of getting tails.

**Solution:**

a) To find the experimental probability of getting head, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{27}{50}$

Let's find the theoretical probability of getting head.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{2}$

b) To find the experimental probability of getting tail, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{23}{50}$

Let's find the theoretical probability of tail.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{2}$

2. **Sahar performed an experiment. She rolled a single dice 40 times. The results are represented in the table below.**

| Outcome | Frequency |
|---|---|
| 1 | 8 |
| 2 | 6 |
| 3 | 7 |
| 4 | 9 |
| 5 | 4 |
| 6 | 6 |

a) Calculate the experimental probability of getting a 5 and compare it with the theoretical probability of getting a 5.
b) Calculate the experimental probability of getting an odd number and compare it with the theoretical probability of getting an odd number.
c) Calculate the experimental probability of getting a number less than 2 and compare it with the theoretical probability of getting a number less than 2.

**Solution:**

a) To find the experimental probability of getting 5, use the formula:
Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{4}{40} = \frac{1}{10}$
Let's find the theoretical probability of getting 5.
Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{6}$

b) To find the experimental probability of getting an odd number, use the formula:
Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{19}{40}$
Let's find the theoretical probability of getting an odd number.
Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{3}{6} = \frac{1}{2}$

c) To find the experimental probability of getting less than 2, use the formula:
Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{8}{40} = \frac{1}{5}$
Let's find the theoretical probability of getting less than 2.
Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{6}$

3. **Sehrish performed an experiment. She had a bag of buttons containing 3 red, 4 yellow, 6 blue and 5 green buttons. She picked one button at a time without looking into the bag and recorded the outcome. She put the button back and in the same way, she repeated this experiment and recorded the results in a table.**

| Outcome | Frequency |
|---|---|
| Red | 10 |
| Yellow | 8 |
| Blue | 10 |
| Green | 12 |
| | $\sum$ = 40 |

Find the experimental probability of the following events:
a) Picking a red button
b) Picking a blue button
c) Not picking a green button
d) Picking a yellow button
Compare the results with the theoretical probabilities of these events.

**Solution:**

a) To find the experimental probability of picking a red button, use the formula:
Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{10}{40} = \frac{1}{4}$
Let's find the theoretical probability of picking a red button.
Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{3}{18} = \frac{1}{6}$

b) To find the experimental probability of picking a blue button, use the formula:
Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{10}{40} = \frac{1}{4}$
Let's find the theoretical probability of picking a blue button.
Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{6}{18} = \frac{1}{3}$

c) To find the experimental probability that not picking a green button, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{28}{40} = \frac{7}{10}$

Let's find the theoretical probability that not picking a green button.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{13}{18}$

d) To find the experimental probability of picking a yellow button, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{8}{40} = \frac{1}{5}$

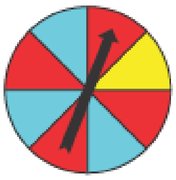
Let's find the theoretical probability of picking a yellow button.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{4}{18} = \frac{2}{9}$

By comparison, we can see that there is more difference between the experimental and theoretical probabilities.

**4. Ali performed an experiment. He spun the spinner 45 times and recorded the outcomes.**

| Outcome | Frequency |
|---|---|
| Red | 18 |
| Yellow | 12 |
| Blue | 15 |
| | $\sum = 45$ |



Calculate the probabilities and complete the table.

| Events | Theoretical probability | Experimental probability |
|---|---|---|
| a) Arrow landed on red | | |
| b) Arrow landed on blue | | |
| c) Arrow landed on yellow | | |

**Solution:**

| Events | Theoretical probability | Experimental probability |
|---|---|---|
| a) Arrow landed on red | $\frac{4}{8} = \frac{1}{2}$ | $\frac{18}{45} = \frac{2}{5}$ |
| b) Arrow landed on blue | $\frac{3}{8}$ | $\frac{15}{45} = \frac{1}{3}$ |
| c) Arrow landed on yellow | $\frac{1}{8}$ | $\frac{12}{45} = \frac{4}{15}$ |

## Unit Check 9

**1. Encircle the correct option.**

a) The presentation of frequency distribution in vertical rectangle bars with no gap is called:

i) bar graph ii) multiple bar graph
**iii) histogram** iv) block graph

b) The width of each bar in a histogram represents the:

**i) size of class** ii) frequency of class
iii) area of class iv) height of class

c) Which event is not affected by the previous event?

**i) independent** ii) overlapping
iii) mutually exclusive iv) impossible

d) The formula to find the standard deviation is:

**i)** $\sqrt{\frac{(x-\bar{x})^2}{n}}$ ii) $\sqrt{\frac{(y-\bar{x})^2}{n}}$ iii) $\sqrt{\frac{(x+\bar{x})^2}{n}}$ iv) $\sqrt{\frac{(x-\bar{x})^2}{2n}}$

e) The difference between the highest and the lowest value of the data is called:

i) variance ii) class boundary

**iii) range** iv) standard deviation

2. **Fill in the blanks.**
   a) **Discrete data** is in the form of a whole number.
   b) The length of different iron rods is an example of the **Continuous** data.
   c) Two events that cannot occur at the same time are called **mutually exclusive** events.
   d) For mutually exclusive events, P(A∩B) = **0**.
   e) When a coin is tossed 6 times, the theoretical probability of getting tails is $\frac{1}{2}$.
3. **Calculate the standard deviation for the data: 4, 9, 5, 1, 2 and 8.**

**Solution:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 4 + 9 + 5 + 1 + 2 + 8 = 29

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{29}{6}$ = 4.83

Find the difference of the mean and each data value. Then take its square.

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^2$ |
|---|---|---|
| 4 | 4 – 4.83 = – 0.83 | 0.69 |
| 9 | 9 – 4.83 = 4.17 | 17.39 |
| 5 | 5 – 4.83 = 0.17 | 0.03 |
| 1 | 1 – 4.83 = – 3.83 | 14.67 |
| 2 | 2 – 4.83 = – 2.83 | 8.01 |
| 8 | 8 – 4.83 = 3.17 | 10.05 |
| $\Sigma$ | | 50.84 |

Take the square of each deviation from the mean. This will result in positive numbers. Then find the sum of these squares.

Putting the values in the formula, we get

Standard Deviation = S = $\sqrt{\frac{\Sigma\,(x-\bar{x})^2}{n}}$

S = $\sqrt{\frac{50.84}{6}}$

S = $\sqrt{8.47}$

S = 2.91

So, the standard deviation of the given data is 2.91 (correct to 2 decimal points).

4. **Calculate the variance and standard deviation for the given data: 3, 5, 7, 11, 13, 17, 19 and 23.**

**Solution:**

**Variance:**

Find the mean. Add up all the data values, then divide them by the number of values.

Sum = $\Sigma x$ = 3 + 5 + 7 + 11 + 13 + 17 + 19 + 23 = 98

Mean = $\bar{x}$ = $\frac{Sum\ of\ values}{Number\ of\ values}$ = $\frac{\Sigma x}{x}$ = $\frac{98}{8}$ = 12.25

Now, find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 12.25, take away 12.25 from each value.

Take the square of each deviation from the mean, i.e., $(x-\bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x-\bar{x}$ | $(x-\bar{x})^2$ |
|---|---|---|
| 3 | 3 – 12.25 = – 9.25 | 85.56 |
| 5 | 5 – 12.25 = – 7.25 | 52.56 |
| 7 | 7 – 12.25 = – 5.25 | 27.56 |
| 11 | 11 – 12.25 = – 1.25 | 1.56 |
| 13 | 13 – 12.25 = 0.75 | 0.56 |
| 17 | 17 – 12.25 = 4.75 | 22.56 |
| 19 | 19 – 12.25 = 6.75 | 45.56 |
| 23 | 23 – 12.25 = 10.75 | 115.56 |
| $\Sigma$ | | 351.48 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x-\bar{x})^2}{n}$

$S^2 = \frac{351.48}{8}$

$S^2 = 43.94$

This is the required variance for the given data values.

**Standard Deviation:**

Putting the values in the formula, we get

Standard Deviation = S = $\sqrt{\frac{\Sigma\,(x-\bar{x})^2}{n}}$

S = $\sqrt{\frac{351.48}{8}}$

S = $\sqrt{43.94}$

S = 6.63

So, the standard deviation of the given data is 6.63 (correct to 2 decimal points).

5. **Calculate the range for the following observations.**

   45.2, 67.9, 88.5, 33.2, 11.6, 99.2, 101.5

   **Solution:**

   In the give data, the greatest value is 101.5 and the smallest value is 11.6.

   $x_{max}$ = 101.5, $x_{min}$ = 11.6

   Range = $x_{max} - x_{min}$

   = 101.5 – 11.6

   = 89.90

   So, the range of this data is 89.90.

6. **Calculate the variance of the data.**

   **32, 47, 49, 43, 39, 28, 35, 45, 41, 50, 47, 43, 49.**

   **Solution:**

   Find the mean. Add up all the data values, then divide them by the number of values.

   Sum = $\Sigma x$ = 32 + 47 + 49 + 43 + 39 + 28 + 35 + 45 + 41 + 50 + 47 + 43 + 49 = 548

Mean = $\bar{x} = \frac{Sum\ of\ values}{Number\ of\ values} = \frac{\Sigma x}{x} = \frac{548}{13}$ = 42.15

Now, find the difference between the mean and each of the values. Subtract the mean from each value to get the deviations from the mean. Since $\bar{x}$ = 42.15, take away 42.15 from each value.

Take the square of each deviation from the mean, i.e., $(x - \bar{x})^2$. This will result in positive values. Then find the sum of the squared values.

| $x$ | $x - \bar{x}$ | $(x - \bar{x})^2$ |
|---|---|---|
| 32 | 32 – 42.15 = – 10.15 | 103.02 |
| 47 | 47 – 42.15 = 4.85 | 23.52 |
| 49 | 49 – 42.15 = 6.85 | 46.92 |
| 43 | 43 – 42.15 = 0.85 | 0.72 |
| 39 | 39 – 42.15 = – 3.15 | 9.92 |
| 28 | 28 – 42.15 = – 14.15 | 200.22 |
| 35 | 35 – 42.15 = – 7.15 | 51.12 |
| 45 | 45 – 42.15 = 2.85 | 8.12 |
| 41 | 41– 42.15 = – 1.15 | 1.32 |
| 50 | 50 – 42.15 = 7.85 | 61.62 |
| 47 | 47 – 42.15 = 4.85 | 23.52 |
| 43 | 43 – 42.15 = 0.85 | 0.72 |
| 49 | 49 – 42.15 = 6.85 | 46.92 |
| $\Sigma$ | | 577.66 |

Putting the values in the formula, we get

$S^2 = \frac{\Sigma\,(x - \bar{x})^2}{n}$

$S^2 = \frac{577.66}{13}$

$S^2$ = 44.44

This is the required variance for the given data values.

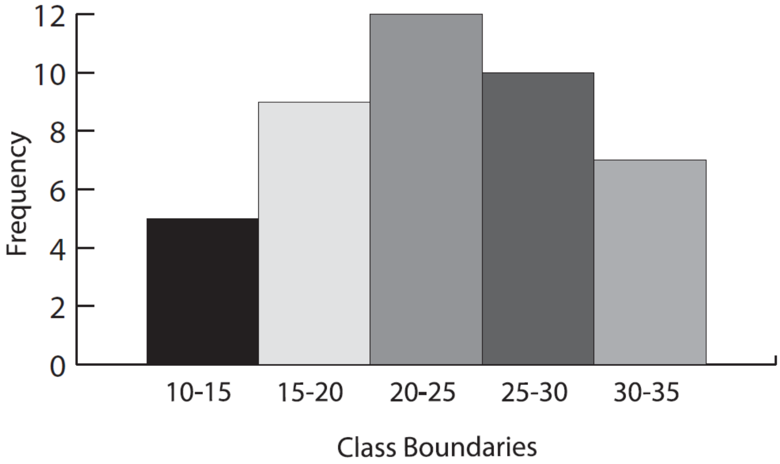
7. **Construct a histogram for the given frequency distribution.**

| Class Interval | 10 – 15 | 15 – 20 | 20 – 25 | 25 – 30 | 30 – 35 |
|---|---|---|---|---|---|
| frequency | 5 | 9 | 12 | 10 | 7 |

**Solution:**

| Class Boundaries | Frequency |
|---|---|
| 10 – 15 | 5 |
| 15 – 20 | 9 |
| 20 – 25 | 12 |
| 25 – 30 | 10 |
| 30 – 35 | 7 |
| | $\Sigma f$ = 43 |

**Histogram:**



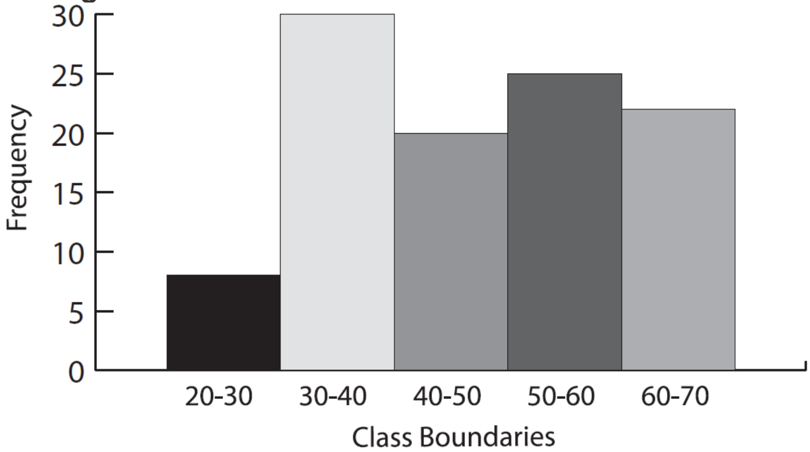

8. **The following data shows the ages of different people who came for the issuance of a smart ID card on a certain day. Draw a histogram for this data.**

| Class Interval | 20 – 30 | 30 – 40 | 40 – 50 | 50 – 60 | 60 – 70 |
|---|---|---|---|---|---|
| frequency | 8 | 30 | 20 | 25 | 22 |

**Solution:**

| Class Boundaries | Frequency |
|---|---|
| 20 – 30 | 8 |
| 30 – 40 | 30 |
| 40 – 50 | 20 |
| 50 – 60 | 25 |
| 60 – 70 | 22 |
| | $\Sigma f = 105$ |

**Histogram:**




9. **The given data is the temperature (in °C) of Lahore in 30 days of May:**
31, 33, 35, 37, 32, 31, 31, 34, 38, 40, 30, 32, 32, 33, 34, 34, 33, 30, 30, 35, 34, 29, 42, 39, 42, 40, 39, 42, 40, 41
Draw a frequency polygon.
**Solution:**
First, arrange the given data in ascending order for convenience.
29, 30, 30, 30, 31, 31, 31, 32, 32, 32, 33, 33, 33, 34, 34, 34, 35, 35, 37, 38, 39, 39, 40, 40, 40, 41, 42, 42, 42
Highest number = 42

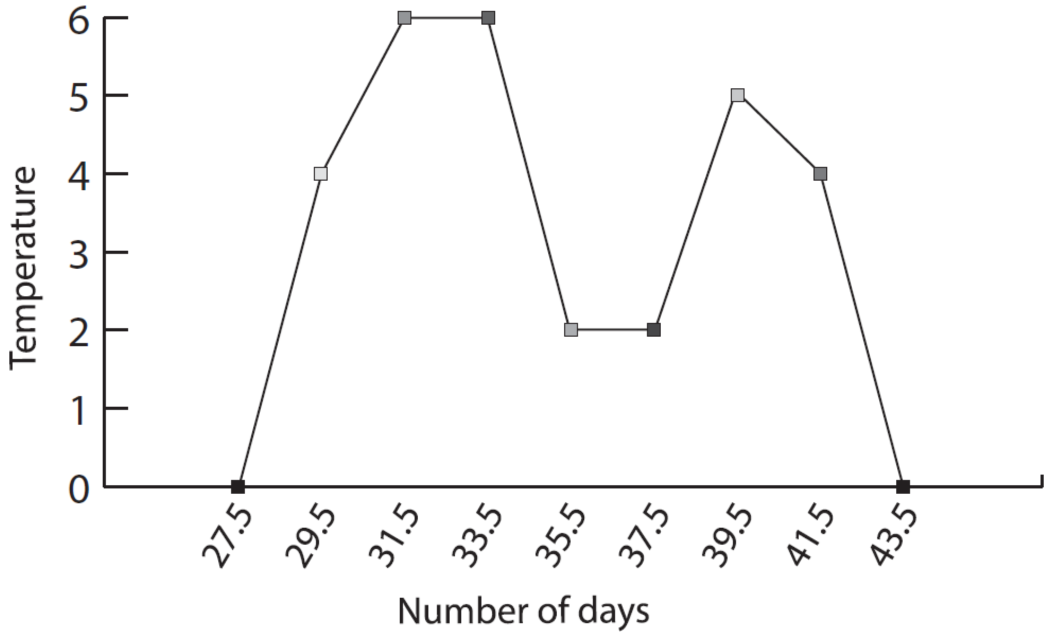
Lowest number = 29
Decide on the number of classes and the size of the class interval.
Let's take 7 classes with size 2.

- First, add two extra classes, one before 29 and one after 42, of the same width but with 0 frequency.
- Next, find the class boundaries for each interval.
- Find the midpoint (or class mark) for each class.

| Class Intervals | Class Boundaries | Mid-Points | Tally | Frequency (f) |
|---|---|---|---|---|
| 27 – 28 | 26.5 – 28.5 | 27.5 | – | 0 |
| 29 – 30 | 28.5 – 30.5 | 29.5 | IIII | 4 |
| 31 – 32 | 30.5 – 32.5 | 31.5 | ~~IIII~~ I | 6 |
| 33 – 34 | 32.5 – 34.5 | 33.5 | ~~IIII~~ I | 6 |
| 35 – 36 | 34.5 – 36.5 | 35.5 | II | 2 |
| 37 – 38 | 36.5 – 38.5 | 37.5 | II | 2 |
| 39 – 40 | 38.5 – 40.5 | 39.5 | ~~IIII~~ | 5 |
| 41 – 42 | 40.5 – 42.5 | 41.5 | IIII | 4 |
| 43 – 44 | 42.5 – 44.5 | 43.5 | – | 0 |
| | | | | $\Sigma f = 29$ |

- Mark midpoints (class marks) along the x-axis and frequency along the y - axis using the appropriate scale.
- Plot all the midpoints (class marks) against the corresponding frequencies.

**Polygon:**




10. **A single dice is rolled. What is the probability of rolling an even or odd number?**
**Solution:**
Let
A: Even numbers = {2, 4, 6} = 3
B: Odd numbers = {1, 3, 5} = 3
The events A and B are mutually exclusive as these two events cannot happen at the same time. Either even or odd.

The intersection of these two events A and B is the empty set. So, the probability of occurring at the same time is zero.
P (A and B) = P(A∩B) = 0 (as the two events cannot have any common element).
There are total 6 outcomes.
P (A or B) = P(A∪B)
$= P(A) + P(B)$
$= \frac{3}{6} + \frac{3}{6}$
$= \frac{3+3}{6}$
$= \frac{6}{6}$
$= 1$
So, the probability of rolling either an even or an odd number when rolling a fair six-sided dice is 1 (or 100%).

11. **Two dice are rolled. What is the probability of getting 6 on both dice?**
**Solution:**
When two dice are thrown, the following results are obtained.

| . | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1,1) | (1,2) | (1,3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

Total number of outcomes = 36
Number of favourable outcomes = (6, 6) = 1
Probability of an event = $\frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$
$= \frac{1}{36}$
So, the probability of getting a factor of 6 on both dice when rolling two dice is $\frac{1}{36}$.

12. **Ahmad is rolling a single dice. What is the probability that it shows 3 or 6?**
**Solution:**
Let
A: Even showing 3 = 1
B: Event showing 6 = 1
The events A and B are mutually exclusive as these two events cannot happen at the same time. Either showing 3 or 6.
The intersection of these two events A and B is the empty set. So, the probability of occurring at the same time is zero.
P (A and B) = P(A∩B) = 0 (as the two events cannot have any common element).
There are total 6 outcomes.
P (A or B) = P(A∪B)

$= P(A) + P(B)$
$= \frac{1}{6} + \frac{1}{6}$
$= \frac{1+1}{6}$
$= \frac{2}{6}$
$= \frac{1}{3}$

So, the probability that the dice shows either 3 or 6 when rolling a fair six-sided dice is $\frac{1}{3}$.

13. **In a box there are 4 blue beads, 6 red beads and 2 yellow beads. Sara picked a bead randomly and recorded the result. Then she put the bead back in the bag. She again picked a bead randomly. Tell if the events are independent or not. Explain your answer. Also calculate the probability that the first bead is blue and the second bead is yellow.**

**Solution:**

Since we put the first bead back in the bag before the second draw, the number of beads and their colors remain the same for the second draw. Therefore, the probability of picking a yellow bead on the second draw does not depend on whether a blue bead was picked on the first draw. Hence, the events are independent.

Now, we find the probability of independent events.

Total beads = 4 (blue) + 6 (red) + 2 (yellow) = 12

Probability of blue = $\frac{Number\ of\ blue\ beads}{Total\ beads} = \frac{4}{12} = \frac{1}{3}$

Probability of yellow beads = $\frac{Number\ of\ \text{yellow beads}}{Total\ beads} = \frac{2}{12} = \frac{1}{6}$

We know that

P (blue and yellow) = P (B ∩ Y)
$= P(B) \times P(Y)$
$= \frac{1}{3} \times 6$
$= \frac{1}{18}$

So, the probability that the first bead is blue, and the second bead is yellow is $\frac{1}{18}$.

14. **Zaman performed an experiment. He rolled a single dice 30 times. The results are represented in the table below.**

| Outcome | Frequency |
|---|---|
| 1 | 5 |
| 2 | 4 |
| 3 | 3 |
| 4 | 6 |
| 5 | 5 |
| 6 | 7 |

a) Calculate the experimental probability of getting a 3 and compare it with the theoretical probability of getting a 3.
b) Calculate the experimental probability of getting an even number and compare it with the theoretical probability of getting an even number.
c) Calculate the experimental probability of getting a number less than 5 and compare it with the theoretical probability of getting a number less than 5.

**Solution:**

a) To find the experimental probability of getting 3, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{3}{30} = \frac{1}{10}$

Let's find the theoretical probability of getting 3.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{1}{6}$

b) To find the experimental probability of getting an even number, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{17}{30}$

Let's find the theoretical probability of getting an even number.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{3}{6} = \frac{1}{2}$

c) To find the experimental probability of getting less than 5, use the formula:

Experimental probability = $\frac{Number\ of\ times\ an\ event\ occurs}{Total\ number\ of\ trials} = \frac{18}{30} = \frac{3}{5}$

Let's find the theoretical probability of getting less than 5.

Theoretical probability of the event = $\frac{Number\ of\ favourable\ outcomes}{Total\ number\ of\ possible\ outcomes} = \frac{4}{6} = \frac{2}{3}$